

# Beamforming on the MISO interference channel with multi-user decoding capability

K. M. Ho<sup>†</sup>, D. Gesbert<sup>†</sup>, E. Jorswieck<sup>\*</sup>, R. Mochaourab<sup>\*</sup>

<sup>†</sup>EURECOM, 2229 route des crêtes ,

BP 193 F-06560 Sophia-Antipolis cedex

Email: {hokm, David.Gesbert}@eurecom.fr

<sup>\*</sup> Dresden University of Technology Communications Laboratory,

Georg Schumann Str. 11, D-01062 Dresden/Germany

Email: {jorswieck,mochaourab}@ifn.et.tu-dresden.de

**Abstract**—This paper considers the multiple-input-single-output interference channel (MISO-IC) in which transmitters and receivers share the same time and frequency resources. We consider receivers with interference decoding capability (IDC) so that the interference signal can be decoded and subtracted from the received signal. On the MISO-IC with single user decoding, transmit beamforming vectors are designed to mitigate interference at the receivers. With IDC, receivers can potentially decode interference which yields a higher data rate. Yet, decoding interference pose a rate constraint on the interferer and in turn on the sum rate of the system. This brings some interesting questions: when should the Tx's mitigate interference and when should Tx's amplify interference? Under what situations should Tx's change from mitigating interference to amplifying interference? We answer these questions in this paper.

## I. INTRODUCTION

The capacity region of the two-user SISO-IC has been studied extensively [1]–[7], although the general capacity region is not fully known, except for special cases, e.g. the low and strong interference regime.

To extend the above results, the authors [8], [9] study the capacity region of the vector Gaussian interference channel in the weak interference regime. Results show that treating interference as noise in the weak interference regime achieves capacity. Apart from the capacity of the IC, the frontier of the achievable rate region assuming linear precoders, also known as the Pareto boundary, holds importance to the understanding of the IC. Any rate points on the Pareto boundary are operating points such that it is impossible to increase one user's

rate without decreasing the others. Assuming perfect CSIT, the Pareto boundary of the SISO-IC and MISO-IC with single user detection (SUD) are characterized in [10], [11] respectively. In [12], the authors extended the results to partial CSIT. In this paper, we assume simple single user encoding transmitters and interference decoding capability at receivers, which yield a simpler achievable rate region comparing to the Han-Kobayashi scheme [13]. This allows us to study the effects of transmit beamforming on the achievable rate region and to characterize the Pareto boundary. We limit ourselves to the two transmitter-receiver (Tx-Rx) pairs interference channel with interference decoding capability (IDC), each receiver can choose to decode interference (D) or treat interference as noise (N). The main contributions of this paper are the following.

We formulate the achievable rate region of the 2-user MISO-IC-IDC which is a region achieved by varying transmit powers and beamforming vectors in Section III. Then, we show that the achievable rate region of the MISO-IC-IDC is a union of four rate regions of different decoding structures (e.g. Rx 1, 2 decode interference or treat interference as noise). In Section V, we characterize the boundaries of rate regions of each decoding structures and therefore characterize the Pareto boundary of the MISO-IC-IDC. As an application of the Pareto boundary characterization, we characterize the maximum sum rate points and the conditions in which MRT strategies are sum rate optimal [14]. Due to space limit, we do not include the results here, for details please refer to [14]. However, in Section VI, we use these results to develop a simple suboptimal algorithm that performs close to the maximum sum rate point, whose computation is NP-hard [15].

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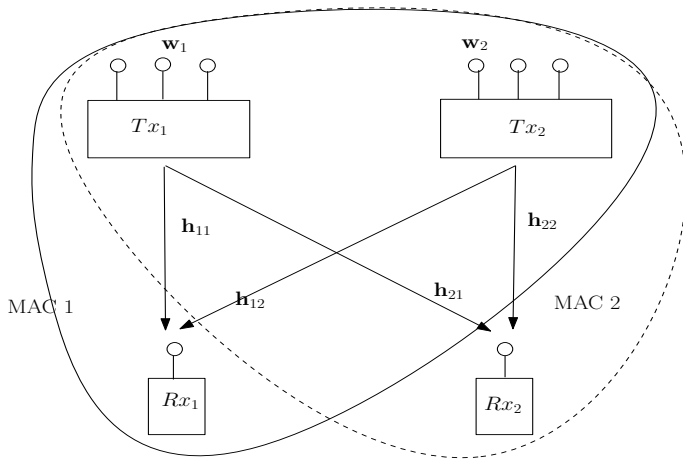


Fig. 1. The 2-user MISO-IC where Txs are quipped with 3 antennas.

## II. CHANNEL MODEL

We assume a simple system of two transmitter-receiver (Tx-Rx) pairs in which each Tx has  $N$  antennas and each Rx has only one antenna. This results in a two-user Multiple-Input-Single-Output Interference Channel (MISO-IC), which is illustrated in Fig. 1 as an example with  $N = 3$ . We assume linear pre-coders and the Txs use the same Gaussian codebooks and therefore the Rxs, if the channel qualities allow, can decode the interference and subtract it from the received signal. Also, we assume that the interference is successfully decoded if the rate of the interference signal is smaller than the Shannon capacity of the interference channel.

Denote the transmit beamforming vector of Tx  $i$  by  $\mathbf{w}_i$  and the channel from Tx  $i$  to Rx  $\bar{i}$ , where  $i, \bar{i} \in \{1, 2\}, \bar{i} \neq i$ ,  $\mathbf{h}_{i\bar{i}} \in \mathbb{C}^{N \times 1}$ . Note that the channel gains are i.i.d complex Gaussian coefficients with zero mean and unit variance. The received signal at Rx  $i$  is therefore

$$y_i = \mathbf{h}_{i\bar{i}}^H \mathbf{w}_i x_i \sqrt{P_i} + \mathbf{h}_{i\bar{i}}^H \mathbf{w}_{\bar{i}} x_{\bar{i}} \sqrt{P_{\bar{i}}} + n_i. \quad (1)$$

The noise  $n_i$  is a complex Gaussian random variable with zero mean and unit variance.  $P_i$  is the transmit power at each Tx and we assume the same power constraint for both Txs,  $P_i \leq P^*$ . The symbol  $x_i$  is the transmit symbol at Tx  $i$  with unit power. The transmit beamformer has unit norm  $\|\mathbf{w}_i\| = 1$ . Denote the hypersphere of dimension  $N$  in the complex space with unit radius by  $\mathcal{S}$ :  $\mathcal{S} = \{\mathbf{w} \in \mathbb{C}^{N \times 1} : \|\mathbf{w}\| = 1\}$ ,  $\mathbf{w}_i \in \mathcal{S}$ . We define here the projector matrices which will be referenced to later:

$$\Pi_{ij} = \frac{\mathbf{h}_{ij} \mathbf{h}_{ij}^H}{\|\mathbf{h}_{ij}\|^2} \quad (2)$$

$$\Pi_{ij}^\perp = \mathbf{I} - \Pi_{ij}. \quad (3)$$

## III. ACHIEVABLE RATE REGION

We assume simple matched filter decoders and propose the following four decoding schemes, similar to the SISO case [1]. We define the following important quantities:

$$\begin{aligned} C_1(\mathbf{w}_1, P_1) &\triangleq \log_2(1 + |\mathbf{h}_{11}^H \mathbf{w}_1|^2 P_1), \\ C_2(\mathbf{w}_2, P_2) &\triangleq \log_2(1 + |\mathbf{h}_{22}^H \mathbf{w}_2|^2 P_2), \\ D_1(\mathbf{w}_1, \mathbf{w}_2, P_1, P_2) &\triangleq \log_2\left(1 + \frac{|\mathbf{h}_{11}^H \mathbf{w}_1|^2 P_1}{|\mathbf{h}_{12}^H \mathbf{w}_2|^2 P_2 + 1}\right), \\ D_2(\mathbf{w}_1, \mathbf{w}_2, P_1, P_2) &\triangleq \log_2\left(1 + \frac{|\mathbf{h}_{22}^H \mathbf{w}_2|^2 P_2}{|\mathbf{h}_{21}^H \mathbf{w}_1|^2 P_1 + 1}\right), \\ T_2(\mathbf{w}_1, \mathbf{w}_2, P_1, P_2) &\triangleq \log_2\left(1 + \frac{|\mathbf{h}_{12}^H \mathbf{w}_2|^2 P_2}{|\mathbf{h}_{11}^H \mathbf{w}_1|^2 P_1 + 1}\right), \\ T_1(\mathbf{w}_1, \mathbf{w}_2, P_1, P_2) &\triangleq \log_2\left(1 + \frac{|\mathbf{h}_{21}^H \mathbf{w}_1|^2 P_1}{|\mathbf{h}_{22}^H \mathbf{w}_2|^2 P_2 + 1}\right). \end{aligned}$$

$C_1$  and  $C_2$  are the *single user rates*, the largest rate user 1 and 2 can achieve without the influence of interference.  $D_1$  and  $D_2$  are the rates of decoding desired signal while treating interference as thermal noise and  $T_1$  and  $T_2$  are the rate of decoding interference while treating desired signals as noise.

If both receivers decode interference, user  $i$  must transmit at a rate that ensures interference decoding at Rx  $\bar{i}$ , thus we have the following:

$$\begin{aligned} R_1 &\leq \min\{C_1(\mathbf{w}_1, P_1), T_1(\mathbf{w}_1, \mathbf{w}_2, P_1, P_2)\} \\ R_2 &\leq \min\{C_2(\mathbf{w}_2, P_2), T_2(\mathbf{w}_1, \mathbf{w}_2, P_1, P_2)\}. \end{aligned} \quad (4)$$

Denote the rate region with interference decoding at both receivers by (5) (shown at the top of next page).

*Remark 1:* For each selected pair of transmit beamformers, a corresponding rate region which satisfies the inequalities (5) is obtained. The achievable rate region  $\mathcal{R}^{dd}$  is defined as the union of all regions achieved by all possible transmit beamformers.

On the other hand, if both Rxs choose to treat interference as noise, we obtain (6). If Rx 1 decodes interference but Rx 2 treats interference as noise, Tx 2 transmits a rate that ensure interference decoding at Rx 1 as described in (7). Similarly, exchanging the role of Tx 1 and 2, we have (8).

The achievable rate region of the MISO-IC with interference decoding capability is therefore the union of the above regions:

$$\mathcal{R} = \mathcal{R}^{nn} \cup \mathcal{R}^{dd} \cup \mathcal{R}^{dn} \cup \mathcal{R}^{nd}. \quad (9)$$

*Definition 1:* Denote the set of points on the Pareto boundary by  $\mathcal{B}(\mathcal{R})$ . If the rate pair  $(r_1, r_2) \in \mathcal{R}$  is on the boundary,  $(r_1, r_2) \in \mathcal{B}(\mathcal{R})$ , then there does not exist

$$\mathcal{R}^{dd} = \bigcup_{\substack{\mathbf{w}_1, \mathbf{w}_2 \in \mathcal{S} \\ 0 \leq P_1, P_2 \leq P^*}} \left\{ (R_1, R_2) \leq \left( \min \left\{ C_1(\mathbf{w}_1, P_1), T_1(\mathbf{w}_1, \mathbf{w}_2, P_1, P_2) \right\}, \min \left\{ C_2(\mathbf{w}_2, P_2), T_2(\mathbf{w}_1, \mathbf{w}_2, P_1, P_2) \right\} \right) \right\} \quad (5)$$

$$\mathcal{R}^{nn} = \bigcup_{\substack{\mathbf{w}_1, \mathbf{w}_2 \in \mathcal{S} \\ 0 \leq P_1, P_2 \leq P^*}} \left\{ (R_1, R_2) \leq \left( D_1(\mathbf{w}_1, \mathbf{w}_2, P_1, P_2), D_2(\mathbf{w}_1, \mathbf{w}_2, P_1, P_2) \right) \right\} \quad (6)$$

$$\mathcal{R}^{dn} = \bigcup_{\substack{\mathbf{w}_1, \mathbf{w}_2 \in \mathcal{S} \\ 0 \leq P_1, P_2 \leq P^*}} \left\{ (R_1, R_2) \leq \left( C_1(\mathbf{w}_1, P_1), \min \left\{ D_2(\mathbf{w}_1, \mathbf{w}_2, P_1, P_2), T_2(\mathbf{w}_1, \mathbf{w}_2, P_1, P_2) \right\} \right) \right\} \quad (7)$$

$$\mathcal{R}^{nd} = \bigcup_{\substack{\mathbf{w}_1, \mathbf{w}_2 \in \mathcal{S} \\ 0 \leq P_1, P_2 \leq P^*}} \left\{ (R_1, R_2) \leq \left( \min \left\{ D_1(\mathbf{w}_1, \mathbf{w}_2, P_1, P_2), T_1(\mathbf{w}_1, \mathbf{w}_2, P_1, P_2) \right\}, C_2(\mathbf{w}_1, P_1) \right) \right\} \quad (8)$$

a rate pair  $(r'_1, r'_2) \geq (r_1, r_2)$ , with one strict inequality. By (9),

$$\mathcal{B}(\mathcal{R}) \subset \mathcal{B}(\mathcal{R}^{nn}) \cup \mathcal{B}(\mathcal{R}^{dd}) \cup \mathcal{B}(\mathcal{R}^{dn}) \cup \mathcal{B}(\mathcal{R}^{nd}) \quad (10)$$

*Definition 2:* Denote the set of beamforming vectors and power allocations that achieve the rate boundaries  $\mathcal{B}(\mathcal{R}^{xy})$  for  $x, y = \{n, d\}$ , the solution set of  $\mathcal{B}(\mathcal{R}^{xy})$ , termed as  $\Omega^{xy}$ .

$$\Omega^{xy} = \left\{ (\mathbf{w}_1, \mathbf{w}_2, P_1, P_2) : \right. \\ \left. (R_1(\mathbf{w}_1, \mathbf{w}_2, P_1, P_2), R_2(\mathbf{w}_1, \mathbf{w}_2, P_1, P_2)) \in \mathcal{B}(\mathcal{R}^{xy}) \right\} \quad (11)$$

Similarly, the solution set of  $\mathcal{B}(\mathcal{R})$  is  $\Omega$ .

$$\Omega = \left\{ (\mathbf{w}_1, \mathbf{w}_2, P_1, P_2) : \right. \\ \left. (R_1(\mathbf{w}_1, \mathbf{w}_2, P_1, P_2), R_2(\mathbf{w}_1, \mathbf{w}_2, P_1, P_2)) \in \mathcal{B}(\mathcal{R}) \right\} \quad (12)$$

In the following sections, we study the Pareto boundary in terms of power allocation and transmit beamforming vectors in different decoding structure namely  $\mathbf{R}^{nd}$  and  $\mathbf{R}^{dd}$ .  $\mathbf{R}^{dn}$  is symmetric to  $\mathbf{R}^{nd}$  and is therefore omitted here.  $\mathbf{R}^{nn}$  is treated in [11].

#### IV. THE PARETO BOUNDARY CHARACTERIZATION

In this section, we characterize the Pareto boundary by characterizing the boundaries of different decoding structures. In Section IV-A, we compute the solution sets of the boundary  $\mathcal{B}(\mathcal{R}^{nd})$  whereas in Section IV-B, we compute the solution sets of the boundary  $\mathcal{B}(\mathcal{R}^{dd})$ .

##### A. The Pareto boundary characterization in $\mathbf{R}^{nd}$

With decoding structure  $\mathbf{R}^{nd}$ , Rx 1 treats interference as noise and Rx 2 decodes and subtracts the interference signal from the received signal before decoding the desired signal. By the definition of rate pair  $(R_1, R_2)$  in Eq. (8), the Pareto boundary is the solution of the following optimization problem

$$\begin{aligned} & \max_{\mathbf{w}_1, \mathbf{w}_2, P_1, P_2} \min \{T_1(\mathbf{w}_1, \mathbf{w}_2, P_1, P_2), D_1(\mathbf{w}_1, \mathbf{w}_2, P_1, P_2)\} \\ & \text{subject to } C_2(\mathbf{w}_2, P_2) = r_2, \\ & \quad \|\mathbf{w}_1\| = 1, \|\mathbf{w}_2\| = 1, \\ & \quad P_1 \leq P^*, P_2 \leq P^*. \end{aligned} \quad (13)$$

*Lemma 1:* The optimization problem (13) has the solution  $\Omega^1$ ,

$$\Omega^1 = \{\mathbf{w}_1 \in \mathcal{W}_1, P_1 = P^*\} \quad (14)$$

where  $\mathcal{W}_1$  is defined in (19).

*Proof:* see [14]. ■

If we reverse the optimization order, we have

$$\begin{aligned} & \max_{\mathbf{w}_2, P_2} C_2(\mathbf{w}_2, P_2), \\ & \text{subject to } T_1(\mathbf{w}_1, \mathbf{w}_2, P_1, P_2) \geq r_1, \\ & \quad D_1(\mathbf{w}_1, \mathbf{w}_2, P_1, P_2) \geq r_1, \\ & \quad \mathbf{w}_1 \in \mathcal{W}_1, \|\mathbf{w}_2\| \leq 1, \\ & \quad P_1 = P^*, 0 \leq P_2 \leq P^*, \end{aligned} \quad (15)$$

for some positive value  $r_1$ .

*Lemma 2:* The optimization problem (15) has the solution  $\Omega^2$ ,

$$\Omega^2 = \{\mathbf{w}_2 \in \mathcal{W}_2, 0 \leq P_2 \leq P^*\} \quad (16)$$

where  $\mathcal{W}_2$  is defined in (20).

*Proof:* see [14]. ■

Lemma 1 gives the solution set of  $\mathbf{w}_1$ , for arbitrary fixed  $\mathbf{w}_2$ , which attain the Pareto boundary of  $\mathcal{R}^{nd}$ . On the other hand, lemma 2 gives the solution set of  $\mathbf{w}_2$  for arbitrary fixed  $\mathbf{w}_1$ . We combine both results and obtain the following theorem.

*Theorem 1:* The Pareto boundary  $\mathcal{B}(\mathcal{R}^{nd})$  is attained by the solution set  $\Omega^{nd}$

$$\Omega^{nd} = \{\mathbf{w}_1 \in \mathcal{W}_1, \mathbf{w}_2 \in \mathcal{W}_2, P_1 = P^*, 0 \leq P_2 \leq P^*\} \quad (17)$$

where  $\mathcal{W}_1, \mathcal{W}_2$  are defined in (19) and (20).

*Proof:* Note that  $\Omega^1 \cup \Omega^2 \subset \Omega^{nd}$ . From Lemma 1 and 2,  $\Omega^1 \cup \Omega^2$  attains Pareto boundary  $\mathcal{Q}^1 \cup \mathcal{Q}^2 \supset \mathcal{B}(\mathcal{R}^{nd})$ . Thus, solution set  $\Omega^{nd}$  attains Pareto boundary  $\mathcal{B}(\mathcal{R}^{nd})$ . ■

Note that in Thm. 1 we computed the set of beamformers that attain the Pareto boundary in  $\mathbf{R}^{nd}$ . This parameterization allows us to represent the beamforming vectors with positive real scalars  $0 \leq \lambda_1, \lambda_2 \leq 1$ . By varying  $\lambda_1, \lambda_2$  from zero to one and  $P_2$  from zero to  $P^*$ , we obtain all beamforming vectors that may attain the Pareto boundary. Intuitively, it means that the Pareto boundary attaining beamforming vectors exist only in a two-dimensional subspace, spanned by the direct channel and the interference channel, in a  $N$ -dimensional signal space.

In the next section, we investigate the Pareto boundary attaining beamforming vectors in decoding structure  $\mathbf{R}^{dd}$ .

### B. The Pareto boundary characterization in $\mathbf{R}^{dd}$

*Theorem 2:* The Pareto boundary  $\mathcal{B}(\mathcal{R}^{dd})$  is attained by solution set

$$\Omega^{dd} = \{0 \leq P_1, P_2 \leq P^*, \mathbf{w}_1 \in \mathcal{V}_1, \mathbf{w}_2 \in \mathcal{V}_2\}. \quad (18)$$

where  $\mathcal{V}_1, \mathcal{V}_2$  are defined in (21) and (22).

*Proof:* see [14]. ■

Note that the solution sets  $\mathcal{W}_i$  and  $\mathcal{V}_i$  are different as  $\mathcal{W}_i$  is a set of beamforming vectors spanned by  $\frac{\Pi_{ji}^+ \mathbf{h}_{ii}}{\|\Pi_{ji}^+ \mathbf{h}_{ii}\|}$  and  $\frac{\Pi_{ji}^- \mathbf{h}_{ii}}{\|\Pi_{ji}^- \mathbf{h}_{ii}\|}$  whereas  $\mathcal{V}_i$  is a set of beamforming vectors spanned by  $\frac{\Pi_{ii} \mathbf{h}_{ji}}{\|\Pi_{ii} \mathbf{h}_{ji}\|}$  and  $\frac{\Pi_{ii}^+ \mathbf{h}_{ji}}{\|\Pi_{ii}^+ \mathbf{h}_{ji}\|}$ .

## V. THE PARETO BOUNDARY OF MISO-IC-IDC

The Pareto Boundary is attained if at least one of the boundaries of the decoding structures is attained. Thus, we have the following solutions sets that attain the Pareto boundary.

$$\Omega = \Omega^{nd} \cup \Omega^{dd}. \quad (23)$$

Note that the solution set of  $\mathbf{R}^{nn}$  is a subset of  $\Omega^{nd}$  [11]. By reversing the role of Tx 1 and 2, we see that the solution set of  $\mathbf{R}^{dn}$  is  $\Omega^{nd}$  except with  $0 \leq P_1 \leq P^*$  which is included in  $\Omega^{dd}$ .

## VI. A SIMPLE TRANSMIT STRATEGY

In this Section, we propose a simple suboptimal transmission strategy. This transmission strategy is inspired by the parameterization of each decoding structure. We propose to select only one beamforming vector in each solution set. Given the channel states information, we compare the sum rate performance of these four beamforming vectors and choose the beamforming vector and the corresponding decoding structure which achieves the highest sum rate.

- $\mathbf{R}^{nn}$ :  $\mathbf{w}_1 = \frac{\Pi_{21}^+ \mathbf{h}_{11}}{\|\Pi_{21}^+ \mathbf{h}_{11}\|}, \mathbf{w}_2 = \frac{\Pi_{12}^+ \mathbf{h}_{22}}{\|\Pi_{12}^+ \mathbf{h}_{22}\|}$ .
- $\mathbf{R}^{nd}$ :  $\mathbf{w}_1 = \frac{\mathbf{h}_{21}}{\|\mathbf{h}_{21}\|}, \mathbf{w}_2 = \frac{\mathbf{h}_{22}}{\|\mathbf{h}_{22}\|}$ .
- $\mathbf{R}^{dn}$ :  $\mathbf{w}_1 = \frac{\mathbf{h}_{11}}{\|\mathbf{h}_{11}\|}, \mathbf{w}_2 = \frac{\mathbf{h}_{12}}{\|\mathbf{h}_{12}\|}$ .
- $\mathbf{R}^{dd}$ :  $\mathbf{w}_1 = \frac{\mathbf{h}_{21}}{\|\mathbf{h}_{21}\|}, \mathbf{w}_2 = \frac{\mathbf{h}_{12}}{\|\mathbf{h}_{12}\|}$ .
- TDMA: a time sharing scheme between single user points and therefore  $\mathbf{w}_i = \frac{\mathbf{h}_{ii}}{\|\mathbf{h}_{ii}\|}$ .

## VII. SIMULATION RESULTS

In this section, we demonstrate that the proposed parameterization allows us to design beamforming vectors which attain the Pareto boundaries. We also plotted the corresponding MRT strategies and maximum sum rate points, in Section VII-A. In Section VII-B, we observe the change of sum rate optimal decoding structure when the strength of the interference channel increases. We compare the sum rate performance between the optimal sum rate and the proposed simple algorithm.

### A. The Pareto boundary in different decoding structure

In Fig. 2 and 3 we plot the achievable rate region of the decoding structure  $\mathbf{R}^{nd}$  and  $\mathbf{R}^{dd}$  respectively. The number of transmit antennas is three and the SNR is set at 0dB. The channel coefficients for this particular channel realization are :  $\mathbf{h}_{11} = [0.3776 + 0.8444i, -1.0265 + 0.3100i, 0.2292 + 0.6424i]^T$ ,  $\mathbf{h}_{22} = [-0.1445 - 0.0385i, -1.2045 - 0.1070i, 0.9119 - 0.3682i]^T$ ,  $\mathbf{h}_{12} = [1.0156 + 0.6832i, 0.6064 - 0.2969i, 0.1510 + 0.8155i]^T$  and  $\mathbf{h}_{21} = [-0.1735 + 0.5270i, 0.6659 + 0.3887i, -1.6426 - 0.4348i]^T$ . We vary the parameters  $\lambda_1, \lambda_2$  and the transmit power from zero to  $P^*$  in order to generate the beamforming vectors in the proposed solution sets.

$$\mathcal{W}_1 = \left\{ \mathbf{w}_1 : \mathbf{w}_1 = \sqrt{\lambda_1} \frac{\Pi_{21} \mathbf{h}_{11}}{\|\Pi_{21} \mathbf{h}_{11}\|} + \sqrt{1 - \lambda_1} \frac{\Pi_{21}^\perp \mathbf{h}_{11}}{\|\Pi_{21}^\perp \mathbf{h}_{11}\|} ; 0 \leq \lambda_1 \leq 1 \right\} \quad (19)$$

$$\mathcal{W}_2 = \left\{ \mathbf{w}_2 : \mathbf{w}_2 = \sqrt{\lambda_2} \frac{\Pi_{12} \mathbf{h}_{22}}{\|\Pi_{12} \mathbf{h}_{22}\|} + \sqrt{1 - \lambda_2} \frac{\Pi_{12}^\perp \mathbf{h}_{22}}{\|\Pi_{12}^\perp \mathbf{h}_{22}\|} ; 0 \leq \lambda_2 \leq 1 \right\} \quad (20)$$

$$\mathcal{V}_1 = \left\{ \mathbf{w}_1 : \mathbf{w}_1 = \sqrt{\lambda_1} \frac{\Pi_{11} \mathbf{h}_{21}}{\|\Pi_{11} \mathbf{h}_{21}\|} + \sqrt{1 - \lambda_1} \frac{\Pi_{11}^\perp \mathbf{h}_{21}}{\|\Pi_{11}^\perp \mathbf{h}_{21}\|} ; 0 \leq \lambda_1 \leq 1 \right\} \quad (21)$$

$$\mathcal{V}_2 = \left\{ \mathbf{w}_2 : \mathbf{w}_2 = \sqrt{\lambda_2} \frac{\Pi_{22} \mathbf{h}_{12}}{\|\Pi_{22} \mathbf{h}_{12}\|} + \sqrt{1 - \lambda_2} \frac{\Pi_{22}^\perp \mathbf{h}_{12}}{\|\Pi_{22}^\perp \mathbf{h}_{12}\|} ; 0 \leq \lambda_2 \leq 1 \right\} \quad (22)$$

### B. The simple algorithm

In this section, we assume a symmetric channel [16] in which the direct channels,  $\mathbf{h}_{ii}$ , are i.i.d complex Gaussian vector channels. The interference channel  $\mathbf{h}_{ji}$  has a projection angle  $\theta_i$  with the direct channel  $\mathbf{h}_{ii}$ :  $|\mathbf{h}_{ji}^H \mathbf{h}_{ii}| = \|\mathbf{h}_{ii}\| \|\mathbf{h}_{ji}\| \cos(\theta_i)$ . Moreover, we assume that the strength of the interference channel is  $\alpha$  times of that of the direct channel:  $\|\mathbf{h}_{ji}\|^2 = \alpha \|\mathbf{h}_{ii}\|^2$  where  $\alpha \in \mathbb{R}^+$ . In Fig. 4 and 5, we plotted the maximum sum rate achieved by different decoding structure and compare it with the proposed simple algorithm when the strength of interference channel,  $\sqrt{\alpha}$ , increases. In both figures, we see that when the interference is weak, it is sum rate optimal to treat interference as noise and when the interference strength increases, sum rate can be increased by allowing one of the Rx to decode interference and in the strong interference regime, both Rxs. decoding interference achieves the highest sum rate. Depending on the channel coefficients, TDMA may outperform  $\mathbf{R}^{nn}$  and  $\mathbf{R}^{dd}$  in the medium interference regime. Note that the computation of the maximum sum rate point is NP-hard. However, we see the the proposed simple algorithm achieves nice sum rate performance with only five choices of beamforming vectors.

## VIII. CONCLUSION

We proposed and formulated the achievable rate region and the Pareto boundary of the MISO-IC-IDC. We characterized the Pareto boundary in terms of beamforming vectors and power allocation. The Pareto boundary attaining beamforming vectors are parameterized by two real valued scalars that take values from zero to one. As an application of this parameterization, we compute the maximum sum rate point and compare with a simple suboptimal algorithm that takes only five beamforming vectors of choice. In symmetric channels, we show that the sum rate optimal decoding structures changes from treating interference as noise to TDMA to decoding interference when the strength of interference increases.

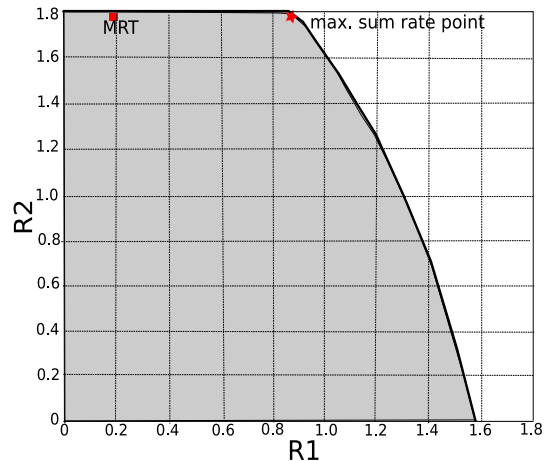


Fig. 2. Achievable rate region of  $\mathbf{R}^{nd}$ : proposed parameterization achieves the Pareto Boundary at SNR 0dB.

The suboptimal algorithm performs nicely according to simulations.

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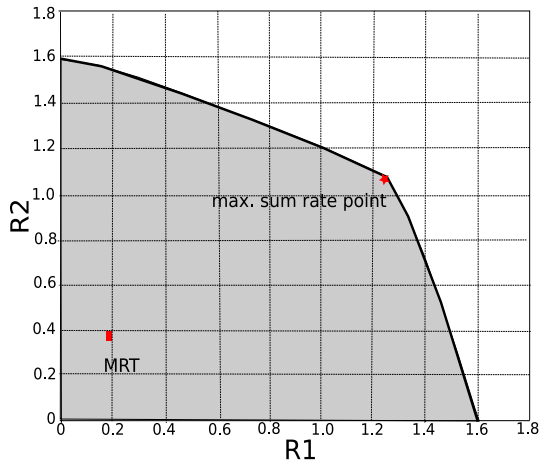


Fig. 3. Achievable rate region of  $\mathbf{R}^{dd}$ : proposed parameterization achieves the Pareto Boundary at SNR 0dB.

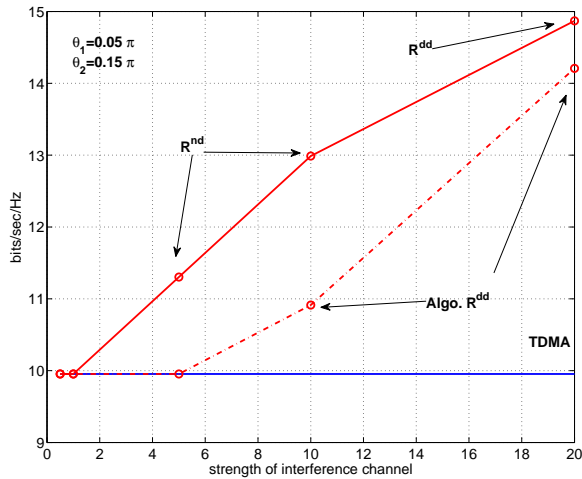


Fig. 4. Sum rate optimal decoding structures when the strength of interference channel increases.

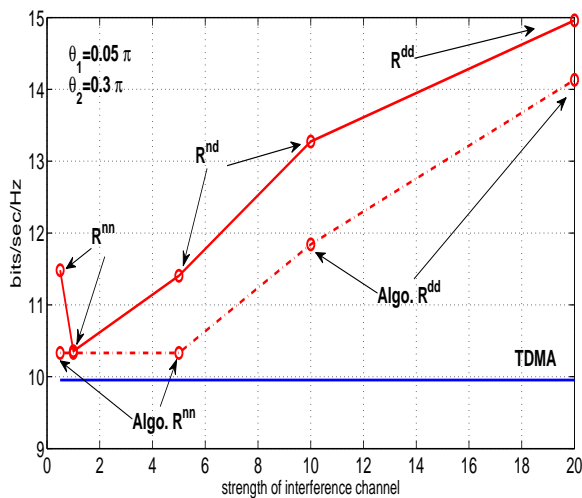


Fig. 5. Sum rate optimal decoding structures when the strength of interference channel increases.

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