Network Coded Multi-way Relaying with Iterative Decoding

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Abstract— **This paper investigates a communication system where three terminals exchange their information with each other with the help of a relay. One joint network and channel coding (JNCC) scheme (based on iterative soft information exchange between channel and network decoder) and one separate network and channel coding scheme based on turbo codes are proposed. The outage behavior of the presented systems are compared with two reference systems, where no network coding (NC) is applied. It is shown that the network coding approaches allow the system to gain diversity for higher rates than the schemes without NC. Moreover, JNCC outperforms the separate approach by exploiting the redundancy provided by the network code.**

I. INTRODUCTION

In wireless communication, fading can have an important influence on the system performance. Diversity techniques are known as a good way to combat fading in such systems. In this work, we use relaying as a diversity technique and use network coding [1] to achieve a diversity gain with a higher spectral efficiency.

We consider a multi-way relay channel where three terminals want to exchange their independent information packets with the help of a relay, for example in a video conference. The transmissions of the nodes are orthogonalized in time. Each terminal receives the signals of the other terminals and the signal transmitted from the relay. The relay uses a decode-and-forward scheme and transmits network encoded version of its received packets. We investigate the performance of such a system in a Rayleigh-fading environment and evaluate the ability to achieve cooperative diversity [2]. We present two practical network code designs based on turbo codes, one joint network and channel coding scheme based on iterative soft information exchange between channel and network decoder and one scheme with separate network and channel coding. We compare their performance analytically and via simulations with reference systems where no network coding is applied.

The multi-way relay channel extends the multi-way channel [3], [4] by introducing a relay and the two-way relay channel (TWRC). The use of network coding for an error-free TWRC was studied in [5]–[7]. Performance bounds for the TWRC with two time phases were considered in [8]–[12]. Practical code designs were proposed in [13]. Performance bounds for the TWRC with three time-phases are considered in [12], [14], [15], practical code designs were proposed in [14], [16]. Comparison of both schemes were given in [12], [17] regarding the achievable rates and in [18] regarding the outage behavior. The multi-way relay channel is also considered in [19] and [20]. Contrary to our model, the direct link is not considered in these works. Moreover, full-duplex relaying is assumed in [19] and analog network coding with multiple antennas is assumed in [20] whereas our work assumes a half-duplex relay that uses digital network coding. In [21] the same system is analyzed under AWGN channel conditions, whereas our paper considers Rayleigh fading channel and focuses on the diversity.

This paper is organized as follows: Section II gives an overview about the system and channel model. In Section III, two different network code designs are presented. Section IV introduces the theoretical calculations about the outage behavior of the presented schemes. In Section V we show our simulation results and discuss these results. Finally, Section VI concludes this paper.

II. SYSTEM MODEL

The investigated system consists of three terminals A, B and C. Each terminal wants to share its information with the other two terminals. A relay node also receives the transmitted signals of each terminal and forwards them to other terminals. The messages of terminals A, B and C are segmented into packets $\mathbf{u}_\mathrm{A}$, $\mathbf{u}_\mathrm{B}$ and $\mathbf{u}_\mathrm{C}$ of K_A, K_B and K_C information bits, respectively. Moreover, each packet contains a cyclic redundancy check (CRC) bits for error detection.

A. Coding Schemes

1) Four-Phase Scheme - Network Coding: Fig. 1 depicts four-phase scheme. Each user has its own time slots to broadcast its signal. Terminals A, B and C encode their information packets $\mathbf{u}_\mathrm{A}$, $\mathbf{u}_\mathrm{B}$ and $\mathbf{u}_\mathrm{C}$ into the symbol blocks $\mathbf{x}_\mathrm{A}$, $\mathbf{x}_\mathrm{B}$ and $\mathbf{x}_\mathrm{C}$ with lengths M_A, M_B and M_C and broadcast them in the first three phases with lengths θ_A, θ_B and θ_C, respectively. Each transmission are received by other terminals and by the relay. After the first three phases, the relay decodes the information packets and obtains the estimates $\tilde{\mathbf{u}}_\mathrm{A}$, $\tilde{\mathbf{u}}_\mathrm{B}$ and $\tilde{\mathbf{u}}_\mathrm{C}$. If the relay can decode the estimates correctly, it network encodes the estimates to the symbol block $\mathbf{x}_\mathrm{R}$ of length M_R and broadcasts it to the other terminals during the fourth phase with length θ_R and with $\theta_\mathrm{A}+\theta_\mathrm{B}+\theta_\mathrm{C}+\theta_\mathrm{R}=1$. Each terminal decodes after the fourth phase the information packets of the other terminals by considering the transmissions received directly from other terminals and received from the relay. If the relay cannot decode all estimates correctly, it remains silent.

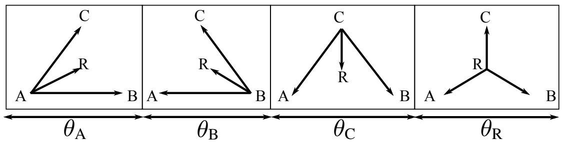

Fig. 1. Four-phase scheme.

2) Six-Phase Scheme - Routing: This reference system describes relaying without network coding. In this case, all terminals transmit their information packets as explained in the previous scheme. Again, the relay decodes the information packets and obtains the estimates $\tilde{\mathbf{u}}_\mathrm{A}$, $\tilde{\mathbf{u}}_\mathrm{B}$ and $\tilde{\mathbf{u}}_\mathrm{C}$. After successfully decoding, it reencodes and punctures these estimates into symbol blocks $\mathbf{x}_\mathrm{RA}$, $\mathbf{x}_\mathrm{RB}$ and $\mathbf{x}_\mathrm{RC}$ of lengths M_RA, M_RB and M_RC with the same channel code as the terminals and broadcasts them in the remaining three phases with lengths θ_RA, θ_RB and θ_RC, respectively and with $\theta_\mathrm{A} + \theta_\mathrm{B} + \theta_\mathrm{C} + \theta_\mathrm{RA} + \theta_\mathrm{RB} + \theta_\mathrm{RC} = 1$. Such distributed channel codes were investigated for one-way relaying in [22]–[25]. It can be said that this scheme is a special case of the previous scheme, where $\theta_\mathrm{R} = \theta_\mathrm{RA} + \theta_\mathrm{RB} + \theta_\mathrm{RC}$.

3) Three-Phase Scheme - Without Relay: This scheme describes the reference system without relay. In this case, each terminal decodes the information packets of the other terminals by only using the direct links from other terminals. This scheme can also be described as a special case of the first scheme, where $\theta_\mathrm{R} = 0$.

B. Channel Model

For the investigated communication system we assume Rayleigh-fading for the links between each node. Moreover, we assume that the fading coefficient do not change during the transmission of one block. The physical distance between terminals A, B and C are equal and it is given by d_t and the distance between the terminals and the relay is given by d_r. We assume that $d_r < d_t$ and hence the links between the terminals and the relay are less influenced by the pathloss. For simplicity, we assume a scenario with symmetric channels and data rates, nevertheless our scheme can be applied to asymmetric scenarios as well. The following equation can be written for describing the input-output relationship of the channel between the nodes *i* and *j*:

$$\mathbf{y}_{i,j} = h_{i,j} \cdot \mathbf{x}_i + \mathbf{z}_{i,j}, \tag{1}$$

where $i, j \in \{\text{A,B,C,R}\}$, $\mathbf{y}_{i,j}$ contains the received samples after the matched filter, $\mathbf{z}_{i,j}$ is the complex gaussian noise and $h_{i,j}$ is the channel coefficient between the nodes *i* and *j*, which can be formulated as:

$$h_{i,j} = a_{i,j} \cdot \sqrt{\left(d_0/d_{i,j}\right)^n}. \tag{2}$$

Here, $a_{i,j}$ corresponds to the Rayleigh coefficients, which is modeled as zero-mean, independently distributed circularly symmetric complex Gaussian random variable and $d_{i,j}$ is the distance between nodes *i* and *j*, d_0 is the reference distance and n is the pathloss exponent. We also assume that the channels are reciprocal, i.e. $h_{i,j} = h_{j,i}$.

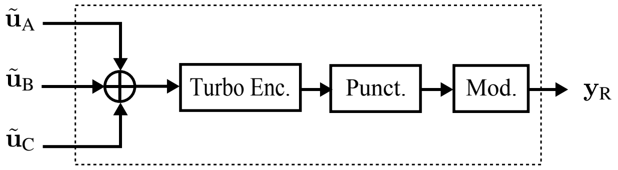

Fig. 2. Encoder at the relay for the system with separate network-channel coding.

III. DESIGN OF NETWORK CODE

In this section, we propose two practical code designs based on turbo codes for the previously described four-phase scheme. In what follows, a symmetric system structure is assumed, where $K_\mathrm{A} = K_\mathrm{B} = K_\mathrm{C}$ and $M_\mathrm{A} = M_\mathrm{B} = M_\mathrm{C}$.

A. Separate Channel and Network Coding

In this approach, network coding is performed in the network layer whereas channel coding is performed separately in the physical layer to guarantee reliable point-to-point links. The terminals firstly channel encode their information packets using a conventional turbo code, then perform puncturing to obtain the desired transmission rate and finally modulate and transmit their coded packets. These packets arrive at the other terminals and at the relay. The relay demodulates and decodes the received signal to obtain the estimates $\tilde{\mathbf{u}}_\mathrm{A}$, $\tilde{\mathbf{u}}_\mathrm{B}$ and $\tilde{\mathbf{u}}_\mathrm{C}$. If all the packets are decoded successfully at the relay, the relay performs network coding by applying a bit-wise modulo-2 addition of the estimated information bits in $\tilde{\mathbf{u}}_\mathrm{A}$, $\tilde{\mathbf{u}}_\mathrm{B}$ and $\tilde{\mathbf{u}}_\mathrm{C}$. Then, the network encoded block can be channel encoded using the same turbo code as the terminals and then punctured to obtain the desired transmission rate. After puncturing, the obtained block can be modulated and broadcasted in the time interval assigned to the relay. The encoder at the relay is depicted in Figure 2.

Each terminal can decode the received signals transmitted directly from other terminals and perform a CRC check. If both packets are decoded correctly, there is no need to use the network encoded packet from the relay, but if one of the directly received packets cannot be decoded successfully, then the packet from the relay can be used to retrieve the other packet. In this case, firstly the packet from the relay should be channel decoded. After channel decoding, the obtained sequence contains the modulo-2 addition of information bits of the three terminals. Since each terminal knows its own information bits apriori, each terminal can decode the missing information packet by applying a modulo-2 addition with the successfully decoded packet, its own information packet and the packet received from the relay. If none of the directly received packets are decoded correctly in this approach, the packet sent from relay cannot be used to retrieve any missing packets. As the physical layer has not to be modified compared to a point-to-point communication, the separation of network coding and channel coding allows a simpler implementation than the joint network-channel coding approach described in the next section.

B. Joint Network and Channel Coding

Joint network and channel coding (JNCC) allows to improve the error protection by using the network coded relay transmission as additional redundancy for the decoding at the terminals. The following JNCC-approach is motivated by the code design in [26] where, contrary to our paper, a model with two sources, one relay and one destination was considered.

Each terminal encodes its packets as explained in the previous section with a turbo code. After estimating the information packets, the relay channel encodes these estimates with the same channel code as the corresponding terminal and punctures the channel coded bits to achieve the desired rate. After interleaving the punctured sequences with different interleavers, a bit-wise modulo-2 addition is applied on the three sequences. This encoder is depicted in Figure 3. In contrast to the previous approach, here the network coding is applied on the code bits, not on the information bits. Moreover, a joint algorithm for network and channel decoding is used at the receiver.

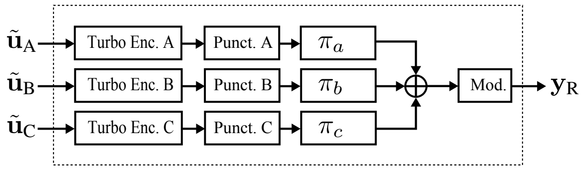

Fig. 3. Encoder at the relay for the system with joint network-channel coding.

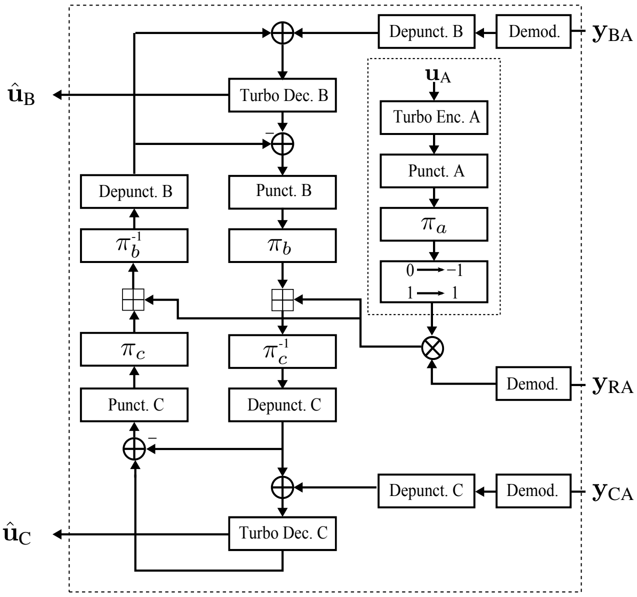

Fig. 4. Decoder at the terminal A for the system with joint network-channel coding.

Figure 4 depicts the joint channel-network decoder at terminal A. In this system, the terminal firstly encodes its own information packet $\mathbf{u}_A$ just as encoded at the terminal. It also demodulates the received signal $\mathbf{y}_{RA}$ from the relay and obtains the LLR values corresponding to $\mathbf{y}_{RA}$. To remove the contributions of $\mathbf{u}_A$, the sign of the LLRs are changed whenever the corresponding bit of the encoded $\mathbf{u}_A$ is 0 and the sequence $\mathbf{L}_{BC}$ is obtained, which contains information about the packets of terminals B and C (please refer to [27] for more about LLR arithmetic). After that, the iterative decoding of the directly received signals $\mathbf{y}_{BA}$ and $\mathbf{y}_{CA}$ can start. For this iterative procedure, firstly $\mathbf{y}_{BA}$ is demodulated, depunctured and decoded. After obtaining the LLRs of the terminal B's code bits at the output of the turbo decoder, it is punctured and interleaved again the same way as it was done at the relay. Now this LLRs can be fed to the *box-plus*[1] operator together with $\mathbf{L}_{BC}$ to remove the contributions of the LLRs of the terminal B from $\mathbf{L}_{BC}$, so that we obtain a sequence containing only apriori information about terminal C. (Note that, this sequence is constructed by removing the contributions of terminals A and B from the relays transmission. The box-plus operation reverses the network coding in LLR domain.) Now, after deinterleaving and depuncturing, similar to the approach given in [26] this information can be fed to the channel decoder together with the LLRs obtained from $\mathbf{y}_{CA}$. After channel decoding of the packets of terminal C, the LLRs of the corresponding code bits can again be processed the same way to be used in the next iteration as apriori information at the channel decoder for the packet of the terminal B. After a fixed number of iterations, the outputs of the channel decoders can be taken as the estimates $\hat{\mathbf{u}}_B$ and $\hat{\mathbf{u}}_C$.

As can be seen from the block diagram of the decoder, the soft values are passing two different interleavers and deinterleavers between the iterations. Hence, these interleavers should be chosen in a way so that the serial combination of one interleaver and another deinterleaver still works as a good interleaver. For example if all the three interleavers are chosen as the same, at the decoders there will not be an interleaving at all, since they will suppress each others effects. We propose the following interleaver design: The first interleaver is chosen as UMTS turbo code [28] interleaver ($\pi_a = \pi$), the second one is constructed by serially combining the first interleaver two times ($\pi_b = \pi\pi$) and the third one does not shuffles its inputs at all ($\pi_c = 1$). The combination of one interleaver and one deinterleaver from this set can build the following mixed interleavers or their inverses:

$$\pi_b \pi_a^{-1} = (\pi\pi) \cdot \pi^{-1} = \pi, \tag{3}$$

$$\pi_a \pi_c^{-1} = \pi \cdot 1^{-1} = \pi, \tag{4}$$

$$\pi_b \pi_c^{-1} = (\pi\pi) \cdot 1^{-1} = \pi\pi. \tag{5}$$

As a result, it can be said that between the iterations the soft values are interleaved and deinterleaved either with π or $\pi\pi$.

IV. OUTAGE BEHAVIOR

In order to evaluate the performance of our code design, we benchmark the performance of the systems with the information-theoretic outage behavior. We define the event OUT as the case, where no reliable communication is possible

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[1] The box-plus operator is analogous to the XOR operation in the LLR domain: LLR$(A \oplus B)$ = LLR$(A) \boxplus$ LLR(B). It can be approximated as $A \boxplus B \approx \text{sign}(A) \cdot \text{sign}(B) \cdot \min\{|A|, |B|\}$ [27].

from at least one of the sources to one of the destinations. We define $\overline{\text{OUT}}$ as the complement of the event OUT and it defines the case, where every terminal can communicate with other terminals (with or without the help of the relay) reliably. In the following, the outage probabilities of the presented systems are given.

A. Three-Phase Scheme - Without Relay

In the system without relay a reliable communication between all terminals is possible, if all following events hold:

$$\begin{aligned} E_{3,1} &: K_{\text{A}} \leq M_{\text{A}} C(\gamma_{\text{AB}}) & E_{3,4} &: K_{\text{B}} \leq M_{\text{B}} C(\gamma_{\text{BA}}) \\ E_{3,2} &: K_{\text{A}} \leq M_{\text{A}} C(\gamma_{\text{AC}}) & E_{3,5} &: K_{\text{C}} \leq M_{\text{C}} C(\gamma_{\text{CA}}) \\ E_{3,3} &: K_{\text{B}} \leq M_{\text{B}} C(\gamma_{\text{BC}}) & E_{3,6} &: K_{\text{C}} \leq M_{\text{C}} C(\gamma_{\text{CB}}) \end{aligned}$$

$C(\gamma_{ij})$ denotes the point-to-point capacity of the channel between the nodes i and j with an SNR value γ_{ij}. In the symmetric case with $K_{\text{A}} = K_{\text{B}} = K_{\text{C}}$, $M_{\text{A}} = M_{\text{B}} = M_{\text{C}}$ and reciprocal channels, $E_{3,4}, E_{3,5}$ and $E_{3,6}$ are identical to the first three events. Hence, the complement of the outage event can be formulated as follows:

$$\overline{\text{OUT}}_{\text{NoRelay}} = E_{3,1} \cap E_{3,2} \cap E_{3,3}. \tag{6}$$

B. Six-Phase Scheme - Routing

The six-phase scheme consists of several one-way relay channels. For example, in order to transmit the information packet of terminal A reliably to terminal B, the following event should hold [29]:

$$E_{6,\text{AB}} = (E_{6,1} \cap E_{6,2}) \cup E_{3,1}, \tag{7}$$

where

$$\begin{aligned} E_{6,1} :\quad K_{\text{A}} \leq &\quad M_{\text{A}} C(\gamma_{\text{AR}}), \\ E_{6,2} :\quad K_{\text{A}} \leq &\quad M_{\text{A}} C(\gamma_{\text{AB}}) + M_{\text{RB}} C(\gamma_{\text{RB}}). \end{aligned}$$

The first part of (7) corresponds to the communication via the relay and the second part corresponds to the direct link between A and B. In order to have a reliable communication from A to B, either the direct link or the link via the relay should allow a reliable communication. The equivalent condition has to be fulfilled for the other links as well. As a result, the complement of the outage event for the six-phase scheme can be written as follows:

$$\overline{\text{OUT}}_{\text{Rout.}} = E_{6,\text{AB}} \cap E_{6,\text{AC}} \cap E_{6,\text{BA}} \cap E_{6,\text{BC}} \cap E_{6,\text{CA}} \cap E_{6,\text{CB}}, \tag{8}$$

where all events are defined analogous to $E_{6,\text{AB}}$.

C. Four-Phase Scheme - Joint Network-Channel Coding

In the four-phase scheme with JNCC, for a reliable communication either all direct links should work, or the following conditions should hold:

$$\begin{aligned} E_{4,1} &: & K_{\text{A}} &\leq M_{\text{A}} C(\gamma_{\text{AR}}) \\ E_{4,2} &: & K_{\text{B}} &\leq M_{\text{B}} C(\gamma_{\text{BR}}) \\ E_{4,3} &: & K_{\text{C}} &\leq M_{\text{C}} C(\gamma_{\text{CR}}) \\ E_{4,4} &: & K_{\text{A}} + K_{\text{C}} &\leq M_{\text{A}} C(\gamma_{\text{AB}}) + M_{\text{C}} C(\gamma_{\text{CB}}) + M_{\text{R}} C(\gamma_{\text{RB}}) \\ E_{4,5} &: & K_{\text{A}} + K_{\text{B}} &\leq M_{\text{A}} C(\gamma_{\text{AC}}) + M_{\text{B}} C(\gamma_{\text{BC}}) + M_{\text{R}} C(\gamma_{\text{RC}}) \\ E_{4,6} &: & K_{\text{B}} + K_{\text{C}} &\leq M_{\text{B}} C(\gamma_{\text{BA}}) + M_{\text{C}} C(\gamma_{\text{CA}}) + M_{\text{R}} C(\gamma_{\text{RA}}) \\ E_{4,7} &: & K_{\text{A}} &\leq M_{\text{A}} C(\gamma_{\text{AB}}) + M_{\text{R}} C(\gamma_{\text{RB}}) \\ E_{4,8} &: & K_{\text{A}} &\leq M_{\text{A}} C(\gamma_{\text{AC}}) + M_{\text{R}} C(\gamma_{\text{RC}}) \\ E_{4,9} &: & K_{\text{B}} &\leq M_{\text{B}} C(\gamma_{\text{BA}}) + M_{\text{R}} C(\gamma_{\text{RA}}) \\ E_{4,10} &: & K_{\text{B}} &\leq M_{\text{B}} C(\gamma_{\text{BC}}) + M_{\text{R}} C(\gamma_{\text{RC}}) \\ E_{4,11} &: & K_{\text{C}} &\leq M_{\text{C}} C(\gamma_{\text{CA}}) + M_{\text{R}} C(\gamma_{\text{RA}}) \\ E_{4,12} &: & K_{\text{C}} &\leq M_{\text{C}} C(\gamma_{\text{CB}}) + M_{\text{R}} C(\gamma_{\text{RB}}) \end{aligned}$$

These conditions can be obtained by extending the results in [12] and [30].

As a result, the complement of the outage event for the network coding case can be written in the symmetric case as follows:

$$\overline{\text{OUT}}_{\text{NC Joint}} = \overline{\text{OUT}}_{\text{NoRelay}} \cup \overline{\text{OUT}}_{\text{WithRelay}}, \tag{9}$$

where $\overline{\text{OUT}}_{\text{WithRelay}}$ is the event that all conditions given in $E_{4,1}$ to $E_{4,12}$ are fulfilled.

D. Four-Phase Scheme - Separate Network and Channel Coding

In this scheme, terminal A can retrieve the information packets of terminals B and C successfully, if it decodes either both packets coming directly from other terminals correctly, or if it decodes one of the packets from direct links and the packet from the relay correctly, i.e. terminal A decodes both information packets successfully, if the following event hold:

$$\begin{aligned} E_{4,A} \quad : \quad & (E_{3,4} \cap E_{3,5}) \cup (E_{3,4} \cap E_{4,RA} \cap E_{4,R}) \cup \\ & (E_{3,5} \cap E_{4,RA} \cap E_{4,R}) \end{aligned} \tag{10}$$

whereas $E_{4,RA}$ and $E_{4,R}$ are defined as follows:

$$\begin{aligned} E_{4,RA} :&\quad \max\{K_{\text{A}}, K_{\text{B}}, K_{\text{C}}\} \leq M_{\text{R}} \cdot C(\gamma_{\text{RA}}), \\ E_{4,R} :&\quad E_{4,1} \cap E_{4,2} \cap E_{4,3}. \end{aligned}$$

Equivalent events $E_{4,B}$ and $E_{4,C}$ can also be defined for terminals B and C. The complement of the outage event for this scheme is defined as:

$$\overline{\text{OUT}}_{\text{NC}} = E_{4,A} \cap E_{4,B} \cap E_{4,C}. \tag{11}$$

E. Maximum Achievable Rate for Binary Channel Input

A system is considered to achieve diversity gain, if it can tolerate a deep fade on one of its links, i.e. no outage occurs. In case of a deep fade between the terminals A and B, to tolerate $\gamma_{\text{AB}} \rightarrow 0$ the following conditions should hold:

$$\theta_{\text{A}} \leq \frac{K_{\text{A}}}{M}, \theta_{\text{B}} \leq \frac{K_{\text{B}}}{M}, \theta_{\text{C}} \leq \frac{K_{\text{C}}}{M}, \theta_{\text{R}} \leq \frac{\max(K_{\text{A}}, K_{\text{B}}, K_{\text{C}})}{M},$$

where $M = M_{\text{A}} + M_{\text{B}} + M_{\text{C}} + M_{\text{R}}$. In symmetric case, this conditions can be formulated together by considering the main condition $\theta_{\text{A}} + \theta_{\text{B}} + \theta_{\text{C}} + \theta_{\text{R}} = 1$ as $4K_{\text{A}}/M \leq 1$. Since the sumrate R_{sum} of the whole system can be written as $3K_{\text{A}}/M$, the following condition should hold for obtaining diversity in such a system:

$$R_{sum} = \frac{K_{\mathrm{A}} + K_{\mathrm{B}} + K_{\mathrm{C}}}{M} \leq \frac{3}{4}. \tag{12}$$

In [18] the maximum achievable sumrate for obtaining diversity in a symmetric setup with two-way relay channel is given as 2/3 and for the system with routing as 1/2, whereas in our system the diversity can be gained for rates up to 3/4.

V. RESULTS AND DISCUSSION

In this section, we show the results of our simulations for the comparison of the presented schemes. We compare both of the network coding schemes and the two reference systems without network coding (system without relay and system with routing). We observe the outage behavior of the systems and compare their diversity gains under Rayleigh fading. In our simulations we use a symmetric system setup and take the distance between the terminals as d_0 and the distance between the terminals and the relay as $d_0/\sqrt{3} = 0.57d_0$, such that the locations of the terminals correspond to the corners of a triangle with edges of equal length and the relay is located directly in the middle of this triangle. We take the pathloss exponent as $n = 3.52$ and as a result the average SNRs of the channels between the terminals and the relay are 8.39dB better than the average SNRs of the channels between the terminals. The information packets of each terminal contain 1500 bits and all the terminals use the UMTS turbo code [28] with 8 iterations and BPSK modulation. To obtain the desired length for the transmission block, the parity bits are punctured. For a fair comparison, we choose the total number of transmitted symbols per cycle equal to 6600 for all the schemes. According to that, for the three-phase scheme without relay each terminal transmits $M_{\mathrm{A}} = M_{\mathrm{B}} = M_{\mathrm{C}} = 2200$ symbols during its transmission period. In the network coding approaches, each terminal and the relay transmit 1650 symbols per transmission period, such that $M_{\mathrm{A}} + M_{\mathrm{B}} + M_{\mathrm{C}} + M_{\mathrm{R}} = 6600$. For the routing case, the relay transmits 550 symbols to each terminal ($M_{\mathrm{RA}} = M_{\mathrm{RB}} = M_{\mathrm{RC}} = 550$). The sumrate for all the systems is $(1500 + 1500 + 1500)/6600 = 0.68$. For the joint network and channel coding case, we use 4 iterations at the decoder. The interleavers are chosen as explained in the previous section. We use the UMTS turbo interleaver [28] as the first interleaver. The second one is constructed by serially combining the same interleaver twice and the third one does not shuffle its input at all. We use the channel model explained in Section II-B and take fading coefficients $a_{i,j}$ as random variables with Rayleigh distributed power and mean-power equal to one.

Beside the simulation of the full communication system, we also evaluate numerical results for the achievable outage rates according to the events defined in Section IV. By using the numerically calculated channel capacity values for BPSK, we calculate the probabilities of an outage event by randomly generating Rayleigh distributed fading coefficients and checking how often an outage event occur for different schemes.

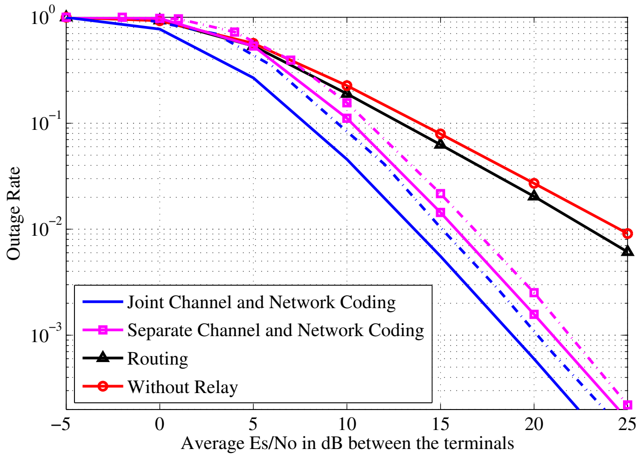

Fig. 5. Solid lines depict the theoretical outage rates. Dashed lines depict the simulation results of the turbo coding schemes.

Figure 5 shows the outage probability of the presented systems. The solid lines show the outage rate based on the results of Section IV and the dashed lines depict the simulated packet error rate of the practical system with turbo code. According to this figure it can be seen that the slopes of the curves corresponding to the network coding cases is approximately -2, which denote a diversity order of 2, whereas the slopes of the curves of the systems with routing and without relay are about -1. This results show that the system with network coding is more robust to the deep fades of the Rayleigh-fading channel and even if one of the links is broken, the system can work without an outage event. Another important issue to note is that this system gains a diversity of order 2 by a sumrate of 0.68, whereas with a two-way relay system the diversity order of 2 can be gained only if the sumrate is lower than 2/3 and for a relay system without network coding the sumrate has to be lower than 1/2 to achieve diversity [18].

If we compare the two network coding approaches, we can observe that joint network and channel coding scheme outperforms the separate network and channel coding case and shows better results than the theoretical limits of the second system because the transmission of the relay is exploited as additional redundancy to support the error protection. However, there is still a gap about 1.7 dB between the theoretical limit and simulation results of the joint network and channel coding.

It is also an important point to mention here that the individual bit error rate curves between the terminals (which are not given in this paper due to the space constraints) for the joint network and channel coding scheme are approximately the same. This shows that our selection of the interleavers at the relay resulted in a symmetric structure so that each decoder (which contains a different combination of the interleavers) works with nearly the same performance.

We also investigated the behavior of the proposed joint network and channel decoding scheme for different number of iterations. In order to concentrate only on the influence of

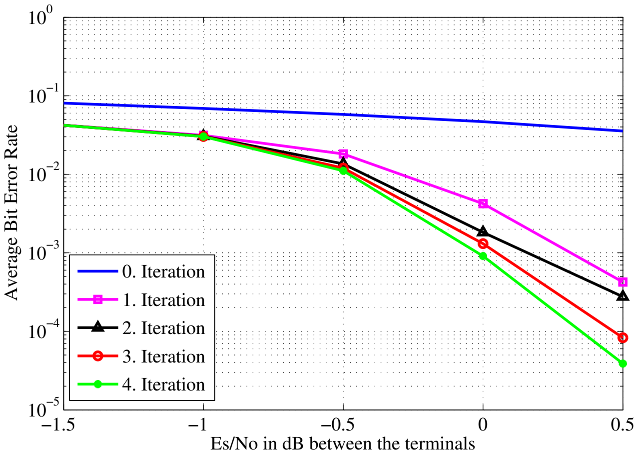

Fig. 6. Bit error rates of the joint network and channel coding approach at different iterations in case of AWGN channel conditions.

the number of iterations, we simulated the same system under AWGN channel conditions without fading. Figure 6 depicts the average bit error rate of the system at different iterations. As can be seen from the figure, after each iteration the results get better and after 4 iterations a considerable gain is obtained compared to the 0th iteration.

VI. CONCLUSION

We investigated the communication between three terminals with the help of a relay. We proposed two network coding schemes for such a system and compared them with reference systems without relay and without network coding. We showed that by passing soft information iteratively between channel decoders and the network decoder a better performance can be obtained compared to the scheme where network and channel decoding is performed separately. Moreover, we showed that network coding increases the maximum code rate to gain diversity for a binary channel input. Whereas the reference system without network coding is not able to gain diversity for rates larger than 1/2, the proposed system with network coding allows to gain diversity for rates up to 3/4.

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