Cooperative diversity using per-user power control in the MAC channel

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Abstract

We consider a multiple access MAC fading channel with two users communicating with a common destination, where each user mutually acts as a relay for the other one as well as wishes to transmit his own information as opposed to having dedicated relays. We wish to evaluate the usefulness of relaying from the point of view of the system's throughput (sum rate) rather than from the sole point of view of the user benefiting from the cooperation as is typically done. We do this by allowing a trade-off between relaying and fresh data transmission through a resource allocation framework. Specifically, We propose cooperative transmission scheme allowing each user to allocate a certain amount of power for his own transmitted data while the rest is devoted to relaying. The underlying protocol is based on a modification of the so-called non-orthogonal amplify and forward (NAF) protocol [1]. We develop capacity expressions for our scheme and derive the rate-optimum power allocation, in closed form for centralized and distributed frameworks. In the distributed scenario, partially statistical and partially instantaneous channel information is exploited.
The centralized power allocation algorithm indicates that even in a mutual

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cooperation setting like ours, on any given realization of the channel, cooperation is never truly mutual, i.e. one of the users will always allocate zero power to relaying the data of the other one, and thus act selfishly. But in distributed framework, our results indicate that the sum rate is maximized when both mobiles act selfishly.

Key words: Cooperative diversity, NAF protocol, Power allocation

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1. Introduction

In many wireless applications, wireless users may not be able to support multiple antennas due to size, complexity, power, or other constraints. The wireless medium brings along its unique challenges such as fading and multiuser interference, which can be combatted via the concept of cooperative diversity [2, 3, 4, 5]. In traditional cooperative diversity setups, a user is unilaterally designated to act as a relay for the benefit of another one, at least for a given period of time. In certain scenarios, the relay is an actual component of the infrastructure with no own data to be delivered to the network [6, 7, 8, 9]. In cellular-type multiuser networks however, there will be a compromise to strike by all users between transmitting their own information and helping others by relaying their data to the destination [10, 11, 12, 13, 14, 15, 16, 17, 18]. A simplified instance of this scenario is given by a multiple access channel with two or more users trying to reach a common destination (e.g. base station). Since each user wishes to send its own information, it must allocate resource (the total of which is constrained at each user) wisely between its own data transmission and the data it will relay for the benefit of some other user.
In this paper we consider resource control in the form of power allocated by a user across its own data and its relay data. The underlying protocol considered here is similar to the one considered by Azarian et al. in [1], which itself evolved from the early work by Laneman, Tse, and Wornell [2]. There, the authors imposed the half-duplex constraint on the cooperating nodes and proposed several cooperative transmission protocols. All the proposed schemes in [2] used a time-division multiple-access (TDMA) strategy, where the two partners relied on the use of orthogonal signaling to repeat each other's signals. In this work relay and own transmission operations take place in orthogonal resource slots but share a common average power resource where the average is computed over two frames. Note that other (than power)

types of resource division could also be considered, such as bandwidth [17]. Recently non-orthogonal signaling strategies have been proposed, e.g. [1], in which a relay transmits delayed information by a user while this user simultaneously transmit fresh data. In this non-orthogonal amplify-forward (NAF) scheme, the diversity-multiplexing trade-off is studied, showing the superiority of the NAF scheme over the orthogonal counterpart. However in [1] and much previous work, the relay network model is unbalanced in the sense that the transmission of own data by the relay is not considered, and the source node is not invited to act as a relay either. In multiuser networks, it is desirable from a global capacity point of view that each user allocates a fraction of its resource toward cooperation. Just how big is this fraction should be is one of the questions addressed by this paper.

We consider the problem of maximizing the sum rate for this cooperative MAC channel, as function of the power allocation toward own and relay data, given certain knowledge of the channel for both users. We derive the optimum power allocation policy in closed form for certain scenarios of interest. We consider both centralized and distributed cases. In the centralized case first explored in [19], the base initially collects instantaneous CSI from both users and computes the optimum power allocation vector on behalf of the two users.

In the distributed case, it is assumed that the complete fast-varying CSI cannot be exchanged between the users and the base. In that case, the users individually come up with a power allocation strategy based on a mix of local CSI and non-local statistical CSI. We show that in fact, when the optimum policy is used, one of the users always acts completely selfishly. Interestingly, this type of selfish behavior by some users in multiuser cooperative MAC was noted by [20], but in a different context with decode-and-forward signaling. Also the problem of distributed power allocation was addressed in [21], however in this paper the algorithm is used to optimize the BER performance. Furthermore, the power allocation is done across users rather than across relay and data transmission operations.

Then we investigate the system gain (sum rate) of mutual cooperation in two different network geometries. We show the system gain depends on the level of symmetry in the user positions.

This paper is organized as follows. In section 2, we describe the system model. In section 3 the sum-rate expression is derived and the optimal power allocation algorithm is presented for the centralized framework. In section 4, the distributed framework is investigated and the simulation results are presented in section 5. We conclude by section 6.

Notations: All boldface letters indicate vectors (lower case) or matrices (upper case). The operator det( ) is the determinant of matrix, with $(\ )^H$ denoting its conjugate-transpose and $(\ )^*$ denoting its conjugate. $\mathbb{E}[.]$ is the expectation operator.

2. System Model

We consider a two user fading Gaussian Multiple Access Channel (MAC), where both the receiver and the transmitters receive noisy versions of the transmitted messages. Each receiver maintains channel state information and employs coherent detection. The channels between users (inter-user channels) and from each user to the destination (uplink channels) are mutually independent. Time is divided in two consecutive frames. Each frame is further divided in two half-frames T_1 and T_2. We use a combination of TDMA and non-orthogonal signaling: In the first half of frame 1, user 1 sends its first half packet (containing $\frac{N}{2}$ bits) while user 2 listens. In the second half, user 2 relays the overheard data with power level β, while user 1 simultaneously sends fresh information (its second half packet) with power level $1-\alpha$ where α is chosen in $[0, 1]$. In frame 2 we proceed just as in frame 1, but with the roles of user 1 and 2, α, β are reversed. Thus we maintain a constant average power across the two frames, for each user, regardless of the choice of α, β.

2.1. Signal model

The signal received by the common destination during the first frame (first and second half) is given by,

$$\begin{cases} y_1(n) = h_{01}x_1(n) + z_0(n) \\ y_1(n+\frac{N}{2}) = \sqrt{1-\alpha}h_{01}x_1(n+\frac{N}{2}) + \sqrt{\beta}h_{02}A_1\left[h_{21}x_1(n) + w_2(n)\right] + z_0(n+\frac{N}{2}) \end{cases} \tag{1}$$

During the second frame, the received signal is:

$$\begin{cases} y_2(n) = h_{02}x_2(n) + z_0(n) \\ y_2(n+\frac{N}{2}) = \sqrt{1-\beta}h_{02}x_2(n+\frac{N}{2}) + \sqrt{\alpha}h_{01}A_2\left[h_{12}x_2(n) + w_1(n)\right] + z_0(n+\frac{N}{2}) \end{cases} \tag{2}$$

where $n = 1, .., \frac{N}{2}$ and h_{ij} captures the effects of fading between transmitter j and receiver i.

Thus, in (1) and (2), α and β can be seen as *cooperation levels* for user 1 and user 2 respectively. $x_{j\in\{1,2\}}(n) \in \mathcal{C}$ is the n^{th} coded symbol, $w_{i\in\{1,2\}}(n)$ and $z_0(n)$ are respectively the noise sample (of variance $N_{i\in\{1,2\}}$) observed by the transmitter $j \in \{1,2\}$ and the noise sample (of variance N_0) observed by the destination. h_{21} and h_{12} represent the inter-user channel gains, and h_{01} and h_{02} denote the user-destination channel gains, which are maintained constant during $T_1 + T_2$. $A_1 \leq \sqrt{\frac{P_2}{|h_{21}|^2 P_1 + N_2}}$ and $A_2 \leq \sqrt{\frac{P_1}{|h_{12}|^2 P_2 + N_1}}$ are the relay repetition gains, where $P_{j\in\{1,2\}}$ is the sample energy. Clearly, the choice $\alpha = 0$, $0 < \beta < 1$, would result in a classical 3 node relay scenario with user 1 acting as a source of information and user 2 being a source as well as a relay node. We remark that (1) and (2) are reduced to equations of an orthogonal direct transmission (non-cooperative protocol) if $\alpha = \beta = 0$, and to an amplify-and-forward protocol if $\alpha = \beta = 1$ [2].
We hereby focus our attention on how the relay operation can improve the *sum rate* of the cooperative MAC system, by proper choices of α and β. We argue that sum rate (rather than the traditional diversity-oriented measures) is a valuable performance metric when degrees of freedom for diversity can be acquired at other layers of the protocol stack, such as via scheduling.

3. Centralized power allocation

3.1. Analysis of sum rate

In the proposition below, we assume a central control unit (located e.g at the base) with knowledge of all CSI, we develop the expression for the sum rate for the above protocol and power allocation system in a way similar to developments by Laneman et al. and others.

Proposition 1. For the Gaussian memoryless multiple-access channel with user cooperation, if the rate pair (R1,R2) is achievable, then the sum-rate $R1 + R2 \leq I_{\alpha,\beta}$ where

$$\begin{aligned} I_{\alpha,\beta} \triangleq & log_2\left[1 + \gamma_{01} + (1-\alpha)\frac{K_1}{l_1(\beta)} + f(\beta\gamma_{02}, \gamma_{21})\right] \\ & + log_2\left[1 + \gamma_{02} + (1-\beta)\frac{K_2}{l_2(\alpha)} + f(\alpha\gamma_{01}, \gamma_{12})\right] \end{aligned} \quad (3)$$

where

$$\begin{array}{l} K_1 = [\gamma_{01}^2 + \gamma_{01}]\,[\gamma_{21}+1] \\ K_2 = [\gamma_{02}^2 + \gamma_{02}]\,[\gamma_{12}+1] \\ l_1(\beta) = 1 + \gamma_{21} + \beta\gamma_{02} \\ l_2(\alpha) = 1 + \gamma_{12} + \alpha\gamma_{01} \\ f(x,y) = \frac{xy}{x+y+1} \end{array} \tag{4}$$

and γ_{ij} is defined as $|h_{ij}|^2 \frac{P_j}{N_i}$ where P_j is the power of the transmitted signal from user j, N_i is the noise power at the receiver i and $i, j \in \{1, 2\}$.
Proof : Please refer to the Appendix.
Note that the expression above requires channel information at the receiver but not the transmitter. However the optimization with respect to power control coefficients α and β, will require full channel knowledge. We can consider in the sequel that $P_1 = P_2 = P$ and $\gamma_{21} = \gamma_{12} = \gamma$ since the same frequency is used in both directions of inter-user communication.

3.2. Optimization of relay power allocation

We now address the problem of optimizing the power allocated by each user toward either transmission of its own data or relay data. The objective function taken here is the multiuser sum rate defined in eq. (3). We start by characterizing the sum rate in some border points of the power region. The lemma below comes handy in the more general characterization of the optimal power allocation policy.

Lemma 1. We characterize the sum-rate over the feasible power allocation region by :

$$\begin{cases} I_{\alpha,0} > I_{\alpha,1} & \forall\ \alpha \in [0,1] \\ I_{0,\beta} > I_{1,\beta} & \forall\ \beta \in [0,1] \end{cases} \tag{5}$$

Proof : Please refer to the Appendix.
This shows that, from the point of view of global throughput performance, the system would rather have all users act selfishly.

3.2.1. Power Allocation Algorithm

We now proceed to give a complete characterization of the optimal power allocation policy for an arbitrary realization of the multiuser channels.

Proposition 2. The optimal power allocation which maximizes the sum-rate (3) is given by,

1. $\alpha = \alpha_* \neq 0$ and $\beta = 0$ if $\begin{cases} \gamma > \gamma_{02}^2 + \gamma_{02} \\ \gamma_{01} > \frac{(1+\gamma_{02})^2(1+\gamma)}{\gamma-(\gamma_{02}^2+\gamma_{02})} - 1 \end{cases}$
2. $\alpha = 0$ and $\beta = \beta_* \neq 0$ if $\begin{cases} \gamma > \gamma_{01}^2 + \gamma_{01} \\ \gamma_{02} > \frac{(1+\gamma_{01})^2(1+\gamma)}{\gamma-(\gamma_{01}^2+\gamma_{01})} - 1 \end{cases}$
3. $\alpha = 0$ and $\beta = 0$ if neither condition above is met.

where optimal values α_*, β_* are detailed in the appendix, and shown below.
Proof : Please refer to the Appendix.
Interpretations: We remark that zero or at most one user out of the two cooperates with the other one. Hence the two users will never both take the role of relay on a given channel realization. In fact the user with "worse" channel conditions always acts selfishly and concentrates all its power for its own data, while the other user will graciously help the selfish user or possible be itself selfish also. Of course, the roles of selfish users and cooperative users will be alternating randomly as the channel changes, so that in the long run both users are going to participate in the cooperation at some points and benefit from it at some other points. Interestingly, this result deprives the otherwise appealing concept of *mutual* cooperation from much of its sense. However a truly mutual cooperation remains possible on the basis of averaging across many realization of the fading channel.
We now examine the interesting particular case of instantaneously symmetric channel:

Lemma 2. In the particular case, when $\gamma = \gamma_{01} = \gamma_{02}$, the two users act selfishly.
Algorithm 1 : Power allocation with instantaneous CSI.
The implementation of the algorithm below requires a centralized power allocation procedure done by e.g. the base.
The following intermediate quantities are computed:
$A_1 = K_1\gamma_{01}^2(1+\gamma+\gamma_{02})$
$A_2 = K_1\gamma_{01}(1+\gamma)(1+\gamma+\gamma_{02})$
$C = K_1\left[\gamma\frac{K_1}{\gamma_{01}} - \frac{K_2}{\gamma_{02}}(1+\gamma) - K_2(2+\gamma+\gamma_{01})\right]$
$A_1' = K_2\gamma_{02}^2(1+\gamma+\gamma_{01})$
$A_2' = K_2\gamma_{02}(1+\gamma)(1+\gamma+\gamma_{01})$

$C' = K_2 \left[\gamma \frac{K_2}{\gamma_{02}} - \frac{K_1}{\gamma_{01}}(1+\gamma) - K_1(2+\gamma+\gamma_{02})\right]$
cond1 $= \frac{K_2}{1+\gamma}$, condp1 $= \frac{K_1}{1+\gamma}$, cond2 $= \frac{(1+\gamma_{02})^2(1+\gamma)}{\gamma-(\gamma_{02}^2+\gamma_{02})} - 1$ and condp2 $= \frac{(1+\gamma_{01})^2(1+\gamma)}{\gamma-(\gamma_{01}^2+\gamma_{01})} - 1$.
if $\gamma >$ cond1 & $\gamma_{01} >$ cond2, **then**
$\alpha_* = -\frac{A_2}{A_1} + \sqrt{\frac{C}{A_1} + \left(\frac{A_2}{A_1}\right)^2}$
user 1 cooperates with a level given by α_* and resulting in a sum-rate of $I_{\alpha_*,0}$.
else
if $\gamma >$ condp1 & $\gamma_{02} >$ condp2, **then**
$\beta_* = -\frac{A'_2}{A'_1} + \sqrt{\frac{C'}{A'_1} + \left(\frac{A'_2}{A'_1}\right)^2}$
user 2 cooperates with a level given by β_* and resulting in a sum-rate of I_{0,β_*}.
else
Decision : No cooperation, $[\alpha_*, \beta_*] = [0, 0]$, sum-rate $= I_{0,0}$.

4. Distributed cooperative power allocation

In the distributed framework, each node has a hybrid channel state information. So instead of considering the global knowledge of the instantaneous channel realizations, each mobile has only a local CSI knowledge i.e, each mobile has the perfect knowledge of its links with the base station and the other mobile, but only a statistical knowledge of the link between the other mobile and the base station.

In this framework, each user is optimizing on an individual basis the amount of power allocated to relaying the other user data. We draw the reader's attention on the fact that, although the users optimize their power allocation in a distributed manner, they will do so in a cooperative fashion since each user has the average sum rate as an objective function to maximize rather than its own individual rate. To perform the optimization, user 1 (resp. user 2) must estimate the complete vector $[\alpha, \beta]$, following which it implements a power control based on α (resp. β) while the other variable plays an auxiliary role only.

Before proceeding to give the optimal distributed power allocation solution, we provide a characterization of the objective function that each user sets out to maximize. Let $\bar{I}_i$ the expected sum rate seen by mobile i. By construction, the expected sum rate is function of the local CSI, known deterministically by mobile (i), and averaged over all realizations of the channel gains which are non-locally observable by this mobile. In this paper, the example of distributed scenario considered assumes that mobile i has instantaneous knowledge of γ_{0i} and γ, while only the statistics of $\gamma_{0j}, j \neq i$ are known to this mobile. Thus we can define:

$$\bar{I}_1(\alpha,\beta) = \mathbb{E}_{\gamma_{02}}\left[I_{\alpha,\beta}\right] \tag{6a}$$
$$\bar{I}_2(\alpha,\beta) = \mathbb{E}_{\gamma_{01}}\left[I_{\alpha,\beta}\right] \tag{6b}$$

where $\mathbb{E}$ is the expectation operator.
In order to seek the optimal distributed power allocation, we start by developing the expressions in (6).

Lemma 3. For the mobile 1, $\bar{I}_1(\alpha,\beta)$ is defined $\forall\alpha$ as

$$\begin{cases} \log_2(a) - \Phi(0) - \log_2(l_2(\alpha)), if\ \beta = 0 \\ -\left[\frac{\exp\left(\frac{a}{b(1)}\right)Ei\left(-\frac{a}{b(1)}\right)-\ln(a)}{\ln(2)}\right] + \left[\frac{\exp\left(\frac{1}{\xi(1)}\right)Ei\left(-\frac{1}{\xi(1)}\right)}{\ln(2)}\right] - \left[\frac{\exp\left(\frac{c}{d(1)}\right)Ei\left(-\frac{c}{d(1)}\right)-\ln(c)}{\ln(2)}\right] \\ -\log_2(l_2(\alpha)), if\ \beta = 1 \\ -\left[\frac{\exp\left(\frac{a}{b(\beta)}\right)Ei\left(-\frac{a}{b(\beta)}\right)-\ln(a)}{\ln(2)}\right] + \left[\frac{\exp\left(\frac{1}{\xi(\beta)}\right)Ei\left(-\frac{1}{\xi(\beta)}\right)}{\ln(2)}\right] - \log_2(l_2(\alpha)) - \Phi(\beta), if\ 0 < \beta < 1 \end{cases} \tag{7}$$

where $l_2(\alpha)$ has been previously defined and

$$\begin{aligned} &a = (1+\gamma_{01})(1+(1-\alpha)\gamma_{01}) \\ &\xi(\beta) = \tfrac{\beta\overline{\gamma}_{02}}{1+\gamma} \\ &b(\beta) = \xi(\beta)(1+\gamma+\gamma_{01}) \\ &c = (1+\gamma)(1+\alpha\gamma_{01}) \\ &d(\beta) = \left[(2-\beta)(1+\gamma)+\alpha\gamma_{01}\right]\overline{\gamma}_{02} \\ &f(\beta) = (1-\beta)(1+\gamma)\left(\overline{\gamma}_{02}\right)^2 \\ &\Delta(\beta) = \left[d(\beta)\right]^2 - 4cf(\beta) \\ &\Lambda_1(\beta) = \tfrac{d(\beta)-\sqrt{\Delta(\beta)}}{2f(\beta)} \\ &\Lambda_2(\beta) = \tfrac{d(\beta)+\sqrt{\Delta(\beta)}}{2f(\beta)} \\ &\Phi(\beta) = \left[\tfrac{\exp(\Lambda_1(\beta))Ei(-\Lambda_1(\beta))}{\ln(2)}\right] + \left[\tfrac{\exp(\Lambda_2(\beta))Ei(-\Lambda_2(\beta))}{\ln(2)}\right] - \log_2(c) \end{aligned} \tag{8}$$

where $Ei(.)$ is the exponential integral defined as $Ei(x) = \int_{-\infty}^{x} \frac{\exp(t)}{t}dt$. The notation of the dependence on α in (8) is omitted.
We deduce the relation for mobile 2 by changing γ_{01} in (7) by γ_{02}, $\overline{\gamma}_{02}$ in (7)

by $\overline{\gamma}_{01}$, α by β and vice versa.
Proof : Please refer to the Appendix.

4.1. Optimal Distributed Power Allocation

Taking (6a) and (6b) respectively, for mobile 1 and 2 as the objective functions, the optimal distributed power allocation problem can be stated as

$$\begin{cases} (\alpha_*^1, \beta_*^1) = \arg\max_{\alpha,\beta} \bar{I}_1(\alpha, \beta) \\ (\alpha_*^2, \beta_*^2) = \arg\max_{\alpha,\beta} \bar{I}_2(\alpha, \beta) \end{cases} \tag{9}$$

where mobile 1 (resp. mobile 2) is mainly concerned with α_*^1 (resp. β_*^2). Therefore the distributed power allocation vector resulting from our scheme will be (α_*^1, β_*^2). Due to the complex expressions in (7), it appears difficult at first glance to give a closed-form solution for the optimal power allocation vector. However, the following result helps reach a surprisingly simple solution to our problem.

Proposition 3. $\forall \alpha_0 \neq 0$, $\forall \beta_0 \neq 0$, (α_0, β_0) cannot be an optimal solution for power allocation for mobile 1 nor mobile 2.
Proof : Please refer to the Appendix.
Consequently, in a distributed scenario with hybrid CSI at the users, we find that from the point of view of each mobile, at least one of the two users should *not* cooperate. In order to determine which one shall cooperate, mobile 1 considers the two following cases: either it shall not cooperate, or the other mobile shall not cooperate. In each case, a power allocation coefficient is determined. Thus, for mobile 1, the following problem is solved:

$$\begin{cases} \alpha_*^1 = \arg\max_{\alpha} \bar{I}_1(\alpha, 0) \\ \beta_*^1 = \arg\max_{\beta} \bar{I}_1(0, \beta) \end{cases} \tag{10}$$

and for mobile 2, the following problem is solved :

$$\begin{cases} \alpha_*^2 = \arg\max_{\alpha} \bar{I}_2(\alpha, 0) \\ \beta_*^2 = \arg\max_{\beta} \bar{I}_2(0, \beta) \end{cases} \tag{11}$$

The following lemma helps further simplify the problem:

Lemma 4. In eq. (10) $\alpha_*^1 = 0$ and in eq. (10) $\beta_*^2 = 0$.
Proof : Please refer to the Appendix.

Proposition 4. In the distributed power allocation problem written in eq. (9), the optimal strategy is for each user to act selfishly.
Proof : Lemma 4 indicates that from the point of view of mobile 1 (resp. mobile 2), the optimal power allocation strategy takes the form $(0, \beta_*^1)$ (resp. the form $(\alpha_*^2, 0)$, hence neither mobile is going to cooperate. Note that this result is valid for any individual value of the average SNR on all links, this for any position of the nodes. Our results mean that, in order to gain from relaying in terms of sum rate, access to instantaneous knowledge of all channel links is crucial.
We provide examples of this behavior through 3D plots of the sum rate for particular instances of the channels. As depicted in figures 5 and 6, mobile 1 has $(0, 0.447)$ as optimal power allocation, while $(0, 0)$ as optimal power allocation for mobile 2. Therefore the distributed algorithm allocates zero power for each mobile to relaying the data of the other one, and thus act selfishly.

5. Simulation results

5.1. Channel Model

We model all channels as Rayleigh block flat fading with additive white Gaussian noise. Channel coefficients h_{ij} are modeled as zero-mean, circularly symmetric complex Gaussian random variables with different variances. Noises are modeled as zero-mean mutually independent, circularly-symmetric, complex Gaussian random sequences with variance N_0.

5.2. Network Geometry

We anticipate that cooperation will perform differently as function of the positions of the users with respect to destination. We study two particular different network geometries, denoted by *symmetric* and *asymmetric* or *linear*, depicted respectively by figures. 2 and 3. In the asymmetric case, we model the path-loss, i.e. the mean channel powers σ_{ij}^2, as a function of the relative relay position d by

$$\sigma_{01}^2 = 1 \ , \ \sigma_{12}^2 = d^{-\nu} \ , \ \sigma_{02}^2 = (1-d)^{-\nu} \tag{12}$$

where ν is the path loss exponent and $0 < d = d_{12} < 1$. The distances are normalized by the distance d_{01}. In these coordinates, the user 1 can be located at (0,0), and the destination can be located at (1,0), without loss of generality. User 2 is located at (d,0) [22]. In the symmetric case, all channels are drawn with same unit-variance.

5.3. Simulation Results

We report results for path loss exponent $\nu = 4$ and we model all channels as Rayleigh block flat fading with additive white Gaussian noise. Figs. 7-10 show the outage capacity behavior for the new cooperation scheme with centrally optimized power allocation, compared with the regular MAC channel with no cooperation. We look at the sum rate performance but also plot for information the corresponding individual user rate performance. We take $SNR = \frac{P}{N_0} = 10dB$. In the symmetric case , Fig. 7 shows a marginal improvement in sum rate due to cooperation in the MAC channel, although the worst case behavior of the individual user rate is improved (for low outage probabilities, less than 0.04). This is due to the fact that in many cases for the symmetric network both users will tend to act selfishly (as per our result in Proposition 2), and when they don't, the gains made by the user benefiting from cooperation tend to be offset by the losses incurred by the relaying user (the roles of benefitor and relayer alternating randomly with new channel realizations).

Figs. 8-10 show the simulation results for an asymmetric (linear) network when user 2 is located between user 1 and the destination at (0.1,0), (0.5,0) and (0.9,0) respectively. The gains due to centrally optimized power allocation in the cooperation are clearly more significant for the user further away from the base. However this gain also translates into a sum-rate (system) gain. For instance when $d = 0.1$, the sum-rate benefits from cooperation by 0.33 $bit/s/Hz$ and the user 1 benefits by up to 1 $bit/s/Hz$. User 2 which is closer to the destination than user 1, still benefits on average, but to a lesser extent.

When user 2 is located close to the destination ($d = 0.9$) both the sum-rate and user 1's rate benefit from the cooperation, while user 2 almost never uses user 1 as a relay. Still, this user has negligible loss of rate in relaying because the amount of power it allocates to relaying user 1's data corresponds a very small rate loss to him, while a significant gain to user 1 who undergoes severe channel conditions.

The figures 4-6 serve to illustrate the shape of the sum-rate, function of the power allocation coefficients in both centralized and distributed frameworks. For SNR = 10 dB and when mobile 2 is placed at $(0.1, 0)$, fig.4 gives an example where mobile 2 must allocate a fraction $\beta_* = 0.446$ to relay information for mobile 1, while mobile 1 allocates zero power to relaying as this maximizes the sum-rate in centralized framework.

In the distributed framework, we plot the expected sum rates "seen" respectively by user 1 and 2 in Fig. 5 and 6. In each case, the expected sum rate is maximized locally by assigning zero power to relaying, resulting in a completely selfish behavior by both users, as predicted by our Proposition 4.

6. conclusion

We have addressed the problem of mutual cooperation between data-carrying users in a wireless MAC channel. Via a power allocation framework, we proposed a scheme allowing each user to balance transmission of own data with a relaying operation, so as to maximize the sum rate received at the base station. We have characterized analytically the optimum cooperation levels, and we showed that cooperation is never mutual on an instantaneous basis as at most one user acts as a relay for the other but never both at the same time.
In the case where gathering complete CSIT is problematic we have addressed the problem of distributed power allocation in the cooperative MAC channel, where the users adjust their cooperation power levels as function of a mix of local channel state information and statistical non-local channel information. We showed that, unlike in the centralized case, both users should act selfishly in the distributed framework.

7. appendix

In this appendix, we collect all the proofs.

7.1. Proof of Proposition 1

For simplicity, we formulate (1) and (2) respectively as

$$\underbrace{\begin{bmatrix} y_1(n) \\ y_1(n+\frac{N}{2}) \end{bmatrix}}_{\mathbf{y}_1} = \underbrace{\begin{bmatrix} h_{01} & 0 \\ \sqrt{\beta}A_1 h_{02} h_{21} & \sqrt{1-\alpha}h_{01} \end{bmatrix}}_{\mathbf{M}_1} \underbrace{\begin{bmatrix} x_1(n) \\ x_1(n+\frac{N}{2}) \end{bmatrix}}_{\mathbf{x}_1} + \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ \sqrt{\beta}A_1 h_{02} & 0 & 1 \end{bmatrix}}_{\mathbf{B}_1} \underbrace{\begin{bmatrix} w_2(n) \\ z_0(n) \\ z_0(n+\frac{N}{2}) \end{bmatrix}}_{\mathbf{z}_1} \quad (13)$$

and

$$\underbrace{\begin{bmatrix} y_2(n) \\ y_2(n+\frac{N}{2}) \end{bmatrix}}_{\mathbf{y}_2} = \underbrace{\begin{bmatrix} h_{02} & 0 \\ \sqrt{\alpha}A_2h_{01}h_{12} & \sqrt{1-\beta}h_{02} \end{bmatrix}}_{\mathbf{M}_2} \underbrace{\begin{bmatrix} x_2(n) \\ x_2(n+\frac{N}{2}) \end{bmatrix}}_{\mathbf{x}_2} + \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ \sqrt{\alpha}A_2h_{01} & 0 & 1 \end{bmatrix}}_{\mathbf{B}_2} \underbrace{\begin{bmatrix} w_1(n) \\ z_0(n) \\ z_0(n+\frac{N}{2}) \end{bmatrix}}_{\mathbf{z}_2} \quad (14)$$

and, without loss of generality, we compute the maximum average mutual information during $T_1 + T_2$.

$$\begin{aligned} I(\mathbf{x}_1, \widetilde{\mathbf{y}}_1) &= I(\mathbf{x}_1; \mathbf{M}_1) + I(\mathbf{x}_1; \mathbf{y}_1/\mathbf{M}_1) \\ &= I(\mathbf{x}_1; \mathbf{y}_1/\mathbf{M}_1) \\ &\leq \log_2 \det(\mathbf{I}_2 + \mathbf{M}_1\mathbf{\Lambda}_{\mathbf{x}_1}\mathbf{M}_1^H\mathbf{\Sigma}_{\mathbf{n}_1}^{-1}) \end{aligned} \quad (15)$$

where $\mathbf{n}_1 = \mathbf{B}_1\mathbf{z}_1$ and $\mathbf{\Lambda}_{\mathbf{x}_1} = \mathbf{E}(\mathbf{x}_1\mathbf{x}_1^H) = P_1\mathbf{I}_2$.
Therefore, $\mathbf{\Sigma}_{\mathbf{n}_1} = \mathbf{B}_1\mathbf{E}(\mathbf{z}_1\mathbf{z}_1^H)\mathbf{B}_1^H$ and equal to

$$\mathbf{\Sigma}_{\mathbf{n}_1} = \begin{bmatrix} N_0 & 0 \\ 0 & N_0 + \beta(A_1)^2|h_{02}|^2N_2 \end{bmatrix} \quad (16)$$

$$\mathbf{M}_1\mathbf{M}_1^H = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \quad (17)$$

where

$$\begin{aligned} A &= |h_{01}|^2 \\ B &= \sqrt{\beta}A_1h_{01}(h_{02})^*(h_{21})^* \\ C &= \sqrt{\beta}A_1(h_{01})^*h_{02}h_{21} \\ D &= (1-\alpha)|h_{01}|^2 + \beta(A_1)^2|h_{02}h_{21}|^2 \end{aligned} \quad (18)$$

Therefore

$$\begin{aligned} &\log_2 \det(\mathbf{I}_2 + P_1\mathbf{M}_1\mathbf{M}_1^H\mathbf{\Sigma}_{\mathbf{n}_1}^{-1}) = \log_2 \Bigg[1 + |h_{01}|^2\frac{P_1}{N_0} \\ &+ \frac{(1-\alpha)|h_{01}|^2P_1 + \beta(A_1)^2|h_{02}h_{21}|^2P_1}{N_0 + N_2\beta(A_1)^2|h_{02}|^2} \\ &+ \frac{(1-\alpha)|h_{01}|^4P_1^2}{N_0(N_0 + N_2\beta(A_1)^2|h_{02}|^2)}\Bigg] \end{aligned} \quad (19)$$

and after substitutions and algebraic manipulations, we obtain

$$\log_2 \det(\mathbf{I}_2 + P_1 \mathbf{M}_1 \mathbf{M}_1^H \mathbf{\Sigma}_{\mathbf{n}_1}^{-1}) = \log_2 \left[1 + \gamma_{01} + (1-\alpha)\frac{K_1}{l_1(\beta)} + f(\beta\gamma_{02}, \gamma_{21}) \right] \tag{20}$$

so, (3) is straightforward.

7.2. Proof of Lemma 1

We show without loss of generalities that $I_{\alpha,0} > I_{\alpha,1}$.

$$I_{\alpha,0} = \log_2 \left[1 + \gamma_{01} + (1-\alpha)\frac{K_1}{l_1(0)} \right] + \log_2 \left[1 + \gamma_{02} + \frac{K_2}{l_2(\alpha)} + f(\alpha\gamma_{01}, \gamma) \right] \tag{21}$$

$$I_{\alpha,1} = \log_2 \left[1 + \gamma_{01} + (1-\alpha)\frac{K_1}{l_1(1)} + f(\gamma_{02}, \gamma) \right] + \log_2 \left[1 + \gamma_{02} + f(\alpha\gamma_{01}, \gamma) \right] \tag{22}$$

In order to demonstrate that $I_{\alpha,0} > I_{\alpha,1}$ it is enough to show that

$$\underbrace{\left[1 + \gamma_{01} + (1-\alpha)\frac{K_1}{l_1(0)} \right] \left[1 + \gamma_{02} + \frac{K_2}{l_2(\alpha)} + f(\alpha\gamma_{01}, \gamma) \right]}_{\pi_1} > \underbrace{\left[1 + \gamma_{01} + (1-\alpha)\frac{K_1}{l_1(1)} + f(\gamma_{02}, \gamma) \right] \left[1 + \gamma_{02} + f(\alpha\gamma_{01}, \gamma) \right]}_{\pi_2} \tag{23}$$

where

$$\begin{cases} \pi_1 = (1+\gamma_{01})(1+\gamma_{02}) + (1+\gamma_{01})\frac{K_2}{l_2(\alpha)} + (1+\gamma_{01}) f(\alpha\gamma_{01}, \gamma) \\ \qquad + (1-\alpha)\frac{K_1}{l_1(0)}(1+\gamma_{02}) + (1-\alpha)\frac{K_1}{l_1(0)}\frac{K_2}{l_2(\alpha)} + (1-\alpha)\frac{K_1}{l_1(0)} f(\alpha\gamma_{01}, \gamma) \\ \pi_2 = (1+\gamma_{01})(1+\gamma_{02}) + (1+\gamma_{01}) f(\alpha\gamma_{01}, \gamma) + (1-\alpha)\frac{K_1}{l_1(1)}(1+\gamma_{02}) \\ \qquad + (1-\alpha)\frac{K_1}{l_1(1)} f(\alpha\gamma_{01}, \gamma) + f(\gamma_{02}, \gamma)(1+\gamma_{02}) + f(\gamma_{02}, \gamma) f(\alpha\gamma_{01}, \gamma) \end{cases} \tag{24}$$

First we have $l_1(0) < l_1(1)$, so $\frac{1}{l_1(0)} > \frac{1}{l_1(1)}$. Therefore

$$\begin{cases} (1-\alpha)\frac{K_1}{l_1(0)}(1+\gamma_{02}) > (1-\alpha)\frac{K_1}{l_1(1)}(1+\gamma_{02}) \\ (1-\alpha)\frac{K_1}{l_1(0)} f(\alpha\gamma_{01}, \gamma) > (1-\alpha)\frac{K_1}{l_1(1)} f(\alpha\gamma_{01}, \gamma) \end{cases} \tag{25}$$

and after some manipulations, we obtain

$$(1-\alpha)\frac{K_1}{l_1(0)}\frac{K_2}{l_2(\alpha)} + (1+\gamma_{01})\frac{K_2}{l_2(\alpha)} = (1+\gamma_{01})\frac{K_2}{l_2(\alpha)} \left[1 + (1-\alpha)\gamma_{01} \right] \tag{26}$$

and

$$f(\gamma_{02},\gamma)(1+\gamma_{02})+f(\gamma_{02},\gamma)f(\alpha\gamma_{01},\gamma)=\frac{\gamma K_2}{(1+\gamma+\gamma_{02})l_2(\alpha)}+\frac{\alpha\gamma\gamma_{01}\gamma_{02}}{l_2(\alpha)} \quad (27)$$

we remember that $K_2-\gamma_{02}^2\gamma=\gamma_{02}(1+\gamma+\gamma_{02})$, therefore

$$f(\gamma_{02},\gamma)(1+\gamma_{02})+f(\gamma_{02},\gamma)f(\alpha\gamma_{01},\gamma)=\frac{K_2}{l_2(\alpha)}\frac{\gamma(1+\alpha\gamma_{01})}{(1+\gamma+\gamma_{02})}-\frac{\alpha\gamma_{01}\gamma^2\gamma_{02}^2}{(1+\gamma+\gamma_{02})l_2(\alpha)} \quad (28)$$

We can say that $0<\frac{\gamma}{1+\gamma+\gamma_{02}}<1$ and $0<1+\alpha\gamma_{01}\leq(1+\gamma_{01})$. So, $\frac{K_2}{l_2(\alpha)}\frac{\gamma(1+\alpha\gamma_{01})}{(1+\gamma+\gamma_{02})}\leq(1+\gamma_{01})\frac{K_2}{l_2(\alpha)}$ then $\frac{K_2}{l_2(\alpha)}\frac{\gamma(1+\alpha\gamma_{01})}{(1+\gamma+\gamma_{02})}\leq(1+\gamma_{01})\frac{K_2}{l_2(\alpha)}[1+(1-\alpha)\gamma_{01}]$. Therefore we can say finally that

$$\frac{K_2}{l_2(\alpha)}\frac{\gamma(1+\alpha\gamma_{01})}{(1+\gamma+\gamma_{02})}-\alpha\frac{\gamma_{01}\gamma^2\gamma_{02}^2}{(1+\gamma+\gamma_{02})l_2(\alpha)}\leq(1+\gamma_{01})\frac{K_2}{l_2(\alpha)}[1+(1-\alpha)\gamma_{01}] \quad (29)$$

and from (25),(26), and (29) we conclude that $I_{\alpha,1}<I_{\alpha,0}$.
The relation $I_{1,\beta}<I_{0,\beta}$ is straightforward.

7.3. Proof of Proposition 2

In order to seek (α_*,β_*) for which $I_{\alpha,\beta}$ is maximized,

$$(\alpha_*,\beta_*)=\arg\max_{\alpha,\beta\in[0,1]} I_{\alpha,\beta} \quad (30)$$

we must solve this system of equations :

$$\begin{cases}\frac{\partial I_{\alpha,\beta}}{\partial\alpha}=0\\ \frac{\partial I_{\alpha,\beta}}{\partial\beta}=0\end{cases} \quad (31)$$

The partial derivatives of $I_{\alpha,\beta}$, $\frac{\partial I_{\alpha,\beta}}{\partial\alpha}$ and $\frac{\partial I_{\alpha,\beta}}{\partial\beta}$ respectively to α and β give

$$\frac{\partial I_{\alpha,\beta}}{\partial\alpha}=\frac{1}{\ln(2)}\left[\frac{\frac{-K_1}{l_1(\beta)}}{1+\gamma_{01}+(1-\alpha)\frac{K_1}{l_1(\beta)}+f(\beta\gamma_{02},\gamma)}+\frac{(1-\beta)K_2\frac{\frac{-\partial l_2(\alpha)}{\partial\alpha}}{[l_2(\alpha)]^2}+\frac{\gamma\gamma_{01}(1+\gamma)}{[l_2(\alpha)]^2}}{1+\gamma_{02}+(1-\beta)\frac{K_2}{l_2(\alpha)}+f(\alpha\gamma_{01},\gamma)}\right] \quad (32)$$

and

$$\frac{\partial I_{\alpha,\beta}}{\partial\beta}=\frac{1}{\ln(2)}\left[\frac{\frac{-K_2}{l_2(\alpha)}}{1+\gamma_{02}+(1-\beta)\frac{K_2}{l_2(\alpha)}+f(\alpha\gamma_{01},\gamma)}+\frac{(1-\alpha)K_1\frac{\frac{-\partial l_1(\beta)}{\partial\beta}}{[l_1(\beta)]^2}+\frac{\gamma\gamma_{02}(1+\gamma)}{[l_1(\beta)]^2}}{1+\gamma_{01}+(1-\alpha)\frac{K_1}{l_1(\beta)}+f(\beta\gamma_{02},\gamma)}\right] \quad (33)$$

after some simplifications, α_* and β_* are determined as solutions of

$$\left\{\begin{array}{l} A_1\alpha^2 + 2\alpha A_2 - C - B_2\beta - B_1\beta^2 = 0 \\ A_1^{'}\beta^2 + 2\beta A_2^{'} - C^{'} - B_2^{'}\alpha - B_1^{'}\alpha^2 = 0 \end{array}\right. \tag{34}$$

where

$$\begin{array}{l} A_1 = K_1\gamma_{01}^2(1+\gamma+\gamma_{02}) \\ A_2 = K_1\gamma_{01}(1+\gamma)(1+\gamma+\gamma_{02}) \\ B_1 = K_2\gamma_{01}\gamma_{02}(1+\gamma+\gamma_{01}) \\ B_2 = K_1K_2(2+\gamma+\gamma_{01}) + \gamma_{01}\gamma_{02}(1+\gamma+\gamma_{01})(\gamma(1+\gamma)-K_2) \\ C = K_1\left[\gamma\frac{K_1}{\gamma_{01}} - \frac{K_2}{\gamma_{02}}(1+\gamma) - K_2(2+\gamma+\gamma_{01})\right] \end{array} \tag{35}$$

and

$$\begin{array}{l} A_1^{'} = K_2\gamma_{02}^2(1+\gamma+\gamma_{01}) \\ A_2^{'} = K_2\gamma_{02}(1+\gamma)(1+\gamma+\gamma_{01}) \\ B_1^{'} = K_1\gamma_{01}\gamma_{02}(1+\gamma+\gamma_{02}) \\ B_2^{'} = K_1K_2(2+\gamma+\gamma_{02}) + \gamma_{01}\gamma_{02}(1+\gamma+\gamma_{02})(\gamma(1+\gamma)-K_1) \\ C^{'} = K_2\left[\gamma\frac{K_2}{\gamma_{02}} - \frac{K_1}{\gamma_{01}}(1+\gamma) - K_1(2+\gamma+\gamma_{02})\right] \end{array} \tag{36}$$

therefore, the system (31) becomes

$$\left\{\begin{array}{l} \frac{\widetilde{\alpha}^2}{B_1} - \frac{\widetilde{\beta}^2}{A_1} = \kappa_1 \\ \frac{\widetilde{\beta}^2}{B_1^{'}} - \frac{\widetilde{\alpha}^2}{A_1^{'}} = \kappa_2 \end{array}\right. \tag{37}$$

where

$$\left\{\begin{array}{l} \widetilde{\alpha} = \alpha + \frac{A_2}{A_1} \\ \widetilde{\beta} = \beta + \frac{B_2}{2B_1} \end{array}\right. \tag{38}$$

and

$$\left\{\begin{array}{l} \kappa_1 = \frac{C}{A_1B_1} + \frac{1}{B_1}\left(\frac{A_2}{A_1}\right)^2 - \frac{1}{A_1}\left(\frac{B_2}{2B_1}\right)^2 \\ \kappa_2 = \frac{C^{'}}{A_1^{'}B_1^{'}} + \frac{1}{B_1^{'}}\left(\frac{A_2^{'}}{A_1^{'}}\right)^2 - \frac{1}{A_1^{'}}\left(\frac{B_2^{'}}{2B_1^{'}}\right)^2 \end{array}\right. \tag{39}$$

In (37), we have two equations of hyperboles. When we replace $\widetilde{\alpha}$ in the second equation by its expression derived from the first one in order to solve this system we obtain

$$\widetilde{\beta}^2\left(\frac{1}{B_1^{'}} - \frac{A_1}{A_1^{'}B_1}\right) = \underbrace{\kappa_2 + \frac{B_1}{A_1^{'}}\kappa_1}_{\neq 0} \tag{40}$$

and because we have

$$\frac{B_1}{A_1} = \frac{A_1'}{B_1'} \tag{41}$$

it is straightforward that there are no solutions, graphically traduced by the no intersection between these hyperboles where eq. (41) shows the equality of the slopes of the asymptotes, unless on the plans $\mathcal{P}_{\alpha,0} = \{\beta = 0, \forall\alpha\}$, $\mathcal{P}_{\alpha,1} = \{\beta = 1, \forall\alpha\}$, $\mathcal{P}_{0,\beta} = \{\alpha = 0, \forall\beta\}$ and $\mathcal{P}_{1,\beta} = \{\alpha = 1, \forall\beta\}$.
Using Lemma 1, we are interested only by $I_{\alpha,0}$ and $I_{0,\beta}$. Therefore at most one user cooperate, so

$$\begin{cases} \alpha_* = \arg\max_{\alpha\in[0,1]} I_{\alpha,0} \\ \beta_* = \arg\max_{\beta\in[0,1]} I_{0,\beta} \end{cases} \tag{42}$$

The derivatives of $I_{\alpha,0}$ and $I_{0,\beta}$, $\frac{dI_{\alpha,0}}{d\alpha}$ and $\frac{dI_{0,\beta}}{d\beta}$ give

$$\begin{cases} \widetilde{\alpha}^2 = \frac{C}{A_1} + \left(\frac{A_2}{A_1}\right)^2 \\ \widetilde{\beta}^2 = \frac{C'}{A_1'} + \left(\frac{A_2'}{A_1'}\right)^2 \end{cases} \tag{43}$$

Therefore, α_* exists when

$$\begin{cases} \frac{C}{A_1} + \left(\frac{A_2}{A_1}\right)^2 > 0 \\ -\left(\frac{A_2}{A_1}\right) + \sqrt{\frac{C}{A_1} + \left(\frac{A_2}{A_1}\right)^2} \in]0,1] \end{cases} \tag{44}$$

and it becomes easy to lead to

$$\begin{cases} \gamma > \gamma_{01}^2 + \gamma_{01} \\ \gamma_{02} > \frac{(1+\gamma_{01})^2(1+\gamma)}{\gamma-(\gamma_{01}^2+\gamma_{01})} - 1 \end{cases} \tag{45}$$

and the same method is intended for β_*.

7.4. Proof of Lemma 3

We start by some mathematical analysis and according to [23]

$$\int_0^\infty \log_2(A + B\lambda)\exp(-\lambda)\,d\lambda = -\left[\frac{\exp\left(\frac{A}{B}\right) Ei\left(-\frac{A}{B}\right)}{\ln(2)}\right] + \log_2(A) \tag{46}$$

where $Ei(.)$ is the exponential integral defined as $Ei(x) = \int_{-\infty}^{x} \frac{\exp(t)}{t} dt$, and

$$\int_0^\infty \log_2\left(A + B\lambda + C\lambda^2\right) \exp\left(-\lambda\right) d\lambda = -\left[\frac{\Upsilon - \ln(A)}{\ln(2)}\right] \tag{47}$$

where

$$\begin{array}{l} \Upsilon = \exp\left(R_1\right) Ei\left(-R_1\right) + \exp\left(R_2\right) Ei\left(-R_2\right) \\ R_1 = \frac{B - \sqrt{B^2 - 4AC}}{2C} \\ R_2 = \frac{B + \sqrt{B^2 - 4AC}}{2C} \end{array} \tag{48}$$

$\overline{I}_{\gamma_{02}}(\alpha, \beta)$ will have different forms according to the value of β, and the notation of the dependence on α in (49) is omitted. Therefore if:

$\beta = 0$, $I_{\alpha,0} = \log_2(a) + \log_2(c + d(0)\lambda + f(0)\lambda^2) - \log_2(l_2(\alpha))$

$\beta = 1$, $I_{\alpha,1} = \log_2(a + b(1)\lambda) - \log_2(1 + \xi(1)\lambda) + \log_2(c + d(1)\lambda) - \log_2(l_2(\alpha))$

else , $I_{\alpha,\beta} = \log_2(a + b(\beta)\lambda) - \log_2(1 + \xi(\beta)\lambda) + \log_2(c + d(\beta)\lambda + f(\beta)\lambda^2) - \log_2(l_2(\alpha))$

where

$$\begin{array}{l} a = (1 + \gamma_{01})(1 + (1 - \alpha)\gamma_{01}) \\ \xi(\beta) = \frac{\beta\overline{\gamma}_{02}}{1+\gamma} \\ b(\beta) = \xi(\beta)(1 + \gamma + \gamma_{01}) \\ c = (1 + \gamma)(1 + \alpha\gamma_{01}) \\ d(\beta) = \left[(2 - \beta)(1 + \gamma) + \alpha\gamma_{01}\right]\overline{\gamma}_{02} \\ f(\beta) = (1 - \beta)(1 + \gamma)\left(\overline{\gamma}_{02}\right)^2 \\ \lambda = \frac{\gamma_{02}}{\overline{\gamma}_{02}} \end{array} \tag{49}$$

So using the relations (46) and (47) we will have the result in Lemma 1.

7.5. Proof of Proposition 3

A subtle technical point is under which conditions the first order derivative of $\mathbb{E}_{\gamma_{02}}\left[I_{\alpha,\beta}\right]$ can be taken inside the expectation operator :

$$\begin{cases} \frac{\partial}{\partial\alpha}\mathbb{E}_{\gamma_{02}}\left[I_{\alpha,\beta}\right] = \mathbb{E}_{\gamma_{02}}\left[\frac{\partial I_{\alpha,\beta}}{\partial\alpha}\right] \\ \frac{\partial}{\partial\beta}\mathbb{E}_{\gamma_{02}}\left[I_{\alpha,\beta}\right] = \mathbb{E}_{\gamma_{02}}\left[\frac{\partial I_{\alpha,\beta}}{\partial\beta}\right] \end{cases} \tag{50}$$

For simplicity, and without loss of generality, we consider mobile 1.

Theorem 1. If the following two conditions hold at a point α (resp β), then $\bar{I}_1(\alpha,\cdot)$ (resp $\bar{I}_1(\cdot,\beta)$) is differentiable at α (resp β) and equations (50) hold [24]:
(i) The function $I_{\cdot,\beta}$ (resp $I_{\alpha,\cdot}$) is differentiable at α (resp β) w.p.l.
(ii) There exists a positive valued random variable $K(\gamma_{02})$ such that $\mathbb{E}_{\gamma_{02}}(K(\gamma_{02}))$ is finite and the inequality

$$|I_{\alpha_1,\beta} - I_{\alpha_2,\beta}| \leq K(\gamma_{02})|\alpha_1 - \alpha_2| \tag{51}$$

resp.

$$|I_{\alpha,\beta_1} - I_{\alpha,\beta_2}| \leq K(\gamma_{02})|\beta_1 - \beta_2| \tag{52}$$

holds w.p.l for all α_1,α_2 (resp β_1,β_2) in a neighborhood of α (resp. β).
Note $I_{\cdot,\beta}$ satisfies the conditions (i) and (ii).
Now, suppose that $\alpha_0 \neq 0$ and $\beta_0 \neq 0$, therefore (α_0, β_0) is an optimal power allocation for mobile 1 if

$$\begin{cases} \frac{\partial}{\partial\alpha}\mathbb{E}_{\gamma_{02}}\left[I_{\alpha,\beta}\right]|_{\alpha=\alpha_0} = 0 \\ \frac{\partial}{\partial\beta}\mathbb{E}_{\gamma_{02}}\left[I_{\alpha,\beta}\right]|_{\beta=\beta_0} = 0 \end{cases} \tag{53}$$

Therefore the equations(6) become

$$\begin{cases} \int \frac{\partial I_{\alpha,\beta}}{\partial\alpha}|_{\alpha=\alpha_0} p_{\gamma_{02}} d\gamma_{02} = 0 \\ \int \frac{\partial I_{\alpha,\beta}}{\partial\beta}|_{\beta=\beta_0} p_{\gamma_{02}} d\gamma_{02} = 0 \end{cases} \tag{54}$$

But (α_0, β_0) cannot maximize $I_{\alpha,\beta}$ (see Proposition 2), therefore $\frac{\partial I_{\alpha,\beta}}{\partial\alpha}|_{\alpha=\alpha_0} \neq 0$ or $\frac{\partial I_{\alpha,\beta}}{\partial\beta}|_{\beta=\beta_0} \neq 0$, and γ_{02} is exponentially distributed ($p_{\gamma_{02}}(\gamma_{02}) > 0\forall\gamma_{02}$) so this leads to a contradiction with (54) because at least one among the two equations cannot be held.

7.6. Proof of Lemma 4

Suppose that $\alpha_0 \neq 0$, therefore α_0 will be allocated to mobile 1 if

$$\frac{\partial \overline{I}_{\gamma_{02}}(\alpha, 0)}{\partial\alpha}|_{\alpha=\alpha_0} = 0 \tag{55}$$

Therefore, we derive the following expression of $\frac{\partial \overline{I}_{\gamma_{02}}(\alpha,0)}{\partial\alpha}$

$$-\frac{1}{\ln(2)}\left[\frac{\gamma_{01}}{1+(1-\alpha)\gamma_{01}} + \frac{\gamma_{01}}{1+\gamma+\alpha\gamma_{01}}\right] - \frac{\partial\Phi(0)}{\partial\alpha} \tag{56}$$

Using the relation below

$$\frac{\partial \left(\exp(\varepsilon)Ei(-\varepsilon)\right)}{\partial \alpha} = \left(\frac{\partial \varepsilon}{\partial \alpha}\right)\left[\exp(\varepsilon)Ei(-\varepsilon) + \frac{\partial \varepsilon}{\partial \alpha}\right] \tag{57}$$

Replacing ε by $\Lambda_1(0)$ and $\Lambda_2(0)$ in (55), it becomes easy to show that

$$\frac{\partial \overline{I}_{\gamma_{02}}(\alpha, 0)}{\partial \alpha} < 0, \quad \forall \alpha \neq 0 \tag{58}$$

and because $\overline{I}_{\gamma_{02}}(\alpha, 0)$ is differentiable in 0, decreases in $[0, 1]$, therefore $\alpha_*^1 = 0$.

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Table 1: Power allocation coefficients over two frames for 2 users transmitting to a base using TDMA scheme. Power levels are used to either send own or relay data. T_1 (resp.T_2) is first (resp. second) half of the frame.

	T_1	T_2	T_1	T_2
user 1	1	$1-\alpha$	0	α
user 2	0	β	1	$1-\beta$

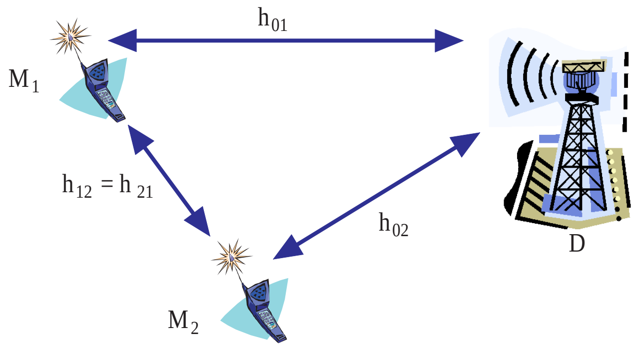

Figure 1: Cellular model.

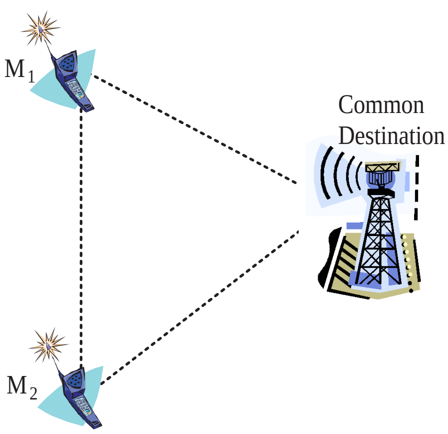

Figure 2: Symmetric network.

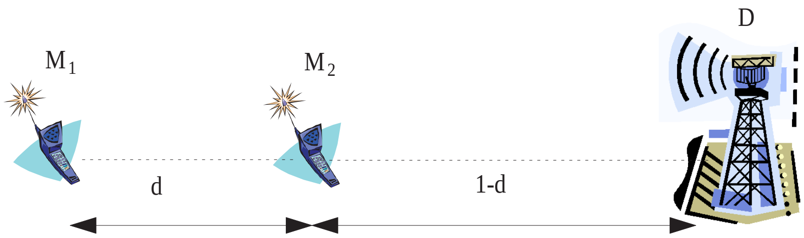

Figure 3: Asymmetric (or linear) network.

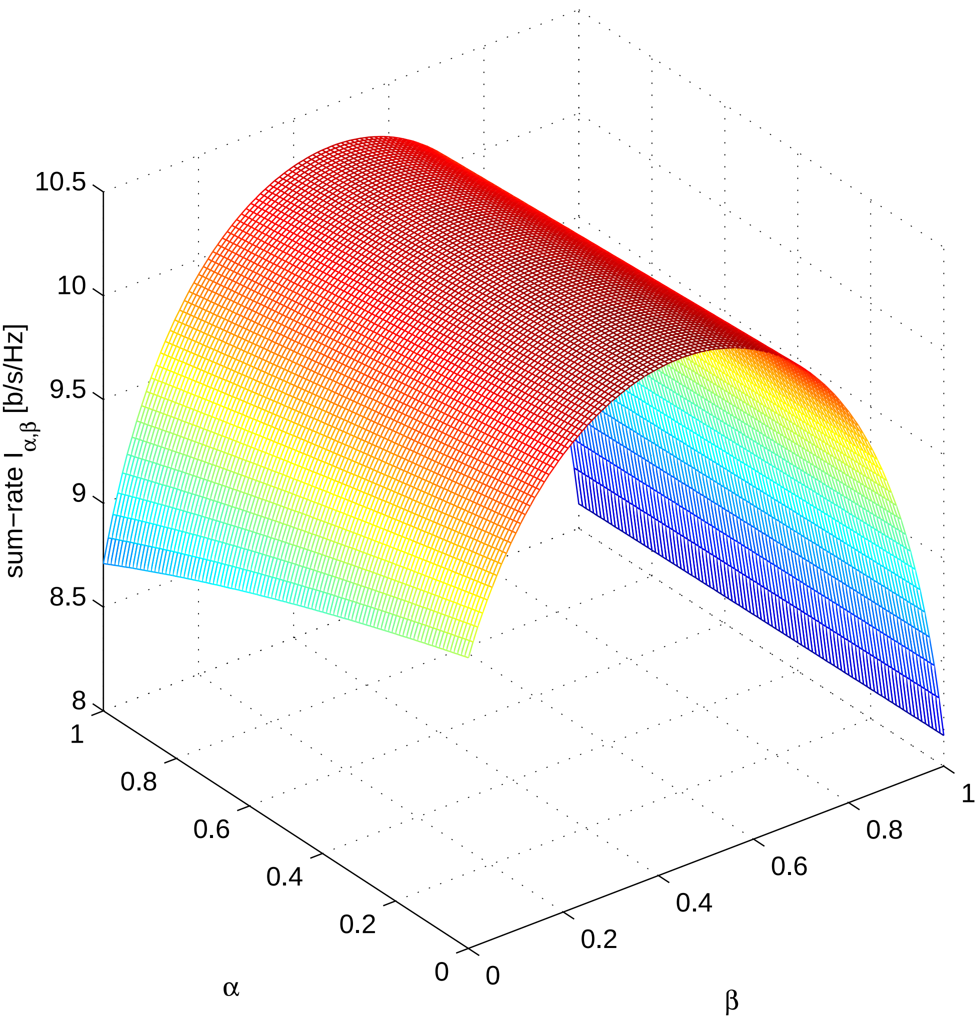

Figure 4: The sum-rate, in centralized case and linear network, when the mobile 2 is located at (0.1,0) and SNR equal to 10 dB. The centralized algorithm gives (0,0.446) as optimal power allocation indicating that only mobile 2 will cooperate.

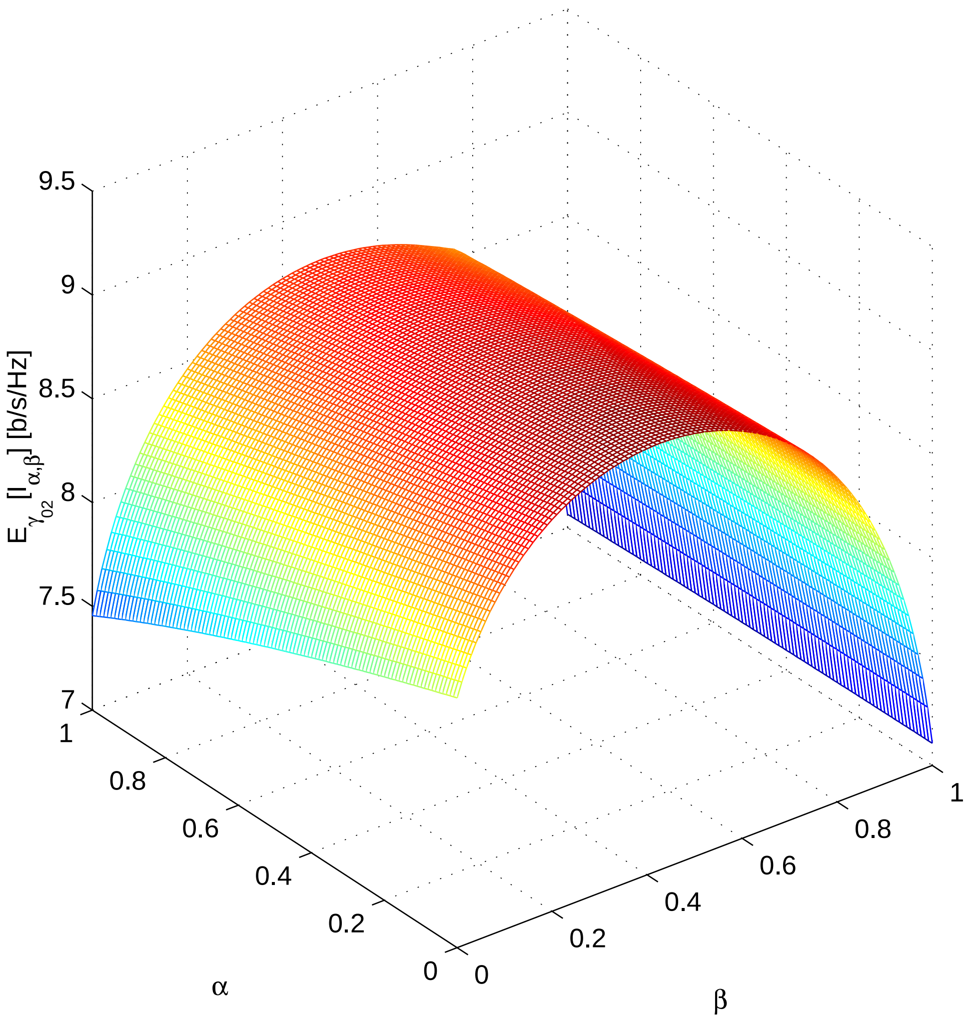

Figure 5: Expected sum-rate seen by mobile 1 (distributed processing) when the mobile 2 is located at (0.1,0) in linear network and SNR equal to 10 dB. The relations in (10) give (0,0.447) as optimal power allocation for mobile 1, therefore it doesn't cooperate.

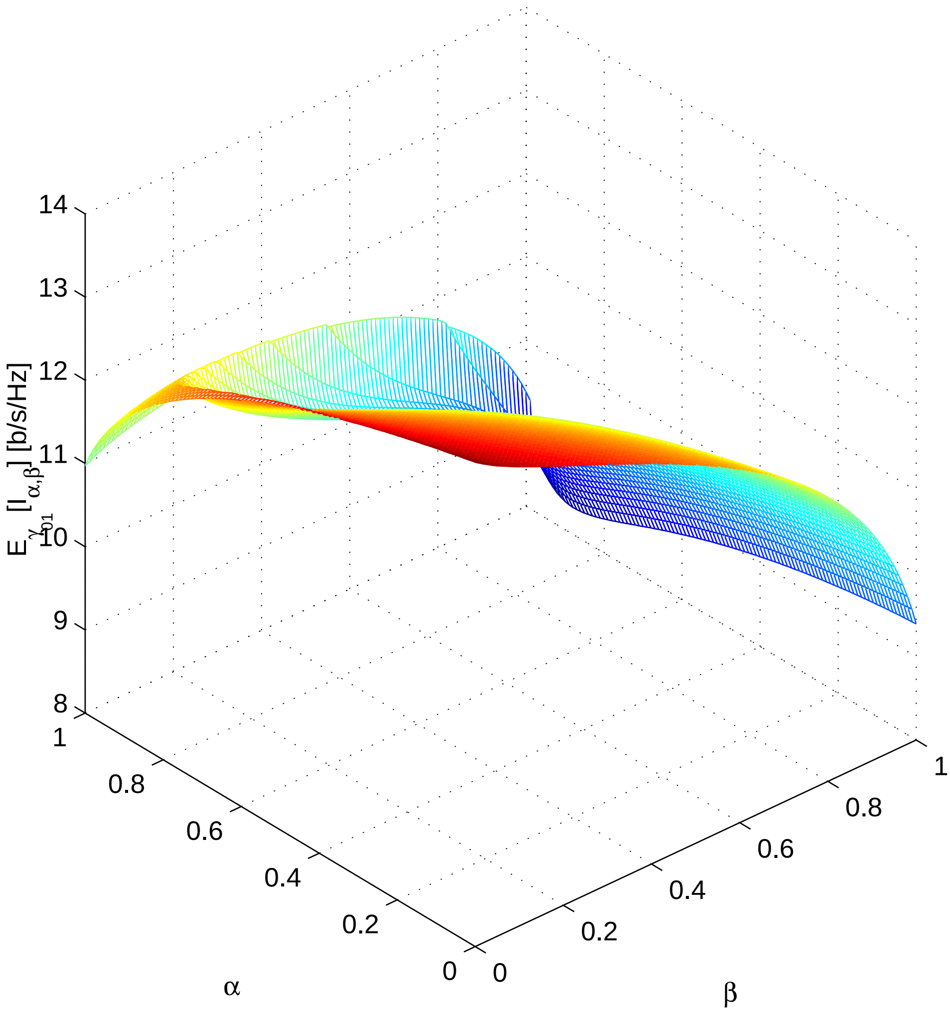

Figure 6: Expected sum-rate seen by mobile 2 (distributed processing) when it is located at (0.1,0) in linear network and SNR equal to 10 dB. The relations in (11) give (0,0) as optimal power allocation for mobile 2, therefore it should not cooperate.

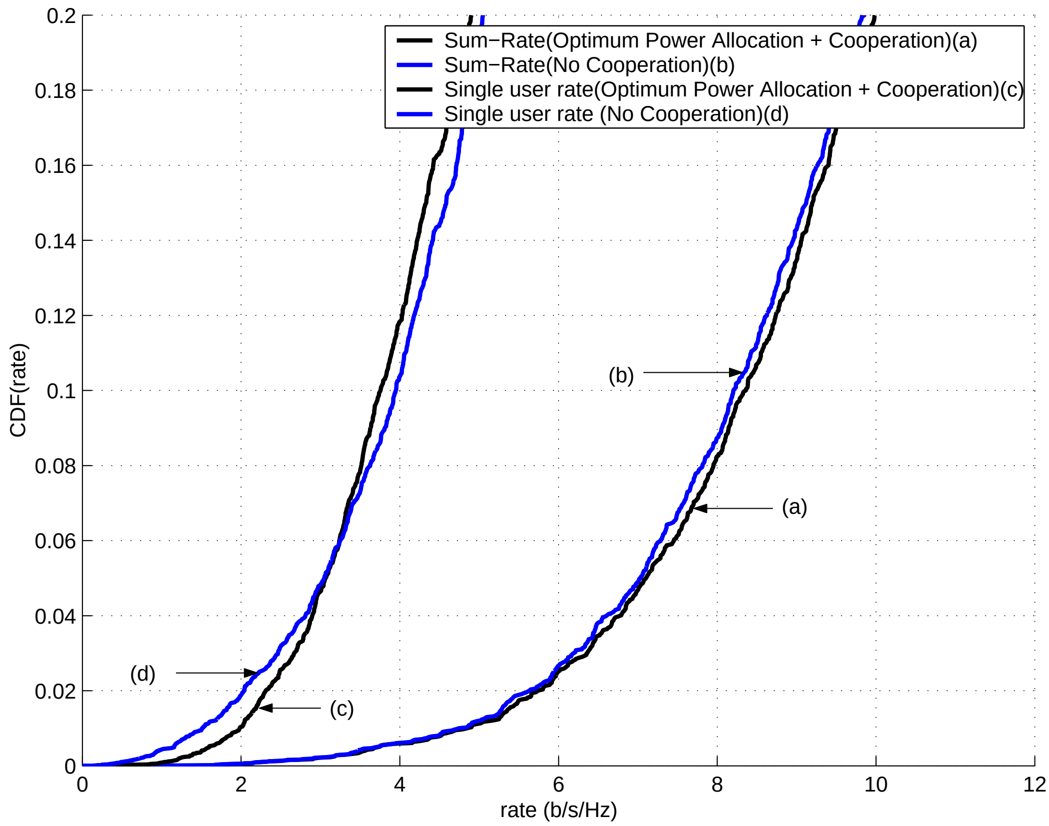

Figure 7: Outage capacity in centralized case and symmetric network in which we consider equal channel gains ($\sigma^2 = 1$) : In this situation, individual users's worst-case rates are slightly improved, while the sum rate is almost unchanged.

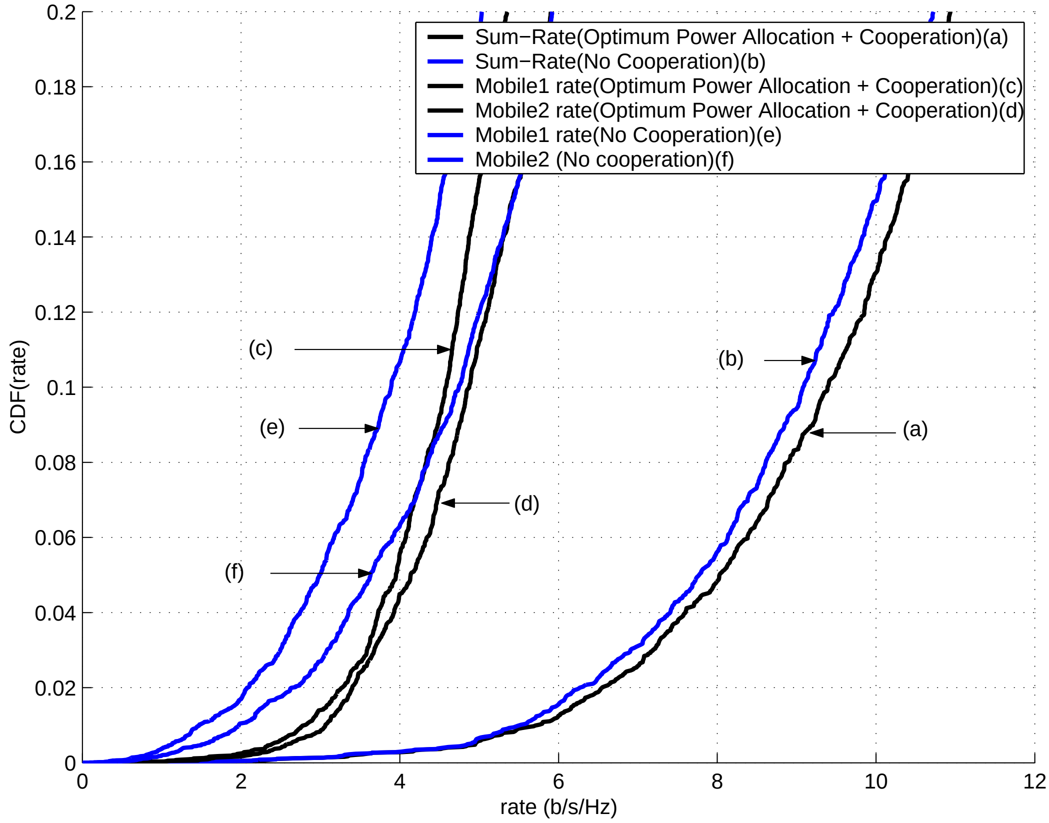

Figure 8: Outage capacity in centralized case and asymmetric network, with user 2 located at (0.1,0), i.e, close to the user 1; Both users as well as the sum rate benefit from cooperation, especially mobile 1.

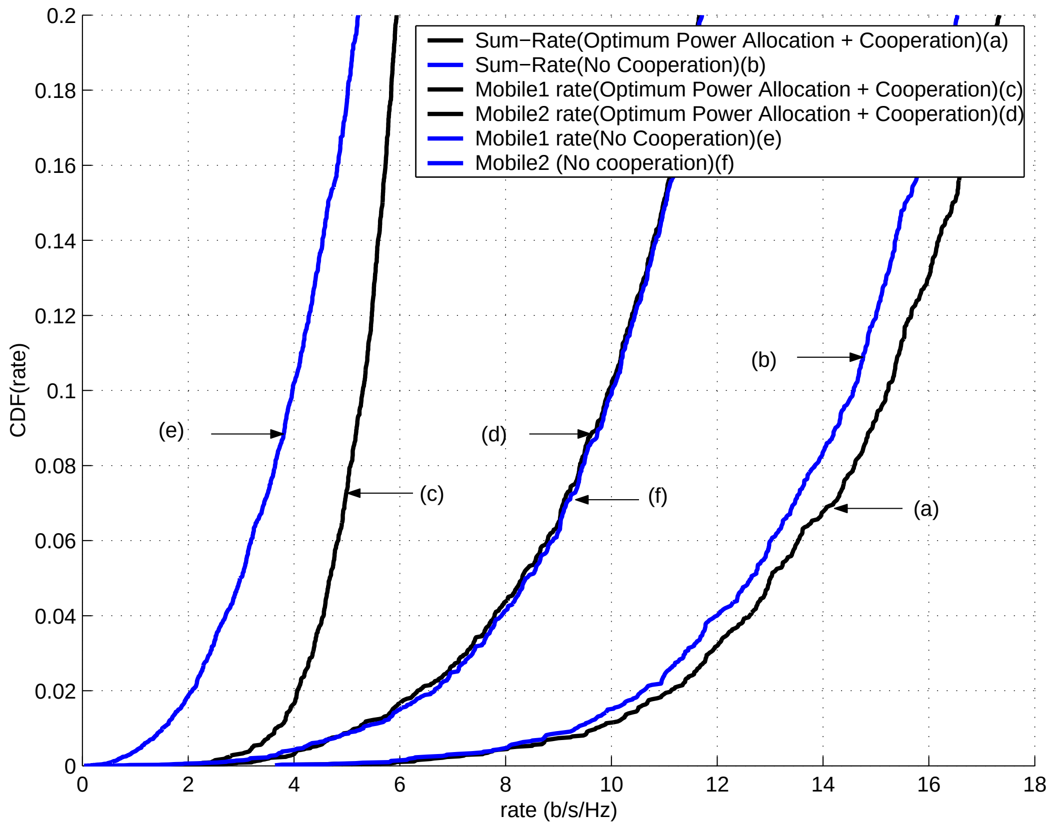

Figure 9: Outage capacity in centralized case and asymmetric network, with user 2 located at (0.5,0), i.e, halfway between user 1 and destination; Only user 1 benefits from cooperation, however the sum rate is also improved.

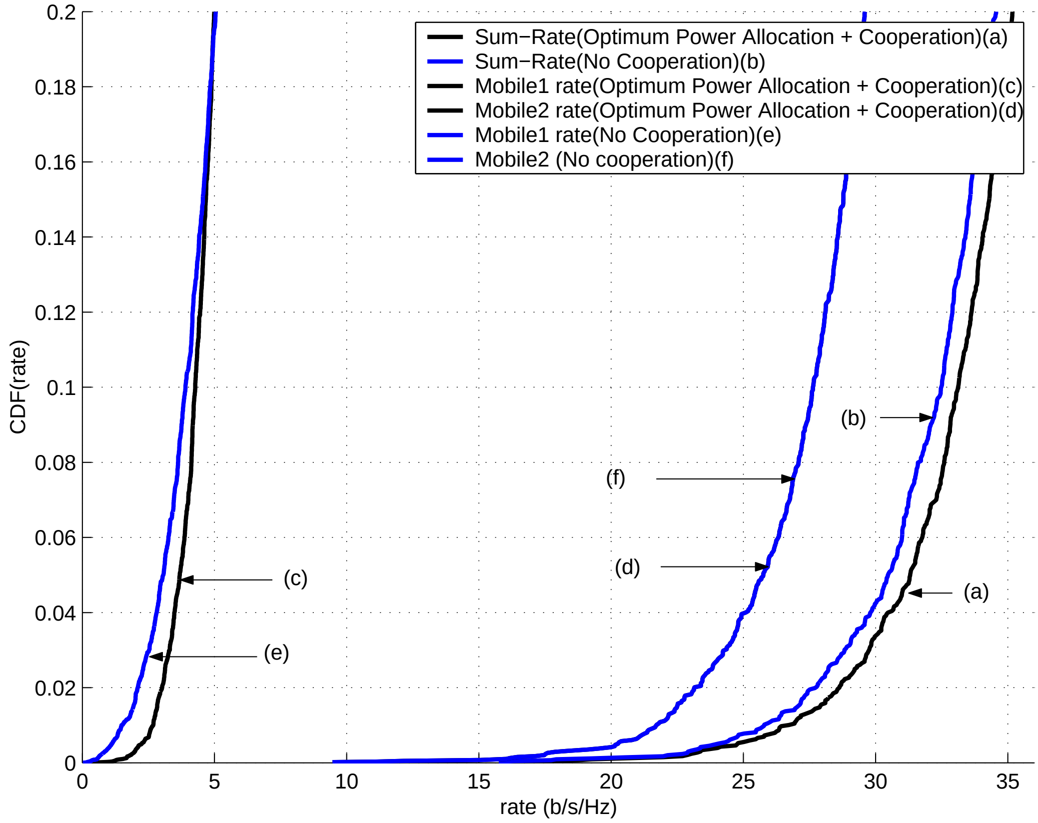

Figure 10: Outage capacity in centralized case and asymmetric network with user 2 located at (0.9,0), i.e, close to the destination; both the sum-rate and user 1's rate benefit from the cooperation, while user 2 almost never uses user 1 as a relay.