Impact of Imperfections on Detectors for Interference Suppression

Rizwan Ghaffar, Raymond Knopp
Eurecom, 2229 route des Crêtes B.P.193
06904 Sophia Antipolis Cedex FRANCE
Email: rizwan.ghaffar@eurecom.fr, raymond.knopp@eurecom.fr

***Abstract*—In this paper, we carry out the performance analysis of recently proposed low complexity max log MAP detector for interference suppression under the realistic imperfections of correlation and non Gaussian alphabets. We also incorporate MMSE detector in our analysis taking into account non Gaussian alphabets however intricacy in the analysis of correlated case restricts our analysis to the case of iid fading. Employing *moment generating function* (MGF)-based approach, we derive upper bounds of coded pairwise error probability (PEP) and study the effect of non Gaussian interference and correlation. The novelty of this contribution is the performance analysis taking into account discrete alphabets as the existing work is based on the unrealistic Gaussian assumption for these alphabets.**

I. Introduction

The objective of achieving higher spectral efficiency in the upcoming wireless standards as 3GPP LTE [1] is leading to the employment of multi-element antenna arrays (MEA) at the mobile station (MS) in addition to existing multi antenna base stations (BSs) thereby exploiting the spatial dimension. The same aspiration is advocating the frequency reuse factor of one in future mobile systems [1] but that would lead to an interference limited system. In such a scenario, the optimal use of receive diversity at the MS to either achieve multiplexing gain or to cope with the interference is still an open question. For the latter case, researchers are focusing on interference alignment, interference mitigation and interference suppression to diminish, manage or exploit these interferers. In this paper we focus on interference suppression and carry out the performance analysis of low complexity max log MAP detector [2] and linear MMSE detector for bit interleaved coded modulation (BICM) MIMO OFDM based cellular system.

For performance analysis, naive assumption of independent and identically distributed (iid) fading channel may not be true in most of the real world scenarios [3]. Some impairments of the radio propagation channel may lead to a substantial degradation in the performance. The limitations on the performance are primarily imposed by the limited number of multipath components or scatterers and antenna spacing. In a typical cellular scenario, the inadequate antenna separation affects MS as the components of antenna array may be separated by a distance less than half of the communication wavelength thereby leading them to be correlated. On the other hand, BS antennas are well separated but lack of scatterers around BSs due to their elevated positions may instigate them as well to be correlated.

In this paper, we investigate the effects of MS antennas correlation and the BSs correlation combined with the strength and the rate of interference on coded pairwise error probability (PEP) of the low complexity max log MAP detector. With the analysis of MMSE detector available in the literature based on the unrealistic assumption of Gaussianity for interference [4], we extend it to analyzing its performance taking into account the discrete constellation interference. However intricacy in handling correlation restricts our analysis to the case of iid fading. Using the *moment generating function* (MGF) approach associated with the quadratic form of a complex Gaussian random variable, we derive analytical expressions for upper bounds on coded PEP of low complexity max log MAP detector and linear MMSE detector in the presence of interference. We demonstrate the strength of our new analytical PEP upper bounds by simulating the performance of a MS in the presence of one interferer of varying alphabet size using the low complexity max log MAP detector and linear MMSE detector.

Regarding notations, we will use lowercase or uppercase letters for scalars, lowercase boldface letters for vectors and uppercase boldface letters for matrices. $(.)^T$, $(.)^*$ and $(.)^\dagger$ indicate transpose, conjugate and conjugate transpose operations respectively. $|.|$ and $\|.\|$ indicate norm of scalar and vector while $abs(.)$ denotes absolute value. The notation $E(.)$ denotes the mathematical expectation while $Q(y) = \frac{1}{\sqrt{2\pi}} \int_y^\infty e^{-x^2/2} dx$ denotes the Gaussian Q-function. $\mathbf{A}_{M \times N}$ indicates a matrix $\mathbf{A}$ with M rows and N columns whereas $\text{vec}(\mathbf{A})$ denotes the vectorization operator which stacks the columns of $\mathbf{A}$. The matrix $\mathbf{I}_n$ is the $n \times n$ identity matrix and the element at the i-th row and j-th column of matrix $\mathbf{A}$ is denoted as $\mathbf{A}(i,j)$.

The paper is divided into six sections. In section II we define the system model while section III discusses the PEP analysis of low complexity max log MAP detector. Section IV contains the PEP analysis of MMSE detector which is followed by simulation results and conclusions.

II. System Model

We consider the downlink of a single frequency reuse cellular system with n_r antennas at the MS and 2 BSs using antenna cycling for transmission with each stream being transmitted by one antenna in any dimension. We assume that two spatial streams arrive at the MS as $\underline{\mathbf{x}}_1$ (desired stream) and $\underline{\mathbf{x}}_2$ (interference stream). x_1 is the symbol of $\underline{\mathbf{x}}_1$ over a signal set χ_1 and x_2 is the symbol of $\underline{\mathbf{x}}_2$ over signal set χ_2. During the transmission at BS-1, code sequence $\mathbf{c}_1$ is interleaved by π_1 and then is mapped onto the signal sequence $\underline{\mathbf{x}}_1 \in \chi_1$. Bit interleaver for the first stream can be modeled as $\pi_1 : k^{'} \rightarrow (k, i)$ where $k^{'}$ denotes the original ordering of the coded bits $c_{k'}$ of first stream, k denotes the time ordering of the signal $x_{1,k}$ and i indicates the position of the bit $c_{k'}$ in the symbol $x_{1,k}$. Assuming an ideal OFDM system, transmission at the k-th frequency tone can be expressed as::-

$$\begin{aligned} \mathbf{y}_k &= \mathbf{h}_{1,k}x_{1,k} + \mathbf{h}_{2,k}x_{2,k} + \mathbf{z}_k, \qquad k = 1, 2, \cdots, T \\ &= \mathbf{H}_k\mathbf{x}_k + \mathbf{z}_k \end{aligned} \tag{1}$$

where $\mathbf{H}_k = [\mathbf{h}_{1,k}\ \mathbf{h}_{2,k}]$ i.e. the channel at k-th frequency tone and $\mathbf{x}_k = [x_{1,k}\ x_{2,k}]^T$. Each subcarrier corresponds to a symbol from a constellation map χ_1 for first stream and χ_2 for second stream. $\mathbf{y}_k, \mathbf{z}_k \in \mathbb{C}^{n_r}$ are the vectors of received symbols and circularly symmetric complex white Gaussian noise of double-sided power spectral density $N_0/2$ at n_r receive antennas. $\mathbf{h}_{1,k} \in \mathbb{C}^{n_r}$ is the vector characterizing flat fading channel response from first transmitting antenna to n_r receive antennas at k-th subcarrier. The complex symbols $x_{1,k}$ and $x_{2,k}$ of two streams are assumed to be independent with variances σ_1^2 and σ_2^2 respectively. The channels at different subcarriers are also assumed to be independent.

Correlation Structure. The entries of the channel matrix are assumed to be circularly symmetric complex Gaussian random variables with zero-mean and unit variance so their magnitudes exhibit Rayleigh distribution. The average Frobenius norm is equal to $n_r \times n_t$ (this is not the case for the instantaneous Frobenius norm). $\mathbf{H}_k(i,j)$ is the complex path gain between j-th BS and i-th antenna of MS at k-th frequency tone and has the following covariance structure:

$$E\left[\mathbf{H}_k(p,j)\,\mathbf{H}_k(q,l)^*\right] = \mathbf{\Psi}_T(j,l)\,\mathbf{\Psi}_R(p,q) \tag{2}$$

where $\mathbf{\Psi}_R$ and $\mathbf{\Psi}_T$ are $n_r \times n_r$ receive and $n_t \times n_t$ transmit correlation matrices. The correlation matrix being Hermitian leads to orthogonal eigenvectors and being positive semidefinite leads to non-negative eigenvalues. So the square root of the correlation matrix is found by eigenvalue decomposition of the matrix $\mathbf{\Psi}_R = \mathbf{E}\mathbf{\Lambda}\mathbf{E}^{-1}$ where $\mathbf{E}$ is the matrix of eigenvectors and $\mathbf{\Lambda}$ is the diagonal matrix with the eigenvalues on the diagonal. Eigenvectors being orthogonal leads to $\mathbf{\Psi}_R = \mathbf{E}\mathbf{\Lambda}\mathbf{E}^\dagger$ and accordingly the square root of the correlation matrix is

$$\mathbf{\Psi}_R^{1/2} = \mathbf{E}\sqrt{\mathbf{\Lambda}}\mathbf{E}^\dagger \tag{3}$$

So $\mathbf{\Psi}_R = \mathbf{\Psi}_R^{1/2}\mathbf{\Psi}_R^{1/2} = \mathbf{\Psi}_R^{\dagger/2}\mathbf{\Psi}_R^{1/2} = \mathbf{\Psi}_R^{1/2}\mathbf{\Psi}_R^{\dagger/2}$.

Since, for Rayleigh fading, second-order statistics fully describe the multi-antenna channel, a general model is

$$\mathrm{vec}(\mathbf{H}_k) = \mathbf{\Psi}^{\frac{1}{2}}\,\mathrm{vec}(\mathbf{W}_k) \tag{4}$$

where $\mathbf{W}_k$ is the spatially white (Rayleigh iid) MIMO channel; $\mathbf{\Psi}$ is the covariance matrix defined as $\mathbf{\Psi} = E\left\{\mathrm{vec}(\mathbf{H}_k)\,\mathrm{vec}(\mathbf{H}_k)^\dagger\right\}$. As per *Kronecker correlation model* [5], the correlation of the vectorized channel matrix can be written as the Kronecker product of transmit and receive correlation i.e. $\mathbf{\Psi} = \mathbf{\Psi}_T^T \otimes \mathbf{\Psi}_R$. So $\mathbf{H}_k$ can therefore be factorized in the form

$$\mathbf{H}_k = \mathbf{\Psi}_R^{1/2}\mathbf{W}_k\mathbf{\Psi}_T^{1/2} \tag{5}$$

This channel matrix represents the *Kronecker correlation model* [5] since the correlation of the vectorized channel matrix can be written as the Kronecker product of transmit and receive correlation.

Let us now focus on the structure of correlation matrix. We consider single-parameter exponential correlation matrix model. For this model, the components of $\mathbf{\Psi}_R$ are given by

$$\begin{aligned} \mathbf{\Psi}_R(p,q) &= \rho^{abs(q-p)}, & p \le q \\ &= \left(\rho^{abs(q-p)}\right)^*, & p > q \end{aligned}$$

where ρ is the (complex) correlation coefficient of neighboring receive branches with $|\rho| \le 1$. Similarly the components of $\mathbf{\Psi}_T$ are given by

$$\begin{aligned} \mathbf{\Psi}_T(j,l) &= \tau^{abs(l-j)}, & j \le l \\ &= \left(\tau^{abs(l-j)}\right)^*, & j > l \end{aligned}$$

where τ is the (complex) correlation coefficient of two BSs with $|\tau| \le 1$. Obviously, this may not be an accurate model for some real-world scenarios but it has been shown that this model can approximate the correlation in a uniform linear array under rich scattering conditions [6] with correlation decreasing with increasing distance between the antennas.

Detectors. For low complexity max log MAP detection [2], the bit metric for the first stream in its full form is given as:-

$$\lambda_1^i(\mathbf{y}_k, c_{k'}) \approx \min_{x_1 \in \chi_{1,c_{k'}}^i, x_2 \in \chi_2} \frac{1}{N_0}\left\|\mathbf{y}_k - \mathbf{h}_{1,k}x_1 - \mathbf{h}_{2,k}x_2\right\|^2$$

where $\chi_{1,c_{k'}}^i$ denotes the subset of the signal set $x_1 \in \chi_1$ whose labels have the value $c_{k'} \in \{0,1\}$ in the position i.

For linear MMSE detection, MMSE filter [3] for the detection of first stream is given as

$$\mathbf{h}_{1,k}^{MMSE} = \left(\mathbf{h}_{1,k}^\dagger\mathbf{R}_{2,k}^{-1}\mathbf{h}_{1,k} + \sigma_1^{-2}\right)^{-1}\mathbf{h}_{1,k}^\dagger\mathbf{R}_{2,k}^{-1} \tag{6}$$

where $\mathbf{R}_{2,k} = \sigma_2^2\mathbf{h}_{2,k}\mathbf{h}_{2,k}^\dagger + N_0\mathbf{I}$. The application of this MMSE filter yields

$$\begin{aligned} y_k &= \alpha_k x_{1,k} + \beta_k x_{2,k} + \mathbf{h}_{1,k}^{MMSE}\mathbf{z}_k & (7) \\ &= \alpha_k x_{1,k} + z_k & (8) \end{aligned}$$

where $\alpha_k = \mathbf{h}_{1,k}^{MMSE}\mathbf{h}_{1,k}$ and $\beta_k = \mathbf{h}_{1,k}^{MMSE}\mathbf{h}_{2,k}$. Based on the Gaussian assumption of post detection interference, the bit metric for the $c_{k'}$ bit on first stream is given as

$$\lambda_1^i(\mathbf{y}_k, c_{k'}) \approx \min_{x_1 \in \chi_{1,c_{k'}}^i}\left[\frac{1}{N_k}\left|y_k - \alpha_k x_1\right|^2\right] \tag{9}$$

where $N_k = \mathbf{h}_{1,k}^{MMSE}\mathbf{R}_{2,k}\mathbf{h}_{1,k}^{MMSE^\dagger}$.

III. PEP ANALYSIS - MAX LOG MAP DETECTOR

The conditional PEP i.e. $P\left(\mathbf{c}_1 \rightarrow \hat{\mathbf{c}}_1 | \overline{\overline{\mathbf{H}}}\right) = \mathcal{P}^{\hat{\mathbf{c}}_1}_{\mathbf{c}_1|\overline{\overline{\mathbf{H}}}}$ of low complexity max log MAP detector [2] is given as:-

$$\mathcal{P}^{\hat{\mathbf{c}}_1}_{\mathbf{c}_1|\overline{\overline{\mathbf{H}}}} = P\left(\sum_{k'} \min_{x_1\in\chi^i_{1,c_{k'}}, x_2\in\chi_2} \frac{1}{N_0}\left\|\mathbf{y}_k - \mathbf{h}_{1,k}x_1 - \mathbf{h}_{2,k}x_2\right\|^2 \geq \sum_{k'} \min_{x_1\in\chi^i_{1,\hat{c}_{k'}}, x_2\in\chi_2} \frac{1}{N_0}\left\|\mathbf{y}_k - \mathbf{h}_{1,k}x_1 - \mathbf{h}_{2,k}x_2\right\|^2\right) \tag{10}$$

where $\overline{\overline{\mathbf{H}}} = [\mathbf{H}_1 \cdots \mathbf{H}_N]$ i.e. the complete channel for the transmission of codeword $\mathbf{c}_1$. For the worst case scenario once $d\left(\mathbf{c}_1 - \hat{\mathbf{c}}_1\right) = d_{free}$, the inequality on the right hand side of (10) shares the same terms on all but d_{free} summation points for which $\hat{c}_{k'} = \bar{c}_{k'}$ where $\bar{(.)}$ denotes the binary complement. Let

$$\tilde{x}_{1,k}, \tilde{x}_{2,k} = \arg\min_{x_1\in\chi^i_{1,c_{k'}}, x_2\in\chi_2} \frac{1}{N_0}\left\|\mathbf{y}_k - \mathbf{h}_{1,k}x_1 - \mathbf{h}_{2,k}x_2\right\|^2$$
$$\hat{x}_{1,k}, \hat{x}_{2,k} = \arg\min_{x_1\in\chi^i_{1,\bar{c}_{k'}}, x_2\in\chi_2} \frac{1}{N_0}\left\|\mathbf{y}_k - \mathbf{h}_{1,k}x_1 - \mathbf{h}_{2,k}x_2\right\|^2 \tag{11}$$

As $x_{1,k}$ and $x_{2,k}$ are the transmitted symbols so $\left\|\mathbf{y}_k - \mathbf{h}_{1,k}x_{1,k} - \mathbf{h}_{2,k}x_{2,k}\right\|^2 \geq \left\|\mathbf{y}_k - \mathbf{h}_{1,k}\tilde{x}_{1,k} - \mathbf{h}_{2,k}\tilde{x}_{2,k}\right\|^2$. The conditional PEP is given as

$$\begin{aligned}
\mathcal{P}^{\hat{\mathbf{c}}_1}_{\mathbf{c}_1|\overline{\mathbf{H}}} &\leq P\left(\sum_{k,d_{free}} \frac{1}{N_0}\left\|\mathbf{y}_k - \mathbf{h}_{1,k}x_{1,k} - \mathbf{h}_{2,k}x_{2,k}\right\|^2 \geq \sum_{k,d_{free}} \frac{1}{N_0}\left\|\mathbf{y}_k - \mathbf{h}_{1,k}\hat{x}_{1,k} - \mathbf{h}_{2,k}\hat{x}_{2,k}\right\|^2\right) \\
&= P\left(\sum_{k,d_{free}} \frac{1}{N_0} 2\Re\left(\mathbf{z}_k^\dagger \mathbf{H}_k\left(\hat{\mathbf{x}}_k - \mathbf{x}_k\right)\right) \geq \sum_{k,d_{free}} \frac{1}{N_0}\left\|\mathbf{H}_k\mathbf{x}_k - \mathbf{H}_k\hat{\mathbf{x}}_k\right\|^2\right) \\
&= Q\left(\sqrt{\sum_{k,d_{free}} \frac{1}{2N_0}\left\|\mathbf{H}_k\left(\hat{\mathbf{x}}_k - \mathbf{x}_k\right)\right\|^2}\right) \\
&= Q\left(\sqrt{\frac{1}{2N_0}\text{vec}\left(\overline{\mathbf{H}}^\dagger\right)^\dagger \boldsymbol{\Delta}\text{vec}\left(\overline{\mathbf{H}}^\dagger\right)}\right)
\end{aligned} \tag{12}$$

where $\boldsymbol{\Delta} = \mathbf{I}_{n_r} \otimes \mathbf{D}\mathbf{D}^\dagger$ while $\mathbf{D}_{2K\times K} = \text{diag}\left\{\hat{\mathbf{x}}_1 - \mathbf{x}_1, \hat{\mathbf{x}}_2 - \mathbf{x}_2, \cdots, \hat{\mathbf{x}}_{k,dfree} - \mathbf{x}_{k,d_{free}}\right\}$ so $\mathbf{D}\mathbf{D}^\dagger$ is a $2K \times 2K$ block diagonal matrix with real entries on the main diagonal. Note that $\overline{\mathbf{H}} = [\mathbf{H}_1 \cdots \mathbf{H}_K]$ where $K = d_{free}$. Using the Chernoff bound $Q(x) \leq \frac{1}{2}\exp\left(\frac{-x^2}{2}\right)$, the conditional PEP can be written as:-

$$\mathcal{P}^{\hat{\mathbf{c}}_1}_{\mathbf{c}_1|\overline{\mathbf{H}}} \leq \frac{1}{2}\exp\left(-\frac{1}{4N_0}\text{vec}\left(\overline{\mathbf{H}}^\dagger\right)^\dagger \boldsymbol{\Delta}\text{vec}\left(\overline{\mathbf{H}}^\dagger\right)\right) \tag{13}$$

Note that $\overline{\mathbf{H}} = \boldsymbol{\Psi}_R^{1/2}\left[\mathbf{W}_1 \cdots \mathbf{W}_K\right]\left(\mathbf{I}_K \otimes \boldsymbol{\Psi}_T^{1/2}\right) = \boldsymbol{\Psi}_R^{\frac{1}{2}}\overline{\mathbf{W}}_{n_r\times 2K}\left(\mathbf{I}_K \otimes \boldsymbol{\Psi}_T^{1/2}\right)$. Using the Kronecker product identity $\text{vec}\left(\mathbf{AXB}\right) = \left(\mathbf{B}^T \otimes \mathbf{A}\right)\text{vec}\left(\mathbf{X}\right)$ we get

$$\begin{aligned}
\text{vec}\left(\overline{\mathbf{H}}^\dagger\right) &= \text{vec}\left(\left(\mathbf{I}_K \otimes \boldsymbol{\Psi}_T^{\dagger/2}\right)\overline{\mathbf{W}}^\dagger_{2k\times n_r}\boldsymbol{\Psi}_R^{\dagger/2}\right) \\
&= \left(\boldsymbol{\Psi}_R^{\frac{*}{2}} \otimes \left(\mathbf{I}_K \otimes \boldsymbol{\Psi}_T^{\dagger/2}\right)\right)\text{vec}\left(\overline{\mathbf{W}}^\dagger\right)
\end{aligned} \tag{14}$$

Developing (13) further on the lines of [7]:-

$$\begin{aligned}
\mathcal{P}^{\hat{\mathbf{c}}_1}_{\mathbf{c}_1|\overline{\mathbf{H}}} &\leq \frac{1}{2}\exp\left(-\frac{1}{4N_0}\text{vec}\left(\overline{\mathbf{H}}^\dagger\right)^\dagger \boldsymbol{\Delta}\text{vec}\left(\overline{\mathbf{H}}^\dagger\right)\right) \\
&= \frac{1}{2}\exp\left(-\frac{1}{4N_0}\text{vec}\left(\overline{\mathbf{W}}^\dagger\right)^\dagger \left(\boldsymbol{\Psi}_R\right)^T \otimes \left(\mathbf{I}_K \otimes \boldsymbol{\Psi}_T^{1/2}\right)\mathbf{D}\mathbf{D}^\dagger\left(\mathbf{I}_K \otimes \boldsymbol{\Psi}_T^{\dagger/2}\right)\text{vec}\left(\overline{\mathbf{W}}^\dagger\right)\right)
\end{aligned} \tag{15}$$

where we have used the identities $\left(A \otimes B\right)^\dagger = A^\dagger \otimes B^\dagger$ and $\left(A \otimes C\right)\left(B \otimes D\right) = AB \otimes CD$. As the argument of exponential in (15) is the Hermitian quadratic form of a Gaussian random variable so using its MGF [3] gives

$$\begin{aligned}
\mathcal{P}^{\hat{\mathbf{c}}_1}_{\mathbf{c}_1} &\leq \frac{1}{2\det\left(\mathbf{I} + \frac{1}{4N_0}\left(\boldsymbol{\Psi}_R\right)^T \otimes \left(\mathbf{I}_K \otimes \boldsymbol{\Psi}_T^{1/2}\right)\mathbf{D}\mathbf{D}^\dagger\left(\mathbf{I}_K \otimes \boldsymbol{\Psi}_T^{\dagger/2}\right)\right)} \\
&\overset{a}{=} \frac{1}{2\det\left(\mathbf{I} + \frac{1}{4N_0}\left(\boldsymbol{\Psi}_R\right)^T \otimes \mathbf{D}\mathbf{D}^\dagger\left(\mathbf{I}_K \otimes \boldsymbol{\Psi}_T\right)\right)} \\
&\overset{b}{=} \frac{1}{2\prod_{k=1}^{2d_{free}}\prod_{l=1}^{n_r}\left(1 + \frac{1}{4N_0}\lambda_l\left(\boldsymbol{\Psi}_R\right)\lambda_k\left(\mathbf{D}\mathbf{D}^\dagger\left(\mathbf{I}_K \otimes \boldsymbol{\Psi}_T\right)\right)\right)} \\
&\overset{c}{\leq} \frac{1}{2}\prod_{k=1}^{d_{free}}\prod_{l=1}^{\kappa}\frac{4N_0}{\lambda_l\left(\boldsymbol{\Psi}_R\right)\lambda_k\left(\mathbf{D}\mathbf{D}^\dagger\left(\mathbf{I}_K \otimes \boldsymbol{\Psi}_T\right)\right)} \\
&\overset{d}{=} \frac{1}{2}\prod_{k=1}^{d_{free}}\prod_{l=1}^{\kappa}\frac{4N_0}{\lambda_l\left(\boldsymbol{\Psi}_R\right)\theta_k\left\|\hat{\mathbf{x}}_k - \mathbf{x}_k\right\|^2}
\end{aligned} \tag{16}$$

(a) involves the commutative identity for the product of block diagonal matrices, while in (b) we have used the identity that for square matrices $\mathbf{A}$ and $\mathbf{B}$ of size m and n respectively with the eigenvalues $\lambda_1\left(\mathbf{A}\right), \cdots, \lambda_m\left(\mathbf{A}\right)$ and $\lambda_1\left(\mathbf{B}\right), \cdots, \lambda_n\left(\mathbf{B}\right)$, the eigenvalues of $\mathbf{A} \otimes \mathbf{B}$ are

$$\lambda_l\left(\mathbf{A}\right)\lambda_k\left(\mathbf{B}\right) \qquad l = 1, \cdots, m, \; k = 1, \cdots, n. \tag{17}$$

For (c) we have used the identity that $\text{rank}\left(\mathbf{A}, \mathbf{B}\right) \leq \min\left(\text{rank}\left(\mathbf{A}\right), \text{rank}\left(\mathbf{B}\right)\right)$. The eigenvalues of $\left(\mathbf{D}\mathbf{D}^\dagger\right)_{2K\times 2K}$ are

$$\lambda_k\left(\mathbf{D}\mathbf{D}^\dagger\right) = \begin{cases} \left\|\hat{\mathbf{x}}_k - \mathbf{x}_k\right\|^2 & \text{for} \quad k = 1, \cdots, d_{free} \\ 0 & \text{for} \quad k = d_{free}+1, \cdots, 2d_{free} \end{cases}$$

while the eigenvalues of $\mathbf{I}_K \otimes \boldsymbol{\Psi}_T$ are

$$\lambda_k\left(\mathbf{I}_K \otimes \boldsymbol{\Psi}_T\right) = \begin{cases} \lambda_1\left(\boldsymbol{\Psi}_T\right) & \text{for} \quad k = 1, \cdots, d_{free} \\ \lambda_2\left(\boldsymbol{\Psi}_T\right) & \text{for} \quad k = d_{free}+1, \cdots, 2d_{free} \end{cases}$$

where we have used the identity $\text{eig}\left(\mathbf{A} \otimes \mathbf{B}\right) = \text{eig}\left(\mathbf{A}\right) \otimes \text{eig}\left(\mathbf{B}\right)$. Note that $\kappa = \text{rank}\left(\boldsymbol{\Psi}_R\right)$. In (d) we have used the lemma 1 of [8] i.e.

$$\begin{aligned}
\lambda_k\left(\mathbf{D}\mathbf{D}^\dagger\left(\mathbf{I}_K \otimes \boldsymbol{\Psi}_T\right)\right) &= \theta_k\lambda_k\left(\mathbf{D}\mathbf{D}^\dagger\right) \\
&= \theta_k\left\|\hat{\mathbf{x}}_k - \mathbf{x}_k\right\|^2
\end{aligned} \tag{18}$$

such that for each k, there exists a positive real number θ_k such that $\lambda_1\left(\boldsymbol{\Psi}_T\right) \leq \theta_k \leq \lambda_2\left(\boldsymbol{\Psi}_T\right)$. Note that $\left\|\hat{\mathbf{x}}_k - \mathbf{x}_k\right\|^2 \geq$

$d_{1,\min}^2 + d_{2,\min}^2$ if $\hat{x}_{2,k} \neq x_{2,k}$ and $\|\hat{\mathbf{x}}_k - \mathbf{x}_k\|^2 \geq d_{1,\min}^2$ if $\hat{x}_{2,k} = x_{2,k}$. There exists $2^{d_{free}}$ possible vectors of $\left[\hat{x}_{2,1}, \cdots, \hat{x}_{2,d_{free}}\right]^T$ basing on the binary criteria that $\hat{x}_{2,k}$ is equal or not equal to $x_{2,k}$. We call these events as ξ_i where $i = 1, \cdots, 2^{d_{free}}$. Consider the event ξ_m where amongst d_{free} terms of $\|\hat{\mathbf{x}}_k - \mathbf{x}_k\|^2$, m terms have $\hat{x}_{2,k} \neq x_{2,k}$. Let the product of θs for these m terms is denoted as $[\theta]^m$ and the product of θs for the remaining $(d_{free} - m)$ terms is denoted as $[\theta]^{m'}$. Conditioned on this event ξ_m, we have

$$\prod_{k=1}^{d_{free}} \prod_{l=1}^{\kappa} \frac{\lambda_l(\mathbf{\Psi}_R)\,\theta_k \|\hat{\mathbf{x}}_k - \mathbf{x}_k\|^2}{4N_0}$$
$$\geq \left(\left(d_{1,\min}^2 + d_{2,\min}^2\right)^m [\theta]^m\right)^\kappa \left(\left(d_{1,\min}^2\right)^{(d_{free}-m)} [\theta]^{m'}\right)^\kappa \times$$
$$\left(\prod_{l=1}^{\kappa} (\lambda_l(\mathbf{\Psi}_R))^{d_{free}}\right)\left(\frac{1}{4N_0}\right)^{\kappa d_{free}}$$

So PEP is given as

$$\mathcal{P}_{\hat{\mathbf{c}}_1}^{\mathbf{c}_1}|\xi_m \leq \frac{1}{2}\left(\frac{1}{\left(d_{1,\min}^2 + d_{2,\min}^2\right)^m [\theta]^m}\right)^\kappa \left(\frac{1}{\left(d_{1,\min}^2\right)^{(d_{free}-m)} [\theta]^{m'}}\right)^\kappa$$
$$\left(\prod_{l=1}^{\kappa} \frac{1}{(\lambda_l(\mathbf{\Psi}_R))^{d_{free}}}\right)(4N_0)^{\kappa d_{free}}$$

whereas probability of this event $\xi_m$is given as

$$P(\xi_m) = \left(P\left(\hat{x}_{2,k} \neq x_{2,k}\right)\right)^m \left(1 - P\left(\hat{x}_{2,k} \neq x_{2,k}\right)\right)^{d_{free}-m} \tag{19}$$

$P(\hat{x}_{2,k} \neq x_{2,k})$ is the uncoded probability that the output of max log MAP detector $\hat{x}_{2,k}$ is not equal to the actual transmitted symbol $x_{2,k}$ and has been derived in the Appendix. Taking into account all these cases combined with their corresponding probabilities, the PEP is upper bounded as

$$\mathcal{P}_{\hat{\mathbf{c}}_1}^{\mathbf{c}_1} \leq \frac{1}{2}\left(\frac{4N_0}{\sigma_1^2 \breve{d}_{1,\min}^2}\right)^{\kappa d_{free}} \left(\prod_{l=1}^{\kappa} \frac{1}{(\lambda_l(\mathbf{\Psi}_R))^{d_{free}}}\right)\left(\frac{1}{[\theta]^{d_{free}}}\right)^\kappa \times$$
$$\left(\sum_{j=0}^{d_{free}} \mathrm{C}_j^{d_{free}} \frac{\left(P\left(\hat{x}_{2,k} \neq x_{2,k}\right)\right)^j \left(1 - P\left(\hat{x}_{2,k} \neq x_{2,k}\right)\right)^{d_{free}-j}}{\left(1 + \frac{\sigma_2^2 \breve{d}_{2,\min}^2}{\sigma_1^2 \breve{d}_{1,\min}^2}\right)^{j\kappa}}\right) \tag{20}$$

where $[\theta]^{d_{free}}$ indicates the product $\theta_1\theta_2\cdots\theta_{d_{free}}$ and $d_{j,\min}^2 = \sigma_j^2 \breve{d}_{j,\min}^2$ with $\breve{d}_{j,\min}^2$ being the normalized minimum distance of the constellation χ_j for $j = \{1, 2\}$ and $\mathrm{C}_j^{d_{free}}$ is the binomial coefficient.

Interestingly, the overall diversity depends only on the rank of $\mathbf{\Psi}_R$ and d_{free} and not on the rank of $\mathbf{\Psi}_T$. However the coding gain depends both on the eigenvalues of $\mathbf{\Psi}_R$ and $\mathbf{\Psi}_T$. The coding gain increases as the interference gets stronger relative to the desired stream i.e. $\sigma_2^2 > \sigma_1^2$ or the rate of interference decreases relative to the desired stream i.e. $\breve{d}_{2,\min}^2 > \breve{d}_{1,\min}^2$. It has been shown in the Appendix that $P(\hat{x}_{2,k} \neq x_{2,k})$ depends on the eigenvalues of receive correlation matrix in addition to its dependence on the strength of desired signal and interference.

IV. PEP Analysis - MMSE Detector

Now we analyze MMSE in the presence of non Gaussian interference i.e. $\beta_k x_{2,k}$ and iid fading. Conditional PEP for MMSE basing on Gaussian assumption of interference is given as

$$\mathcal{P}_{\hat{\mathbf{c}}_1|\mathbf{H}}^{\mathbf{c}_1} = P\left(\sum_{k'} \min_{x_1 \in \chi_{1,c_{k'}}^i} \frac{|y_k - \alpha_k x_1|^2}{N_k} \geq \sum_{k'} \min_{x_1 \in \chi_{1,\hat{c}_{k'}}^i} \frac{|y_k - \alpha_k x_1|^2}{N_k}\right)$$

Replacing y_k by (7), conditional PEP is upper bounded as

$$\mathcal{P}_{\hat{\mathbf{c}}_1|\mathbf{H}}^{\mathbf{c}_1} \leq Q\left(\sqrt{\sum_{k,d_{free}} \frac{\alpha_k^2}{2N_k}\left\{|x_{1,k} - \hat{x}_{1,k}|^2\right\}} + \frac{\sum_{k,d_{free}} \frac{2\alpha_k}{N_k} \Re\left(x_{2,k}^* \beta_k^* (x_{1,k} - \hat{x}_{1,k})\right)}{\sqrt{\sum_{k,d_{free}} \frac{2\alpha_k^2}{N_k} |x_{1,k} - \hat{x}_{1,k}|^2}}\right) \tag{21}$$

Using the bounds $Q(a+b) \leq Q(a_{\min} - b_{\max})$, $\Re\left(\mathbf{a}^\dagger \mathbf{b}\right) \leq \|\mathbf{a}\|\,\|\mathbf{b}\|$ and the Jensen's inequality i.e. $E\left(\sqrt{X}\right) \leq \sqrt{E(X)}$

$$\mathcal{P}_{\hat{\mathbf{c}}_1|\mathbf{H}}^{\mathbf{c}_1} \leq Q\left(\sqrt{\frac{d_{1,\min}^2}{2} \sum_{k,d_{free}} \mathbf{h}_{1,k}^\dagger \mathbf{R}_{2,k}^{-1} \mathbf{h}_{1,k}} - \frac{2 d_{1,\max} d_{x_2,\max} \sqrt{\sum_{k,d_{free}} \mathbf{h}_{1,k}^\dagger \mathbf{R}_{2,k}^{-1} \mathbf{h}_{2,k} \mathbf{h}_{2,k}^\dagger \mathbf{R}_{2,k}^{-1} \mathbf{h}_{1,k}}}{\sqrt{2 d_{1,\min}^2 \sum_{k,d_{free}} \mathbf{h}_{1,k}^\dagger \mathbf{R}_{2,k}^{-1} \mathbf{h}_{1,k}}}\right)$$

where $d_{1,\max}$ and $d_{x_2,\max}$ are the maximum distance of constellation χ_1 and the maximum distance of a constellation point of χ_2 from the origin respectively. Maximizing the numerator of the second argument of the Q function by aligning $\mathbf{h}_{1,k}$ with the eigenvector of $\mathbf{R}_{2,k}^{-1} \mathbf{h}_{2,k} \mathbf{h}_{2,k}^\dagger \mathbf{R}_{2,k}^{-1}$ corresponding to its only non zero eigenvalue $\|\mathbf{h}_{2,k}\|^2 \left(\sigma_2^2 \|\mathbf{h}_{2,k}\|^2 + N_0\right)^{-2}$, we get :-

$$\mathcal{P}_{\hat{\mathbf{c}}_1|\mathbf{H}}^{\mathbf{c}_1} \leq Q\left(\sqrt{\frac{d_{1,\min}^2}{2} \sum_{k,d_{free}} \mathbf{h}_{1,k}^\dagger \mathbf{R}_{2,k}^{-1} \mathbf{h}_{1,k}} - \frac{2 d_{1,\max} d_{x_2,\max} \sqrt{\sum_{k,d_{free}} \|\mathbf{h}_{1,k}\|^2 \|\mathbf{h}_{2,k}\|^2 \left(\sigma_2^2 \|\mathbf{h}_{2,k}\|^2 + N_0\right)^{-2}}}{\sqrt{2 d_{1,\min}^2 \sum_{k,d_{free}} \mathbf{h}_{1,k}^\dagger \mathbf{R}_{2,k}^{-1} \mathbf{h}_{1,k}}}\right)$$

Using Chernoff bound and bounding one exponential by 1

$$\mathcal{P}_{\hat{\mathbf{c}}_1|\mathbf{H}}^{\mathbf{c}_1} \leq \frac{1}{2} \exp\left(-\frac{d_{1,\min}^2}{4} \sum_{k,d_{free}} \mathbf{h}_{1,k}^\dagger \mathbf{R}_{2,k}^{-1} \mathbf{h}_{1,k}\right) \times$$
$$\exp\left(d_{1,\max} d_{x_2,\max} \sqrt{\sum_{k,d_{free}} \|\mathbf{h}_{1,k}\|^2 \|\mathbf{h}_{2,k}\|^2 \left(\sigma_2^2 \|\mathbf{h}_{2,k}\|^2 + N_0\right)^{-2}}\right)$$

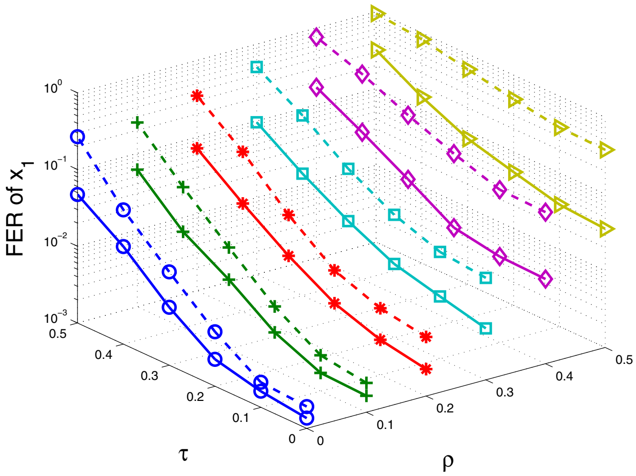

Fig. 1. Desired stream x_1 is QAM16 while interference stream x_2 is QPSK. Continuous lines indicate low complexity max log MAP detection while dashed lines indicate linear MMSE detection. SNR for max log MAP detection is 6.6 dB while for MMSE detection is 8.65 dB.

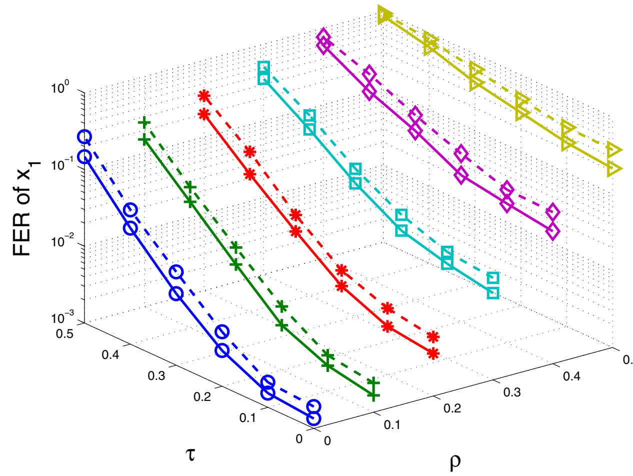

Fig. 2. Both desired stream x_1 and interference stream x_2 are QAM16. Continuous lines indicate low complexity max log MAP detection while dashed lines indicate linear MMSE detection. SNR for max log MAP detection is 6.6 dB while for MMSE detection is 8.65 dB.

where one negative exponential has been upper bounded by 1. Maximizing the second exponential for $\mathbf{h}_{2,k}$ yields

$$\mathcal{P}^{\hat{\mathbf{c}}_1}_{\mathbf{c}_1|\mathbf{H}} \leq \frac{1}{2}\exp\left(-\Psi_{x_1}\sum_{k,d_{free}}\mathbf{h}^{\dagger}_{1,k}\mathbf{R}^{-1}_{2,k}\mathbf{h}_{1,k}\right)\exp\left(\Psi_{h_2}\sqrt{\sum_{k,d_{free}}\|\mathbf{h}_{1,k}\|^2}\right) \qquad (22)$$

where $\Psi_{h_2} = \frac{d_{1,max}d_{x_2,\max}\|\mathbf{h}_{2,\min}\|}{\left(\sigma_2^2\|\mathbf{h}_{2,\min}\|^2+N_0\right)}$ and $\Psi_{x_1} = \frac{d^2_{1,\min}}{4}$. $\|\mathbf{h}_{2,\min}\|$ is the minimum norm of $\mathbf{h}_{2,k}$. The fact that conditioned on $\underline{\mathbf{h}}_2$, the covariance of two random variables (exponentials) in (22) is negative leads to their joint expectation being upper bounded by the product of their individual expectations. Averaging first over $\underline{\mathbf{h}}_1$ followed by $\underline{\mathbf{h}}_2$ yields

$$\begin{aligned}\mathcal{P}^{\hat{\mathbf{c}}_1}_{\mathbf{c}_1} \leq & \left(\frac{4N_0}{\sigma_1^2\breve{d}^2_{1,\min}}\right)^{d_{free}(n_r-1)}\left(\frac{4}{\sigma_1^2\breve{d}^2_{1,\min}}\right)^{d_{free}} \times \\ & E_{\underline{\mathbf{h}}_2}\left[\prod_{l=1}^{d_{free}}\left(\sigma_2^2\|\mathbf{h}_{2,l}\|^2+N_0\right)\left\{\Phi\left(\frac{d_{free}n_r}{2},\frac{1}{2},\frac{\Psi^2_{h_2}}{2}\right)+\right.\right. \\ & \left.\left.\Psi_{h_2}\sqrt{2}\frac{\Gamma\left(\frac{d_{free}n_r+1}{2}\right)}{\Gamma\left(\frac{d_{free}n_r}{2}\right)}\Phi\left(\frac{d_{free}n_r+1}{2},\frac{3}{2},\frac{\Psi^2_{h_2}}{2}\right)\right\}\right] \end{aligned} \qquad (23)$$

where $\Gamma(.)$ is the gamma function and $\Phi(\alpha,\gamma,z)$ is the confluent hypergeometric function. For the second exponential, we have used the MGF of Chi distributed RV with $d_{free}n_r$ degrees of freedom. The expectation w.r.t $\underline{\mathbf{h}}_2$ seemingly does not have closed from solution. (23) shows the loss of one order of diversity which is similar to the case of Gaussian assumption for PDI.

V. Simulation Results

We consider 2 BSs each using BICM OFDM system for downlink transmission using the punctured rate $1/2$ turbo code[1] of 3GPP LTE [1]. We consider an ideal OFDM based system (no ISI) and analyze the system in frequency domain. We assume antenna cycling at the BS and receive diversity at the MS with two antennas. The resulting SIMO channel at each sub carrier from BS to MS has correlated Gaussian matrix entries (receive correlation) while we also assume two SIMO channels to be correlated (transmit correlation). However we consider channels at different subcarriers to be independent. For the structure of correlation matrix, we consider exponential correlation matrix model. Perfect channel state information (CSI) is assumed at the MS while BSs have no CSI. Furthermore, all mappings of coded bits to QAM symbols use Gray encoding. We consider linear MMSE detector and the low complexity max log MAP detector. Figs. 1 and 2 show the frame error rates (FERs) of desired stream for the frame size of 1056 information bits. The effects of transmit and receive correlation along with the rate of interference stream have been isolated in these simulations. To achieve this, the strengths of desired stream and interference stream are kept same (Cell Edge case) while FERs of two detectors have been approximately equated once there is no correlation. Then as the correlation (both transmit and receive) get stronger, the degrading effect on the performance of both the detectors in the presence of different interferences is compared. In the case of equal rate streams i.e. desired and interference streams belong to same constellation, the degrading effect of both transmit and receive correlation on MMSE and low complexity max log MAP detection is approximately same. However once

[1]The LTE turbo decoder design was performed using the coded modulation library www.iterativesolutions.com

the interference has lower rate compared to the desired stream, the degrading effect of enhanced correlation is quite reduced on max log MAP detector as compared to MMSE detector. The degradation of MMSE performance with increase in transmit and receive correlation is independent of the rate of interference. The SNR gap between the max log MAP detection and MMSE detection for the same FER (at zero correlation) widens as rate of interference stream decreases relative to the desired stream. This can be attributed to the ability of max log MAP detector to partially decode interference once the lower rate or the higher strength of interference permits its partial decoding [3]. This partial decoding capability of max log MAP detector reduces with increased correlation especially once interference has comparable rate relative to the desired stream. MMSE does not benefit from exploiting interference as it is based on the attenuation of interference strength and the subsequent assumption of Gaussianity for its behavior.

VI. Conclusions

We have focused in this paper on the effects of correlation (receive and transmit) and interference rate on the performance of low complexity max log MAP detector. We have also derived PEP bounds for MMSE detector for the case of iid fading considering discrete alphabets. We have shown that the degrading effect of both transmit and receive correlation is less for max log MAP detector as compared to MMSE detector in the case when interference because of its relative rate allows its partial decoding. The degradation of MMSE performance with enhanced correlation is independent of the rate of interference.

Acknowledgments

Eurecom's research is partially supported by its industrial partners: BMW, Bouygues Telecom, Cisco Systems, France Télécom, Hitachi Europe, SFR, Sharp, ST Microelectronics, Swisscom, Thales. The research work leading to this paper has also been partially supported by the European Commission under the IST FP7 research network of excellence NEWCOM++.

Appendix

Considering the definition of $\hat{x}_{2,k}$ in (11), it can be expanded as:-

$$\hat{x}_{2,k} = \arg \min_{x_1 \in \chi^{i}_{1_{\bar{c}_k}'}, x_2 \in \chi_2} \left[\frac{1}{N_0} \left\{ \left\| \mathbf{h}_{1,k} \left(x_{1,k} - x_1\right) + \mathbf{z}_k \right\|^2 + \left\| \mathbf{h}_{2,k} \left(x_{2,k} - x_2\right) \right\|^2 + 2\Re \left(\mathbf{h}_{1,k} \left(x_{1,k} - x_1\right) + \mathbf{z}_k \right)^{\dagger} \mathbf{h}_{2,k} \left(x_{2,k} - x_2\right) \right\} \right]$$

The last two terms will be zero if $\hat{x}_{2,k} = x_{2,k}$. Conditioning it on x_1, the probability $P\left(\hat{x}_{2,k} \neq x_{2,k} | \mathbf{h}_{1,k}, \mathbf{h}_{2,k}, x_1\right) = \mathcal{P}^{\hat{x_2}}_{x_2|\mathbf{H}_k, x_1}$ is given as

$$= P\left(-2\Re\left(\left(\mathbf{h}_{1,k} \mathrm{x}_{1,k} + \mathbf{z}_k\right)^{\dagger} \mathbf{h}_{2,k} \mathrm{x}_{2,k}\right) \geq \left\|\mathbf{h}_{2,k} \mathrm{x}_{2,k}\right\|^2 | \mathbf{H}_k, x_{1,k}\right)$$
$$= Q\left(\sqrt{\frac{\left\|\mathbf{h}_{2,k} \mathrm{x}_{2,k}\right\|^2}{2N_0}} + \sqrt{\frac{2}{N_0}} \Re\left(\frac{\left(\mathbf{h}_{1,k} \mathrm{x}_{1,k}\right)^{\dagger} \mathbf{h}_{2,k} \mathrm{x}_{2,k}}{\sqrt{\left\|\mathbf{h}_{2,k} \mathrm{x}_{2,k}\right\|^2}}\right)\right)$$

where $\mathrm{x}_{j,k}$ denotes $(x_{j,k} - x_j)$. Using the relation $Q\left(a + b\right) \leq Q\left(a_{min} - |b_{max}|\right)$ and $\Re\left(\mathbf{a}^{\dagger} \hat{\mathbf{b}}\right) \leq \|\mathbf{a}\|$ where $\hat{\mathbf{b}}$ is the unit vector we get

$$\mathcal{P}^{\hat{x_2}}_{x_2|\mathbf{H}_k} \leq \frac{1}{2} \exp\left(-\frac{\left\|\mathbf{h}_{2,k}\right\|^2 d^2_{2,\min}}{4N_0} - \frac{\left\|\mathbf{h}_{1,k}\right\|^2 d^2_{1,\max}}{N_0} + \frac{\left\|\mathbf{h}_{2,k}\right\| \left\|\mathbf{h}_{1,k}\right\| d_{2,\min} d_{1,\max}}{N_0}\right) \tag{24}$$

Considering the norms of $\mathbf{h}_{1,k}$ and $\mathbf{h}_{2,k}$ we make two non-overlapping regions as $\left(\|\mathbf{h}_{2,k}\| \geq \|\mathbf{h}_{1,k}\|\right)$ and $\left(\|\mathbf{h}_{2,k}\| < \|\mathbf{h}_{1,k}\|\right)$ with the corresponding probabilities as $\mathcal{P}^{<}_{\mathbf{h}_1}$ and $\mathcal{P}^{>}_{\mathbf{h}_1}$. Note that in the first region $\|\mathbf{h}_{2,k}\| \|\mathbf{h}_{1,k}\| \leq \|\mathbf{h}_{2,k}\|^2$ while for second region $\|\mathbf{h}_{2,k}\| \|\mathbf{h}_{1,k}\| < \|\mathbf{h}_{1,k}\|^2$. So

$$\mathcal{P}^{\hat{x_2}}_{x_2|\mathbf{H}_k} \leq \frac{1}{2}\left[\exp\left(-\left\|\mathbf{h}_{2,k}\right\|^2 \frac{d^2_{2,\min} - 4d_{2,\min}d_{1,\max}}{4N_0}\right) \exp\left(-\frac{\left\|\mathbf{h}_{1,k}\right\|^2 d^2_{1,\max}}{N_0}\right) \mathcal{P}^{<}_{\mathbf{h}_1} + \exp\left(-\frac{\left\|\mathbf{h}_{2,k}\right\|^2 d^2_{2,\min}}{4N_0}\right) \exp\left(-\left\|\mathbf{h}_{1,k}\right\|^2 \frac{d^2_{1,\max} - d_{2,\min}d_{1,\max}}{N_0}\right) \mathcal{P}^{>}_{\mathbf{h}_1}\right]$$

We upperbound both the probabilities i.e. $\mathcal{P}^{<}_{\mathbf{h}_1}$ and $\mathcal{P}^{>}_{\mathbf{h}_1}$ by 1. Taking expectation over $\mathbf{h}_{2,k}$ conditioned on $\mathbf{h}_{1,k}$ and then subsequently taking expectation over $\mathbf{h}_{1,k}$ yields:-

$$\begin{aligned}
\mathcal{P}^{\hat{x_2}}_{x_2} \leq & \frac{1}{2} E_{\mathbf{h}_1}\left[\left(\frac{4N_0}{d^2_{2,\min} - 4d_{2,\min}d_{1,\max}}\right)^{\kappa} \exp\left(-\frac{\left\|\mathbf{h}_{1,k}\right\|^2 d^2_{1,\max}}{N_0}\right) \prod_{l=1}^{\kappa} \frac{1}{\lambda_l(\mathbf{\Psi}_R)} + \left(\frac{4N_0}{d^2_{2,\min}}\right)^{\kappa} \exp\left(-\left\|\mathbf{h}_{1,k}\right\|^2 \frac{d^2_{1,\max} - d_{2,\min}d_{1,\max}}{N_0}\right) \prod_{l=1}^{\kappa} \frac{1}{\lambda_l(\mathbf{\Psi}_R)}\right] \\
\leq & \frac{1}{2}\left(\frac{4N_0}{\sigma_2^2 \breve{d}^2_{2,\min}}\right)^{\kappa} \left(\frac{N_0}{\sigma_1^2 \breve{d}^2_{1,\max}}\right)^{\kappa} \prod_{l=1}^{\kappa} \frac{1}{\left(\lambda_l\left(\mathbf{\Psi}_R\right)\right)^2} \times \left(\frac{1}{\left(1 - \frac{4\sigma_1 \breve{d}_{1,\max}}{\sigma_2 \breve{d}_{2,\min}}\right)^{\kappa}} + \frac{1}{\left(1 - \frac{\sigma_2 \breve{d}_{2,\min}}{\sigma_1 \breve{d}_{1,\max}}\right)^{\kappa}}\right)
\end{aligned} \tag{25}$$

where we have used the MGF of Hermitian quadratic form of a Gaussian random variable while writing $\|\mathbf{h}_{j,k}\|^2 = \mathbf{h}^{\dagger}_{j,k} \mathbf{I}_{n_r} \mathbf{h}_{j,k}$ where $\mathbf{h}_{j,k} \sim \mathcal{CN}\left(\mathbf{0}, \mathbf{\Psi}_R\right)$. Interestingly transmit correlation being between $\mathbf{h}_{1,k}$ and $\mathbf{h}_{2,k}$ does not appear in the above expression. This expression shows the dependence on the interference strength, SNR and the correlation.

References

[1] LTE, *Requirements for Evolved UTRA (E-UTRA) and Evolved UTRAN (E-UTRAN)*. 3GPP TR 25.913 v.7.3.0, 2006.

[2] R. Ghaffar and R. Knopp, "Interference Suppression for Next Generation Wireless Systems," in *IEEE 69-th Vehicular Technology Conference VTC-Spring 2009*, Barcelona, Apr. 2009.

[3] ——, "Spatial Interference Cancellation and Pairwise Error Probability Analysis," in *IEEE International Conference on Communications, ICC 2009*, June 2009.

[4] J. Winters, "Optimum combining in digital mobile radio with cochannel interference," *IEEE Journal on Selected Areas in Communications*, vol. 2, no. 4, pp. 528–539, Jul 1984.

[5] C. Oestges, "Validity of the kronecker model for mimo correlated channels," in *VTC Spring*, 2006, pp. 2818–2822.

[6] S. Loyka, "Channel capacity of MIMO architecture using the exponential correlation matrix," *IEEE Communications Letters,*, vol. 5, no. 9, pp. 369–371, Sep 2001.

[7] A. Hedayat, H. Shah, and A. Nosratinia, "Analysis of space-time coding in correlated fading channels," *IEEE Transactions on Wireless Communications*, vol. 4, no. 6, pp. 2882–2891, 2005.

[8] H. Bolcskei and A. J. Paulraj, "Performance of spacetime codes in the presence of spatial fading correlation," in *Asilomar Conference on Signals, Systems and Computers, 2000*, pp. 687–693.