Optimal MU-MIMO precoder with MISO decomposition approach

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Abstract—This paper proposes a new iterative implementation algorithm for sum-rate maximization in a Multiuser MIMO system (MU-MIMO). The proposed algorithm is based on joint precoder and decoder optimization. For that we considered the best existing precoder design algorithm for a MISO multiuser system proposed in [1]. This algorithm is based on the Lagrangian sum-rate maximization procedure. For the receiving part, an optimal receiver is designed based on the system throughput maximization derived in the general case from the sum-rate expression given for a MU-MIMO broadcast channel. To link these two optimal algorithms, we use an iterative procedure transforming the MU-MIMO channel for each iteration into a MU-MISO channel trough virtual channel calculations. Finally to validate our prosed solution we compare it with an existing MMSE based iterative optimization algorithm. This algorithm proposed in [2] is based on an MMSE as well at the transmission side than at the receiving side. The obtained results demonstrate significant gains without introducing neither supplementary complexity nor resource needs.

Index Terms—Multi-user; MIMO; Broadcast channel; Capacity; SVH; Iterative; MMSE.

I. INTRODUCTION

Multiuser MIMO (MU-MIMO) downlink system known in the information theory as the broadcast channel system represents today one of the most important research fields in wireless communications because of the potential for improving both reliability and capacity of the system. Some theoretical analysis of the capacity demonstrated that the capacity of a broadcast MU-MIMO channel can be achieved by applying a Dirty-Paper Coding (DPC) [3], [4] algorithm as a pre-coder. Nevertheless, a DPC precoding is difficult to compute and is high resource consuming. Some suboptimal linear algorithms with low implementation complexity exist and can be divided into two families: the iterative [2], [5] and the closed form solutions [6]–[8].

In the case of a MU-MIMO system, the precoder completely defines the system performance when only one receive antenna is used at each receiver side. The performance of a MU-MIMO system is measured by the total Sum-Rate and will be given in Section III. On the contrary, when multiple antennas are used at the receiver, the system performance depends also on the receiver structure. The optimum precoder depends on the structure of the receiver and vice versa the optimum receiver depends on the structure of the precoder at the transmission. That is why extracting the full performance of a MU-MIMO system requires the use of some iterative algorithms.

In this paper we are going to focus on the iterative linear solutions to be able to fully exploit the degrees of freedom at the transmission and the reception. In fact using a non iterative linear solution that is a one formula based algorithm provides a fast solution, but makes it difficult to cancel out all the interference created by the other users especially when the number of total transmitted streams is getting closer to the number of transmitting antennas.

Different iterative solutions exist and use different precoder and receiver structures in an iterative way to reduce the interuser interference and enhance the system performances. In this paper, an SVH (Stojnic, Vikalo, Hassibi) precoder combined with an MSR (Maximum Sum-Rate) receiver is proposed and is compared to an MMSE MMSE iterative algorithm given in [2]. The choice of the SVH as an alternative precoding technique for iterative solution is based on the fact that the SVH algorithm is believed to be the optimal mathimatical solution for the MU-MISO (with one antenna at receiver side) problem.

In next section, a model for the considered system is presented, followed by a detailed description of the proposed iterative algorithm and the employed receiver structure. In section IV, the simulation conditions and the obtained results are detailed and discussed. Finally some conclusions are given in the last section.

II. SYSTEM MODEL

Lets consider in our study a multi-user MIMO communication system with N_T transmission antennas at the base station and K different users with N_{R_k} receiving antennas for each user k. Such a system is represented on figure 1.

We assume that the base station has a perfect knowledge of the channel state information (CSI) of all K users. Let S_k a $Q_k \times 1$ vector representing the transmitted data symbols for user k where Q_k is the number of transmission streams for the same user. In our paper we are interested in the case of one stream per user $Q_k=1$.

The total transmit power at the base station is supposed to be constant and equal to P_T . The noise variance N_0 is equal to 1. For the channel part, H_k denotes the MIMO channel for user k which is a $N_{R_k} \times N_T$ matrix.



Fig. 1. System model.

III. SVH/MSR ITERATIVE ALGORITHM

This Section gives a description of our iterative algorithm and is organized as follows. We first start by presenting the general optimum receiving structure considering any precoder at the transmitter. After that we present the precoder which is an SVH-based precoder. And finally we present the iterative algorithm combining these two optimal algorithms for joint optimization.

A. Receiver deseign

Given a precoder T_k and, as long as the inter-user interference exists at the receiver side, we need to optimize the receiver structure to maximize the sum-rate for the considered MU-MIMO system. To build our MSR receiver, we focused on the sum-rate expression that we try to maximize. We consider the SR (Sum-Rate) expression given by (1) according to [9]– [11] and we look at it as a function of the receiver D_k . Having only one stream per user, the receiver is a simple vector of size $1 \times N_{R_k}$. R_{S_k} is the covariance matrix of the transmitted data S_k . In this case, R_{S_k} is a scalar as only one stream is transmitted to each user.

$$SR = \sum_{k=1}^{K} log_2 \left(1 + \frac{D_k H_k T_k R_{S_k} T_k^H H_k^H D_k^H}{D_k \left(\Upsilon_k + N_0 I\right) D_k^H} \right)$$
(1)

where $\Upsilon_k = H_k \sum_{j=1, j \neq k}^{K} T_j R_{S_j} T_j^H H_k^H$ represents the interference generated by the other users and collected by user k.

Maximizing the sum-rate of (1) with respect to the receiver filter D_k becomes equivalent to optimizing r_k :

$$r_{k} = \log_{2} \left(1 + \frac{D_{k}H_{k}T_{k}R_{S_{k}}T_{k}^{H}H_{k}^{H}D_{k}^{H}}{D_{k}\left(\Upsilon_{k} + N_{0}I\right)D_{k}^{H}} \right)$$
(2)

Thus maximizing the throughput for one stream can be done by finding the best solution of D_k . In fact, by applying the result of [12], the optimal solution for our maximization problem is given by the generalized eigenvector corresponding to the largest eigenvalue of the matrix pair

$$\left(H_{k}T_{k}R_{S_{k}}T_{k}^{H}H_{k}^{H}, H_{k}\sum_{j=1, j\neq k}^{K}T_{j}R_{S_{j}}T_{j}^{H}H_{k}^{H}+N_{0}I\right),$$

which is also the eigenvector corresponding to the largest eigenvalue of ψ , which is defined as:

$$\psi = \left(\sum_{j=1, j \neq k}^{K} H_k T_j R_{S_j} T_j^H H_k^H + N_0 I \right)^{-1} H_k T_k R_{S_k} T_k^H H_k^H$$
(3)

So finally our optimal receiver maximizing the system total sum-rate is given by equation (4).

$$D_{MSR,k}^{H} = \zeta_m\left(\psi\right),\tag{4}$$

where $\zeta_m(X)$ represents the largest eigenvector of X. The largest eigenvector is defined as the eigenvector corresponding to the largest eigenvalue of X. Given the structure of ψ , $\zeta_m(\psi)$ is, in the case of a single stream per user as considered in this paper, of the form $y^* = \frac{y}{\|y\|}$ where

$$y = T_k^H H_k^H (\sum_{j=1}^K H_k T_j R_{S_j} T_j^H H_k^H + N_0 I)^{-1}$$

is the MMSE receiver and y^* becomes the normalized MMSE receiver.

B. SVH algorithm

In this section the so called SVH algorithm presented in [1] under the method 2.1 is detailed. This algorithm is giving the optimal solution derived for the quasiconvex-demonstrated optimization problem. In fact, the paper solves it exactly using the bisection method and gives the mathematical expression for the precoder. After that it proposes a practical iterative algorithm presented under the method 2.1 generating the optimal precoder for a MU-MISO system. The proposed algorithm can be expressed by using the equation system given by (5) for a MU-MISO system.

$$F^{iter_{SVH}} = diag\left(f_1, \cdots, f_K\right) \tag{5a}$$

$$G^{iter_{SVH}} = diag\left(g_1, \cdots, g_K\right) \tag{5b}$$

$$\left(\begin{array}{c}T^{iter_{SVH}} = \frac{\alpha H^{T} G^{iter_{SVH}}}{\left(\frac{N_{0}}{P_{T}} tr\left(F^{iter_{SVH}}\right)I + H^{H} F^{iter_{SVH}}H\right)} (5c)\right)$$

 α is a scalar factor to respect the total power constraint:

$$P_T = tr\left(T^{iter_{SVH}}\left(T^{iter_{SVH}}\right)^H\right) \tag{1}$$

The diagonal elements f_k and g_k are given by:

$$\int f_k = \frac{num_k}{den_k (den_k + num_k)}$$
(2a)

$$g_k = \frac{\left(HT^{iter_{SVH}-1}\right)_{kk}}{den_k}$$
(2b)

where

$$num_k = \left| \left(HT^{iter_{SVH}-1} \right)_{kk} \right|^2 \tag{1}$$

and

$$den_{k} = \sigma^{2} tr \left(T^{iter_{SVH}-1} \left(T^{iter_{SVH}-1} \right)^{H} \right) + \sum_{n=1,n\neq k}^{K} \left| \left(HT^{iter_{SVH}-1} \right)_{kn} \right|^{2}$$
(2)

Here $T^{iter_{SVH}} = \left[\left(T_1^{iter_{SVH}} \right), \cdots, \left(T_K^{iter_{SVH}} \right) \right]$ and $H = \left[H_1^T, \cdots, H_K^T \right]^T$.

The iterative algorithm consists in initializing the $F^{iter_{SVH}}$ and $G^{iter_{SVH}}$ matrices with I and to calculate the corresponding precoder T. The algorithm then iterates by computing the new F and G corresponding to the last precoder. The new precoder is then calculated in function of these obtained Fand G. The system converges when it is stabilized meaning that the obtained value for the precoder no longer changes $|SR^{iter_{SVH}} - SR^{iter_{SVH}-1}| < \varepsilon_{SVH}$. The end of the algorithm can also be controlled by fixing the number of iterations. SR is calculated according to (1) with $D_k = 1$.

C. Iterative algorithm

In this subsection we are going to propose the iterative solution. The algorithm is based on the association of the two previously described algorithms namely, the SVH precoder (described in subsection B) and the MSR decoder (given in subsection A). These two algorithm are combined in an iterative way by means of a virtual channel given in (3).

$$H_k^{iter} = D_{MSR,k}^{iter-1} H_k \tag{3}$$

The obtained channel through this transformation becomes a $1 \times N_T$ MISO channel. Under this condition, the SVH algorithm described later as **Algorithm 1** can be applied providing the optimal solution for each of the MU-MISO subproblems. The precoder is then calculated in an iterative way and used to compute the optimal MU-MIMO decoder for each user. Finally what we get is an general iterative procedure evolving through the virtual channel updates and an internal iterative procedure (the SVH 2.1 algorithm) aiming at calculating the optimal MISO precoder for each user. The iterative SVH procedure is defined as follows.

Algorithm 1

Step 1/ Initialize $F^{iter,0}$ and $G^{iter,0}$ matrices to I and calculate the first precoder $T^{iter,iter_{SVH}}$ using equation (5c) of the system (5).

Step 2/ Calculate the two vectors *num* and *den* given respectively by equation (1) and (2).

Step 3/ Calculate the new precoding matrix $T^{iter,iter_{SVH}}$ with (5c) after evaluating matrices $F^{iter,iter_{SVH}}$ and $G^{iter,iter_{SVH}}$ using respectively equations (5a) and (5b).

Step 4/ Repeat steps 2/ and 3/ until a maximal number of iterations $iter_{SVH} = iter_{SVH max}$ is reached or the system is stabilized. The stability of the system is reached when $|SR^{iter,iter_{SVH}} - SR^{iter,iter_{SVH}-1}| < \varepsilon_{SVH}$.

The global iterative algorithm is then defined as a succession of 6 steps for each user. All users are treated at an iteration considering the precoders and the channels of all other ones. The precoders have a double indexation. The first index is for the external iterative process while the second one is for the internal algorithm method 2.1 from SVH.

Algorithm 2

Step 1/ First we perform a first iteration using a linear

closed form precoder that we note $T_k^{0,0}$ like an SJNR precoder as defined in [8], [13] using equation (4) and we calculate the optimal receiver $D_{MSR}^{0,0}$ using the expression given in (4).

$$T_k^{0,0} = \sqrt{P_k} \zeta_m \left[\left(\sum_{j=1, j \neq k}^K H_j^H H_j + \frac{N_0}{P_k} I \right)^{-1} H_k^H H_k \right]$$
(4)

Step 2/ We change the transmission channel H_k with a virtual one equivalent to the cascade of the real transmission channel and the calculated receiver. The new channel is given by (3).

Step 3/ Once we get our MISO equivalent channel we enter the SVH optimization procedure and the new precoder $T_k^{iter,iter_{SVH}}$ is calculated using the new channel H_k^{iter} according to the iterative process described in **Algorithm 1**.

Step 4/ We compute the new optimal receiver $D_{MSR,k}^{iter}$ using equation (4) with the new precoder $T_k^{iter,iter_{SVH}}$.

Step 5/ We evaluate the total sum-rate for the obtained system $T_k^{iter,iter_{SVH}}$ and $D_{MSR,k}^{iter}$ using equation (1).

Step 6/ Repeat Step 2/ to Step 5/ until the algorithm converges. The convergence is determined either by the stabilization of the total sum-rate obtained when $|SR^{iter} - SR^{iter+1}| < \varepsilon$ or when the predefined maximum number of iterations of the external loop equal to $iter_{max}$ is attained. Here ε is a prefixed threshold defining the convergence.

IV. SIMULATIONS AND RESULTS

In all our simulations, we consider that we have only one stream per user $Q_k = 1$ and the number of receiving antennas is the same for all users $N_{R_k} = N_R = 4$ or 2. We choose a Rayleigh fading channel $H_k = (h_{i,j}^k)_{1 \le i \le N_R, 1 \le j \le N_T}$ such as $E ||h_{i,j}^k||^2 = 1$. The simulation generates 10000 independent channel realizations for each user. To generate the total throughput of the system, we perform an average over all channel realizations on the quantity SR given in equation (1). For the SJNR precoder, we distribute the energy equally over all considered users according to $P_k = P_T \nearrow K$. The two convergence control parameters for both algorithms ε_{SVH} , ε are fixed and equal to 0.001. In all the following, N_{iter} represents the number of iterations in the external loop defined as $iter_{max}$ in **Algorithm 2**.

Figure 2 presents for a fixed number of total iterations $iter_{max}=N_{iter}=50$ the proposed algorithm with different values of the maximal number of authorized iterations in method 2.1. We consider $iter_{SVH_{max}} = N_{SVH} = 1, 2, 5$. A fourth curve is added describing the algorithm when the number of iterations is left free. Meaning that the algorithm is run until the convergence of method 2.1 is achieved. This figure shows that the convergence and the performance evaluation of the system requires no more than two iterations. In fact the 3 first curves shown on the figure "TXSVH; RXMSR; $N_{SVH} = 5$, $N_{iter} = 50$ " and "TXSVH; RXMSR; $N_{SVH} = 50$ " are almost the same. The performance improvement that can be get with no iteration limits compared to the algorithm with



Fig. 2. Throughput in function of $iter_{SVH_{max}}$ for $N_T = N_R = K = 4$.

 $iter_{SVH_{max}}=5$ respectively $iter_{SVH_{max}}=2$ is roughly around 10^{-4} respectively 10^{-3} bits/Hz/s. This demonstrates that not more than two iterations are required for the convergence of the internal MISO optimization method. On the other hand, looking at the curve named "TXSVH; RXMSR; $N_{SVH} =$ 1; $N_{iter} = 50^{\circ}$ with a number of internal iterations equal to 1, demonstrates a very low sum-rate for the system. In fact, if we analyze the algorithm for $iter_{SVH_{max}} = 1$, we can see that it represents indeed the MMSE/MSR algorithm equivalent to the MMSE/MMSE one described in [2] with a normalized receiver. This shows the importance of performing the optimization of the sum-rate for the virtual MU-MISO system. These observations demonstrate that the proposed algorithm offers much better performances just by adding one loop for the transmitter optimization procedure (method 2.1). Compared to the existing iterative solutions, this increase in performances is achieved by introducing low extra complexity and very little computational delay.

Figure 3 looks at the influence of the total number of iterations for the proposed algorithm. To do that, a fixed number of iterations $iter_{SVH_{max}}=2$ is considered (we showed through the analysis of the previous figure that no further iterations are required) and now the external maximal number of iterations is changed. The obtained curves show that the total throughput of the system is slightly increasing at low SNRs and that it is getting higher as the SNR is increasing. This shows the importance of introducing the receiver structure in the optimization procedure proving the fact that joint optimization is required for MU-MIMO systems to be able to get the best out of it. It can also be concluded that although the optimization procedure considers, at each iteration, a MU-MISO channel optimization, the proposed solution evolves toward better performances exploiting the diversity offered by the MIMO channel. Moreover, the additional gain obtained



Fig. 3. Throughput in function of $iter_{max}$ for $N_T=4$, $N_R=4$, K=4.

is decreasing with the increasing number of iterations. This demonstrates the convergence property of the optimization process performed by the proposed algorithm.

Once we have proven that the algorithms works fine, we must look at its' performances compared to the existing iterative algorithms. In fact, figure 4 represents a comparison of our iterative algorithm with a MMSE/MMSE iterative algorithm proposed in [2], [14] referred to as the *original* one and with a modified version of it that is called normalized MMSE/MMSE. This algorithm uses a MMSE precoder and a MMSE receiver at each iteration. The original paper proposes an initialization with $D_k^0 = I_{Q_k \times N_{R_k}} = I_{1 \times N_R}$, where $I_{1 \times N_R}$ has only a one in the first position and zeros elsewhere. The modified version called normalized, is introducing a normalization factor applied at the receiver and has been introduced to compare it with the proposed MSR receiver that we showed in section III.A to be equivalent to a normalized MMSE. For these simulations, $N_{iter_{max}}$ is considered to be constant and equals 25 and 50. The results are plotted for two cases. The first one is for $N_T=2, N_R=2, K=2$ and the second for $N_T=4, N_R=4, K=4$. Analysing the obtained curves, it can be seen that for the same number of iteration and an equivalent level of complexity, the new algorithm outperforms clearly the MMSE/MMSE one even by using the optimal receiver. Further more, by comparing curves "TXSVH, RXMSR, N_{SVH}=2, N_{iter}=25" and "TXSVH, RXMSR, N_{SVH}=2, N_{iter}=50" the obtained performances are very close as only a slight decrease of the sum-rate is noted despite a division by 2 of the number of external iterations $N_{iter_{max}}$ performed by Algorithm 2. In addition to that, these two curves remain always better than "TXMMSE, RXMMSEOriginal, $N_{iter} = 50$ " and "TXMMSE, RXNormalizedMMSE, $N_{iter} = 50$ " representing respectively the existing MMSE/MMSE iterative algorithm and the MMS/MMSE one using a normalized receiver



Fig. 4. Throughput for $N_T = N_R = K = 2$ and $N_T = N_R = K = 4$.

both of them running over $N_{iter_{max}} = 50$ iterations. This result shows, the stability, and the strength of our proposed algorithm and demonstrate a much faster convergence speed. We also present the cooperative (i.e. single user MIMO on the overall channel $H^T = [H_1^T \cdots H_K^T]$) curves as a benchmark of the system. The cooperative curves are the highest upper bound of the considered system as it considers perfect cooperation between all users. Comparing the SVH/MSR iterative algorithm with the cooperative curves demonstrate that the SVH/MSR curves remain parallel at low transmit powers but saturates at high SNR when the system dimensions grows. This shows that our proposed algorithm is able to exploits better the diversity offered by the system especially at low SNRs. The obtained sum-rates are nevertheless lower than those obtained with the cooperative system and can be easily explainable by the fact that receiver of user k does not know any information about other users. This prevents the algorithm from completely eliminating the inter-user interference in an efficient way.

V. CONCLUSION

In this paper, a novel iterative joint optimization procedure for sum-rate maximization is proposed. We introduce a new iterative procedure which combines iteratively a MISO optimized precoder defined and derived by SVH in [1] with the MSR receiver that was calculated from the sum-rate expression maximization. We showed throughout the realized simulations that the presented algorithm was converging toward a better system throughput. Comparisons done with an existing MMSE/MMSE iterative solution given in [2], [14] and with an ameliorated version of it, showed better performances, faster convergence with lower complexity for our algorithm.

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