Receiver and Transmitter Iterative Optimization Using Maximum Sum-Rate Criterion for Multi-User MIMO Systems

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Abstract—In this paper we present a new practical method for sum-rate maximization for a Multi-User MIMO system. Through this work, we first establish a general formula for the optimal receiver derived from the sum-rate expression called MSR (Maximum Sum Rate) receiver. The obtained MSR receiver is a linear function of the used linear transmitter. In a second step, we introduce this receiver in an iterative algorithm to derive the corresponding precoder. The proposed iterative algorithm is based on optimizing the MSR receiver for the receiving part and an SJNR (Signal to Jamming plus Interference Ratio) precoder at the transmitter. Simulation results are compared to two multi-user schemes among the existing ones. The first comparison is done with the closed form SJNR algorithm given in [1] and the second one is done with an iterative scheme based on an MMSE receiver and an MMSE precoder given in [2,12]. These comparisons highlight the gains obtained from both the MSR receiver structure and our iterative procedure.

I. INTRODUCTION

Multiuser MIMO (MU-MIMO) downlink system known in the information theory as the broadcast channel system represents today one of the most important research fields in wireless communications because of the potential for improving both reliability and capacity of the system. Some theoretical analysis of the capacity demonstrated that the capacity of a broadcast MU-MIMO channel can be achieved by applying a Dirty-Paper Coding (DPC) [3, 4] algorithm as a pre-coder. Nevertheless, a DPC precoding is difficult to compute and is high resource consuming. Some suboptimal linear algorithms with low implementation complexity exist and can be divided into two families: the iterative [2, 5] and the closed form solutions [1, 6, 7].

In the case of a MU-MIMO system, the precoder completely defines the system performance when only one receive antenna is used at each receiver side. The performance of a MU-MIMO system is measured by the total Sum-Rate and will be given in Section III. On the contrary, when multiple antennas are used at the receiver, the system performance depends also on the receiver structure. The optimum precoder depends on the structure of the receiver and vice versa the optimum receiver depends on the structure of the precoder at the transmission. That is why extracting

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the full performance of a MU-MIMO system requires the use of some iterative algorithms.

In this paper we are going to focus on the iterative linear solutions to be able to fully exploit the degrees of freedom at the transmission and the reception. In fact using a non iterative linear solution that is a one formula based algorithm provides a fast solution, but makes it difficult to cancel out all the interference created by the other users especially when the number of total transmitted streams is getting closer to the number of transmitting antennas.

Different iterative solutions exist and use different precoder and receiver structures in an iterative way to reduce the inter-user interference and enhance the system performances. In this paper, an SJNR combined with an MSR receiver is proposed and is compared to an MMSE MMSE iterative algorithm given in [2] and to a modified version of this same algorithm. The choice of the SJNR as an alternative precoding technique for iterative solution is based on one of our previous studies evaluating the performances of this algorithm. Indeed, compared to the Per User MMSE linear precoder [6], the SJNR has demonstrated higher performances.

In next section, a model for the considered system is presented, followed by a detailed description of our proposed receiver structure and iterative algorithm. In section IV, the simulation conditions and the obtained results are detailed and discussed.

II. SYSTEM MODEL

Lets consider in our study a multi-user MIMO communication system with N_T transmission antennas at the base station and K different users with N_{R_k} receiving antennas for each user k. Such a system is represented on figure 1. We assume that the base station has a perfect knowledge of the channel state information (CSI) of all K users. Let S_k a $Q_k \times 1$ vector representing the transmitted data symbols for user k where Q_k is the number of transmission streams for the same user. In our paper we are interested in the case of one stream per user $Q_k = 1$.

The total transmit power at the base station is supposed to be constant and equal to P_T . The noise variance N_0 is equal to 1. For the channel part, H_k denotes the MIMO channel for user k which is a $N_{R_k} \times N_T$ matrix.

III. SJNR/MSR ITERATIVE ALGORITHM

This Section gives a description of our iterative algorithm and is organized as follows. We first start by defining the optimum receiver structure given any precoder at the

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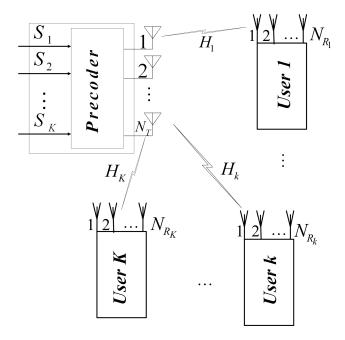


Fig. 1. System model.

transmitter. After that we present the precoder which is a maximum SJNR based precoder. And finally we present the iterative algorithm for joint optimization.

A. Receiver deseign

Given a precoder T_k and as long as the inter-user interference exists at the receiver side, we need to optimize the receiver structure to maximize the sum-rate of the MU-MIMO system. To build our MSR receiver, we focused on the sum-rate expression that we try to maximize. We consider the SR (Sum-Rate) expression given in (1) [8, 9, 10] and we look at it as a function of the receiver D_k . Having only one stream per user, the receiver is a simple vector of size $1 \times N_{R_k}$. R_{S_k} is the covariance matrix (in this case a scalar) of the transmitted data S_k .

$$SR = \sum_{k=1}^{K} log_2 \left(1 + \frac{D_k H_k T_k R_{S_k} T_k^H H_k^H D_k^H}{D_k \left(\Upsilon_k + N_0 I \right) D_k^H} \right)$$
(1)

where $\Upsilon_k = H_k \sum\limits_{j=1,j \neq k}^K T_j R_{S_j} T_j^H H_k^H$ represents the interference generated by the other users and collected by user k.

Maximizing the sum-rate in (1) with respect to the receiver filter D_k becomes equivalent to optimizing r_k :

$$r_{k} = log_{2} \left(1 + \frac{D_{k}H_{k}T_{k}R_{S_{k}}T_{k}^{H}H_{k}^{H}D_{k}^{H}}{D_{k}\left(\Upsilon_{k} + N_{0}I\right)D_{k}^{H}} \right)$$
 (2)

Thus maximizing the throughput for one stream can be done by finding the best solution of D_k . In fact, by applying the result of [11], the optimal solution for our maximization problem is given by the generalized eigenvector corresponding to the largest eigenvalue of the matrix pair

$$\left(H_k T_k R_{S_k} T_k^H H_k^H , H_k \sum_{j=1, j \neq k}^K T_j R_{S_j} T_j^H H_k^H + N_0 I\right),$$

which is also the eigenvector corresponding to the largest eigenvalue of ψ , which is defined as:

$$\psi = \left(\sum_{j=1, j \neq k}^{K} H_k T_j R_{S_j} T_j^H H_k^H + N_0 I\right)^{-1} H_k T_k R_{S_k} T_k^H H_k^H \tag{3}$$

So finally our optimal receiver maximizing the system total sum-rate is given by equation (4).

$$D_{MSR\ k}^{H} = \zeta_m\left(\psi\right),\tag{4}$$

where $\zeta_m(X)$ represents the largest eigenvector of X. The largest eigenvector is defined as the eigenvector corresponding to the largest eigenvalue of X. It must be noted that in the case of an hermitian semidefinite positive matrix the eigen decomposition is equivalent to an SVD(Singular Value Decomposition) and that the generated singular values are in an decreasing order. Given the structure of ψ , $\zeta_m(\psi)$ is, in the case of a single stream per user considered in this paper, of the form $y^* = \frac{y}{\|y\|}$ where

$$y = T_k^H H_k^H (\sum_{j=1}^K H_k T_j R_{S_j} T_j^H H_k^H + N_0 I)^{-1}$$

is the MMSE receiver and that y^* is the normalized MMSE receiver.

B. SJNR algorithm

Now we shall consider the design of the transmit filters T_k under the total transmit power constraint $\sum\limits_{k=1}^K P_k = P_T$. Here $P_k = trace\left(T_k R_{S_k} T_k^H\right)$ denotes the transmitted power for user k

We consider the Signal to Jamming plus Noise Ratio (SJNR) defined as the signal power over the total power of interference caused by the user k and received by the other mobiles introduced in [1] given by expression (5):

$$SJNR_{k} = \frac{T_{k}^{H} H_{k}^{H} H_{k} T_{k}}{T_{k}^{H} \sum_{j=1, j \neq k}^{K} H_{j}^{H} H_{j} T_{k} + N_{0} I}$$
 (5)

A solution to maximize the SJNR for the different users has been proposed in [1]. They demonstrate that the generalized eigenvalue of the SJNR expression is the optimal solution. The precoder for user k is therefore given by the expression of equation (6).

$$T_{k} = \sqrt{P_{k}} \zeta_{m} \left[\left(\sum_{j=1, j \neq k}^{K} H_{j}^{H} H_{j} + \frac{N_{0}}{P_{k}} I \right)^{-1} H_{k}^{H} H_{k} \right]$$
(6)

As a receiver [1] proposes a matched filter (MF) given by equation (7).

$$D_k = \frac{\left(H_k T_k\right)^H}{\|H_k T_k\|},\tag{7}$$

where ||X|| is the norm of vector X.

C. Iterative algorithm

In this subsection we are going to describe the iterative solution. We associate the precoder described in subsection B of this section and the decoder of subsection A iteratively by introducing a virtual channel (8).

$$H_k^{iter} = D_{MSR,k}^{iter-1} H_k \tag{8}$$

The obtained precoder used in the iterative algorithm becomes as given in (9).

$$T_{k}^{iter} = \sqrt{P_{k}} \zeta_{m} \left[\sum_{\substack{j=1\\j\neq k}}^{K} (H_{j}^{iter})^{H} H_{j}^{iter} + \frac{N_{0}}{P_{k}} I \right] (H_{k}^{iter})^{H} H_{k}^{iter}$$
(9)

The iterative algorithm is then defined as a succession of 6 steps for each user. All users are treated at an iteration considering the precoders and the channels of all other ones.

Step 1/ First we perform a first iteration using a linear precoder T_k^0 like an SJNR precoder according to equation (6) and we calculate the optimal receiver using our expression given in (4).

Step 2/ We change the transmission channel H_k with a virtual one equivalent to the cascade of the real transmission channel and the calculated receiver. The new channel is given by (8).

Step 3/ The new precoder T_k^{iter} is calculated using the new channel H_k^{iter} according to equation (9).

Step 4/ We compute the new optimal receiver $D_{MSR,k}^{iter}$ using equation (4) with the new precoder T_k^{iter} replacing T_k .

Step 5/ We evaluate the total sum-rate for the obtained system T_k^{iter} and $D_{MSR,k}^{iter}$ using equation (1).

Step 6/ Repeat steps 2/ to 5/ until the algorithm converges. The convergence is determined either by the stabilization of the total sum-rate obtained when $|SR^{iter} - SR^{iter+1}| < \varepsilon$ or when the predefined maximum number of iterations equal to $iter_{max}$ is attained.

IV. SIMULATIONS AND RESULTS

In all our simulations, we consider that we have only one stream per user $Q_k=1$ and the number of receiving antennas is the same for all users $N_{R_k}=N_R=2$ or 4. We choose a Rayleigh fading channel $H_k=(h_{i,j}^k)_{1\leq i\leq N_R, 1\leq j\leq N_T}$ such as $E\|h_{i,j}^k\|^2=1$. The simulation generates 10000 independent channel realizations for each user. To generate the total throughput of the system, we perform an average over all channel realizations on the quantity SR given in equation (1). For the SJNR precoder, we distribute the energy equally over all considered users according to $P_k=P_T/K$, unless otherwise stated. The convergence control parameter ϵ is fixed to 0.001.

Figures 2 compares the SJNR precoder with different receiver implementations. The presented curves are

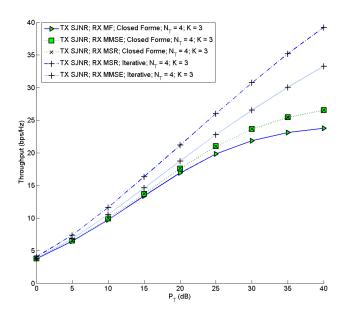


Fig. 2. Throughput in function of transmit power for $N_T=4,\ N_R=2$ and K=3.

generated using $N_T=4$ transmitting antennas and $N_R=2$ receiving antennas. The number of considered users is K=3. We present 3 different closed form receivers applied to the system with an SJNR precoder. The first one is the basic MF (Matched Filter) receiver. The second one is an MMSE (Minimum Mean Square Error) receiver defined by equation (10) according to [2,12,13]. The last closed form receiver is the MSR receiver defined by (4). We also plot curves for two iterative procedures: the first one is our proposed iterative procedure described in Section III with a SJNR precoder and our MSR receiver. The second one is a variant of our iterative algorithm where we just replace the receiver by the MMSE receiver. The maximum number of possible iteration $iter_{max}$ is fixed to 50.

$$D_k = T_k^H H_k^H \left(H_k \left(\sum_{i=1}^K T_i T_i^H \right) H_k^H + N_0 I \right)^{-1}$$
 (10)

The simulations results demonstrate that the closed form receiver is saturating at high transmit power whatever the receiver structure is. It also shows that in the case of non iterative precoding, the MMSE receiver is equivalent to our MSR one and gives better performance than an MF receiver.

However, using an iterative algorithm to calculate the couple precoder/receiver we note significant improvement for both used reception techniques (MMSE and MSR). Comparing curves "TX SJNR RX MMSE; Iterative" and "TX SJNR RX MSR; Iterative" shows that the MSR receiver offers much more gain. For example, at 30 dB it offers 6 extra bits/s/Hz than the MMSE receiver can offer. This shows the importance of normalization of the receiver when the transmitter is designed by the max SJNR criterion given in (9). The iterative procedure also eliminates the saturation of the system as the curve has nearly a linear behaviour at high transmit powers. The saturation observed

especially for all non iterative algorithms is mainly caused by the interference between the different users.

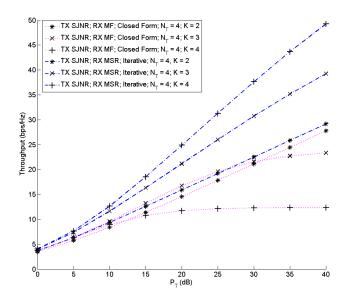


Fig. 3. Throughput in function of transmit power for $N_T=4$, $N_R=2$ and $K=\{2,3,4\}$ for iterative and non iterative algorithms.

Figure 3 represents a comparison between the iterative and the non iterative algorithms for different number of served users. Looking at the non iterative algorithm the "TX SJNR RX MF" ones, we see that the total sum-rate of the system is not proportional to the total number of users. In fact, the first curve studying the case of two users with 4 transmitting antennas at the base station shows a linear curve containing no visible saturation. By adding one extra user to the system, the total sum rate increases for low transmit powers. But the slope of the sum-rate gets weaker and describes a saturation of the system sum-rate at 23.3 bps/Hz. At very high transmit power, the system gets even worse than the two users case. The last represented curve for the closed form case is simulated for 4 served users. We see that the saturated system is offering very bad performances. The saturation is already appearing at very low transmit power; and the maximum attainable value for the sum-rate is around 12.3 bps/Hz much worse than the 3 user case and even the 2 user one.

On the other hand, the iterative solution with an MSR receiver shows much higher performances. But the most important is that even when the system is fully charged, the performance does not present any saturation. Furthermore, the performance and the slopes of our simulated curves are constantly increasing with the number of users; this demonstrates that the iterative algorithm succeeds in reducing the interference part created by the other users and makes it possible to serve the new ones increasing by the way the total sum-rate of the system. We should note that although we used the closed form SJNR/MF (the originally proposed version in [1]) as a basis of comparison, the same saturation phenomenon occurs for the closed forme

SJNR/MMSE. In fact, at 40 dB, the SJNR/MMSE achieves a throughput of 27.5, 26.0, 15.1 bits/s/Hz for K = 2,3 and 4.

Figure 4 is a comparison of our iterative algorithm

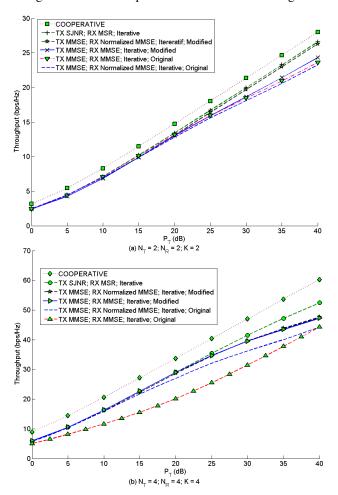


Fig. 4. Comparing throughputs for MMSE/MMSE ans SJNR/MSR iterative algorithms for $N_T=2,\ N_R=2,\ K=2$ and $N_T=4,\ N_R=4,\ K=4$.

with a MMSE/MMSE iterative algorithm proposed in [2, 12]. This algorithm uses a MMSE precoder and a MMSE receiver at each iteration. The original paper proposes an initialization with $D_k^0=I_{Q_k\times N_{R_k}}=I_{1\times N_R}$, where $I_{1\times N_R}$ has only a one in the first position and zeros elsewhere. This algorithm gives poor performances. We have improved it by changing the initialization by using the MMSE receiver (10) calculated with the closed form precoder given by [6]. The curves "TX MMSE; RX MMSE; Iterative; Original" and "TX MMSE; RX MMSE; Iterative; Modified" represent respectively the original MMSE/MMSE iterative algorithm presented in [2] and the modified one. The curves "TX MMSE; RX Normalized MMSE" represent another variant of the MMSE receiver which has been normalized. The normalized receiver version of MMSE gives almost the same performances as the MMSE of [2] for the case $N_T = 2, N_R = 2 and K = 2$ but introduces important gains when the system dimensions increase as observed for the case $N_T = 4$, $N_R = 4$ and K = 4. For these simulations, we consider the case where $N_T = 4$, $N_R = 4$ and K = 4 and

the case of $N_T=2$, $N_R=2$ and K=2. The maximum number of possible iteration $iter_{max}$ is fixed to 50 in the case of 4 users and to 25 in the case of two users. Despite our efforts to improve the MMSE/MMSE iterative algorithm of [2] the obtained results are always worse than SJNR/MSR iterative algorithm described in Section III. These curves confirm the superiority of our iterative solution even in the high transmit power region compared to the modified MMSE/MMSE iterative algorithm. We also observe that the obtained gain is increasing with the number of served users.

We also present the cooperative (i.e. single user MIMO on the overall channel $H^T = \begin{bmatrix} H_1^T \cdots H_K^T \end{bmatrix}$) curves as a benchmark of the system. The cooperative curves are the highest upper bound of the considered system as it considers perfect cooperation between all users. Comparing the SJNR/MSR iterative algorithm with the cooperative curves demonstrate that the two curves remain almost parallel even at high transmit powers. But, our proposed algorithm presents lower sum-rates. The last observation is easily explainable by the fact that receiver of user k does not have the information about the other users and can not therefore eliminate the inter-user interference as well as the cooperative.

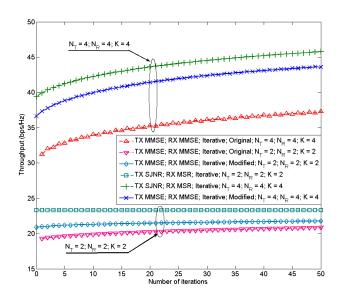


Fig. 5. Throughput in function of iterations for P_T = 35 dB, N_T = 2, N_R = 2, K = 2 and N_T = 4, N_R = 4, K = 4.

Figure 5 presents the evolution of the total sum-rate in function of the number of iterations for a total transmit power of 35 dB. The simulations are done for two cases: $N_T=4,\ N_R=4,\ K=4$ and $N_T=2,\ N_R=2,\ K=2$. These curves confirm that our SJNR/MSR iterative algorithm converges to a higher total sum-rate for the system as, for example in the first case, it offers 3 extra bits/s/Hz compared to the modified MMSE/MMSE algorithm and 10 extra bits/s/Hz compared to the original MMSE/MMSE presented in [2]. On the other hand, the curves can show that our proposed SJNR/MSR iterative algorithm converges much faster than both MMSE/MMSE iterative algorithms.

In fact for the same total sum-rate of the MU-MIMO system, our algorithm needs 27 iterations less than the modified MMSE/MMSE algorithm. For example, if we consider the value of the sum-rate achieved by the modified MMSE/MMSE at the 50th iteration, it gives us at 35 dB an average value around 44 bps/Hz. The same value for the sum-rate is achieved by our algorithm in 23 iterations. This represents almost a gain of 50%. Compared to the original MMSE/MMSE algorithm, the gain of the proposed iterative algorithm is much more important. Indeed, the final value of the MMSE/MMSE achieved after 50 iteration is already exceeded since the first iteration with our proposed SJNR/MSR. We also point out the importance of the initialization in the MMSE/MMSE iterative algorithm. In fact, just by changing the initialization of the algorithm proposed in [2] we increased the sum-rate by 7 bits/s/Hz and reduced the convergence time by nearly 96% as the same sum-rate is achieved by the modified version in 2 iterations and requires 50 for the original one.

Finally, we consider the power distribution optimization to maximise the sum-rate for the case of $N_T = 4$, $N_R = 2$ and K = 4. To do so, an exhaustive search over the power distribution set $\Pi = \{P_1, P_2 \cdots, P_K\}$ is performed by finely discretizing the power value of each user and respecting the total power constraint $\sum_{k=1}^{K} P_k = P_T$. We considered two optimization scenarios. The first one corresponds to executing the power optimization (PO) at the last iteration of our iterative algorithm. The second scenario applies the power optimization at the 5 last iterations. For all the simulations, the unique stop iteration condition is the maximum number of iterations $iter_{max}$, fixed to 50. The results of these simulations are plotted in Figure 6. The curves with equal power distribution and with power optimization are very close. The power optimization has almost no impact on the throughput. Indeed, the main gain is achieved through the iterative procedure using the equal power distribution. The gain of power optimization at the last iteration (or at the last five iterations) is around 0.08 bps/Hz (or 0.189 bps/Hz) at $P_T = 0 dB$ which corresponds to a gain of 1,98% (or 4.031%) compared to the equal power distributed algorithm; and it gets weaker when the transmit power increases, for example at $P_T = 10$ dB the gain is only 0.0128 bps/Hz (or 0.0128 bps/Hz) corresponding to a gain of 0.1% (or 0.1%) compared to the equal power distributed algorithm. This demonstrates that implementing a sophisticated power optimization procedure at each iteration is not necessary and that we can use the equal power distribution for the iterative SJNR/MSR procedure optionally followed by a power optimization at the last iteration to enhance a little the sum-rate in the low transmit power region $P_T < 10 \, dB$. Some fast optimization algorithms can be used instead of the exhaustive search.

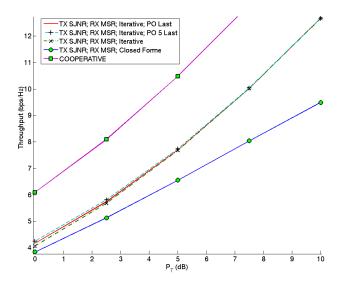


Fig. 6. Impact of power optimization on the throughput for $N_T=4$, $N_R=2$ and K=4.

V. CONCLUSION

In this paper, a receiver design based on the total sum-rate maximization is proposed. We introduce a new iterative procedure which combines iteratively an SJNR precoder with the MSR receiver. We compared our iterative solution to a linear closed form transmitter precoding vectors design method defined in [1] and to an iterative linear MMSE/MMSE algorithm given in [2,12]. The comparison confirms that our solution outperforms the existing algorithms as it achieves a higher total sum-rate of the system and offers a much faster average convergence compared to the existing iterative solution.

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