

# Adaptive Interference Suppression for DS-CDMA in Multipath Channels\*

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## ABSTRACT

The decentralized minimum mean square error-zero forcing (MMSE-ZF) receiver which can be implemented as a pre-combining interference canceler followed by coherent combining lends itself to a particular disjoint adaptation of the interference canceling filter and the channel. A least mean square (LMS) based adaptation of this receiver is presented in this paper. The quadratic cost-function viz. the output variance is quadratic in coefficients of the interference canceling filter leading to global convergence. On the other hand, the channel coefficients are separately optimized based upon the interference canceler. We explore decision directed strategies to improve the performance of the overall receiver. It is shown that significant performance gains can be achieved if decisions are reused in a soft fashion to influence the adaptation procedure.

## 1. INTRODUCTION

Linear receivers [1] for DS-CDMA systems seem to offer an alternative to the RAKE receiver in mobile cellular networks, due to their affordable complexity compared to other schemes. Among linear receiver more in the spirit of the RAKE receiver are the decentralized minimum output energy (MOE) receiver [2] [3], the directly estimated MMSE receiver [4], and the projection receiver [5], where, individualized single-user baseband receivers are obtained for the desired user irrespective of the nature of interfering users or their origin. No assumption is made about the structure of the cyclostationary interference except that it is uncorrelated with the desired user's signal. These receivers are obtained essentially from *blind* criteria exploiting the second order statistics of the received signal and *a priori* information of the desired user's timing and spreading sequence. Batch processing complexity is usually considered prohibitive for most real-time applications.

Of particular interest for stochastic gradient based adaptive implementation are the MOE receiver of [3], where two algorithms for the adaptation of the receiver coefficients and the channel impulse response are introduced. The drawback of both is the interdependence of the two entities to be adapted, which makes it difficult to choose adaptation step sizes,  $\mu$ , for the two. Furthermore as pointed out there, the presence of local minima cannot *a priori* be dismissed. One particular implementation of the MMSE-ZF receiver [5] which is the pre-combining interference canceler followed by coherent combining lends itself to a particular disjoint adaptation of the interference canceling filter and the channel. We

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shall investigate, in this paper, an LMS based adaptation of this receiver. It is shown that the quadratic cost-function viz. the estimation error (co)-variance at the output of the bank of correlators is quadratic in coefficients of the interference canceling filter leading to guaranteed global convergence. The channel coefficients, on the other hand, are separately optimized based upon the knowledge of the interference canceler, assuming that interference has already been done away with. We explore the particularly attractive case of sparse channels and present a decision directed strategy to improve the quality of the IC filter. It is shown that significant performance gains can be achieved if decisions are reused in a soft fashion to influence the adaptation procedure.

## 2. MULTIUSER DATA MODEL

Fig. 1 shows the baseband signal model. The  $K$  users are assumed to transmit linearly modulated signals over a linear multipath channel with additive white Gaussian noise. It is assumed that the receiver employs  $M$  sensors to receive the mixture of signals from all users. The receiver front-end is a low-pass filter with sufficiently large bandwidth. The continuous time signal received at the  $m$ th sensor can be written in baseband notation as

$$y^m(t) = \sum_{k=1}^K \sum_n a_k(n) g_k^m(t - nT) + v^m(t), \quad (1)$$

where  $a_k(n)$  are the transmitted symbols from the user  $k$ ,  $T$  is the common symbol period,  $g_k^m(t)$  is the overall channel impulse response (including the spreading sequence, and the transmit and receive filters) for the  $k$ th user's signal at the  $m$ th sensor, and  $\{v^m(t)\}$  is the complex circularly symmetric AWGN with power spectral density  $N_0$ . Assuming the  $\{a_k(n)\}$  and  $\{v^m(t)\}$  to be jointly wide-sense stationary, the process  $\{y^m(t)\}$  is wide-sense cyclostationary with period  $T$ . The overall channel impulse response  $g_k^m(t)$ , is the convolution of the spreading code  $c_k$  and

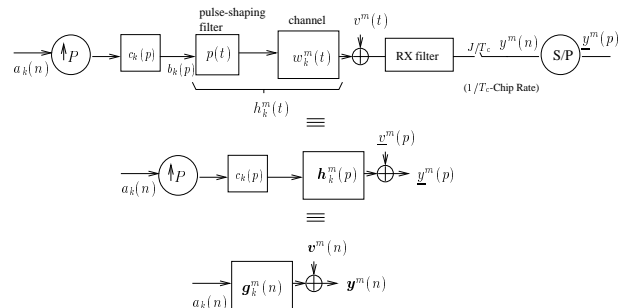


Figure 1. Signal model in continuous and discrete time.

$h_k^m(t)$ , itself the convolution of the chip pulse shape and the actual channel (assumed to comprise of discrete multipath) representing the multipath fading environment. This can be expressed as

$$g_k^m(t) = \sum_{p=0}^{P-1} c_k(p) h_k^m(t - pT_c), \quad (2)$$

where  $T_c$  is the chip duration. The symbol and chip periods are related through the processing gain  $P$ :  $T = PT_c$ . Sampling the received signal  $J$  times the chip rate, we obtain the wide-sense stationary  $PJ \times 1$  vector signal  $\mathbf{y}^m(n)$  at the symbol rate. It is to be noted that the oversampling aspect (with respect to the symbol rate) is inherent to DS-SS systems by their very nature, due to the large (extra) bandwidth and the need to acquire chip-level resolution. This aspect directly translates into space-time diversity and explains the interference cancellation capability of these systems.

We consider the channel delay spread between the  $k$ th user and all of the  $M$  sensors to be of length  $l_k T_c$ . Let  $n_k$  be the chip-delay index for the  $k$ th user:  $\mathbf{h}_k^m(n_k T_c)$  is the first non-zero  $J \times 1$  chip-rate sample of  $h_k^m(t)$ . Let us denote by  $N_k$ , the FIR duration of  $g_k^m(t)$  in symbol periods. It is a function of  $l_k$  and  $n_k$ . We nominate the user 1 as the user of interest and assume that  $n_1 = 0$  (synchronization to user 1). Let  $N = \sum_{k=1}^K N_k$ . The vectorized oversampled signals at  $M$  sensors lead to a discrete-time  $P MJ \times 1$  vector signal at the symbol rate that can be expressed as

$$\begin{aligned} \mathbf{y}(n) &= \sum_{k=1}^K \sum_{i=0}^{N_k-1} \mathbf{g}_k(i) a_k(n-i) + \mathbf{v}(n) \\ &= \sum_{k=1}^K \mathbf{G}_{k,N_k} A_{k,N_k}(n) + \mathbf{v}(n) = \mathbf{G}_N \mathbf{A}_N(n) + \mathbf{v}(n), \quad (3) \end{aligned}$$

$$\mathbf{y}(n) = \begin{bmatrix} \mathbf{y}_1(n) \\ \vdots \\ \mathbf{y}_P(n) \end{bmatrix}, \mathbf{y}_p(n) = \begin{bmatrix} \mathbf{y}_p^1(n) \\ \vdots \\ \mathbf{y}_p^M(n) \end{bmatrix}, \mathbf{y}_p^m(n) = \begin{bmatrix} y_{p,1}^m(n) \\ \vdots \\ y_{p,J}^m(n) \end{bmatrix}$$

$$\begin{aligned} \mathbf{G}_{k,N_k} &= [\mathbf{g}_k(N_k-1) \dots \mathbf{g}_k(0)], \\ \mathbf{G}_N &= [\mathbf{G}_{1,N_1} \dots \mathbf{G}_{K,N_K}] \\ A_{k,N_k}(n) &= [a_k(n-N_k+1) \dots a_k(n)]^T, \\ \mathbf{A}_N(n) &= [A_{1,N_1}^T(n) \dots A_{K,N_K}^T(n)]^T, \quad (4) \end{aligned}$$

and the superscript  $T$  denotes Hermitian transpose. For the user of interest (user 1),  $\mathbf{g}_1(i) = (\mathbf{C}_1(i) \otimes I_{MJ}) \mathbf{h}_1$ , where,  $\mathbf{h}_1$  is the  $l_1 MJ \times 1$  propagation channel vector given by

$$\mathbf{h}_1 = \begin{bmatrix} h_{1,1} \\ \vdots \\ h_{1,l_1} \end{bmatrix}, \mathbf{h}_{1,l} = \begin{bmatrix} h_{1,l}^1 \\ \vdots \\ h_{1,l}^M \end{bmatrix}, \mathbf{h}_{1,l}^m = \begin{bmatrix} h_{1,l}^m(1) \\ \vdots \\ h_{1,l}^m(J) \end{bmatrix},$$

$\otimes$  denotes the Kronecker product, and the matrices  $\mathbf{C}_1(i)$  are shown in fig. 2, where the band consists of the spreading code  $(c_0^H \dots c_{P-1}^H)^H$  shifted successively to the right and down by one position. For the interfering users, we have a similar setup except that owing to asynchrony, the band in fig. 2 is shifted down  $n_k$  chip periods and is no longer coincident with the top left edge of the box. We denote by  $\mathbf{C}_1$ , the concatenation of the code matrices given above for user 1:  $\mathbf{C}_1 = [\mathbf{C}_1^H(0) \dots \mathbf{C}_1^H(N_1-1)]^H$ . Let us now examine the structure of the inter-symbol interference (ISI)

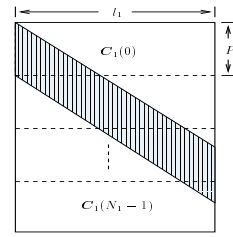


Figure 2. The Code Matrix  $\mathbf{C}_1$ .

induced in the model due to the multipath propagation. To this end, we stack  $L$  successive  $\mathbf{y}(n)$  vectors in a super vector

$$\mathbf{Y}_L(n) = \mathcal{T}_L(\mathbf{G}_N) \mathbf{A}_{N+K(L-1)}(n) + \mathbf{V}_M(n), \quad (5)$$

where,  $\mathcal{T}_L(\mathbf{G}_N) = [\mathcal{T}_L(\mathbf{G}_{1,N_1}) \dots \mathcal{T}_L(\mathbf{G}_{K,N_K})]$  and  $\mathcal{T}_L(\mathbf{x})$  is a banded block Toeplitz matrix with  $L$  block rows and  $[\mathbf{x} \mathbf{0}_{p \times (L-1)}]$  as first block row ( $p$  is the number of rows in  $\mathbf{x}$ ), and  $\mathbf{A}_{N+K(L-1)}(n)$  is the concatenation of user data vectors ordered as  $[A_{1,N_1+L-1}^T(n), A_{2,N_2+L-1}^T(n), \dots, A_{K,N_K+L-1}^T(n)]^T$ . We refer to  $\mathcal{T}_L(\mathbf{G}_{k,N_k})$  as the channel convolution matrix for the  $k$ th user.

Consider the noiseless received signal shown in fig. 3 for user 1. Due to the limited delay spread, the effect of a particular symbol,  $a_1(n-d)$ , propagates to the next  $N_1-1$  symbol periods, rendering the channel a moving average process of order  $N_1-1$ . For the other users, the matrices  $\mathcal{T}_L(\mathbf{G}_{k,N_k})$ , where  $k \neq 1$ , have a similar structure and can be viewed as being superimposed over the channel matrix  $\mathcal{T}_L(\mathbf{G}_{1,N_1})$  in fig. 3. Same applies for the data vectors  $A_{k,N_k+L-1}(n)$ ,  $\forall k \neq 1$ . The overall effect of the ISI and the MAI is therefore that of engendering the shaded triangles in the figure, lying within the region of interest at the  $n$ th instant, which need to be removed from  $\mathbf{Y}_{N_1}$ .

## 2.1. Sparseness of the Channel

The propagation channel can be sparse, which corresponds to a set of delayed echos (paths). In this case it is advantageous to exploit the non FIR nature of the channel (zero taps). This reduces the number of parameters to be estimated, equaling only the number of non-zero taps. If the channel is sparse, we shall consider that the code matrix  $\mathbf{C}_1$  includes the pulse shaping filter and is no longer regularly banded.

## 3. THE MMSE-ZF/PROJECTION RECEIVER

In the multiuser problem given in (5), there exists a multitude of possible zero-forcing constraints, ranging from zero MAI only, or zero ISI only, to zero forcing for both MAI and ISI, which we

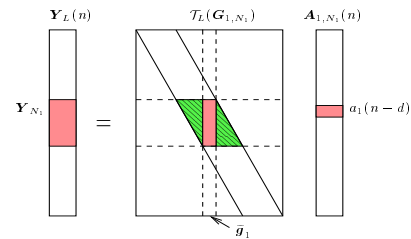


Figure 3. The ISI for desired user.

shall consider here. For the purpose of our problem, let us consider the ZF or the zero-distortion constraint, which can be written as,

$$\mathbf{F}^H \mathcal{T}(\mathbf{G}_N) = \mathbf{e}_d^T, \quad (6)$$

where,  $\mathbf{e}_d^T = [0 \cdots 0 | \overbrace{0 \cdots 0}^d 1 0 \cdots 0 | 0 \cdots 0]$ , with  $d$  the "equalization" delay for the desired user.

Considering all user symbols  $a_k(n)$  to be uncorrelated, the received signal covariance matrix can be written as  $\mathbf{R}_{YY} = \sigma_a^2 \mathcal{T} \mathcal{T}^H + \sigma_v^2 \mathbf{I}$ , where  $\mathcal{T}$  replaces  $\mathcal{T}(\mathbf{G}_N)$  to simplify the notation. The MMSE-ZF receiver is by definition the solution to the MMSE criterion under the ZF constraint, which can be written as

$$\begin{aligned} \min_{\mathbf{F}: \mathbf{F}^H \mathcal{T} = \mathbf{e}_d^T} \mathbf{F}^H \mathbf{R}_{YY} \mathbf{F} &= \sigma_a^2 + \min_{\mathbf{F}: \mathbf{F}^H \mathcal{T} = \mathbf{e}_d^T} \mathbf{F}^H \mathbf{R}_{VV} \mathbf{F} \\ &\Rightarrow \min_{\mathbf{F}: \mathbf{F}^H \mathcal{T} = \mathbf{e}_d^T} \mathbf{F}^H \mathbf{F} \end{aligned} \quad (7)$$

Let us further express the receiver vector  $\mathbf{F}$  as

$$\mathbf{F} = \mathcal{T} \mathbf{F}_1 + \mathcal{T}^\perp \mathbf{F}_2, \quad (8)$$

where,  $\mathcal{T}^\perp$  spans the orthogonal complement of  $\mathcal{T}$  and satisfies  $P_{\mathcal{T}^\perp} = P_{\mathcal{T}}^\perp$ , where  $P_X = X(X^H X)^{-1} X^H$  is the projection operator onto the column space of the matrix  $X$ . From the ZF constraint,  $\mathbf{F}^H \mathcal{T} = \mathbf{e}_d^T = \mathbf{F}_1^H \mathcal{T}^H \mathcal{T}$ , and therefore,

$$\mathbf{F}_1 = (\mathcal{T}^H \mathcal{T})^{-1} \mathbf{e}_d. \quad (9)$$

Hence,  $\mathbf{F} = \mathcal{T}(\mathcal{T}^H \mathcal{T})^{-1} \mathbf{e}_d + \mathcal{T}^\perp \mathbf{F}_2$ , where  $\mathbf{F}_2$  is the unconstrained part which becomes zero upon solving the minimization problem in (7). Thus  $\mathbf{F} = \mathcal{T}(\mathcal{T}^H \mathcal{T})^{-1} \mathbf{e}_d$ , and we can write the MMSE-ZF criterion as:

$$\min_{\mathbf{F}: \mathbf{F}^H \mathcal{T} = \mathbf{e}_d^T} \mathbf{F}^H \mathbf{R}_{YY} \mathbf{F} = \sigma_a^2 + \sigma_v^2 \mathbf{e}_d^T (\mathcal{T}^H \mathcal{T})^{-1} \mathbf{e}_d. \quad (10)$$

The ZF solution in the noiseless case gives the distortionless response for the desired user's signal. It was shown in [5], that the MMSE-ZF receiver could be obtained by doing unbiased MOE on noiseless (denoised) statistics.

#### 4. CONNECTIONS BETWEEN LINEAR RECEIVERS

We can classify the unbiased linear MOE<sup>1</sup> receiver in terms of the other optimization criteria as indicated in the following proposition. It was shown in [5] that the minimum mean-squared error (MMSE), and the minimum output energy (MOE) are interchangeable criteria under the unbiased constraint, and are equivalent to the maximization of the output SINR.

$$\arg \min_{\mathbf{F}: \mathbf{F}^H \tilde{\mathbf{g}}_1 = 1} \text{MSE}_{\text{unbiased}} = \arg \min_{\mathbf{F}: \mathbf{F}^H \tilde{\mathbf{g}}_1 = 1} \text{OE} = \arg \max_{\mathbf{F}} \text{SINR}, \quad (11)$$

<sup>1</sup>a derivative of the minimum variance distortionless response (MVDR) method, and a particular instance of the linearly constrained minimum-variance (LCMV) criterion

#### 4.1. Unbiased MOE via the Generalized Sidelobe Canceler

The generalized sidelobe canceler (GSC) [6], is a particular implementation of the LCMV beamformer. Hence, the unbiased MOE criterion, which itself is a particular instance of the LCMV approach can be implemented in the GSC fashion as elucidated in the following. Let us denote by

$$\mathbf{T}_1 = [\mathbf{0} \quad \mathbf{C}_1^H \quad \mathbf{0}] \otimes \mathbf{I}_{MJ}, \text{ \& } \mathbf{T}_2 = \begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_1^\perp & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \end{bmatrix} \otimes \mathbf{I}_{MJ}, \quad (12)$$

the partial signature of the desired user and its orthogonal complement employed, respectively, in the upper and lower branches of the GSC, as shown in fig. 4.  $\mathbf{C}_1^{\perp H}$  is the orthogonal complement of  $\mathbf{C}_1$ , the tall code matrix given in section 2 ( $\mathbf{C}_1^{\perp H} \mathbf{C}_1 = \mathbf{0}$ ). Then,  $\mathbf{C}_1^H \mathbf{Y}_{N_1} = \mathbf{T}_1 \mathbf{Y}_L$  and the matrix  $\mathbf{T}_2$  acts as a blocking transformation for all components of the signal of interest. Note that  $P_{\mathbf{T}_1^H} + P_{\mathbf{T}_2^H} = \mathbf{I}$ , where,  $P_X$  is the projection operator (projection on the column space of  $X$ ). Then the LCMV problem can be written as

$$\begin{aligned} \min_{\mathbf{F}: \mathbf{F}^H \mathbf{T}_1^H = (\mathbf{h}_1^H \mathbf{h}_1)^{-1} \mathbf{h}_1^H} \mathbf{F}^H \mathbf{R}_{YY}^d \mathbf{F} \\ = \min_{\substack{\mathbf{F}: \mathbf{F}^H \mathbf{T}_1^H \mathbf{h}_1 = 1 \\ \mathbf{F}^H \mathbf{T}_1^H \mathbf{h}_1^\perp = 0}} \mathbf{F}^H \mathbf{R}_{YY}^d \mathbf{F}, \end{aligned} \quad (13)$$

where,  $[\mathbf{h}_1 \quad \mathbf{h}_1^\perp]$  is a square non-singular matrix, and  $\mathbf{h}_1^H \mathbf{h}_1^\perp = \mathbf{0}$ . Note that in the LCMV problem (GSC formulation) there is a number of constraints to be satisfied. However, imposing the second set of constraints, namely  $\mathbf{F}^H \mathbf{T}_1^H \mathbf{h}_1^\perp = \mathbf{0}$  has no consequence because the criterion automatically leads to their satisfaction once,  $\text{span}\{\mathbf{R}_{YY}^d\} \cap \text{span}\{\mathbf{T}_1^H\} = \text{span}\{\mathbf{T}_1^H \mathbf{h}_1\}$ , i.e., when the intersection of the signal subspace and the subspace spanned by the columns of  $\mathbf{T}_1^H$  is one dimensional.

The matrix  $\mathbf{T}_1$  is nothing but a bank of correlators matched to the  $l_1$  delayed multipath components of user 1's code sequence. Note that the main branch in fig. 4 by itself gives an unbiased response for the desired symbol,  $a_1(n-d)$ , and corresponds to the (normalized) coherent RAKE receiver. For the rest, we have an estimation problem, which can be solved in the least squares sense, for some matrix  $\mathbf{Q}$ . This interpretation of the GSC corresponds to the pre-combining (or pathwise) interference (ISI and MAI) canceling approach (see [5] and references therein).

The vector of estimation errors is given by

$$\mathbf{Z}(n) = [\mathbf{T}_1 - \mathbf{Q} \mathbf{T}_2] \mathbf{Y}_L(n). \quad (14)$$

Since the goal is to minimize the estimation error variances, or in other words, estimate the interference term in the upper branch as closely as possible from  $\mathbf{T}_2 \mathbf{Y}_L(n)$ , the interference cancellation problem settles down to minimization of the trace of the estimation error covariance matrix  $\mathbf{R}_{ZZ}$  for a matrix filter  $\mathbf{Q}$ , which results in

$$\mathbf{Q} = \left( \mathbf{T}_1 \mathbf{R}^d \mathbf{T}_2^H \right) \left( \mathbf{T}_2 \mathbf{R}^d \mathbf{T}_2^H \right)^{-1}, \quad (15)$$

and where,  $\mathbf{R}^d$  is the noiseless (denoised) data covariance matrix,  $\mathbf{R}_{YY}$ , with the subscript removed for convenience. The output  $\mathbf{Z}(n)$  can directly be processed by a multichannel matched filter to get the symbol estimate,  $\hat{a}_1(n-d)$ , the data for the user 1.

$$\hat{a}_1(n-d) = \frac{1}{\tilde{g}_1^H \tilde{g}_1} \mathbf{F}^H \mathbf{Y}_L(n) = \frac{1}{\tilde{g}_1^H \tilde{g}_1} \mathbf{h}_1^H (\mathbf{T}_1 - \mathbf{Q} \mathbf{T}_2) \mathbf{Y}_L(n) \quad (16)$$

The covariance matrix of the prediction errors is then given by

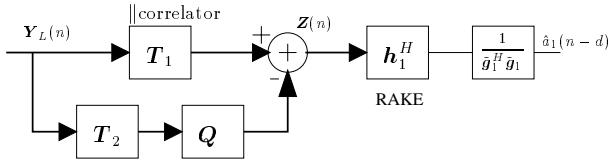


Figure 4. GSC implementation of the MMSE-ZF receiver.

$$\mathbf{R}_{ZZ} = \mathbf{T}_1 \mathbf{R}^d \mathbf{T}_1^H - \mathbf{T}_1 \mathbf{R}^d \mathbf{T}_2^H \left( \mathbf{T}_2 \mathbf{R}^d \mathbf{T}_2^H \right)^{-1} \mathbf{T}_2 \mathbf{R}^d \mathbf{T}_1^H, \quad (17)$$

From the above structure of the interference canceler, we observe that when  $\mathbf{T}_1 (\mathbf{Y}_L - \tilde{\mathbf{g}}_1 a_1(n))$  can be perfectly estimated from  $\mathbf{T}_2 \mathbf{Y}_L$ , the matrix  $\mathbf{R}_{ZZ}$  is rank-1 in the noiseless case! Using this fact, the desired user channel can be obtained (up to a scale factor) as the maximum eigenvector of the matrix  $\mathbf{R}_{ZZ}$ , since  $\mathbf{Z}(n) = (\mathbf{C}_1^H \mathbf{C}_1) \otimes \mathbf{I}_{MJ} \mathbf{h}_1 \tilde{a}_1(n-d)$ . It can further be shown easily that if  $\mathbf{T}_2 = \mathbf{T}_1^\perp$ , then

$$\mathbf{T}_1 \mathbf{R}_{YY}^{-1} \mathbf{T}_1^H = \left( \mathbf{T}_1 \mathbf{T}_1^H \right) \mathbf{R}_{ZZ}^{-1} \left( \mathbf{T}_1 \mathbf{T}_1^H \right), \quad (18)$$

where,  $\mathbf{R}_{ZZ}$  is given by (17), and  $\mathbf{Q}$ , given by (14), is optimized to minimize the estimation error variance.  $\mathbf{R}^d$  replaces  $\mathbf{R}_{YY}$  in the above developments. From this, we can obtain the propagation channel estimate for the desired user,  $\hat{\mathbf{h}}_1$  as  $\hat{\mathbf{h}}_1 = V_{max} \{ (\mathbf{T}_1 \mathbf{T}_1^H)^{-1} \mathbf{R}_{ZZ} (\mathbf{T}_1 \mathbf{T}_1^H)^{-1} \}$ . The above structure results in perfect interference cancellation (both ISI and MAI) in the noiseless case, the evidence of which is the rank-1 estimation error covariance matrix, and a consequent distortionless response for the desired user.

## 5. ADAPTIVE IMPLEMENTATIONS

The GSC formulation of the MMSE-ZF as given in section 4.1, converts the constrained optimization problem (unbiasedness constraint) into an unconstrained one [6]. [3] proposes to adapt the MOE problem in a GSC fashion by splitting it into two optimization problems, one for the interference canceling filter, and the other for the channel impulse response,  $\mathbf{h}_1$ . The problem with such an approach is that the problem becomes that of joint optimization thus rendering it susceptible of falling into local minima. The alternative formulation is that of a *precombining* interference canceler, as shown in fig. 4. The interference canceler operates independently of the channel response. The optimization problem however becomes that of optimizing for a matrix filter,  $\mathbf{Q}$ . The entity that needs to be optimized is the trace of  $\mathbf{R}_{ZZ}$ .

One situation of interest is that of sparse channels where  $\mathbf{T}_1$  contains a small number of non-zero rows, highlighting the fact that only these directions of the correlator bank carry the signal plus interference energy. Note that  $\mathbf{T}_1$  no longer contains the code, but it also contains the contribution of the pulse shaping filter, so that the vector  $\mathbf{h}_1$  is a short vector with the non-zero elements of the sparse channel. If the corresponding rows of  $\mathbf{Q}$  can be assumed to operate independently so as to cancel interference in these directions, they can be adapted independently. Let us denote by  $\mathbf{q}_i$  and  $\mathbf{t}_i$ , the  $i$ th row of the matrix  $\mathbf{Q}$  and  $\mathbf{T}_1$  respectively, then the cost function to be optimized becomes

$$\mathcal{Z}_i = \mathbf{R}_{ZZ}^{ii} = (\mathbf{t}_i - \mathbf{q}_i \mathbf{T}_2) \mathbf{R}^d (\mathbf{t}_i - \mathbf{q}_i \mathbf{T}_2)^H. \quad (19)$$

where,  $i \in \{1, \dots, l_1\}$ , and  $l_1$  is the number of non-zero taps of the sparse channel.  $\mathcal{Z}_i$  is quadratic in  $\mathbf{q}_i$ , and can be optimized in the LMS or the NLMS fashion. It can be noticed that while minimizing  $\mathcal{Z}_i$ , its contribution to the trace of  $\mathbf{R}_{ZZ}$  also gets minimized. Same applies for other  $\mathbf{q}_i$ 's. Then, the update equation will be of the form

$$\mathbf{q}_i(n+1) = \mathbf{q}_i(n) - \mu_q \nabla_{\mathbf{q}_i^*} \mathcal{Z}_i, \quad (20)$$

where,  $\mu_q$  is the step size for the LMS algorithm [7]. The derivative (the gradient) can be computed as

$$\nabla_{\mathbf{q}_i^*} \mathcal{Z}_i = -\mathbf{T}_2 \mathbf{R}^d \mathbf{t}_i^H + \mathbf{T}_2 \mathbf{R}^d \mathbf{q}_i^H, \quad (21)$$

leading to the recursive update equation

$$\mathbf{q}_i(n+1) = \mathbf{q}_i(n) - \mu_q \left[ \mathbf{T}_2 \mathbf{R}^d \mathbf{q}_i^H(n) - \mathbf{T}_2 \mathbf{R}^d \mathbf{t}_i^H \right]. \quad (22)$$

As  $\mathbf{R}_{ZZ}$  is rank-1 in the batch processing mode, the adaptive search for path coefficients  $\mathbf{h}_1$  can then be based upon the maximization of the signal variance,  $\sigma_a^2 (\mathbf{h}_1^H \mathbf{T}_1 \mathbf{T}_1^H \mathbf{h}_1) / \|\tilde{\mathbf{g}}_1\|^2$  at the output of the maximum ratio combiner, resulting in a recursive update as

$$\mathbf{h}_i(n+1) = \mathbf{h}_i(n) + \mu_h [\mathbf{T}_1 - \mathbf{Q}(n) \mathbf{T}_2] \mathbf{R}^d [\mathbf{T}_1 - \mathbf{Q}(n) \mathbf{T}_2]^H. \quad (23)$$

In the above adaptive algorithm,  $\mathbf{R}^d$  is approximated by  $\mathbf{Y}(n) \mathbf{Y}(n)^H - \hat{\sigma}_v^2 \mathbf{I}$ , where  $\hat{\sigma}_v^2$  accounts for the denoising operation.

### 5.1. Hard/Soft Decision Directed Mode

The adaptive interference cancellation scheme can be adapted in a decision directed mode to improve the quality of the filter  $\mathbf{Q}(n)$ . The presence of the signal term in the output of correlators,  $\mathbf{T}_1$ , perturbs the estimation of the IC filters. The hard/soft decision-directed mechanism works by examining at the scaled soft outputs  $\hat{a}(n-d)$  of the receiver (see fig 4). If  $\hat{a}(n-d) \geq \zeta$ , then an update is made by subtracting the contribution of the desired term as  $\mathbf{T}_1 (\mathbf{Y} - \tilde{\mathbf{g}}_1 \hat{a}(n-d))$  from the correlator outputs. Otherwise no update is made. Several more sophisticated schemes are also possible. For example the update can always be made while subtracting the soft output rather than the hard decision.

### 5.2. Delay Tracking

In the above framework,  $\mathbf{T}_1$  is assumed to be fixed. However, the discrete path components of the sparse channel tend to drift from their nominal positions.  $\mathbf{T}_2$  is no longer a strict signal blocker if it is obtained from  $\mathbf{T}_1$  as the orthogonal complement.  $P_{T_1^\perp}^H$  qualifies as a blocking transformation. An update of the  $\mathbf{T}_1$  can be when oversampling the received signal w.r.t. the chip rate is employed. Considering an oversampled version of the pulse-shaping filter,  $p(t)$ , we can write as

$$p\left(k \frac{T_c}{J}\right) \approx p\left(\frac{k T_c}{J}\right) + \tau \dot{p}\left(\frac{k T_c}{J}\right), \quad (24)$$

where  $\dot{p}(t)$  is the first derivative of the pulse shaping filter, and  $\tau$  is a continuous time delay. Hence the positions in  $\mathbf{T}_1$  can be updated as discrete shifts every time a decision is made based upon the values of  $p(k T_c / J)$  and  $\dot{p}(k T_c / J)$  for a given  $k$ .

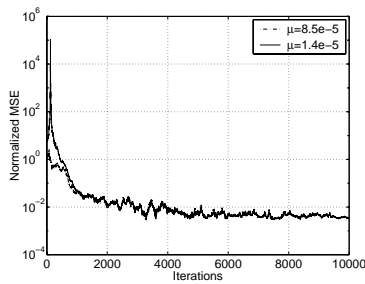


Figure 5. Channel estimation error for the adaptive algorithm  $K = 5$ ,  $P = 16$ ,  $\text{SNR}=25$  dB.

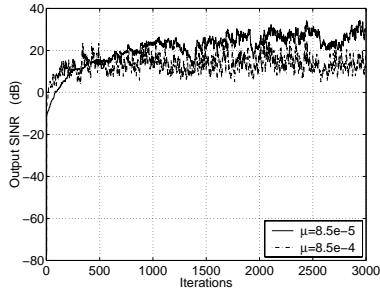


Figure 6. Output SINR for two step sizes  $K = 6$ ,  $P = 16$ ,  $\text{SNR}=25$  dB, near-far conditions.

## 6. NUMERICAL EXAMPLES

We consider  $K = 5$  asynchronous users in the system with a spreading factor of  $P = 16$ . The propagation channel (excluding the transmit and receive filters, which is a raised-cosine pulse, and the effect of which are absorbed in the code convolution matrix  $C_1$ ) for the  $k$ th user is modeled as a sparse channel with  $l_k$  discrete paths spanning a delay spread of  $8 - 21$  chip periods for different  $k$ 's. Mild near-far conditions prevail in that the interfering users are randomly (ranging from 8 to 10 dB) stronger than the user of interest.

In general the normalized LMS (NLMS) results in better convergence due to the gradient noise amplification problem in the original LMS algorithm [7]. We shall however present the LMS adapted version in these simulations. Fig. 5 shows the normalized mean-square error (NMSE)<sup>2</sup> of adaptive channel estimation algorithm. We start with random initialization for the channel taps since the interference canceling filter does not need path amplitudes and phases. Path delays are assumed to be known in these examples.

In fig. 6, we show the convergence of the LMS adaptive algorithm [7] for the two different step sizes. Convergence is guaranteed in all cases due to the quadratic nature of the cost function, once the step size,  $\mu$  lies in the region of interest.

In fig. 7, we show the performance of the decision-directed algorithm. It is seen that the blind algorithm suffers from a saturation effect due to the presence of the desired signal component in the estimation of the interference canceling filter, while removing this over reliable decisions gives significant performance gains.

$${}^2\text{NMSE} = E \frac{\|h_1 - \hat{h}_1\|^2}{\|h_1\|^2} = \frac{1}{L} \sum_{i=1}^L \frac{\|h_1 - \hat{h}_1^{(i)}\|^2}{\|h_1\|^2}$$

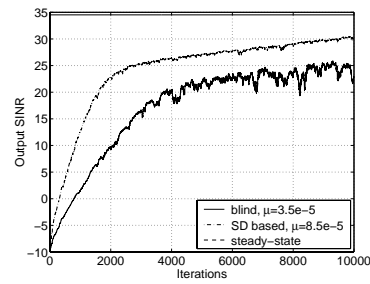


Figure 7. Output SINR for blind and decision-directed algorithms  $K = 6$ ,  $P = 16$ ,  $\text{SNR}=25$  dB.

## 7. CONCLUSIONS

The adaptive receiver presented above distinguishes clearly between two issues, namely channel identification and receiver adaptation, i.e., the interference canceling part of the receiver operate in a fashion that it attempts to cancel the interference independently of the channel parameters apart from the delay. In this respect, it qualifies as a pre-combining interference canceler. The disjointness of the two estimation algorithms leads to global convergence of the two once the system is identifiable in the batch mode. It was also seen that the decision-directed mode of operation results in much improved performance over the blind method. It must however be mentioned that the quality of the blocking transformation is crucial in all cases. If the desired signal component leaks through this branch, performance of the algorithm greatly suffers. Delay tracking is therefore necessary for operation in fading environments.

Simulation examples are presented in the case of joint channel and MOE receiver optimization in [8]. It appears that the choice of step sizes (there are two) in the joint optimization problem is a complicated issue. We tried to compare the performance of our approach with the joint optimization one of [8], however, no definite range of step sizes could be obtained for convergence for the latter case.

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