

Balancing Egoism and Altruism on the single beam MIMO Interference Channel

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Abstract—This paper considers the so-called multiple-input-multiple-output interference channel (MIMO-IC) which has relevance in applications such as multi-cell coordination in cellular networks as well as spectrum sharing in cognitive radio networks among others. We consider a beamforming design framework based on striking a compromise between beamforming gain at the intended receiver (Egoism) and the mitigation of interference created towards other receivers (Altruism). Combining egoistic and altruistic beamforming has been shown previously in several papers to be instrumental to optimizing the rates in a multiple-input-single-output interference channel MISO-IC (i.e. where receivers have no interference canceling capability). Here, by using the framework of Bayesian games, we shed more light on these game-theoretic concepts in the more general context of MIMO channels and more particularly when coordinating parties only have CSI of channels that they can measure directly. This allows us to derive distributed beamforming techniques. We draw parallels with existing work on the MIMO-IC, including rate-optimizing and interference-alignment precoding techniques, showing how such techniques may be improved or re-interpreted through a common prism based on balancing egoistic and altruistic beamforming. Our analysis and simulations currently limited to single stream transmission per user attest the improvements over known interference alignment based methods in terms of sum rate performance in the case of so-called asymmetric networks.

Index Terms—multi-cell, MIMO, distributed beamforming, Pareto boundary, game theory, Bayesian equilibrium, interference channels, distributed bargaining, egoistic, altruistic, interference alignment

I. INTRODUCTION

The mitigation of interference in multi-point to multi-point radio systems is of utmost importance and has relevance in several practical contexts. Among the more popular cases, we may cite the optimization of multi cell MIMO systems with full frequency reuse and cognitive radio scenarios featuring two or more service providers sharing an identical spectrum license on overlapping coverage areas. In all these cases, the system may be modeled as a network of N_c interfering radio links where each link consists of a sender trying to communicate messages to a unique receiver in spite of the interference arising from or created towards other links.

For system limitation or privacy reasons, when the backhaul network cannot support a complete sharing of data symbols across all TxS, the channel remains an interference channel. Coordination in terms of beamforming is required to be decentralized in the sense that global CSIT may not be available everywhere. In the context of distributed beamforming, game

theory appears as a sensible approach as a basis for algorithm design. Recently an interesting game theory framework for beamforming-based coordination was proposed for the MISO case by which the transmitters (e.g. the base stations) seek to strike a compromise between selfishly serving their users while ignoring the interference effects on the one hand, and altruistically minimizing the harm they cause to other non-intended receivers on the other hand. An important result in this area was the characterization of all so-called Pareto rate optimal beamforming solutions for the two-cell case in the form of positive linear combinations of the purely selfish and purely altruistic beamforming solutions [1]–[3] and [4]–[6] in the case of partial CSI. Unfortunately, how or whether at all this analysis can be extended to the context of MIMO interference channels (i.e. where receivers have themselves multiple antennas and interference cancelling capability) remains an open question.

In parallel, coordination on the MIMO interference channel has emerged as a very popular topic in its own right, with several important non-game related contributions shedding light on rate-scaling optimal precoding strategies based on so-called interference alignment, subspace optimization, alternated maximum SINR optimization, [7]–[9] and rate-maximizing precoding strategies [10], [11], to cite just a few examples.

Interference alignment based strategies exhibit the designed feature of rendering interference cancellable (when feasible, according to the available degrees of freedom) at both the transmitter and receiver side. Such a behaviour is optimal in the large SNR region when interference is the key bottleneck. At finite SNR, various strategies exist which aim at maximizing a link quality metric individually over each link, while taking interference into account. This often takes the form of maximizing the link's SINR or minimizing minimum-mean-square-error (MMSE). This approach provides good rates in symmetric networks where all links are subject to impairments (noise, average interference) of similar level. In more general and practical situations however, we argue that a better sum rate may be obtained from a proper and different weighting of the egoistic and altruistic objective at each individual link. This situation is particularly important when more links are subject to statistically stronger interference than others, a case which has so far received little attention and which we shall refer here as asymmetric networks. For this purpose, we suggest to re-visit the problem of coordinated beamforming design by directly building on the game theoretic concept of egoistic and altruistic game equilibria. Because our focus is on scenarios where CSI is not fully available, we consider a class of games suitable to the case of partial information-

This work has been performed in the framework of the European research project SAPHYRE, which is partly funded by the European Union under its FP7 ICT Objective 1.1 - The Network of the Future.

based decision making, called Bayesian games. Note that this is different from the limited CSI feedback scenario studied by previous authors [12] who consider channel quantization requirement as function of SNR. Our approach is two fold, first derive analytically the game equilibria. Second, exploit the obtained equilibria solution into heuristic design of a practical beamforming technique. The behaviour of our solution is then studied both theoretically (large SNR regime) and tested by simulations.

More specifically in this paper, our contributions are as follows:

- We define the egoistic and altruistic objective functions and derive analytically the equilibria of so-called egoistic and altruistic Bayesian games [13].
- Based on the equilibria, we propose a practical distributed beamforming scheme which provides a game-theoretic interpretation of the distributed sum rate maximization problem the MIMO-IC, such as [11].
- The proposed techniques allows a tradeoff between the reduced complexity/feedback and the rate maximization offered by [11].
- We show that our algorithm exhibits the same rate scaling (when SNR grows) as shown by recent interesting interference alignment based methods [7]–[9] which operates on the same feedback assumption as the proposed beamforming scheme. At finite SNR, we show improvements in terms of sum rate, especially in the case of asymmetric networks where interference-alignment methods are unable to properly weigh the contributions on the different interfering links to maximize the sum rate. This situation is particularly relevant. In practical contexts where for complexity limitation reasons only a subset of cells (links) is coordinated across, while other uncoordinated links contribute to additional unequal amounts of unstructural interference.

A. Notations

The lower case bold face letter represents a vector whereas the upper case bold face letter represents a matrix. $(\cdot)^H$ represents the complex conjugate transpose. \mathbf{I} is the identity matrix. $V^{(max)}(\mathbf{A})$ (resp. $V^{(min)}(\mathbf{A})$) is the eigenvector corresponding to the largest (resp. smallest) eigenvalue of \mathbf{A} . \mathcal{E}_B is the expectation operator over the statistics of the random variable B . $\mathbb{S} \setminus \mathbb{B}$ define a set of elements in \mathbb{S} excluding the elements in \mathbb{B} . $Tr(\mathbf{A})$ denotes the trace of matrix \mathbf{A} .

II. BAYESIAN GAMES DEFINITION ON INTERFERENCE CHANNEL

Let $\mathcal{N} = \{1, \dots, N\}$ be a set containing a finite set \mathcal{N}_c , with cardinality $N_c \leq N$, of cooperating transmitters (Tx), also termed as players. From now on, we use players and Tx's interchangeably. We call the set \mathcal{N}_c a coordination cluster and Tx's outside the cluster will contribute to uncontrolled interference. The provided model has general applications in which the Tx's can be base stations in cellular downlink where typically coordination is restricted to a subset of neighbouring

cell sites while more distant sites cannot be coordinated over [14]; nodes in ad-hoc network and cognitive radio.

Each Tx is equipped with N_t antennas and the Rx with N_r antennas. Each Tx communicates with a unique Rx at a time. Tx's are not allowed or able to exchange users' packet (message) information, giving rise to an interference channel over which we seek some form of beamforming-based coordination. The channel from Tx i to Rx j $\mathbf{H}_{ji} \in \mathbb{C}^{N_r \times N_t}$ is given by:

$$\mathbf{H}_{ji} = \sqrt{\alpha_{ji}} \bar{\mathbf{H}}_{ji}, \quad i, j = 1, \dots, N_c \quad (1)$$

Each element in channel matrix $\bar{\mathbf{H}}_{ji}$ is an independent identically distributed complex Gaussian random variable with zero mean and unit variance and α_{ji} denotes the slow-varying shadowing and pathloss attenuation. $\bar{\mathbf{H}}_{ji}$ is circularly symmetric complex gaussian and the probability density is

$$f_{\bar{\mathbf{H}}_{ji}}(\mathbf{H}) = \frac{1}{\pi^{N_t N_r}} \exp(-Tr(\mathbf{H}\mathbf{H}^H)). \quad (2)$$

A. Limited Channel knowledge

Although there may exist various ranges and definitions of local CSI, we assume a standard definition of a quasi-distributed CSI scenario where the devices (Tx and Rx alike) are able to gain knowledge of those local channel coefficients *directly connected* to them, as illustrated in Fig. 1, possibly complemented with some limited non local information (to be defined later).

The set of CSI locally available (resp. not available) at Tx i denoted by \mathbb{B}_i (resp. \mathbb{B}_i^\perp) is denoted by:

$$\mathbb{B}_i = \{\mathbf{H}_{ji}\}_{j=1, \dots, N_c}; \quad \mathbb{B}_i^\perp = \{\mathbf{H}_{kl}\}_{k, l=1, \dots, N_c} \setminus \mathbb{B}_i \quad (3)$$

Similarly, define the set of channels known (resp. unknown) at Rx i denoted by \mathbb{M}_i (resp. \mathbb{M}_i^\perp) as: $\mathbb{M}_i = \{\mathbf{H}_{ij}\}_{j=1, \dots, N_c}; \quad \mathbb{M}_i^\perp = \{\mathbf{H}_{kl}\}_{k, l=1, \dots, N_c} \setminus \mathbb{M}_i$. By construction here, locally available channel knowledge, \mathbb{B}_i , is only known to Tx i but *not* other Tx's. We call this knowledge \mathbb{B}_i the *type* of player (Tx) i , in the game theoretic terminology [13].

In the view of Tx i , the decision to be made shall be based on its type \mathbb{B}_i and its *beliefs* on other Tx's types. Since Tx i does not know other Tx's types, we assume that Tx i has a probability density over the possible values of other players channel knowledge \mathbb{B}_j . For simplicity, we assume that these *beliefs* are symmetric: the probability density of the *gaussian channels* available at Tx i regarding \mathbb{B}_j is the same as the probability density of Tx j over \mathbb{B}_i . The asymmetric path loss attenuations α_{ji} are assumed to be long term statistics and known to the Tx's. And we assume that the channel coefficients in the network are statistically independent from each other. We define here the joint beliefs (probability density) at Tx i :

$$\mu_i = p(\mathbb{B}_i^\perp) = f_{\bar{\mathbf{H}}_{ji}}(\mathbf{H})^{N_c(N_c-1)} = \mu. \quad (4)$$

The Tx index i is dropped because the beliefs are symmetric among Tx's, given the asymmetric path loss coefficients α_{ji} . $p(\cdot)$ is a probability measure and $f_{\bar{\mathbf{H}}_{ji}}(\mathbf{H})$ is density of a complex gaussian channel defined in (2). The second equality relies on

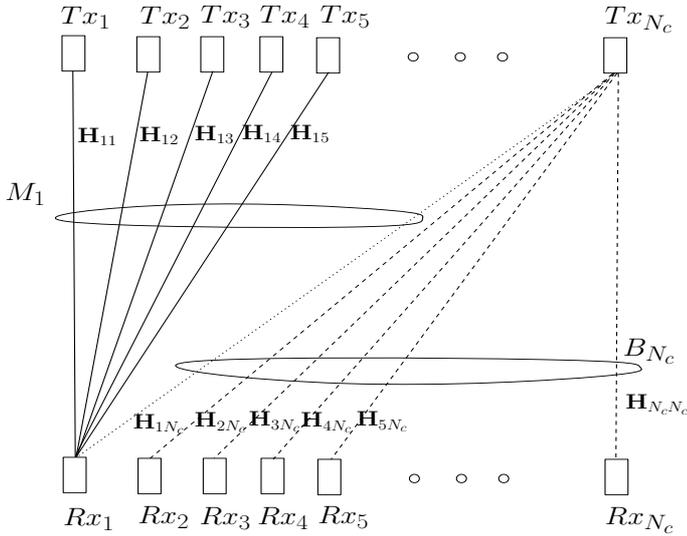


Fig. 1. Limited channel knowledge model: as an illustration, the local CSI available at Tx N_c is shown in dashed lines. The local CSI available at Rx 1 is shown in solid lines.

the assumptions that the channel coefficients from any Tx to any Rx are independent.

Based on its *belief*, Tx i should make a decision, which is to design the transmit beamforming vector, $\mathbf{w}_i \in \mathcal{C}^{N_t \times 1}$. As in several important contributions dealing with coordination on the interference channel [2], [8], [15]–[19], we assume linear beamforming. We call the transmit beamforming vector \mathbf{w}_i an action of Tx i and denote the set of all possible actions by \mathcal{A} at any Tx.

$$\mathcal{A} = \{\mathbf{w} \in \mathcal{C}^{N_t \times 1} : |\mathbf{w}|^2 \leq 1\} \quad (5)$$

The received signal at Rx i is therefore

$$y_i = \mathbf{v}_i^H \mathbf{H}_{ii} \mathbf{w}_i + \sum_{j \neq i} \mathbf{v}_i^H \mathbf{H}_{ij} \mathbf{w}_j + n_i \quad (6)$$

where n_i is a gaussian noise with power σ_i^2 . Note that the noise levels σ_i^2 depend on the link index which was not considered in previous work on transmitter coordination. The Rxs are assumed to employ maximum SINR (Max-SINR) beamforming throughout the paper so as to also maximize the link rates [20]. The receive beamformer \mathbf{v}_i is classically given by:

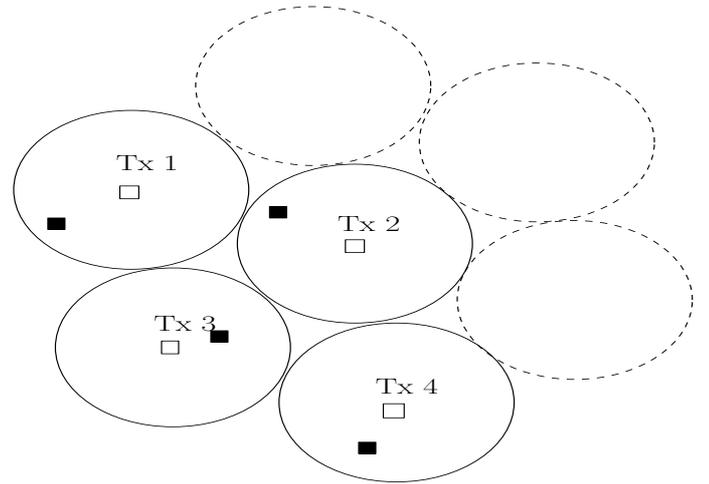
$$\mathbf{v}_i = \frac{\mathbf{C}_{Ri}^{-1} \mathbf{H}_{ii} \mathbf{w}_i}{|\mathbf{C}_{Ri}^{-1} \mathbf{H}_{ii} \mathbf{w}_i|} \quad (7)$$

where \mathbf{C}_{Ri} is the covariance matrix of received interference and noise

$$\mathbf{C}_{Ri} = \sum_{j \neq i} \mathbf{H}_{ij} \mathbf{w}_j \mathbf{w}_j^H \mathbf{H}_{ij}^H P + \sigma_i^2 \mathbf{I}. \quad (8)$$

P is the transmit power.

Importantly, the noise will in practice capture thermal noise effects but also any interference originating from the rest of the network, i.e. coming from transmitters located beyond the coordination cluster. Thus, depending on path loss and shadowing effects, the $\{\sigma_i^2\}$ may be quite different from each



$$N = 7; N_c = 4$$

Fig. 2. This figure illustrates a system of $N = 7$ cells where $N_c = 4$ form a coordination cluster. Empty squares represent transmitters whereas filled squares represent receivers. The noise power (which includes out of cluster interference) undergone in each cell varies from link to link.

other [21]. Fig. 2 illustrates a system of $N = 7$ cells where $N_c = 4$ form a coordination cluster. Note that for simplification of analysis, we consider the sum of uncoordinated source of interference and thermal noise to be spatially white.

Receiver feedback v.s. Reciprocal Channel: In the case of reciprocal channels (TDD), the feedback requirement to obtain \mathbb{B}_i can be replaced by a channel estimation step based on uplink pilot sequences. Additionally, it will be classically assumed that the receivers are able to estimate the covariance matrix of their interference signal, based on, say, transmit pilot sequences.

We can now define the Bayesian game on interference channel as a 5-tuple.

Definition 1:

$$G = \langle \mathcal{N}_c, \mathcal{A}, \{\mathbb{B}_i\}, \mu, \{u_i\} \rangle. \quad (9)$$

μ denotes the *beliefs* of the players and $\{u_i\}$ denotes the utility functions of the players, which can be either egoistic or altruistic.

Specific definitions of u_i will be given in the following sections. The players are assumed to be *rational* as they maximize their own utility based on their *types* and *beliefs*.

Definition 2: A pure-strategy of player i , $s_i : \mathbb{B}_i \rightarrow \mathcal{A}_i$ is a deterministic choice of action given information \mathbb{B}_i of player i .

Definition 3: A strategy profile $\mathbf{s}^* = (s_i^*, s_{-i}^*)$ achieves the Bayesian Equilibrium if s_i^* is the best response of player i given strategy tuple s_{-i}^* for all other players and is characterized by

$$\forall i \quad s_i^* = \arg \max_{\mathcal{A}_i} \mathcal{E}_{\mathbb{B}_i} \{u_i(s_i, s_{-i}^*)\}. \quad (10)$$

Note that, intuitively, the player's strategy is optimized by averaging over the *beliefs* (the distribution of all missing state information) while in a standard game, such expectation is not required.

In the following sections, we derive the equilibria for egoistic and altruistic bayesian games respectively. These equilibria constitute extreme strategies which do not perform optimally in terms of the overall network performance, yet can be exploited as components of a more general beamforming-based coordination technique which is then proposed in section V.

III. BAYESIAN GAMES WITH RECEIVER BEAMFORMER FEEDBACK

We assume that Tx has the local channel state information \mathbb{B}_i and the added knowledge of receive beamformers through a feedback channel. Note that in the case of reciprocal channels, the receive beamformer feedback is *not* required.

A. Egoistic Bayesian Game

Definition 4: Denote the set of transmit beamforming vectors of players $j, j \neq i$, by \mathbf{w}_{-i} . The egoistic utility function for Tx i is defined as its received SINR

$$u_i(\mathbf{w}_i, \mathbf{w}_{-i}) = \frac{|\mathbf{v}_i^H \mathbf{H}_{ii} \mathbf{w}_i|^2 P}{\sum_{j \neq i}^{N_c} |\mathbf{v}_i^H \mathbf{H}_{ij} \mathbf{w}_j|^2 P + \sigma_i^2}. \quad (11)$$

Based on Tx i 's belief, Tx i maximizes the utility function in (11) where \mathbf{v}_i is a known quantity.

Lemma 5: There exist at least one Nash Equilibrium in the egoistic Bayesian Game G (9) with utility function defined in (11).

Proof: \mathcal{A}_i is convex, closed and bounded for all players i and the egoistic utility function $u_i(\mathbf{w}_i, \mathbf{w}_{-i})$ is continuous in both \mathbf{w}_i and \mathbf{w}_{-i} . The utility function is convex in \mathbf{w}_i for any set \mathbf{w}_{-i} . Thus, at least one Bayesian Equilibrium exists [22], [23]. ■

Theorem 6: The best-response strategy of player i in the egoistic Bayesian Game G (9) with utility function (11) is to maximize the utility function based on its belief:

$$\mathbf{w}_i^{Ego} = \arg \max_{\mathcal{E}_{\mathbb{B}_i^\perp}} \{u_i(\mathbf{w}_i, \mathbf{w}_{-i})\}. \quad (12)$$

The best-response strategy of player i is

$$\mathbf{w}_i^{Ego} = V^{(max)}(\mathbf{E}_i) \quad (13)$$

where \mathbf{E}_i denotes the *egoistic equilibrium matrix* for Tx i , given by

$$\mathbf{E}_i = \mathbf{H}_{ii}^H \mathbf{v}_i \mathbf{v}_i^H \mathbf{H}_{ii}.$$

Proof: The knowledge of receive beamformers decorrelates the maximization problem which can be written as

$$\mathbf{w}_i^{Ego} = \arg \max_{|\mathbf{w}_i| \leq 1} \mathcal{E}_{\mathbb{B}_i^\perp} \left\{ \frac{1}{\sum_{j \neq i}^{N_c} |\mathbf{v}_i^H \mathbf{H}_{ij} \mathbf{w}_j|^2 P + \sigma_i^2} \right\} \\ \mathbf{w}_i^H \mathbf{H}_{ii}^H \mathbf{v}_i \mathbf{v}_i^H \mathbf{H}_{ii} \mathbf{w}_i \quad (14)$$

The egoistic-optimal transmit beamformer is therefore the dominant eigenvector of $\mathbf{H}_{ii}^H \mathbf{v}_i \mathbf{v}_i^H \mathbf{H}_{ii}$. ■

B. Altruistic Bayesian Game

Definition 7: The utility of the altruistic game is defined here so as to minimize the sum of interference powers caused to other receivers.

$$u_i(\mathbf{w}_i, \mathbf{w}_{-i}) = - \sum_{j \neq i} |\mathbf{v}_j^H \mathbf{H}_{ji} \mathbf{w}_i|^2 \quad (15)$$

Lemma 8: There exist at least one Nash Equilibrium in the altruistic Bayesian Game G (9) with utility function defined in (15).

Proof: \mathcal{A}_i is convex, closed and bounded for all players i and the altruistic utility function $u_i(\mathbf{w}_i, \mathbf{w}_{-i})$ is continuous in both \mathbf{w}_i and \mathbf{w}_{-i} . The utility function is concave in \mathbf{w}_i for any set \mathbf{w}_{-i} . Thus, at least one Bayesian Equilibrium exists [22], [23]. ■

Theorem 9: Based on belief μ , Tx i seeks to maximize the utility function defined in (15). The best-response strategy is

$$\mathbf{w}_i^{Alt} = V^{(min)} \left(\sum_{j \neq i} \mathbf{A}_{ji} \right) \quad (16)$$

where \mathbf{A}_{ji} denotes the *altruistic equilibrium matrix* for Tx i towards Rx j , defined by $\mathbf{A}_{ji} = \mathbf{H}_{ji}^H \mathbf{v}_j \mathbf{v}_j^H \mathbf{H}_{ji}$.

Proof: Recall the utility function to be $-\sum_{j \neq i} |\mathbf{v}_j^H \mathbf{H}_{ji} \mathbf{w}_i|^2 = -\sum_{j \neq i} \mathbf{w}_i^H \mathbf{A}_{ji} \mathbf{w}_i$. Since \mathbf{v}_j are known from feedback or estimation in reciprocal channels, the optimal \mathbf{w}_i is the least dominant eigenvector of the matrix $\sum_{j \neq i} \mathbf{A}_{ji}$. ■

IV. SUMRATE MAXIMIZATION WITH RECEIVE BEAMFORMER FEEDBACK

From the results above, it can be seen that balancing altruism and egoism for player i can be done by trading-off between setting the beamformer close to the dominant eigenvectors of the egoistic equilibrium \mathbf{E}_i or that of the negative altruistic equilibrium $\{-\mathbf{A}_{ji}\}$ ($j \neq i$) matrices in (16). Interestingly, it can be shown that sum rate maximizing precoding for the MIMO-IC does exactly that. Thus we hereby briefly re-visit rate-maximization approaches such as [11] with this perspective.

Denote the sum rate by $\bar{R} = \sum_{i=1}^{N_c} R_i$ where $R_i = \log_2 \left(1 + \frac{|\mathbf{v}_i^H \mathbf{H}_{ii} \mathbf{w}_i|^2 P}{\sum_{j \neq i}^{N_c} |\mathbf{v}_i^H \mathbf{H}_{ij} \mathbf{w}_j|^2 P + \sigma_i^2} \right)$.

Lemma 10: The transmit beamforming vector which maximizes the sum rate \bar{R} is the dominant eigenvector of a matrix, which is a linear combination of \mathbf{E}_i and \mathbf{A}_{ji} :

$$\left(\mathbf{E}_i + \sum_{j \neq i}^{N_c} \lambda_{ji}^{opt} \mathbf{A}_{ji} \right) \mathbf{w}_i = \mu_{max} \mathbf{w}_i \quad (17)$$

where

$$\lambda_{ji}^{opt} = - \frac{|\mathbf{v}_j^H \mathbf{H}_{jj} \mathbf{w}_j|^2 P}{\sum_{k=1}^{N_c} |\mathbf{v}_j^H \mathbf{H}_{jk} \mathbf{w}_k|^2 P + \sigma_j^2} \frac{\sum_{k=1}^{N_c} |\mathbf{v}_i^H \mathbf{H}_{ik} \mathbf{w}_k|^2 P + \sigma_i^2}{\sum_{k \neq j}^{N_c} |\mathbf{v}_j^H \mathbf{H}_{jk} \mathbf{w}_k|^2 P + \sigma_j^2} \quad (18)$$

and μ_{max} is defined in the proof.

Proof: see appendix VIII-A. ■

Note that the balancing between altruism and egoism in sum rate maximization is done using the dominant eigenvector of a simple *linear combination* of the altruistic and egoistic equilibrium matrices. The balancing parameters, $\{\lambda_{ji}^{opt}\}$, can be shown simply to coincide with the pricing parameters invoked in the iterative algorithm proposed in [11]. Clearly, these parameters plays a key role, however their computation is a function of the *global* channel state information and requires additional message (price) exchange. Instead, we seek below a suboptimal egoism-altruism balancing technique which only requires statistical channel information, while exhibiting the right performance scaling when SNR grows large.

V. A PRACTICAL DISTRIBUTED BEAMFORMING ALGORITHM: DBA

We are proposing the following distributed beamforming algorithm (DBA) where one computes the transmit and receive beamformers iteratively as:

$$\mathbf{w}_i = V^{max} \left(\mathbf{E}_i + \sum_{j \neq i}^{N_c} \lambda_{ji} \mathbf{A}_{ji} \right) \quad (19)$$

$$\mathbf{v}_i = \frac{\mathbf{C}_{Ri}^{-1} \mathbf{H}_{ii} \mathbf{w}_i}{|\mathbf{C}_{Ri}^{-1} \mathbf{H}_{ii} \mathbf{w}_i|} \quad (20)$$

where λ_{ji} shall be made to depend on channel statistics only. At this stage, it is interesting to compare with previous schemes based on interference alignment such as the practical algorithms proposed in [9]. In such schemes, the transmit beamformer \mathbf{w}_i is taken independent of \mathbf{H}_{ii} . Note that here however, \mathbf{w}_i is correlated to the direct channel gain \mathbf{H}_{ii} through the Egoistic matrix \mathbf{E}_i in DBA. The correlation is useful in terms of sum rate as it allows proper weighting between the contributions of the egoistic and altruistic matrices in a link specific manner.

A. The egoism-altruism balancing parameters λ_{ji}

The egoism-altruism balancing parameters λ_{ji} are now found heuristically based on the statistical channel information. Recall from (18) that

$$\lambda_{ji}^{opt} = - \frac{S_j}{S_j + I_j + \sigma_j^2} \frac{S_i + I_i + \sigma_i^2}{I_j + \sigma_j^2} \quad (21)$$

where $S_j = |\mathbf{v}_j^H \mathbf{H}_{jj} \mathbf{w}_j|^2 P$ and $I_j = \sum_{k \neq j}^{N_c} |\mathbf{v}_j^H \mathbf{H}_{jk} \mathbf{w}_k|^2 P$.

Following the principle behind sum rate maximization, we conjecture that at convergence, residual coordinated interference shall be proportionate to the noise and out-of-cluster interference, i.e. $I_j = O(\sigma_j^2)$. Note that this should not be interpreted as an assumption in a proof but rather as a proposed design guideline. Based on this, we propose the following characterization:

$$\lambda_{ji}^{opt} = - \frac{S_j}{S_j + O(\sigma_j^2)} \frac{S_i + O(\sigma_i^2)}{O(\sigma_j^2)}. \quad (22)$$

By Jensen's inequality, a lower bound on the average λ_{ji}^{opt} is found by:

$$\mathcal{E}(\lambda_{ji}^{opt}) \geq - \frac{1}{1 + \frac{O(\sigma_j^2)}{\mathcal{E}S_j}} \frac{1 + \frac{O(\sigma_i^2)}{\mathcal{E}S_i}}{\frac{O(\sigma_j^2)}{\mathcal{E}S_i}}. \quad (23)$$

Although $\mathcal{E}S_i$ is not known explicitly, it is strongly related to the strength of the direct channel $P\alpha_{ii}$. Let $\gamma_i = \frac{P\alpha_{ii}}{\sigma_i^2}$. In order to obtain an exploitable formulation for λ_{ji} , we replace $\mathcal{E}S_i$ by $P\alpha_{ii}$ and $O(\sigma_i^2)$ by σ_i^2 , to derive:

$$\lambda_{ji} = - \frac{1}{1 + \gamma_j^{-1}} \frac{1 + \gamma_i^{-1}}{\frac{\sigma_j^2}{P\alpha_{ii}}}. \quad (24)$$

Interestingly, in the special case where direct channels have the same average strength, we obtain a simple expression

$$\lambda_{ji} = - \frac{1 + \gamma_i^{-1}}{1 + \gamma_j^{-1}} \gamma_j. \quad (25)$$

The above result suggests Tx i to behave more altruistically towards link j when the SNR of link j is high or when the SNR of link i is comparatively lower. This is in accordance with the intuition behind rate maximization over parallel gaussian channels.

DBA iterates between optimizing the transmit and receive beamformers, as summarized in Algorithm 1. Iterating between transmit and receive beamformers is reminiscent of recent interference-alignment based methods [8], [9]. However here, interference alignment is *not* a design criterion. In [8], an improved interference alignment technique based on alternately maximizing the SINR at both transmitter and receiver sides is proposed. In contrast, here the Max-SINR criterion is only used at the receiver side. Although the distinction is unimportant in the large SNR case (see below), it dramatically changes performance in certain situations at finite SNR (see Section VI).

Algorithm 1 DBA

- 1) Initialize beamforming vectors $\mathbf{w}_i, i = 1, \dots, N_c$, to be predefined vectors.
 - 2) For each Rx i , compute $\mathbf{v}_i = \frac{\mathbf{C}_{Ri}^{-1} \mathbf{H}_{ii} \mathbf{w}_i}{|\mathbf{C}_{Ri}^{-1} \mathbf{H}_{ii} \mathbf{w}_i|}$ where \mathbf{C}_{Ri} is computed with \mathbf{w}_i in previous step.
 - 3) For each Tx i , compute $\mathbf{w}_i = V^{max} \left(\mathbf{E}_i + \sum_{j \neq i}^{N_c} \lambda_{ji} \mathbf{A}_{ji} \right)$ where λ_{ji} are computed from statistical parameters (24).
 - 4) Repeat step 2 and 3 until convergence.
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B. Asymptotic Interference Alignment

One important aspect of the algorithm above is whether it achieves the interference alignment in high SNR regime [8]. The following theorem answers this question positively.

Definition 11: Define the set of beamforming vectors solutions in downlink (respectively uplink) interference alignment

to be [8]

$$\mathcal{I}\mathcal{A}^{DL} = \left\{ (\mathbf{w}_1, \dots, \mathbf{w}_{N_c}) : \sum_{k \neq i}^{N_c} \mathbf{H}_{ik} \mathbf{w}_k \mathbf{w}_k^H \mathbf{H}_{ik}^H \text{ is low rank}, \forall i \right\} \quad (26)$$

$$\mathcal{I}\mathcal{A}^{UL} = \left\{ (\mathbf{v}_1, \dots, \mathbf{v}_{N_c}) : \sum_{k \neq i}^{N_c} \mathbf{H}_{ki}^H \mathbf{v}_k \mathbf{v}_k^H \mathbf{H}_{ki} \text{ is low rank}, \forall i \right\}. \quad (27)$$

Thus, for all $(\mathbf{w}_1, \dots, \mathbf{w}_{N_c}) \in \mathcal{I}\mathcal{A}^{DL}$, there exist receive beamformers $\mathbf{v}_i, i = 1, \dots, N_c$ such that the following is satisfied:

$$\mathbf{v}_i^H \mathbf{H}_{ij} \mathbf{w}_j = 0 \quad \forall i, j \neq i. \quad (28)$$

Note that the uplink alignment solutions are defined for a virtual uplink having the same frequency and only appear here as a technical concept helping with the proof.

Theorem 12: Assume the downlink interference alignment set is non-empty (interference alignment is feasible). Denote average SNR of link i by $\gamma_i = \frac{P\alpha_{ii}}{\sigma_i^2}$. Let $\lambda_{ji} = -\frac{1+\gamma_i^{-1}}{1+\gamma_j^{-1}}\gamma_j$, then in the large SNR regime, $P \rightarrow \infty$, any transmit beamforming vector in $\mathcal{I}\mathcal{A}^{DL}$ is a convergence (stable) point of *DBA*.

Proof: see Appendix VIII-B. ■

Note that this does not prove global convergence, but local convergence, as is the case for other IA or rate maximization techniques [8], [9], [11]. Another way to characterize local convergence is as follows: assuming interference alignment is feasible ($\mathcal{I}\mathcal{A}^{DL}$ is non-empty), the first algorithm in [8] was shown to converge to transmit beamformers belonging to $\mathcal{I}\mathcal{A}^{DL}$ and the receivers are based on the minimum eigenvector of the downlink interference covariance matrix, which tends to be low-rank. However, *DBA* selects its receive beamformer from the Max-SINR criterion which, in the large SNR situation, is also identical to selecting receive beamformers in the null space of the interference covariance matrix. Therefore when interference alignment is feasible, the algorithm in [8] and *DBA* coincide at large SNR. This aspect is confirmed by our simulations (see section VI).

VI. SIMULATION RESULTS

In this section, we investigate the sum rate performances of *DBA* in comparison with several related methods, namely the *Max-SINR* method [8], the alternated-minimization (*Alt-Min*) method for interference alignment [9] and the sum rate optimization method (*SR-Max*) [11]. The *SR-Max* method is by construction optimal but is more complex and requires extra sharing or feedback of pricing information among the transmitters. To ensure a fair comparison, all the algorithms in comparisons are initialized to the same solution and have the same stopping condition. The algorithms are considered to reach convergence if the sum rates achieved between successive iterations have difference less than 0.001. We perform sum rate comparisons in both symmetric channels and asymmetric channels where links undergo different levels of out-of-cluster noise. Define the Signal to Interference ratio of link i to be

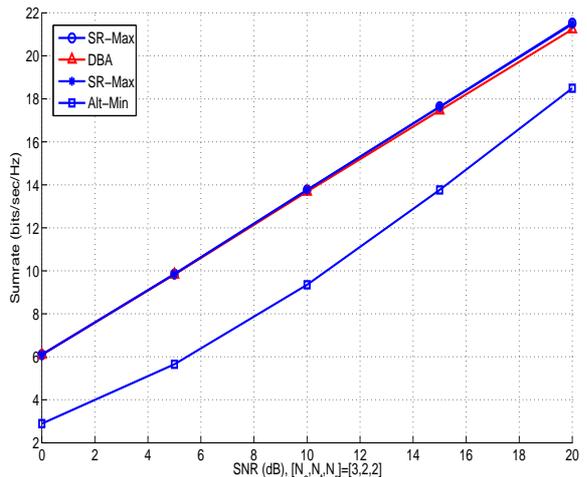


Fig. 3. Sum rate comparison in multi links systems is illustrated with $[N_c, N_t, N_r] = [3, 2, 2]$ with increasing SNR. *DBA*, *SR-Max* and *Max-SINR* achieve very close performance in symmetric networks.

$SIR_i = \frac{\alpha_{ii}}{\sum_{j \neq i}^{N_c} \alpha_{ij}}$. The *SIR* is assumed to be 1 for all links, unless otherwise stated. Denote the difference in SNR between two links in asymmetric channels by ΔSNR . Note that the proposed algorithm is not limited to the following settings, but can be applied to network with arbitrary players and number of antennas.

A. Symmetric Channels

Fig. 3 illustrates the sum rate comparison of *DBA* with *Max-SINR*, *Alt-Min* and *SR-Max* in a system of 3 links and each Tx and Rx have 2 antennas. Since interference alignment is feasible in this case, the sum rate performance of *SR-Max* and *Max-SINR* increases linearly with SNR. *DBA* achieves sum rate performance with the same scaling as *Max-SINR* and *SR-Max* (i.e. multiplexing gain of 3). Therefore these methods seem to perform similarly in symmetric channels.

B. Asymmetric Channels

In the asymmetric system, some links undergo uneven levels of noise and uncontrolled interference. Another aspect is that more links can experience greater path loss or shadowing than others. Here we consider a few typical scenarios for which could constitute asymmetric networks. In Fig. 4, there are 3 links in the system in which the noise and unstructural interference in one of the links are 20dB stronger than the other two links. This set up captures the scenario that one link is at the boundary of the coordination cluster and suffer from strong out-of-cluster noise. The *SIR* of every link is assumed to be 10 dB. In this scenario, *DBA* outperforms interference alignment based methods because they are unable to properly weigh the importance of each link in the overall sum rate. *SR-Max* is by construction sum rate optimal. However, in the asymmetric network, we observe by simulation that the convergence may require more iterations than other algorithms

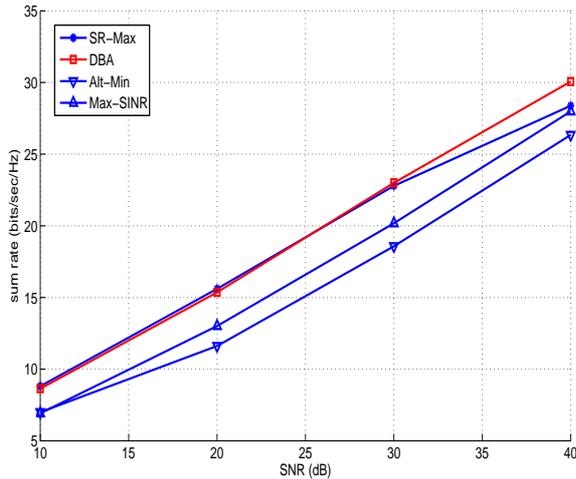


Fig. 4. Sum rate performance for asymmetric channel, with one link under strong noise, is illustrated. The strong noise, from out of cluster interference, is 20dB stronger than other links. *DBA* outperforms standard *IA* methods thanks to a proper balance between egoistic and altruistic beamforming algorithm.

and the increment in sum rate per iteration can be small in some channel realizations.

In Fig. 5, we compare the sum rate performance in the same set up as in Fig. 4, except that the SIR's of the links are $[10, 10, 0.1]$ respectively. Thus, link 3 not only suffers from strong out of cluster noise, but also suffers from strong interference within the cluster. The asymmetry penalizes the *Max-SINR* and interference alignment methods because they are unable to properly weigh the contributions of the weaker link in the sum rate. The *Max-SINR* strategy turns out to make link 3 very egoistic in this example, while its proper behavior should be altruistic. In contrast, *DBA* exploits useful statistical information, allowing weaker link to allocate their spatial degrees of freedom wisely towards helping stronger links and vice versa, yielding a better sum rate for the same feedback budget. The performance is very close to *SR-Max*, with less information exchange.

In Fig. 6, there are 3 links cooperating in the system. Each Tx and Rx has 2 antennas and has 1 stream transmission. The noise at each Rx is the same. The system is asymmetric in a sense that the direct channel gain H_{11} of link 1 is 30dB weaker than other links in the network. This set up models a realistic environment where the user suffers strong shadowing. *DBA* achieves sum rate closed to *SR-Max* and much better than other interference alignment based schemes *Max-SINR* and *Alt-Min*.

VII. CONCLUSION

We model the distributed beamforming optimization problem on MIMO interference channel using the framework of Bayesian Games which allow players to have incomplete information of the game, in this case the channel state information. Based on the incentives of the players, we proposed two games: the Egoistic Bayesian Game (players selfishly maximize its rate) and the Altruistic Bayesian Game

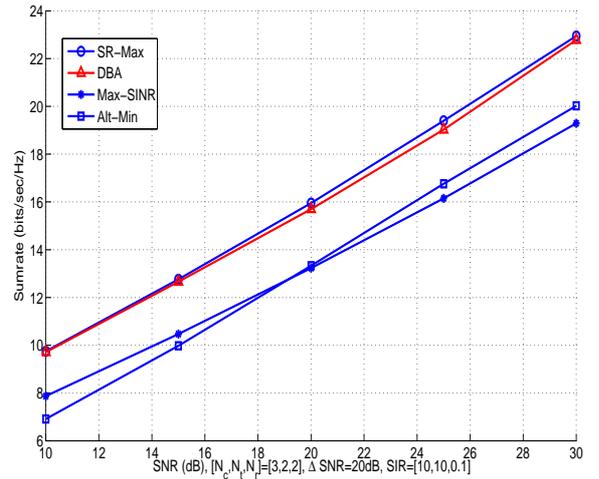


Fig. 5. Sum rate performance for asymmetric channel, with one link under strong interference within the cooperating cluster, is illustrated.

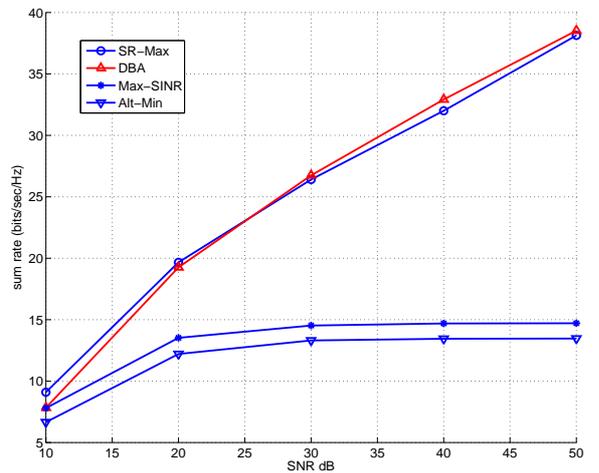


Fig. 6. Sum rate performance for asymmetric channel is illustrated. The direct channel gain of link 1 is 30dB weaker than other links.

(players altruistically minimize interference generated towards other players). We proved the existence of equilibria of such games and the best response strategy of players are computed. Inspired from the equilibria, a beamforming technique based on balancing the egoistic and the altruistic behavior with the aim of maximizing the sum rate is proposed. Such beamforming algorithm exhibits the same optimal rate scaling (when SNR grows) shown by recent iterative interference-alignment based methods. The proposed beamforming algorithm achieves close to optimal sum rate maximization method [11] without additional pricing feedbacks from users and outperform interference alignment based methods in terms of sum rate in asymmetric networks.

VIII. APPENDIX

A. Proof of Lemma 10

Define the largrangian of the sum rate maximization problem for Tx i to be $\mathcal{L}(\mathbf{w}_i, \mu) = \bar{R} - \mu_{max}(\mathbf{w}_i^H \mathbf{w}_i - 1)$. The necessary condition of largrangian $\frac{\partial}{\partial \mathbf{w}_i^H} \mathcal{L}(\mathbf{w}_i, \mu) = 0$ gives: $\frac{\partial}{\partial \mathbf{w}_i^H} R_i + \sum_{j \neq i}^{N_c} \frac{\partial}{\partial \mathbf{w}_i^H} R_j = \mu_{max} \mathbf{w}_i$. With elementary matrix calculus,

$$\frac{\partial}{\partial \mathbf{w}_i^H} R_i = \frac{P}{\sum_{k=1}^{N_c} |\mathbf{v}_i^H \mathbf{H}_{ik} \mathbf{w}_k|^2 P + \sigma_i^2} \mathbf{E}_i \mathbf{w}_i \quad (29)$$

$$\frac{\partial}{\partial \mathbf{w}_i^H} R_j = -\frac{|\mathbf{v}_j^H \mathbf{H}_{jj} \mathbf{w}_j|^2 P}{\sum_{k=1}^{N_c} |\mathbf{v}_j^H \mathbf{H}_{jk} \mathbf{w}_k|^2 P + \sigma_j^2} \frac{P}{\sum_{k \neq j}^{N_c} |\mathbf{v}_j^H \mathbf{H}_{jk} \mathbf{w}_k|^2 P + \sigma_j^2} \mathbf{A}_j \mathbf{w}_i \quad (30)$$

where λ_{ji}^{opt} is a function of all channel states information and beamformer feedback:

$$\lambda_{ji}^{opt} = -\frac{|\mathbf{v}_j^H \mathbf{H}_{jj} \mathbf{w}_j|^2 P}{\sum_{k=1}^{N_c} |\mathbf{v}_j^H \mathbf{H}_{jk} \mathbf{w}_k|^2 P + \sigma_j^2} \frac{\sum_{k=1}^{N_c} |\mathbf{v}_i^H \mathbf{H}_{ik} \mathbf{w}_k|^2 P + \sigma_i^2}{\sum_{k \neq j}^{N_c} |\mathbf{v}_j^H \mathbf{H}_{jk} \mathbf{w}_k|^2 P + \sigma_j^2} \quad (31)$$

Thus, the gradient is zero for any \mathbf{w}_i eigenvector of the matrix shown on the L.H.S. of (17). Among all stable points, the global maximum of the cost function is reached by selecting the dominant eigenvector of $E_i + \sum_{j \neq i} \lambda_{ji} A_{ji}$.

B. Proof of Theorem 12: convergence points of DBA

To prove that interference alignment forms a convergence set of DBA, we will prove that if DBA achieves interference alignment, DBA will not deviate from the solution (stable point).

Assumed interference alignment is reached and let $(\mathbf{w}_1^{IA}, \dots, \mathbf{w}_{N_c}^{IA}) \in \mathcal{IA}^{DL}$ and $(\mathbf{v}_1^{IA}, \dots, \mathbf{v}_{N_c}^{IA}) \in \mathcal{IA}^{UL}$. Let $\mathbf{Q}_i^{DL} = \sum_{k \neq i}^{N_c} \mathbf{H}_{ik} \mathbf{w}_k^{IA} \mathbf{w}_k^{IA,H} \mathbf{H}_{ik}^H$ and $\mathbf{Q}_i^{UL} = \sum_{k \neq i}^{N_c} \mathbf{H}_{ki}^H \mathbf{v}_k^{IA} \mathbf{v}_k^{IA,H} \mathbf{H}_{ki}$.

Given receivers $(\mathbf{v}_1^{IA}, \dots, \mathbf{v}_{N_c}^{IA})$, we compute new transport beamformers. In high SNR regime, $\lambda_{ji} \rightarrow -\infty$ and DBA gives $\mathbf{w}_i = V^{min}(\mathbf{Q}_i^{UL})$ (19). By (26), \mathbf{Q}_i^{UL} is low rank and thus \mathbf{w}_i is in the null space of \mathbf{Q}_i^{UL} . In direct consequence, the conditions of interference alignment (28) are satisfied. Thus, $(\mathbf{w}_1, \dots, \mathbf{w}_{N_c}) \in \mathcal{IA}^{DL}$.

Given transmitters $(\mathbf{w}_1^{IA}, \dots, \mathbf{w}_{N_c}^{IA})$, we compute new receive beamformers. The receive beamformer is defined as $\mathbf{v}_i = \arg \max_{\mathbf{v}_i} \frac{\mathbf{v}_i^H \mathbf{H}_{ii} \mathbf{w}_i^{IA} \mathbf{w}_i^{IA,H} \mathbf{H}_{ii}^H \mathbf{v}_i}{\mathbf{v}_i^H \mathbf{Q}_i^{DL} \mathbf{v}_i}$. Since \mathbf{Q}_i^{DL} is low rank, the optimal \mathbf{v}_i is in the null space of \mathbf{Q}_i^{DL} . Hence, $\mathbf{v}_i \in \mathcal{IA}^{UL}$.

Since both \mathbf{w}_i and \mathbf{v}_i stays within \mathcal{IA}^{DL} and \mathcal{IA}^{UL} , interference alignment is a convergence point of DBA in high SNR.

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