Linear Precoders for Multiuser MIMO for finite constellations and a simplified receiver structure under controlled interference

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***Abstract*—This paper is an extension to the earlier proposed idea of exploiting the discrete constellation alphabets in linear precoding for the downlink of multiuser (MU) MIMO. In the earlier work, it was shown that the transmission of controlled interference to the users bears the potential of exploiting the interference structure in improving the error resilience. In this paper we propose a linear precoding strategy for the downlink of multiuser MIMO on the idea of controlled interference. However under the controlled interference, intuitive optimal receiver is Maximum Likelihood (ML) detector which would require an exhaustive search of the controlled interferers. It would have exponential search complexity in the number of users and their constellation sizes. We further propose in this paper a simplified receiver structure which exploits the interference structure by ML detection but its complexity is equivalent to that of a receiver that confronts no interference (in two user case). Monte Carlo simulations show the improved performance of the proposed linear precoding strategy relative to the existing linear precoding schemes.**

I. Introduction

Spatial dimension surfacing from the usage of multiple antennas promises improved reliability, higher spectral efficiency and the spatial separation of users [1]. In single-user MIMO, the existing precoder design methods based on the spatial dimension can be classified into two groups: (i) diversity oriented designs and (ii) transmission rate oriented designs. The first group usually employs the pairwise error probability analysis technique to maximize the diversity order through the rank criterion. This approach can achieve the steepest asymptotic slope (highest diversity order) on the error probability versus SNR curve, however, it may not obtain the highest possible coding gain. The second group often utilizes the Gaussian-input channel capacity (ergodic capacity and/or outage capacity) as design criteria to optimize the precoders. For the case of multiuser (MU) MIMO broadcast channel, the scarcity of available bandwidth has led the focus of research to the second group of encoders which try to achieve spatial separation of users for maximizing the system capacity based on the Gaussian assumption of alphabets. However, Gaussian inputs are too idealistic to be implemented in practical communication systems, and replacing Gaussian inputs by realistic discrete-constellation inputs for the designed precoders will often lead to significant performance degradation.

For transmission in single-user MIMO Gaussian channel with the transmitter knowing the transfer coefficients between the antennas, optimal strategy for maximizing mutual information in the case of Gaussian assumption for alphabets is the conversion of cross-coupled matrix channels into parallel noninteracting channels by precoders and receive filters. Power is allocated on these parallel channels by classic waterfilling [1]. Although Gaussian inputs are optimum from a mutual information standpoint, they are too idealistic to be implemented in practical communication systems. The reason for this assumption is the convenience in mathematics to derive the elegant capacity formula [1]. For finite discrete inputs, optimal transmission strategy engulfs mercury-waterfilling [2] under the condition of noninteracting channels. The basic idea is not to allocate further power to a channel which is already close to saturation as the maximum mutual information of an M-ary constellation can not exceed $\log_2 M$. However if the noninteracting condition is removed, then the optimal transmission strategy involves precoders that eventually result in cross-coupled effective channels [3] leading to joint detection at the receivers.

For transmission in MU MIMO Gaussian broadcast channel, optimal precoding involves a theoretical pre-interference cancellation technique known as dirty paper coding (DPC) [4]. Due to highly nonlinear nature of signal processing involved in DPC, its practical implementation is far from realizable. Moreover its optimality is constrained to idealistic Gaussian alphabets.

Linear precoding provides an alternative approach for transmission in MU MIMO Gaussian broadcast channel, trading off a reduction in precoder complexity for suboptimal performance. Orthogonalization based schemes use channel inversion (CI) and block diagonalization (BD) to transform the MU downlink into parallel single user systems [5]. However, the performance of CI is degraded with a small number of users when the channel is ill-conditioned. This is because inverting a poorly conditioned matrix unavoidably results in the reduction of effective channel gain which is more prominent in the case of low SNR [6]. To overcome the drawbacks of ZF, channel inversion regularization (CIR) precoding is proposed in [7] which adds a multiple of the identity matrix before channel inversion. In spite of introducing some crosstalk interference from other users, the CIR scheme can effectively increase the sum rate by alleviating the reduction in effective channel gain. Though optimum linear precoders [8] and optimum unitary linear precoders [9] for MU MIMO Gaussian broadcast channel have also been derived in the literature but the complexity associated with their calculation makes them less attractive for practical systems.

Gaussian being the worst case interference, the recommended precoding strategies for such inputs [10] [11] are pre-interference subtraction (DPC), interference cancellation (CI) and interference attenuation (RCI). These strategies therefore lead to simplified receiver structures for the users which is considered as the foremost advantage of precoding but are void of exploiting the interference structure in mitigating its effect. In the real world, inputs must be drawn from discrete constellations (often with very limited peak-to-average ratios) which may significantly depart from Gaussian idealization. The authors in [12] showed by the sum rate analysis that these interferences (discrete constellations) unlike Gaussian case have structures that can be exploited in the detection process. Basing on this notion, the authors argued that the precoders may be designed to manage the interference in a way that this controlled interference can be exploited in the detection process at the receivers however they did not propose a precoding strategy.

In this paper, we have extended the work of [12] restricting ourselves to the case of linear precoders. Our contributions in this paper include the proposition of a linear precoding strategy based on the notion of controlled interference and the proposition of a simplified receiver structure which is able to exploit the structure of these controlled interferences. We confine ourselves to the case of 2 single antenna users assuming that 2 users have been scheduled per time slot. The notion of controlled interference in this context means that the only those number of interferers are transmitted which the user can exploit because of its receiver structure.

The definitions of some symbols and operators used in this paper are listed below.
x: Boldface small symbol represents vector.
H: Boldface capital symbol represents a matrix.
E: Denotes expectation.
$\mathcal{CN}$: A complex circularly symmetric Gaussian random variable.
$\mathbf{I}_n$: $n \times n$ identity matrix
$\mathrm{tr}\,(\mathbf{H})$: Trace of matrix **H**
$(.)_R$: Subscript R represents real part.
$(.)_I$: Subscript I represents imaginary part.
$|.|$: norm of scalar
$\|.\|$: norm of vector
$(.)^T$: Transpose
$(.)^*$: Conjugate
$(.)^\dagger$: Conjugate transpose
log: All logarithms are to the base 2.

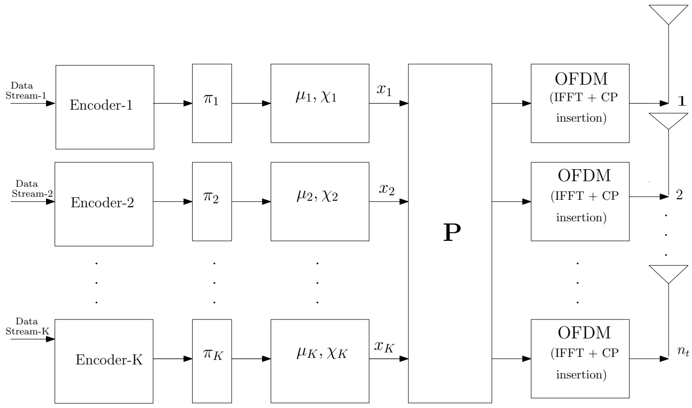

Fig. 1. Block diagram of the transmitter with n_t antennas. π_1 denotes the random interleaver, μ_1 the labeling map, χ_1 the signal set and x_1 the complex symbol for user-1.

II. System Model

Coherent with the next generation wireless systems as LTE [13] and IEEE 802.16m [14] which employ bit interleaved coded modulation (BICM) [15] with orthogonal frequency division multiplexing (OFDM) for downlink transmission, our system model is shown in fig.1. We consider the downlink of a wireless system with n_t transmit antennas at the transmitter while K users have one receive antenna each. We assume that one OFDM symbol has N subcarriers.

After encoding and interleaving, the output bits are mapped onto the tone $x_{k,n}$ using the signal map $\chi_k \subseteq \mathcal{C}$ with a Gray labeling map $\mu_k : \{0,1\}^{\log_2|\chi_k|} \to \chi_k$ where $k = 1, \cdots, K$ and n indicates the subcarrier. It is assumed that an appropriate length of cyclic prefix (CP) is used for each OFDM symbol. By doing so, OFDM converts downlink frequency selective channels into parallel flat fading channels denoted as $\mathbf{h}^\dagger_{k,n} \in \mathbb{C}^{n_t \times 1}$ where $\mathbb{C}^{n_t \times 1}$ denotes the n_t-dimensional complex space. We assume a spatially uncorrelated flat Rayleigh fading channel model so that the elements of $\mathbf{h}^\dagger_{k,n},\ (k = 1, 2 \cdots, K)$ can be modeled as independent identically distributed (i.i.d) zero mean circularly symmetric complex Gaussian (ZMCSCG) random variables with variance of 0.5 per dimension. Each symbol for each tone is then multiplied by the corresponding precoding vector $\mathbf{p}_{k,n}$. Following precoding, OFDM is applied at each transmit antenna.

User scheduling is beyond the scope of this paper rather we just assume that basing on some criteria, two users are scheduled per time slot. Without loss of generality, we assume that the first 2 users are scheduled. Let the precoder matrix be $\mathbf{P}_n = \left[\mathbf{p}_{1,n}\ \mathbf{p}_{2,n}\right]$. Cascading IFFT at the transmitter and FFT at the user with CP extension, transmission at the n-th frequency tone for the first user can be expressed as::-

$$y_{1,n} = \mathbf{h}^\dagger_{1,n}\mathbf{p}_{1,n}x_{1,n} + \mathbf{h}^\dagger_{1,n}\mathbf{p}_{2,n}x_{2,n} + z_{1,n}, \quad n = 1, 2, \cdots, N$$

where $y_{1,n}$ is the received symbol at user-1 and $z_{1,n}$ is ZMCSCG white noise of variance N_0. The complex symbols $x_{1,n}$ and $x_{2,n}$ are also assumed to be independent and of variances

σ_1^2 and σ_2^2 respectively. The transmitter is subjected to an average power constraint $\mathbb{E}\left\|\mathbf{p}_{1,n}x_{1,n}+\mathbf{p}_{2,n}x_{2,n}\right\|^2 \leq P$. This power constraint may be met (long term power constraint) by designing the precoder matrix as $\mathbf{P}/\sqrt{\mathrm{tr}\left(\mathbf{P}^\dagger\mathbf{P}\right)}$ and imposing the constraint that $\sigma_1^2=\sigma_2^2 \leq P$. This may also be met (short term power constraint) by making $\mathbf{p}_{1,n}$ and $\mathbf{p}_{2,n}$ unit norm vectors and imposing the constraint $\sigma_1^2+\sigma_2^2 \leq P$. We assume that the transmitter has perfect knowledge of channel state information of all users (perfect CSIT), and each user knows its own effective channel (scalar coefficient) and that of the other coscheduled user perfectly. This implies that user-1 has perfect knowledge of the coefficients $\mathbf{h}_{1,n}^\dagger\mathbf{p}_{1,n}$ and $\mathbf{h}_{1,n}^\dagger\mathbf{p}_{2,n}$. For channel estimation by the users, transmitter needs to transmit pilot symbols for the symbol intervals equal to the number of co-scheduled users (two). It would enable both the users not only to estimate their own coefficients but also the the coefficient of the other co-scheduled user. For notational convenience, we drop the frequency index for subsequent sections and rewrite the system equation as:-

$$y_1 = \alpha_1 x_1 + \beta_2 x_2 + z_1$$
$$y_2 = \beta_1 x_1 + \alpha_2 x_2 + z_2$$

where α is the effective channel of the desired signal and β is the effective channel of the interferer. We may also denote the concatenation of the channels by $\mathbf{H}^\dagger=[\mathbf{h}_1\ \mathbf{h}_2]$ so $\mathbf{H}$ is the $2\times n_t$ forward channel matrix with k-th row $\mathbf{h}_k^\dagger$ equal to the channel of the k-th user. Basing on this we can rewrite the system equation as

$$\mathbf{y}=\mathbf{HPx}+\mathbf{z} \tag{1}$$

where $\mathbf{x}=[x_1\ x_2]^T$ and $\mathbf{z}=[z_1\ z_2]^T$.

III. Channel Capacity Analysis

For the sake of clarity, we briefly overview the channel capacity analysis of [12]. Sum rate of the downlink channel is given as

$$I(\mathbf{P})=\mu I_1(\mathbf{P})+(1-\mu)I_2(\mathbf{P}) \qquad 0\leq\mu\leq 1 \tag{2}$$

where $I_1(\mathbf{P})$ and $I_2(\mathbf{P})$ are the mutual information of the first and second user respectively and μ is the parameter that defines the power distribution between the two users under the sum power constraint. The mutual information for the first user for finite size QAM constellation with $|\chi_1|=M_1$ takes the form as

$$I(Y;X_1|\alpha_1,\beta_2)=\log M_1-\frac{1}{M_1M_2N_zN_H}\sum_{x_1}\sum_{x_2}\sum_{\mathbf{H}}^{N_H}\sum_{z_1}^{N_z} \times\log\frac{\sum_{x_1'}\sum_{x_2'}\exp\left[-\frac{1}{N_0}\left|\alpha_1x_1+\beta_2x_2+z_1-\alpha_1x_1'-\beta_2x_2'\right|^2\right]}{\sum_{x_2'}\exp\left[-\frac{1}{N_0}\left|\beta_2x_2+z_1-\beta_2x_2'\right|^2\right]} \tag{3}$$

where N_z and N_H are the realizations of noise and channel $\mathbf{H}$ respectively. Mutual information for the user-2 can be calculated in the similar manner.

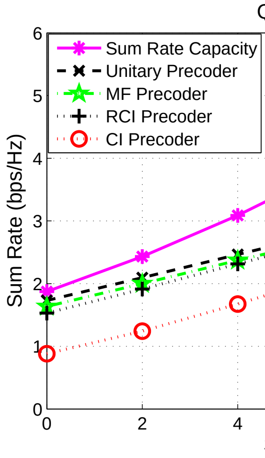

Fig. 2. Both users belong to QPSK constellations.

Fig. 2 shows the sum rate of a broadcast channel with 2 transmit antennas and 2 single antenna users for QPSK. Sum rate capacity (Gaussian broadcast channel) along with the sum rate of unitary, MF, RCI and CI precoders are shown. Power distribution between the two streams i.e. the factor μ is optimized to maximize the sum rate for MF, CI and unitary precoders. Sum rates in the low SNR regime with the unitary and MF based precoders dominate those of CI and CIR precoders which substantiate the argument that the controlled interference may be beneficial relative to canceling or attenuating the MU interference.

However controlled interference eclipses the main advantage of precoding and brings back the complexity to the receivers. We now propose a simplified receiver structure which on one hand exploits the interference structure in mitigating its effect and on the other is as complex as the receiver in interference free scenario.

IV. Receiver Structure

The max log MAP bit metric for bit b of x_1 is given as [15]

$$\begin{aligned}\lambda_1^i(y_1,b)\approx&\min_{x_1\in\chi_{1,b}^i,x_2\in\chi_2}\left|y_1-\alpha_1x_1-\beta_2x_2\right|^2\\=&\min_{x_1\in\chi_{1,b}^i,x_2\in\chi_2}\Big\{|y_1|^2+|\alpha_1x_1\|^2-2\left(y_1\alpha_1^*x_1^*\right)_R\\&+2\left(\rho_{12}x_1^*x_2\right)_R-2\left(y_1\beta_2^*x_2^*\right)_R+|\beta_2x_2|^2\Big\}\end{aligned} \tag{4}$$

where $\chi_{1,b}^i$ denotes the subset of the signal set $x_1\in\chi_1$ whose labels have the value $b\in\{0,1\}$ in the position i and $\rho_{12}=\alpha_1^*\beta_2$ is the cross correlation between the two coefficients. We introduce two terms as outputs of MF i.e. $y_{1,MF}=y_1\alpha_1^*$ and $y_{2,MF}=y_1\beta_2^*$. Breaking some of the terms in their real and imaginary parts, we have

$$\begin{aligned}\lambda_1^i(y_1,b)\approx&\min_{x_1\in\chi_{1,b}^i,x_2\in\chi_2}\Big\{|\alpha_1x_1|^2-2\left(y_{1,MF}x_1^*\right)_R\\&+2\left(p_{12,R}x_{1,R}+p_{12,I}x_{1,I}-y_{2,MF,R}\right)x_{2,R}+|\beta_2|^2x_{2,R}^2\\&+2\left(p_{12,R}x_{1,I}-p_{12,I}x_{1,R}-y_{2,MF,I}\right)x_{2,I}+|\beta_2|^2x_{2,I}^2\Big\}\end{aligned} \tag{5}$$

For x_2 belonging to the equal energy alphabets, the values of $x_{2,R}$ and $x_{2,I}$ which minimize (5) need to be in the opposite directions of $(p_{12,R}x_{1,R}+p_{12,I}x_{1,I}- y_{2,MF,R})$ and $(p_{12,R}x_{1,I}-p_{12,I}x_{1,R}- y_{2,MF,I})$ respectively thereby evading search on alphabets of x_2 and reducing one complex dimension of the system. The bit metric is therefore written as

$$\lambda_1^i(y_1, b) \approx \min_{x_1 \in \chi_{1,b}^i} \Big\{ |\alpha_1 x_1|^2 - 2\left(y_{1,MF}x_1^*\right)_R \\ -2\left|p_{12,R}x_{1,R}+p_{12,I}x_{1,I}- y_{2,MF,R}\right| |x_{2,R}| \\ -2\left|p_{12,R}x_{1,I}-p_{12,I}x_{1,R}- y_{2,MF,I}\right| |x_{2,I}| \Big\} \quad (6)$$

For non equal energy alphabets, it is the minimization problem of a quadratic function again trimming one complex dimension of the system. In that case, the real and imaginary parts of x_2 which minimizes (4) are given as

$$x_{2,R} \rightarrow -\frac{p_{12,R}x_{1,R}+p_{12,I}x_{1,I}-y_{2,MF,R}}{|\beta_2|^2}$$
$$x_{2,I} \rightarrow -\frac{p_{12,R}x_{1,I}-p_{12,I}x_{1,R}-y_{2,MF,I}}{|\beta_2||^2} \quad (7)$$

where $\rightarrow$ indicates the quantization process in which amongst the finite available points, the point closest to the calculated continuous value is selected.

The proposed receiver not only has the complexity equivalent to that of a receiver without interference but has the ability to exploit the structure of one interference in detecting the desired signal. Scheduling and precoding strategies would allow one interference to be transmitted in any dimension and this restrained transmission is termed as the controlled interference.

V. Simulation Results

Figs. 3 and 4 compare the performance of CI, MF (without power optimization), Unitary (based on QR-QL decomposition of the channel), RCI and MF optimized (with power optimization) precoder with the proposed receiver structure. We consider BICM MIMO OFDM based transmission from transmitter equipped with two antennas to the two single antenna users using the rate-$1/2$ punctured turbo code proposed for 3GPP LTE [13] [1] and the *de facto* standard, 64 state $(133, 171)$ rate-$1/2$ convolutional encoder of 802.11n standard [16]. We consider ideal OFDM system (no ISI) and analyze the system in the frequency domain. The channel has iid Gaussian matrix entries with unit variance and is independently generated for each time instant while perfect CSIT at the transmitter is assumed. Furthermore, all mappings of coded bits to QAM symbols use Gray encoding. We focus on frame error rates (FER) while the frame length is fixed to 1056 information bits.

Simulations show that the controlled interference improves the performance relative to attenuating or nulling the MU interference. The suboptimality of CI precoding at low SNRs is quite obvious while the change of slope of the curves for

[1]The LTE turbo decoder design was performed using the coded modulation library www.iterativesolutions.com

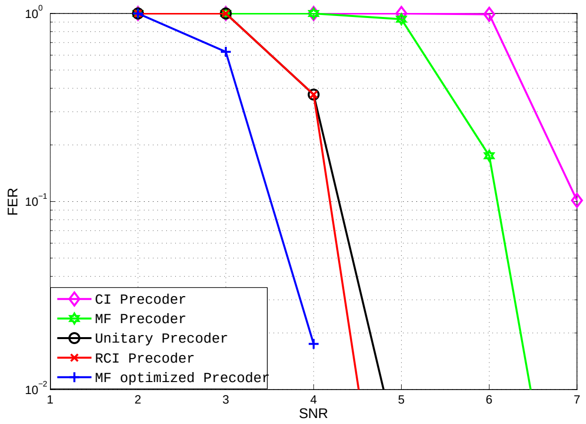

Fig. 3. Downlink channel with $n_t = 2$ and 2 single antenna users. 3GPP LTE punctured rate $1/2$ turbo code is used with maximum of 5 decoding iterations. Modulation alphabet is QPSK.

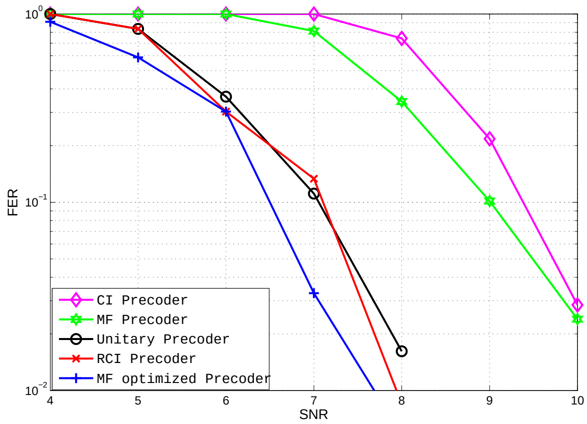

Fig. 4. Downlink channel with $n_t = 2$ and 2 single antenna users. 64 state rate $1/2$ convolutional code is used. Modulation alphabet is QPSK.

CI, RCI and unitary precoding illustrate improved diversity of unitary precoding relative to CI precoding. The improvement in performance can be attributed to the metric (5) which effectively exploits the structure of interference in decoding the desired signal. The relative performance of MF optimized precoder w.r.t CI and RCI precoders is expected to improve for higher order modulations. It is due to the fact that the suboptimality of CI and RCI widens as the constellation proliferates.

VI. Conclusion

In this paper we have shown that the controlled interference in MU linear precoding bears the potential of enhanced sum rate in the low SNR regime. To cater for the resultant additional receiver complexity in the presence of interference, we have proposed a low complexity receiver structure whose complexity in the presence of one interferer is equivalent to a receiver without interference. The proposed receiver on one hand reduces one complex dimension of the system and on the other is able to exploit the interference structure in the decoding process.

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