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Research Article

On Optimum End-to-End Distortion in MIMO Systems

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This paper presents the joint impact of the numbers of antennas, source-to-channel bandwidth ratio, and spatial correlation on the optimum expected end-to-end distortion in an outage-free MIMO system. In particular, based on an analytical expression valid for any SNR, a closed-form expression of the optimum asymptotic expected end-to-end distortion valid for high SNR is derived. It is comprised of the optimum distortion exponent and the multiplicative optimum distortion factor. Demonstrated by the simulation results, the analysis on the joint impact of the optimum distortion exponent and the optimum distortion factor explains the behavior of the optimum expected end-to-end distortion varying with the numbers of antennas, source-to-channel bandwidth ratio, and spatial correlation. It is also proved that as the correlation tends to zero, the optimum asymptotic expected end-to-end distortion in the setting of correlated channel approaches that in the setting of uncorrelated channel. The results in this paper could be performance objectives for analog-source transmission systems. To some extent, they are instructive for system design.

1. Introduction

1.1. Background. It is well known that the functional diagram and the basic elements of a digital communication system can be illustrated by Figure 1 [3]. The source can be either analog (continuous-amplitude) or digital (discrete-amplitude). Whichever is the source, there is always a tradeoff between the efficiency and the reliability. For transmitting a digital sequence, the tradeoff would be between the spectral efficiency (bit/s/Hz) [4] and the error probability. For transmitting a bandlimited analog source, under the assumption of a band-limited white Gaussian source, the tradeoff would be between the source-to-channel bandwidth ratio W_s/W_c (SCBR) [5] and the mean squared error (MSE) [6, 7], that is, the end-to-end distortion.

A point of distinction between digital-source transmission and analog-source transmission is as follows: in digital-source transmission, if the spectral efficiency (bit/s/Hz) is below the upper bound (channel capacity) subject to channel state and the transmitter knows the instantaneous channel state information (CSI) perfectly, the error probability would go to zero, whereas, in analog-source transmission, no matter how good the channel condition and the system are, the end-to-end distortion is nonvanishing, because the entropy of a continuous-amplitude source is infinite and thus the exact recovery of an analog source requires infinite channel capacity [6–9].

Regarding the end-to-end distortion, in [10, 11], Ziv and Zakai investigated the decay of MSE with SNR for the analog-source transmission over a noisy single-input single-output (SISO) channel without any channel knowledge on the transmitter side (CSIT). In [12, 13], Laneman et al. used the *distortion exponent* in the asymptotic expected distortion

$$\Delta \triangleq -\lim_{\rho\to\infty}\frac{\mathrm{ED}(\rho)}{\log\rho} \tag{1}$$

related to SCBR as a metric to compare different source-channel coding approaches for parallel channels. Note that ρ denotes the SNR and ED denotes the expected end-to-end distortion over all possible channel states. Choudhury and Gibson presented the relations between the end-to-end distortion and the outage capacity for AWGN channels [14]. Zoffoli et al. studied the characteristics of the distortions in

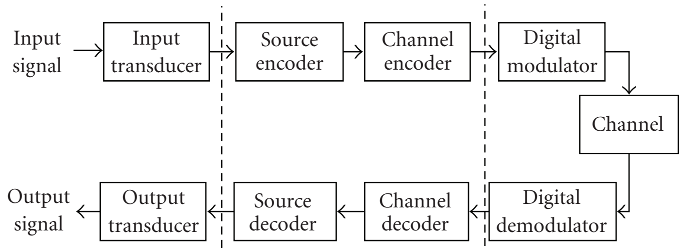


FIGURE 1: Basic elements of a digital communication system.

MIMO systems with different strategies, with and without CSIT [15, 16].

In [17–19], for tandem source-channel coding systems, assuming optimal block quantization and SNR-dependent rate-adaptive transmission as in [20], Holliday and Goldsmith investigated the expected end-to-end distortion for uncorrelated block-fading MIMO channels based on the results in [20–22]. They gave the following upper bound on the total expected distortion (MSE):

$$\mathrm{ED} \le 2^{-(2r/\eta)\log\rho+O(1)} + 2^{-(N_r-r)(N_t-r)\log\rho+o(\log\rho)}, \tag{2}$$

where η is the SCBR, r is the multiplexing gain (the source rate scales like $r\log\rho$), N_t is the number of transmit antennas, and N_r is the number of receive antennas. Considering the asymptotic high SNR regime, they proposed that the multiplexing gain r should satisfy

$$\Delta^*_{\mathrm{sep}} = (N_r - r)(N_t - r) = \frac{2r}{\eta} + o(1), \tag{3}$$

where Δ^*_{sep} is the optimum distortion exponent for tandem source-channel coding systems. The explicit expression of Δ^*_{sep} is given by Theorem 2 in [23]:

$$\begin{aligned}\Delta^*_{\mathrm{sep}}(\eta) &= \frac{2[jd^*(j-1)-(j-1)d^*(j)]}{2+\eta(d^*(j-1)-d^*(j))},\\ \eta &\in \left[\frac{2(j-1)}{d^*(j-1)}, \frac{2j}{d^*(j)}\right)\end{aligned} \tag{4}$$

for $j = 1,\ldots,N_{\min}$ with $N_{\min} = \min\{N_t, N_r\}$ and $d^*(j) = (N_t - j)(N_r - j)$. Note that a factor 2 appears here and there because the source is real whereas the channel is complex.

In [23, 24], assuming an uncorrelated block-fading MIMO channel, perfect CSIT and joint source-channel coding, Caire and Narayanan derived the *optimum distortion exponent:*

$$\Delta^*(\eta) = \sum_{i=1}^{N_{\min}} \min\left\{\frac{2}{\eta}, 2i-1+|N_t-N_r|\right\}, \tag{5}$$

which is larger than Δ^*_{sep}. Concurrently, the same result as (5) was also provided by Gunduz and Erkip [25, 26].

Caire-Narayanan's and Gunduz-Erkip's derivations are extensions to the outage probability analysis in [20]. They jointly considered the MIMO-channel mutual information in bits per channel use (bpcu) [27]:

$$\mathcal{I} = \log\left|\mathbf{I}_{N_t} + \frac{\rho}{N_t}\mathbf{H}\mathbf{H}^\dagger\right|, \tag{6}$$

where $\mathbf{H}$ is the $N_r \times N_t$ complex channel matrix with N_t inputs and N_r outputs, the rate-distortion function for a $\mathcal{N}(0,1)$ source [9]:

$$D(R_s) = 2^{-2R_s}, \tag{7}$$

where R_s is the source rate, and Shannon's rate-capacity inequality for outage-free transmission [7]:

$$R_s \le R_c. \tag{8}$$

1.2. Problem Statement. Nevertheless, there is something more than the distortion exponent in the expected end-to-end distortion. Intuitively, for high SNR, the form of the *asymptotic optimum expected end-to-end distortion* can be written as

$$\mathrm{ED}^*_{\mathrm{asy}} = \mu^*(\rho)\rho^{-\Delta^*}, \tag{9}$$

where the multiplicative *optimum distortion factor* $\mu^*(\rho)$ varies less than exponentially,

$$\lim_{\rho\to\infty}\frac{\log\mu^*(\rho)}{\log\rho} = 0. \tag{10}$$

For an analog-source transmission system, its performance at a high SNR could be measured via the asymptotic expected end-to-end distortion:

$$\mathrm{ED}_{\mathrm{asy}} = \mu(\rho)\rho^{-\Delta}, \tag{11}$$

where the distortion exponent Δ and the distortion factor $\mu(\rho)$ could be obtained analytically.

Obviously, we cannot say that a system achieves the optimum asymptotic expected distortion $\mathrm{ED}^*_{\mathrm{asy}}$ if what it achieves is only the optimum distortion exponent Δ^*. Also, we cannot say that in the regime of practical high SNR, the scheme with a larger distortion exponent must perform better than the other. As illustrated by Figure 2, in the regime of practical high SNR, the effect of the distortion factor must be taken into consideration. In other words, for practical cases, studying only the optimum distortion exponent is insufficient and giving the closed-form expression of $\mathrm{ED}^*_{\mathrm{asy}}$ is more meaningful. Using $\mathrm{ED}^*_{\mathrm{asy}}$ as an objective, via analyzing both Δ^* and $\mu^*(\rho)$, it is possible to design an analog-source transmission system performing better than the existing systems in the regime of practical high SNR.

For deriving $\mathrm{ED}^*_{\mathrm{asy}}$, if we could obtain the analytical expression of ED^* valid for any SNR, then it would be easy to find out the optimum distortion factor $\mu^*(\rho)$ and the optimum distortion exponent Δ^*.

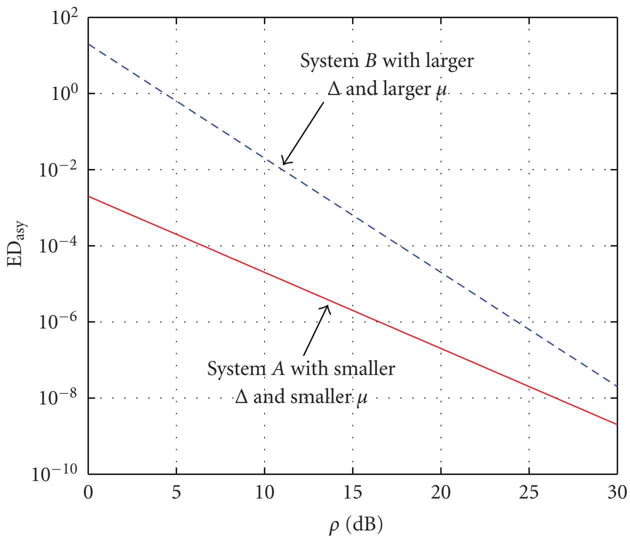


Figure 2: Impact of distortion factor.

1.3. Outline. In this paper, for the cases of spatially uncorrelated channel and correlated channel, we give an analytical expression of the optimum expected end-to-end distortion ED^* in an outage-free MIMO system valid for any SNR, based on which the optimum asymptotic expected end-to-end distortion $\mathrm{ED}^*_{\mathrm{asy}}$ is derived. The simulation results agree with our analysis with the derived results on the joint impact of the numbers of antennas, source-to-channel bandwidth ratio, and spatial correlation.

The remainder of this paper is organized as follows. The system model is given in Section 2. In Section 3, the preliminaries such as the mathematical definitions, properties, and lemmas are presented for deriving the main results in Section 4. Section 5 is dedicated to the simulation results, numerical analysis, and discussions. Finally, the contributions of this paper are concluded in Section 6, with our perspectives on future work.

Throughout the paper, vectors and matrices are denoted by bold characters, $|\mathbf{A}|$ denotes the determinant of matrix $\mathbf{A}$, and $\{a_{ij}\}_{i,j=1,\dots,N}$ is an $N \times N$ matrix with entries a_{ij}, $i, j = 1, \dots, N$. Also, $\mathbb{E}\{\cdot\}$ denotes expectation and, in particular, $\mathbb{E}_x\{\cdot\}$ denotes expectation over the random variable x. The superscript $\dagger$ denotes conjugate transpose. $(a)_n$ denotes $\Gamma(a+n)/\Gamma(a)$. log refers to the logarithm with base 2. Parts of the work in this paper have been presented in [1, 2].

2. MIMO System Model

Assume that a continuous-time white Gaussian source $s(t)$ of bandwidth W_s and source power P_s is to be transmitted over a flat block-fading MIMO channel of bandwidth W_c and the system is working on "short" frames due to strict time delay constraint, that is, no time diversity can be exploited. The transmission system is supposed to be free of outage, for example, the transmitter knows the instantaneous channel capacity by scalar feedback and does joint source-channel coding. Let $\hat{s}(t)$ denote the recovered source at the receiver.

Suppose that a K-to-$(N_t \times T)$ joint source-channel encoder is employed at the transmitter [23], which maps the source block $\mathbf{s}' \in \mathbb{R}^K$ onto channel codewords $\mathbf{X} \in \mathbb{C}^{N_t \times T}$. Herein, the source block $\mathbf{s}'$ is composed of K source samples, N_t is the number of transmit antennas, and T is the number of channel uses for transmitting one block. The corresponding source-channel decoder is a mapping $\mathbb{C}^{N_r \times T} \to \mathbb{R}^K$ that maps the channel output $\mathbf{Y} = \{\mathbf{y}_1, \dots, \mathbf{y}_T\}$ into an approximation $\hat{\mathbf{s}}'$. Assuming that the continuous-time source $s(t)$ is sampled by a Nyquist sampler, $2W_s$ samples per second, and the bandlimited MIMO channel is used as a discrete-time channel at $2W_c$ channel uses per second [9, pages 247–250], we have the SCBR

$$\eta = \frac{W_s}{W_c} = \frac{K}{T}. \tag{12}$$

At the tth channel use, the output of the discrete-time flat block-fading MIMO channel with N_t inputs and N_r outputs is

$$\mathbf{y}_t = \mathbf{H}\mathbf{x}_t + \mathbf{n}_t, \quad t = 1, \dots, T, \tag{13}$$

where $\mathbf{x}_t \in \mathbb{C}^{N_t}$ is the transmitted signal satisfying the long-term power constraint $\mathbb{E}[\mathbf{x}_t^H \mathbf{x}_t] = P$, $\mathbf{H} \in \mathbb{C}^{N_t \times N_t}$ is the channel matrix with entries h_{ij}'s distributed as $\mathcal{CN}(0,1)$, and $\mathbf{n}_t \in \mathbb{C}^{N_t}$ is the additive white noise vector with entries $\mathrm{n}_{t,i}$'s distributed as $\mathcal{CN}(0, \sigma_n^2)$. Note that the SNR per receive antenna is $\rho = P/\sigma_n^2$.

In the case of uncorrelated channel, the h_{ij}'s are independent to each other. In the case of receiver-side spatially correlated channel, we have the correlation matrix $\mathbf{\Sigma} = \mathbb{E}(\mathbf{H}\mathbf{H}^\dagger)$ which is assumed to be a full-rank matrix with distinct eigenvalues $\boldsymbol{\sigma} = \{\sigma_1, \sigma_2, \dots, \sigma_{N_{\min}}\}$, $0 < \sigma_1 < \sigma_2 < \cdots < \sigma_{N_{\min}}$. It can be seen that in the case of uncorrelated channel, $\mathbf{\Sigma}$ is an identity matrix with $\sigma_1 = \sigma_2 = \cdots = \sigma_{N_{\min}} = 1$.

3. Mathematical Preliminaries

The mathematical properties, definitions, and lemmas in this section will be used in the derivations for the main results.

3.1. Mathematical Properties and Definitions. We shall use the integral of an exponential function

$$\int_0^\infty e^{-px} x^{q-1} (1+ax)^{-\nu} dx = a^{-q} \Gamma(q) \Psi\left(q, q+1-\nu, \frac{p}{a}\right),$$
$$\Re\{q\} > 0, \quad \Re\{p\} > 0, \quad \Re\{a\} > 0 \tag{14}$$

as introduced in [28, page 365]. This involves the confluent hypergeometric function

$$\Psi(a, c; x) = \frac{1}{\Gamma(a)} \int_0^\infty e^{-xt} t^{a-1} (1+t)^{c-a-1} dt, \quad \Re\{a\} > 0, \tag{15}$$

which satisfies (with $y = \Psi$)

$$x \frac{d^2 y}{dx^2} + (c-x) \frac{dy}{dx} - ay = 0. \tag{16}$$

Table 1: $\Psi(a, c; x)$ for small x, real c.

c	Ψ
$c > 1$	$x^{1-c}\Gamma(c-1)/\Gamma(a) + o(x^{1-c})$
$c = 1$	$-[\Gamma(a)]^{-1}\log x + o(\lvert\log x\rvert)$
$c < 1$	$\Gamma(1-c)/\Gamma(a-c+1) + o(1)$

Bateman has given a thorough analysis on $\Psi(a, c; x)$ [29, pages 257–261]. In particular, he obtained the expressions on $\Psi(a, c; x)$ for small x as Table 1 shows. In Appendix A, we also state some of his more general results for any x, which we will use for the analysis in the case of spatially correlated MIMO channel.

3.2. Mathematical Lemmas. The proofs of the mathematical lemmas below can be found in Appendices B–H.

Lemma 1. *Define an $m \times m$ full-rank matrix $\mathbf{W}(x)$ whose (i, j)th entry is of the form $c_{ij}x^{\min\{a,i+j\}}$, $c_{ij} \neq 0$, $x, a \in \mathbb{R}^+$, $1 \leqslant i, j \leqslant m$. Then*

$$\lim_{x\to 0}\frac{\log|\mathbf{W}(x)|}{\log x} = \sum_{i=1}^{m}\min\{a, 2i\}. \tag{17}$$

Lemma 2. *Define an $m \times m$ Hankel matrix $\mathbf{W}(x)$ whose (i, j)th entry is of the form $c_{i+j}x^{i+j}$, $c_{i+j} \neq 0$, $x \in \mathbb{R}^+$, $1 \leqslant i, j \leqslant m$. Then, each summand in the determinant of $\mathbf{W}(x)$ has the same degree $m(m+1)$ over x.*

Lemma 3. *Define an $m \times m$ Hankel matrix $\mathbf{W}$ whose (i, j)th entry is $\Gamma(a+i+j-1)$, $1 \leqslant i, j \leqslant m$, $a \in \mathbb{R}$. Then*

$$|\mathbf{W}| = \prod_{k=1}^{m}\Gamma(k)\Gamma(a+k). \tag{18}$$

Lemma 4. *Define an $m \times m$ Hankel matrix $\mathbf{W}$ whose (i, j)th entry is $\Gamma(a+i+j-1)\Gamma(b-i-j+1)$ where $1 \leqslant i, j \leqslant m$, $m \geqslant 2$ and $a, b \in \mathbb{R}$. Then*

$$|\mathbf{W}| = \Gamma(a+1)\Gamma(b-1)\Gamma^{m-1}(a+b) \times \prod_{k=2}^{m}\Gamma(k)\Gamma(a+k)\frac{\Gamma(b-2k+2)\Gamma(b-2k+1)}{\Gamma(a+b-k+1)\Gamma(b-k+1)}. \tag{19}$$

Lemma 5. *Define an $m \times m$ Toeplitz matrix $\mathbf{W}$ whose (i, j)th entry is $\Gamma(a+i-j)$, $1 \leqslant i, j \leqslant m$, $a \in \mathbb{R}$. Then*

$$|\mathbf{W}| = (-1)^{m(m-1)/2}\prod_{k=1}^{m}\Gamma(k)\Gamma(a+k-m). \tag{20}$$

Lemma 6. *Define*

$$f(n) = \prod_{k=1}^{m}\frac{\Gamma(n-m-a+k)}{\Gamma(n-k+1)}, \quad g(n) = n^{am}f(n), \tag{21}$$

subject to $a \in \mathbb{R}^+$, $m, n \in \mathbb{Z}^+$, $n \geq m$, and $n-m+1 \geq a$. Then both $f(n)$ and $g(n)$ are monotonically decreasing.

Lemma 7. *Let $(a)_n$ denote $\Gamma(a+n)/\Gamma(a)$, $a \in \mathbb{R}$, $n \in \mathbb{Z}^+$. Then*

$$(a+1)_n = (-1)^n(-a-n)_n. \tag{22}$$

4. Main Results

4.1. Uncorrelated MIMO Channel

Theorem 1 (Optimum Expected Distortion over an Uncorrelated MIMO Channel). *Assume a continuous-time white Gaussian source $s(t)$ of bandwidth W_s and power P_s to be transmitted over an uncorrelated block-fading MIMO channel of bandwidth W_c. The optimum expected end-to-end distortion is*

$$\mathrm{ED}^*_{\mathrm{unc}}(\eta) = \frac{P_s|\mathbf{U}(\eta)|}{\prod_{k=1}^{N_{\min}}\Gamma(N_{\max}-k+1)\Gamma(N_{\min}-k+1)}, \tag{23}$$

where $\eta = W_s/W_c$ (SCBR), $N_{\min} = \min\{N_t, N_r\}$, $N_{\max} = \max\{N_t, N_r\}$, and $\mathbf{U}(\eta)$ is an $N_{\min} \times N_{\min}$ Hankel matrix whose (i, j)th entry is

$$u_{ij}(\eta) = \left(\frac{\rho}{N_t}\right)^{-d_{ij}}\Gamma\left(d_{ij}\right)\Psi\left(d_{ij}, d_{ij}+1-\frac{2}{\eta}; \frac{N_t}{\rho}\right), \tag{24}$$

where $d_{ij} = i+j+|N_t-N_r|-1$, $1 \leq i, j \leq N_{\min}$, and $\Psi(a, b; x)$ is the Ψ function (see [29, pages 257–261]). This theorem is valid for any SNR.

Proof. The source rate of the source $s(t)$ is

$$R_s = W_s\log\frac{P_s}{D}, \tag{25}$$

where D is the distortion (MSE) [6].

Under the assumption that the transmitter only knows the instantaneous channel capacity R_c, the covariance matrix of the transmitted vector $\mathbf{x}$ at the transmitter is taken to be a scaled identity matrix $P/N_t \cdot \mathbf{I}_{N_t}$. As stated in [27], the mutual information per MIMO channel use is

$$\mathcal{I}(\mathbf{x};\mathbf{y}) = \log\left|\mathbf{I}_{N_r} + \frac{\rho}{N_t}\mathbf{H}\mathbf{H}^\dagger\right|. \tag{26}$$

And as stated in [9, pages 248–250], a channel of bandwidth W_c can be represented by samples taken $1/2W_c$ seconds apart; that is, the channel is used at $2W_c$ channel uses per second as a discrete-time channel. Hence, the channel capacity (bit/second) is

$$R_c = 2W_c\mathcal{I} = 2W_c\log\left|\mathbf{I}_{N_r} + \frac{\rho}{N_t}\mathbf{H}\mathbf{H}^\dagger\right|. \tag{27}$$

Substituting (27) into Shannon's rate-capacity inequality

$$R_s \leq R_c, \tag{28}$$

we get the optimum end-to-end distortion

$$D^*(\eta) = P_s\left|\mathbf{I}_{N_r} + \frac{\rho}{N_t}\mathbf{H}\mathbf{H}^\dagger\right|^{-2/\eta}. \tag{29}$$

Thereby, the optimum expected end-to-end distortion is

$$\mathrm{ED}^*(\eta) = P_s \mathbb{E}_{\mathbf{H}} \left| \mathbf{I}_{N_r} + \frac{\rho}{N_t} \mathbf{H}\mathbf{H}^\dagger \right|^{-2/\eta}, \tag{30}$$

whose form is analogous to the moment generating function of capacity in [30]. By the mathematical results given by Chiani et al. [30] for the expectation over an uncorrelated MIMO Gaussian channel $\mathbf{H}$, we have

$$\mathrm{ED}^*_{\mathrm{unc}}(\eta) = P_s K |\mathbf{U}(\eta)|, \tag{31}$$

where $\mathbf{U}(\eta)$ is an $N_{\min} \times N_{\min}$ Hankel matrix with (i, j)th entry given by

$$u_{ij}(\eta) = \int_0^\infty x^{N_{\max}-N_{\min}+j+i-2} e^{-x} \left(1 + \frac{\rho}{N_t} x\right)^{-2/\eta} dx, \tag{32}$$

$$K = \frac{1}{\prod_{k=1}^{N_{\min}} \Gamma(N_{\max} - k + 1)\Gamma(N_{\min} - k + 1)}. \tag{33}$$

By the integral solution (14), (32) can be written in the analytic form

$$u_{ij}(\eta) = \left(\frac{\rho}{N_t}\right)^{-d_{ij}} \Gamma\left(d_{ij}\right) \Psi\left(d_{ij}, d_{ij} + 1 - \frac{2}{\eta}; \frac{N_t}{\rho}\right). \tag{34}$$

This concludes the proof of the theorem. □

Theorem 1 tells us that the analytical expression of $\mathrm{ED}^*_{\mathrm{unc}}$ is a polynomial in ρ^{-1}. Therefore, for high SNR, the optimum asymptotic expected end-to-end distortion is of the form

$$\mathrm{ED}^*_{\mathrm{asy,unc}} = \mu^*_{\mathrm{unc}}(\eta) \rho^{-\Delta^*_{\mathrm{unc}}(\eta)}, \tag{35}$$

where $\Delta^*_{\mathrm{unc}}(\eta)$ is the *optimum distortion exponent* satisfying

$$\Delta^*_{\mathrm{unc}}(\eta) = -\lim_{\rho \to \infty} \frac{\log \mathrm{ED}^*_{\mathrm{unc}}(\eta)}{\log \rho}, \tag{36}$$

and μ^*_{unc} is the accompanying *optimum distortion factor* satisfying

$$\lim_{\rho \to \infty} \frac{\log \mu^*_{\mathrm{unc}}(\eta)}{\log \rho} = 0. \tag{37}$$

Since $\mathrm{ED}^*_{\mathrm{unc}}$ is concave in the log-log scale and monotonically decreasing with SNR and $\mathrm{ED}^*_{\mathrm{asy,unc}}$ is the tangent of the curve $\mathrm{ED}^*_{\mathrm{unc}}$ at the point where SNR is infinitely high, we see that the asymptotic tangent line $\mathrm{ED}^*_{\mathrm{asy,unc}}$ is always above the curve $\mathrm{ED}^*_{\mathrm{unc}}$; that is, $\mathrm{ED}^*_{\mathrm{asy,unc}}$ is always worse than $\mathrm{ED}^*_{\mathrm{unc}}$.

The closed-form expressions of $\Delta^*_{\mathrm{unc}}(\eta)$ and $\mu^*_{\mathrm{unc}}(\eta)$ are given as follows.

Theorem 2 (Optimum Distortion Exponent over an Uncorrelated MIMO Channel). *The optimum distortion exponent is*

$$\Delta^*_{\mathrm{unc}}(\eta) = \sum_{k=1}^{N_{\min}} \min\left\{\frac{2}{\eta}, 2k - 1 + |N_t - N_r|\right\}. \tag{38}$$

Proof. This optimum distortion exponent appeared already in [23, 25]. However, a different proof is provided here.

Consider $u_{ij}(\eta)$ in Theorem 1. When ρ is large, N_t/ρ is small. We thus refer to Table 1 and see that, for high SNR, $u_{ij}(\eta)$ approaches $e_{ij}(\eta)\rho^{-\Delta_{ij}(\eta)}$ with

$$\begin{gathered} \Delta_{ij}(\eta) = \min\left\{\frac{2}{\eta}, i + j - 1 + |N_t - N_r|\right\}, \\ \lim_{\rho \to \infty} \frac{\log e_{ij}(\eta)}{\log \rho} = 0. \end{gathered} \tag{39}$$

Straightforwardly, in the regime of high SNR, the asymptotic form of $|\mathbf{U}(\eta)|$ can be represented by $|\mathbf{E}(\eta)|\rho^{-\Delta^*_{\mathrm{unc}}(\eta)}$ with

$$\lim_{\rho \to \infty} \frac{\log |\mathbf{E}(\eta)|}{\log \rho} = 0. \tag{40}$$

By Lemma 1, we obtain that

$$\Delta^*_{\mathrm{unc}}(\eta) = \sum_{k=1}^{N_{\min}} \min\left\{\frac{2}{\eta}, 2k - 1 + |N_t - N_r|\right\}. \tag{41}$$

This concludes the proof of this theorem. □

Theorem 3 (Optimum Distortion Factor over an Uncorrelated MIMO Channel). *Define two four-tuple functions $\kappa_l(\beta, t, m, n)$ and $\kappa_h(\beta, t, m, n)$ for $\beta \in \mathbb{R}^+$ and $t \in \{0, \mathbb{Z}^+\}$ as in* (42).

$$\begin{aligned} &\kappa_l(\beta, t, m, n) \\ &= \begin{cases} \Gamma(\beta)^{-1}\Gamma(n - m + 1)\Gamma(\beta - n + m - 1) \\ \quad \times \prod_{k=2}^{t} \Gamma(k)\Gamma(n - m + k) \\ \quad \times \prod_{k=2}^{t} \Gamma(\beta - n + m - 2k + 2) \\ \quad \times \prod_{k=2}^{t} \Gamma(\beta - n + m - 2k + 1) \\ \quad \times \prod_{k=2}^{t} \Gamma(\beta - n + m - k + 1)^{-1} \\ \quad \times \prod_{k=2}^{t} \Gamma(\beta - k + 1)^{-1}, & t > 1, \\ \Gamma(\beta)^{-1}\Gamma(n - m + 1)\Gamma(\beta - n + m - 1), & t = 1, \\ 1 & t = 0, \end{cases} \\ &\kappa_h(\beta, t, m, n) \\ &= \begin{cases} \prod_{k=1}^{t} \Gamma(k)\Gamma(n - m - \beta + k), & t > 0, \\ 1, & t = 0. \end{cases} \end{aligned} \tag{42}$$

The optimum distortion factor $\mu^*_{\mathrm{unc}}(\eta)$ is given as follows.

(1) For $2/\eta \in (0, |N_t - N_r| + 1)$, referred to as the high SCBR regime (HSCBR), the optimum distortion factor is

$$\mu^*_{\text{unc}}(\eta) = P_s N_t^{\Delta^*_{\text{unc}}} \frac{\kappa_h(2/\eta, N_{\min}, N_{\min}, N_{\max})}{\prod_{k=1}^{N_{\min}} \Gamma(N_{\max} - k + 1)\Gamma(N_{\min} - k + 1)}. \quad (43)$$

It decreases monotonically with $N_{\max}$.

(2) For $2/\eta \in (N_t + N_r - 1, +\infty)$, referred to as the low SCBR regime (LSCBR), the optimum distortion factor is

$$\mu^*_{\text{unc}}(\eta) = P_s N_t^{\Delta^*_{\text{unc}}} \frac{\kappa_l(2/\eta, N_{\min}, N_{\min}, N_{\max})}{\prod_{k=1}^{N_{\min}} \Gamma(N_{\max} - k + 1)\Gamma(N_{\min} - k + 1)}. \quad (44)$$

(3) For $2/\eta \in [|N_t - N_r| + 1, N_t + N_r - 1]$, referred to as the moderate SCBR regime (MSCBR), the optimum distortion factor is

$$\mu^*_{\text{unc}}(\eta) = \begin{cases} \kappa_l\left(\dfrac{2}{\eta}, l, N_{\min}, N_{\max}\right)\mathfrak{A}, & \mathfrak{B} \neq 0, \\ \kappa_l\left(\dfrac{2}{\eta}, l-1, N_{\min}, N_{\max}\right)\log\rho\,\mathfrak{A}, & \mathfrak{B} = 0 \end{cases} \quad (45)$$

where

$$\mathfrak{A} = \frac{P_s N_t^{\Delta^*_{\text{unc}}} \kappa_h(2/\eta - 2l, N_{\min} - l, N_{\min}, N_{\max})}{\prod_{k=1}^{N_{\min}} \Gamma(N_{\max} - k + 1)\Gamma(N_{\min} - k + 1)},$$
$$\mathfrak{B} = \mod\left\{\frac{2}{\eta} + 1 - |N_t - N_r|, 2\right\}, \quad (46)$$
$$l = \left\lfloor \frac{2/\eta + 1 - |N_t - N_r|}{2} \right\rfloor.$$

Proof. See Appendix I. □

4.2. Spatially Correlated MIMO Channel

Theorem 4 (Optimum Expected Distortion over a Correlated MIMO Channel). *The optimum expected end-to-end distortion in a system over a spatially correlated MIMO channel is*

$$\text{ED}^*_{\text{cor}}(\eta) = \frac{P_s\,|\,\mathbf{G}(\eta)\,|}{\prod_{k=1}^{N_{\min}} \sigma_k^{|N_t - N_r|+1}\Gamma(N_{\max} - k + 1)\prod_{1\le m<n\le N_{\min}}(\sigma_n - \sigma_m)}, \quad (47)$$

where $\mathbf{G}(\eta)$ is an $N_{\min} \times N_{\min}$ matrix whose (i, j)th entry is given by

$$g_{ij}(\eta) = \left(\frac{\rho}{N_t}\right)^{-d_j}\Gamma\left(d_j\right)\Psi\left(d_j, d_j + 1 - \frac{2}{\eta}; \frac{N_t}{\sigma_i\rho}\right), \quad (48)$$

$d_j = |N_t - N_r| + j$, $\boldsymbol{\sigma} = \{\sigma_1, \sigma_2, \ldots, \sigma_{N_{\min}}\}$ with $0 < \sigma_1 < \sigma_2 < \cdots < \sigma_{N_{\min}}$ denoting the ordered eigenvalues of the correlation matrix $\mathbf{\Sigma}$.

Proof. Following the proof of Theorem 1, by the mathematical results given by Chiani et al. in [30] for a spatially correlated $\mathbf{H}$, we have

$$\text{ED}^*_{\text{cor}}(\eta) = P_s K_{\mathbf{\Sigma}}\,|\,\mathbf{G}(\eta)\,|, \quad (49)$$

where $\mathbf{G}(\eta)$ is an $N_{\min} \times N_{\min}$ matrix with (i, j)th entry given by

$$g_{ij}(\eta) = \int_0^\infty x^{|N_t - N_r|+j-1} e^{-x/\sigma_i}\left(1 + \frac{\rho}{N_t}x\right)^{-2/\eta} dx, \quad (50)$$

$$K_{\mathbf{\Sigma}} = \frac{|\mathbf{\Sigma}|^{-N_{\max}}}{|\mathbf{V}_2(\boldsymbol{\sigma})|\prod_{k=1}^{N_{\min}}\Gamma(N_{\max} - k + 1)}, \quad (51)$$

where $\mathbf{V}_2(\sigma)$ is a Vandermonde matrix given by

$$\mathbf{V}_2(\boldsymbol{\sigma}) \triangleq \mathbf{V}_1\left(-\left\{\sigma_1^{-1}, \ldots, \sigma_{N_{\min}}^{-1}\right\}\right) \quad (52)$$

with the Vandermonde matrix $\mathbf{V}_1(\mathbf{x})$ defined as

$$\mathbf{V}_1(\mathbf{x}) \triangleq \begin{pmatrix} 1 & 1 & \cdots & 1 \\ x_1 & x_2 & \cdots & x_{N_{\min}} \\ \vdots & \vdots & \ddots & \vdots \\ x_1^{N_{\min}-1} & x_2^{N_{\min}-1} & \cdots & x_{N_{\min}}^{N_{\min}-1} \end{pmatrix}. \quad (53)$$

In terms of the property of a Vandermonde matrix [31], the determinant of $\mathbf{V}_2(\boldsymbol{\sigma})$

$$\begin{aligned} |\mathbf{V}_2(\boldsymbol{\sigma})| &= \prod_{1\le m<n\le N_{\min}}\left(-\sigma_j^{-1} + \sigma_i^{-1}\right) \\ &= \prod_{1\le m<n\le N_{\min}} \sigma_m^{-1}\sigma_n^{-1}(\sigma_n - \sigma_m) \\ &= \prod_{k=1}^{N_{\min}} \sigma_k^{1-N_{\min}} \prod_{1\le m<n\le N_{\min}} (\sigma_n - \sigma_m) \\ &= \prod_{k=1}^{N_{\min}} \sigma_k^{1-N_{\min}} |\mathbf{V}_1(\boldsymbol{\sigma})|. \end{aligned} \quad (54)$$

Thereby,

$$K_{\mathbf{\Sigma}} = \frac{1}{\prod_{k=1}^{N_{\min}} \sigma_k^{|N_t - N_r|+1}\Gamma(N_{\max} - k + 1)\prod_{1\le m<n\le N_{\min}}(\sigma_n - \sigma_m)}. \quad (55)$$

In terms of the integral solution (14), (50) can be written in the analytic form

$$g_{ij}(\eta) = \left(\frac{\rho}{N_t}\right)^{-d_j} \Gamma\left(d_j\right) \Psi\left(d_j, d_j + 1 - \frac{2}{\eta}; \frac{N_t}{\sigma_i \rho}\right). \tag{56}$$

This concludes the proof of this theorem. □

Theorem 5 (Optimum Distortion Exponent over a Correlated MIMO Channel). *The optimum distortion exponent Δ^*_{cor} in the case of spatially correlated MIMO channel is the same as the optimum distortion exponent Δ^*_{unc} in the case of uncorrelated MIMO channel, that is,*

$$\Delta^*_{\text{cor}}(\eta) = \Delta^*_{\text{unc}}(\eta) = \sum_{k=1}^{N_{\min}} \min\left\{\frac{2}{\eta}, 2k - 1 + |N_t - N_r|\right\}. \tag{57}$$

Proof. See Appendix J. □

Theorem 6 (Optimum Distortion Factor over a Correlated MIMO Channel). *The optimum distortion factor $\mu^*_{\text{cor}}(\eta)$ is given as follows.*

(1) *For $2/\eta \in (0, |N_t - N_r| + 1)$ (HSCBR), the optimum distortion factor is*

$$\mu^*_{\text{cor}}(\eta) = \prod_{k=1}^{N_{\min}} \sigma_k^{-2/\eta} \mu^*_{\text{unc}}(\eta). \tag{58}$$

(2) *For $2/\eta \in (N_t + N_r - 1, +\infty)$ (LSCBR), the optimum distortion factor is*

$$\mu^*_{\text{cor}}(\eta) = \prod_{k=1}^{N_{\min}} \sigma_k^{-N_{\max}} \mu^*_{\text{unc}}(\eta). \tag{59}$$

(3) *For $2/\eta \in [|N_t - N_r| + 1, N_t + N_r - 1]$ (MSCBR), the optimum distortion factor is*

$$\begin{aligned} \mu^*_{\text{cor}}(\eta) = {} & \frac{(-1)^{l(l-1)/2} |\mathbf{V}_3(\boldsymbol{\sigma})|}{\prod_{k=1}^{N_{\min}} \sigma_k^{|N_t - N_r|+1} \prod_{1 \le m < n \le N_{\min}} (\sigma_n - \sigma_m)} \\ & \times \prod_{k=1}^{N_{\min}-l} \frac{(k)_l}{(|N_t - N_r| - 2/\eta + l + k)_l} \mu^*_{\text{unc}}(\eta), \end{aligned} \tag{60}$$

where $l = \lfloor 2/\eta + 1 - |N_r - N_t|/2 \rfloor$ and each entry of $\mathbf{V}_3(\boldsymbol{\sigma})$ is

$$v_{3,ij} = \sigma_i^{-\min\{j-1, 2/\eta - d_j\}}. \tag{61}$$

Proof. See Appendix K. □

Theorem 7 (Convergence). *As the correlation degree goes to zero, the value of the optimum distortion factor in the setting of correlated channel converges to the value of the optimum distortion factor in the setting of uncorrelated channel,*

$$\lim_{\boldsymbol{\Sigma} \to \mathbf{I}} \mu^*_{\text{cor}}(\eta) = \mu^*_{\text{unc}}(\eta). \tag{62}$$

Proof. See Appendix L. □

5. Numerical Analysis and Discussion

In this section, the examples in various settings are provided. The simulation and numerical results illustrate the foregoing results.

5.1. An Example in the HSCBR Regime over an Uncorrelated MIMO Channel. Figure 3 shows the numerical and simulation results on the optimum expected end-to-end distortions in the outage-free MIMO systems over uncorrelated block-fading MIMO channels in the high SCBR regime and at the high SNR, $\rho = 30$ dB. The number of antennas on one side (either the transmitter side or the receiver side) is fixed to five and the number of antennas on the other side is varying. $\text{ED}^*_{\text{unc,sim}}$, represented by circles in Figure 3(b) denotes the ED^*_{unc} corresponding to (30), evaluated by 10 000 realizations of $\mathbf{H}$.

From Figure 3(b), we see that $\text{ED}^*_{\text{unc,sim}}$ monotonically decreases with the number of antennas on one side, which agrees with our intuition. There is an excellent agreement between $\text{ED}^*_{\text{unc,asy}}$, represented by the dash lines, and $\text{ED}^*_{\text{unc,sim}}$, which indicates that, in the setting when SNR is 30 dB, the behavior of ED^*_{unc} at a high SNR can be explained by studying $\text{ED}^*_{\text{unc,asy}}$.

In Figure 3(a), in terms of Theorem 2, the optimum distortion exponent Δ^*_{unc}, represented by the solid line with circles, increases with $N_{\min}$ and then remains constant when $N_{\min}$ stops increasing, though the number of antennas on one side is increasing. In Figure 3(b), in terms of Theorem 3, μ^*_{unc}, represented by the dot-dash lines, is monotonically decreasing with $N_{\max}$. Therefore, when $N_{\min} \le 5$, ED^*_{unc} is decreasing because Δ^*_{unc} is increasing; although the optimum distortion factor μ^*_{unc} is increasing, the increase of Δ^*_{unc} dominates the tendency of ED^*_{unc} since the SNR is high. When the $N_{\min}$ is fixed to 5, ED^*_{unc} is decreasing because μ^*_{unc} is decreasing, though Δ^*_{unc} keeps constant. In a word, we see that, for high SNR, the decrease of ED^*_{unc} with the number of antennas is due to either the increase of the optimum distortion exponent or the decrease of the optimum distortion factor.

Moreover, from Figure 3, it is seen that the commutation between the numbers of transmit and receive antennas impacts ED^*_{unc}. This impact comes from the effect on the optimum distortion factor μ^*_{unc}. As indicated by the expressions in Theorem 3 and shown in Figure 3(b), between a couple of commutative antenna allocation schemes, $(N_t = N_{\min}, N_r = N_{\max})$ and $(N_t = N_{\max}, N_r = N_{\min})$, the former scheme whose number of transmit antennas is the smaller between the two numbers of antennas suffers less

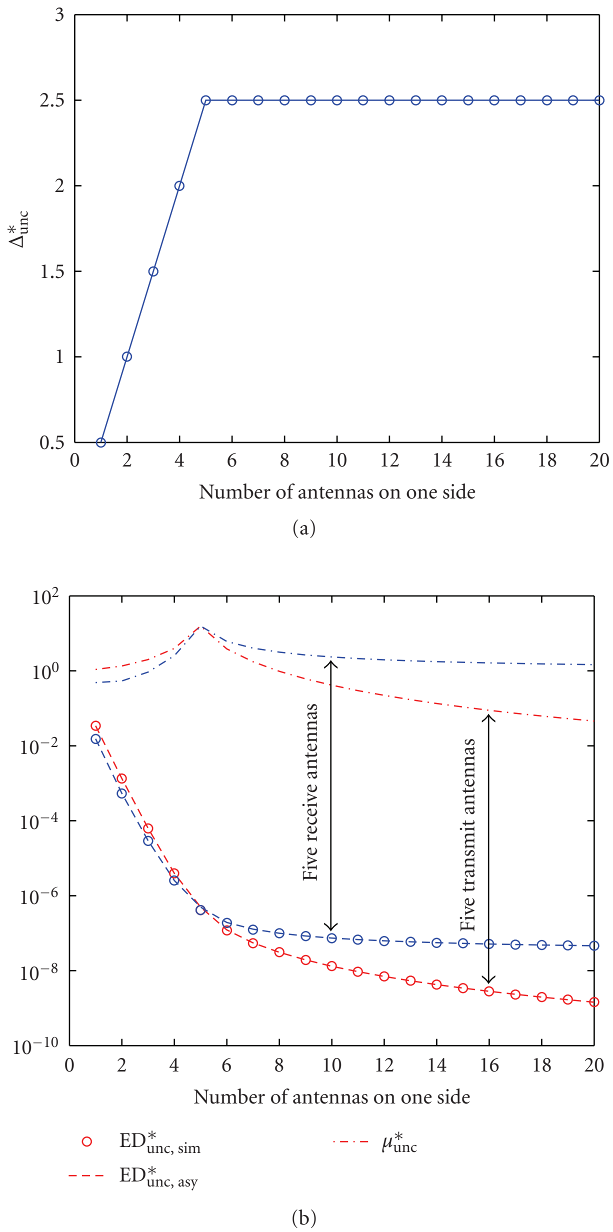


FIGURE 3: Uncorrelated channel, one of (N_t, N_r) is fixed to 5, $\eta = 4$, high SCBR.

distortion than the other. This is reasonable since under a certain total transmit power constraint, the scheme with fewer transmit antennas achieves higher average transmit power per transmit antenna.

5.2. An Example in the MSCBR Regime over an Uncorrelated MIMO Channel. In [15, 16], assuming a $\mathcal{N}(0,1)$ -distributed source and the system bandwidth is normalized to unity, Zoffoli et al. studied the characteristics of the distortions in 2×2 MIMO systems with different space-time coding strategies. In particular, in [16], assuming that the transmitter knows the instantaneous channel capacity and thus the system is free of outage, they compared the strategies with respect to expected distortion and the cumulative density function of distortion. They exhibited that, among REP (repetition), ALM (Alamouti), and SM (spatial multiplexing) strategies, the expected distortion of the ALM strategy is very close to that of the SM strategy.

As Zoffoli et al. derived [16], the expected distortion of the ALM strategy is

$$\text{ED}_{\text{ALM}} = \frac{2}{3} \cdot \frac{\rho[(\rho-4)\rho-4] + 4e^{2/\rho}(3\rho+2)\Gamma(0,2/\rho)}{\rho^5} \tag{63}$$

and the expected distortion of the SM strategy is

$$\begin{aligned} \text{ED}_{\text{SM}} &\\ &= 16\rho^{-6}\left[\rho - (\rho+2)e^{2/\rho}\Gamma(0,2/\rho)\right]^2 \\ &\quad + 8\rho^{-6}\left(\rho - 2e^{2/\rho}\Gamma(0,2/\rho)\right) \\ &\quad \times \left[\rho(\rho+2) - 4(\rho+1)e^{2/\rho}\Gamma(0,2/\rho)\right]. \end{aligned} \tag{64}$$

Note that $\Gamma(a,x)$ denotes the upper incomplete gamma function, $\Gamma(a,x) = \int_x^\infty t^{a-1}e^{-t}dt$. As given in [16], Figure 4(a) shows the difference between the expected distortions of the two strategies in log-lin scale. In log-lin scale, the expected distortion of the ALM strategy is very close to that of the SM strategy in the high SNR regime; that is, $\text{ED}_{\text{ALM}} - \text{ED}_{\text{SM}}$ is very small.

According to the assumption in [16], the SCBR of the systems is one, that is, $\eta = 1$. As $N_t = N_r = 2$, it is seen that, for the systems considered,

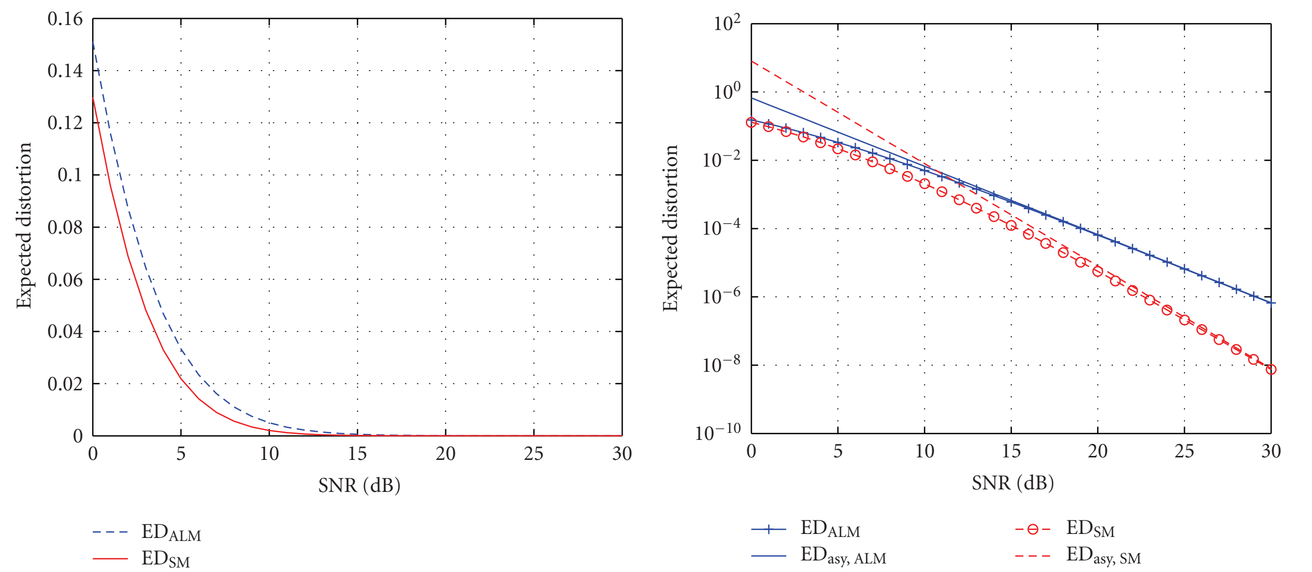
$$|N_t - N_r| + 1 < \frac{2}{\eta} < N_t + N_r - 1, \tag{65}$$

and thus the systems are in the moderate SCBR regime. From the description of SM strategy, it is seen that the expected distortion achieved by SM strategy is the optimum expected distortion for a 2×2 MIMO system with $\eta = 1$, that is, $\text{ED}_{\text{SM}} = \text{ED}^*_{\text{unc}}$. Regarding the asymptotic characteristics, from (63) and (64), we have

$$\begin{aligned} \text{ED}_{\text{asy,ALM}} &= \frac{2}{3}\rho^{-2}, \\ \text{ED}_{\text{asy,SM}} &= \text{ED}^*_{\text{asy,unc}} = 8\rho^{-3}. \end{aligned} \tag{66}$$

The ratio $\text{ED}_{\text{ALM}}/\text{ED}_{\text{SM}}$ is an alternative metric revealing the difference between ED_{ALM} and ED_{SM}, illustrated by Figure 4(b) in log-log scale. We see that in the high SNR regime, although ED_{ALM} approaches ED_{SM} in the linear scale as Figure 4(a) shows, the ratio $\text{ED}_{\text{ALM}}/\text{ED}_{\text{SM}}$ becomes larger and larger as Figure 4(b) shows. It can also be seen that the expected distortions of the ALM and SM strategies are determined by their asymptotic expressions when the SNR's are greater than 13 dB and 20 dB, respectively.

5.3. An Example in the LSCBR Regime over an Uncorrelated MIMO Channel. Figure 5 presents an example when



(a) Log-lin scale

(b) Log-log scale

FIGURE 4: ALM versus SM, uncorrelated channel, $N_t = N_r = 2$, $\eta = 1$, moderate SCBR.

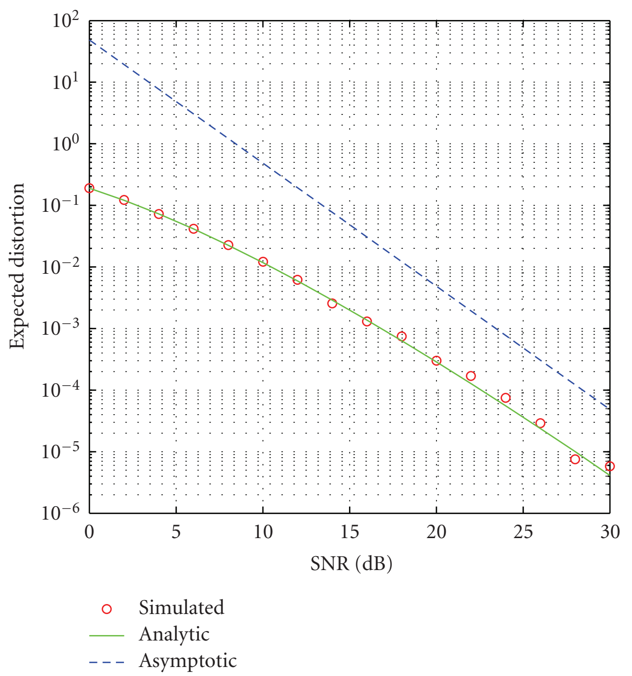


FIGURE 5: Uncorrelated channel, $N_t = 1$, $N_r = 2$, $\eta = 0.99$, low SCBR.

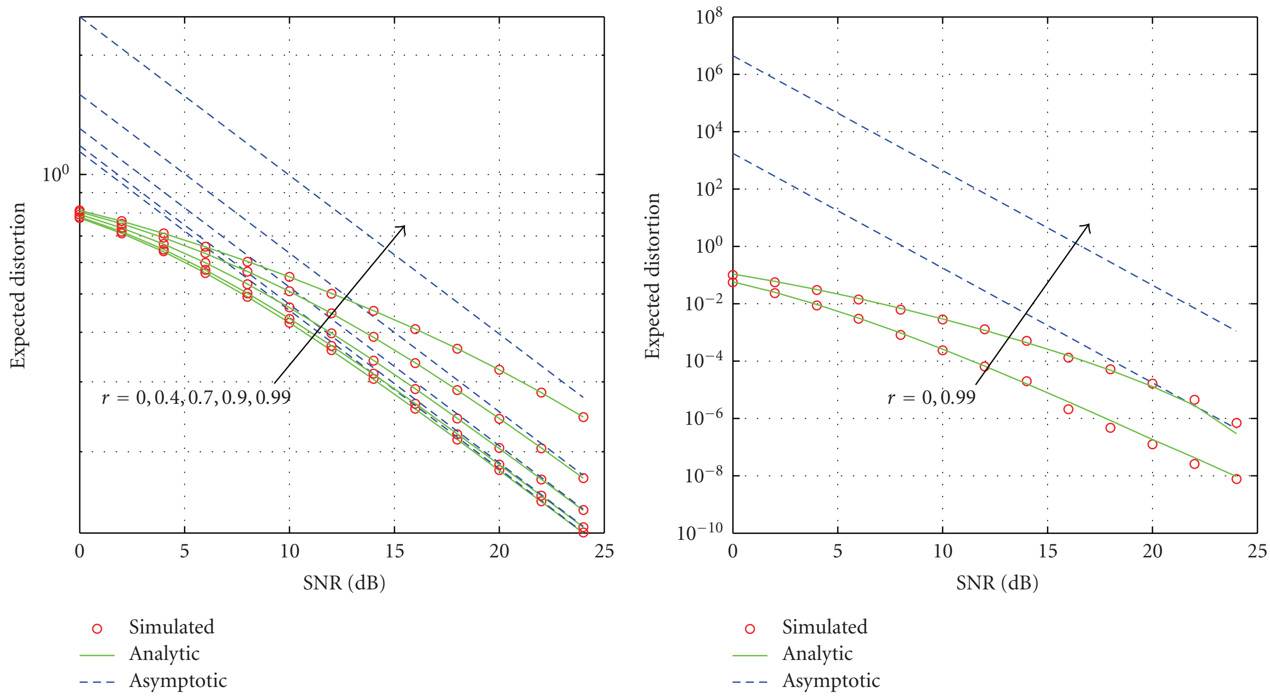
$N_t = 1$, $N_r = 2$ and $\eta = 0.99$. The red circles represent the results of Monte Carlo simulations which are carried out by generating 10 000 realizations of $\mathbf{H}$ and evaluating (30). The blue dash line represents $\mathrm{ED}^*_{\mathrm{asy,unc}}$. The green solid line represents the analytical expression of $\mathrm{ED}^*_{\mathrm{unc}}$ in Theorem 1. It can be seen that the simulated results agree well with our analytical results. The gap between the asymptotic tangent line and the curve of $\mathrm{ED}^*_{\mathrm{unc}}$ implies that, for the systems in the LSCBR regime, more terms in the polynomial of $\mathrm{ED}^*_{\mathrm{unc}}$ are to be analyzed, which is much more complicated than analyzing the asymptotic expression. It is a subject for future research.

5.4. Examples in HSCBR and LSCBR Regimes over a Spatially Correlated MIMO Channel. The analytical framework we derived is general and valid for all correlated cases with distinct (unrepeated) eigenvalues of the correlation matrix $\mathbf{\Sigma}$. To give an example, we consider a well-known correlation model as in [30]: the exponential correlation with $\mathbf{\Sigma} = \{r^{|i-j|}\}_{i,j=1,\ldots,N_r}$ and $r \in (0,1)$ [32].

Figure 6 illustrates the optimum expected end-to-end distortion ED^* on a power-one white Gaussian source transmitted in different correlation scenarios. Red circles represent the results of Monte Carlo simulations which are carried out by generating 10 000 realizations of $\mathbf{H}$ and evaluating (30). Green lines represent the analytical expressions of $\mathrm{ED}^*_{\mathrm{cor}}$ in Theorem 4 and $\mathrm{ED}^*_{\mathrm{unc}}$ in Theorem 1. Blue dashed lines represent the optimum asymptotic expected end-to-end distortion $\mathrm{ED}^*_{\mathrm{asy}}$:

$$\mathrm{ED}^*_{\mathrm{asy}} = \begin{cases} \mu^*_{\mathrm{unc}}\rho^{-\Delta^*_{\mathrm{unc}}}, & r = 0, \\ \mu^*_{\mathrm{cor}}\rho^{-\Delta^*_{\mathrm{cor}}}, & r > 0. \end{cases} \tag{67}$$

In Figure 6(a), we see that there is an agreement between ED^* and $\mathrm{ED}^*_{\mathrm{asy}}$ in the high SNR regime. Corresponding to Theorems 5 and 6, in the high SNR regime, due to the same optimum SNR distortion exponent, the optimum distortions of the systems in different correlation scenarios have the same descendent slopes; the difference comes from different distortion factors which depend on the correlation coefficients. The optimum distortion is increasing with r



(a) $N_t = 4, N_r = 2, \eta = 10$, high SCBR

(b) $N_t = 2, N_r = 2, \eta = 0.6657$, low SCBR

Figure 6: Expected distortions of uncorrelated and correlated channels.

and the line of the uncorrelated case ($r = 0$) is the lowest. For reaching the same optimum expected distortion, there is about 8 dB difference of SNR between the cases of $r = 0.99$ and $r = 0$. This agrees with our intuition that spatial correlation decreases channel capacity.

The impact of correlation can also be seen in Figure 6(b) by the example in the low SCBR regime. There are gaps between the asymptotic lines and the optimum expected distortions for the same reason as for the example in Section 5.3, that more terms in the polynomials are to be analyzed.

6. Conclusion and Future Work

6.1. Conclusion. In this paper, considering transmitting a white Gaussian source $s(t)$ over a MIMO channel in an outage-free system, we have derived the analytical expression of the optimum expected end-to-end distortion valid for any SNR (see Theorems 1 and 4) and the closed-form asymptotic expression of the optimum asymptotic expected end-to-end distortion (see Theorems 2, 3, 5, and 6) comprised of the optimum distortion exponent and the multiplicative optimum distortion factor. By the results on the optimum asymptotic expected end-to-end distortion, we have analyzed the joint impact of the numbers of antennas, source-to-channel bandwidth ratio (SCBR) and spatial correlation on the optimum expected end-to-end distortion. Straightforwardly, our results are bounds for outage-suffered systems and could be the performance objectives for analog-source transmission systems. To some extend, they are instructive for system design.

6.2. Future Work. (i) As we have shown in Figures 5 and 6(b), for a system in the low SCBR regime, there is an apparent gap between $\mathrm{ED}^*_{\mathrm{asy}}$ and ED^* in the practical high SNR regime. The reason that the gap exists is the effect of the other terms in the polynomial expansion of ED^*. Therefore, if the closed-form expression with more terms in the polynomial expansion of ED^* could be derived, the analysis on the behavior of ED^* would be more precise.

(ii) Let us provide an insight into Theorem 2. Define a nonnegative integer m as

$$m = \begin{cases} N_{\min}, & 0 < \dfrac{2}{\eta} < |N_t - N_r| + 1; \\ N_{\min} - \left\lfloor \dfrac{2/\eta + 1 - |N_t - N_r|}{2} \right\rfloor, & \\ & |N_t - N_r| + 1 \le \dfrac{2}{\eta} \le N_t + N_r - 1; \\ 0, & \dfrac{2}{\eta} > N_t + N_r - 1. \end{cases} \tag{68}$$

Then, (38) can be written in the form

$$\Delta^*(\eta) = (N_t - m)(N_r - m) + \frac{2m}{\eta}, \tag{69}$$

which looks analogous to the formula of the Diversity-Multiplexing Tradeoff (DMT) [20] and to the expression of the distortion exponent (3) in tandem source-channel coding systems [19]. Note that (69) has nothing to do with outage since the instantaneous channel capacity is assumed to be known at the transmitter. This intriguing similarity induces us to conjecture that there may be a hidden connection to be explored.

Appendices

A. Some Properties of $\Psi(a,c;x)$

(i) If c is not an integer,

$$\Psi(a,c;x) = \frac{\Gamma(1-c)}{\Gamma(a-c+1)}\Phi(a,c;x) + \frac{\Gamma(c-1)}{\Gamma(a)}x^{1-c}\Phi(a-c+1,2-c;x), \tag{A.1}$$

where $\Phi(a,c;x)$ is another confluent hypergeometric function:

$$\Phi(a,c;x) = \sum_{r=0}^{\infty}\frac{(a)_r}{(c)_r}\frac{x^r}{r!}. \tag{A.2}$$

Note that $(a)_n = \Gamma(a+n)/\Gamma(a)$.

(ii) If c is a positive integer,

$$\begin{aligned}\Psi(a,c;x) &= \frac{(-1)^{n-1}}{n!\Gamma(a-n)}\Bigg[\Phi(a,n+1;x)\log x + \sum_{r=0}^{\infty}\frac{(a)_r}{(n+1)_r} \\ &\qquad \times(\psi(a+r)-\psi(1+r)-\psi(1+n+r))\frac{x^r}{r!}\Bigg] \\ &\quad + \frac{(n-1)!}{\Gamma(a)}\sum_{r=0}^{n-1}\frac{(a-n)_r}{(1-n)_r}\frac{x^{r-n}}{r!} \qquad n=0,1,2,\dots.\end{aligned} \tag{A.3}$$

The last sum is to be omitted if $n = 0$:

(iii)

$$\Psi(a,c;x) = x^{1-c}\Psi(a-c+1,2-c;x). \tag{A.4}$$

Thus, when c is a nonpositive integer, we can obtain the form of $\Psi(a,c;x)$ from (A.3) and (A.4):

$$\begin{aligned}\Psi(a,c;x) &= \frac{(-1)^{-c}}{(1-c)!\Gamma(a)}\Bigg[\Phi(a+1-c,2-c;x)x^{1-c}\log x \\ &\qquad + \sum_{r=0}^{\infty}\frac{(a+1-c)_r}{(2-c)_r}(\psi(a+1-c+r)-\psi(1+r) \\ &\qquad -\psi(2-c+r))\frac{x^{r+1-c}}{r!}\Bigg] + \frac{\Gamma(1-c)}{\Gamma(a+1-c)}\sum_{r=0}^{-c}\frac{(a)_r}{(c)_r}\frac{x^r}{r!}.\end{aligned} \tag{A.5}$$

B. Proof of Lemma 1

We will prove this lemma recursively.

Define $p(n) = \min\{a,n\}$, subject to $a \in \mathbb{R}^+$ and $n \in \mathbb{Z}^+$. If $m_1 - m_2 = n_1 - n_2$, $m_1 > n_1$, and $m_2 > n_2$, then

$$p(m_1) - p(m_2) \le p(n_1) - p(n_2). \tag{B.1}$$

In the case that $m = 2$, by definition,

$$\mathbf{W}_2(x) = \begin{pmatrix} c_{11}x^{p(2)} & c_{12}x^{p(3)} \\ c_{21}x^{p(3)} & c_{22}x^{p(4)} \end{pmatrix}. \tag{B.2}$$

Then

$$|\mathbf{W}_2(x)| = c_{11}c_{22}x^{p(2)+p(4)} - c_{12}c_{21}^2x^{2p(3)}. \tag{B.3}$$

By (B.1),

$$p(2) + p(4) \le 2p(3). \tag{B.4}$$

Consequently, when $m = 2$,

$$\lim_{x\to 0}\frac{\log|\mathbf{W}_2(x)|}{\log x} = p(2)+p(4) = \sum_{i=1}^{2}\min\{a,2i\}. \tag{B.5}$$

Suppose when $m = k-1$, $k \in \mathbb{Z}^+ \cap [3,+\infty)$,

$$\lim_{x\to 0}\frac{\log|\mathbf{W}_{k-1}(x)|}{\log x} = \sum_{i=1}^{k-1}\min\{a,2i\}. \tag{B.6}$$

When $m = k$, $\mathbf{W}_k(x)$ can be written as

$$\begin{pmatrix} \mathbf{W}_{k-1}(x) & \mathbf{b}_k(x) \\ \mathbf{b}_k^T(x) & c_{kk}x^{p(2k)} \end{pmatrix}, \tag{B.7}$$

where the column vector is

$$\mathbf{b}_k(x) = \begin{pmatrix} c_{1k}x^{p(k+1)} \\ \vdots \\ c_{k-1,k}x^{p(2k-1)} \end{pmatrix}. \tag{B.8}$$

Hence, in terms of Schur determinant formula [31],

$$\begin{aligned}\lim_{x\to 0}\frac{\log|\mathbf{W}_k(x)|}{\log x} &= \lim_{x\to 0}\frac{\log\left[|\mathbf{W}_{k-1}(x)|\left|\mathbf{W}_{k-1}^*(x)\right|\right]}{\log x} \\ &= \lim_{x\to 0}\frac{\log|\mathbf{W}_{k-1}(x)|}{\log x} + \lim_{x\to 0}\frac{\log\det\mathbf{W}_{k-1}^*(x)}{\log x},\end{aligned} \tag{B.9}$$

where $\mathbf{W}_{k-1}^*(x)$ is the Schur complement of $\mathbf{W}_{k-1}(x)$,

$$\mathbf{W}_{k-1}^*(x) = c_{2k}x^{p(2k)} - \mathbf{b}_k^T(x)\mathbf{W}_{k-1}^{-1}(x)\mathbf{b}_k(x). \tag{B.10}$$

Since $\mathbf{W}_{k-1}(x)\mathbf{W}_{k-1}^{-1}(x) = \mathbf{I}$, $\mathbf{W}_{k-1}^{-1}(x)$ is of the form

$$\begin{pmatrix} c'_{11}x^{-p(2)} & \cdots & c'_{1k}x^{-p(k)} \\ \vdots & \ddots & \vdots \\ c'_{k1}x^{-p(k)} & \cdots & c'_{k-1,k-1}x^{-p(2k-2)} \end{pmatrix}. \tag{B.11}$$

Consequently,

$$\begin{aligned}\lim_{x\to 0}&\frac{\log\left[\mathbf{b}_k^T(x)\mathbf{W}_{k-1}^{-1}(x)\mathbf{b}_k(x)\right]}{\log x}\\ &= \min\{p(2k-1)-p(k)+p(k+1),\\ &\qquad p(2k-1)-p(k+1)+p(k+2),\dots,\\ &\qquad p(2k-1)-p(2k-2)+p(2k-1)\}\\ &\overset{(a)}{=} p(2k-1)-p(2k-2)+p(2k-1)\\ &\overset{(b)}{\geq} p(2k),\end{aligned} \tag{B.12}$$

where both steps (a) and (b) follow the inequality (B.1). Therefore, by (B.9) and (B.10),

$$\lim_{x\to 0}\frac{\log|\mathbf{W}(x)|}{\log x} = \sum_{i=1}^{k}\min\{a,2i\}, \tag{B.13}$$

which concludes this proof.

C. Proof of Lemma 2

Each summand in $|\mathbf{W}(x)|$, which is a product of the entries $w_{1j_1},\dots,w_{mj_m}$, can be written as

$$x^{\sum_{k=1}^{m}(k+j_k)}\prod_{k=1}^{m}c_{k+j_k}, \tag{C.1}$$

where the numbers $\{j_1, j_2,\dots, j_m\}$ is a permutation of $\{1,2,\dots,m\}$. Then, each summand has the same degree $m(m+1)$, which concludes the proof.

D. Proof of Lemma 3

By definition,

$$\mathbf{W} = \begin{pmatrix}\Gamma(a+1) & \cdots & \Gamma(a+m)\\ \vdots & \ddots & \vdots\\ \Gamma(a+m) & \cdots & \Gamma(a+2m-1)\end{pmatrix}. \tag{D.1}$$

For calculating the determinant of $\mathbf{W}$, we do Gaussian elimination by elementary row operations from bottom to top for obtaining the equivalent upper triangular $\mathbf{L}$ [33]. Below-diagonal entries are eliminated from the first column to the last column.

Let $\mathbf{W}_l$ denote the matrix after the below-diagonal entries of the lth column are eliminated. Then the (i,j)th entry of $\mathbf{W}_l$ subject to $i \geq j > l$ is of the form

$$w_{l,i,j} = \theta_{l,i,j}\Gamma(a+i+j-1-l). \tag{D.2}$$

Hence, after below-diagonal entries of the $(l-1)$th column are eliminated, for the entries subject to $i > l$ and $j = l$,

$$\begin{aligned} w_{l-1,i-1,l} &= \theta_{l-1,i-1,l}\Gamma(a+i-1),\\ w_{l-1,i,l} &= \theta_{l-1,i,l}\Gamma(a+i).\end{aligned} \tag{D.3}$$

Consequently, for eliminating the (i,l)th multiplied entry of $\mathbf{W}_{l-1}$ to obtain $\mathbf{W}_l$, the factor for the row operation in the Gaussian elimination on the ith row

$$c_{l,i} = -\frac{\theta_{l-1,i,l}}{\theta_{l-1,i-1,l}}(a+i-1). \tag{D.4}$$

That is, $w_{l,i,j}$ is obtained as follows:

$$\begin{aligned} w_{l,i,j} &= w_{l-1,i,j} + c_{l,i}w_{l-1,i-1,j}\\ &= \Bigg[\theta_{l-1,i,j}(a+i+j-l-1)\\ &\qquad -\theta_{l-1,i-1,j}\frac{\theta_{l-1,i,l}}{\theta_{l-1,i-1,l}}(a+i-1)\Bigg]\\ &\qquad\times\Gamma(a+i+j-l-1).\end{aligned} \tag{D.5}$$

Comparing the RHS of the above equation to (D.2), we get

$$\begin{aligned}\theta_{l,i,j} &= \theta_{l-1,i,j}(a+i+j-l-1)\\ &\quad -\theta_{l-1,i-1,j}\frac{\theta_{l-1,i,l}}{\theta_{l-1,i-1,l}}(a+i-1).\end{aligned} \tag{D.6}$$

Before doing any operation on $\mathbf{W}$, $\theta_{0,i,j} = 1$. Then, by (D.6), we obtain $\theta_{1,i,j} = j-1$ and $\theta_{2,i,j} = \Gamma(j)/\Gamma(j-2)$. Supposing

$$\theta_{l,i,j} = \frac{\Gamma(j)}{\Gamma(j-l)}, \tag{D.7}$$

then by (D.6) we have

$$\theta_{l+1,i,j} = \frac{\Gamma(j)}{\Gamma(j-l-1)}. \tag{D.8}$$

Therefore, our conjecture is right. Hence,

$$\theta_{i-1,i,i} = \Gamma(i), \tag{D.9}$$

and the ith diagonal entry of $\mathbf{L}$ is

$$w_{i-1,i,i} = \Gamma(i)\Gamma(a+i). \tag{D.10}$$

Consequently,

$$|\mathbf{W}_m| = \prod_{k=1}^{m}\Gamma(k)\Gamma(a+k), \tag{D.11}$$

which concludes this proof.

E. Proof of Lemma 4

This proof is similar to Appendix D.

By definition,

$$\mathbf{W} = \mathbf{A}\cdot\mathbf{B} \tag{E.1}$$

where $\cdot$ denotes Hadamard product,

$$\mathbf{A} = \begin{pmatrix} \Gamma(a+1) & \cdots & \Gamma(a+m) \\ \vdots & \ddots & \vdots \\ \Gamma(a+m) & \cdots & \Gamma(a+2m-1) \end{pmatrix},$$
$$\mathbf{B} = \begin{pmatrix} \Gamma(b-1) & \cdots & \Gamma(b-m) \\ \vdots & \ddots & \vdots \\ \Gamma(b-m) & \cdots & \Gamma(b-2m+1) \end{pmatrix}. \tag{E.2}$$

The (i, j)th entry of $\mathbf{W}_l$ subject to $i \geq j > l$ is of the form

$$w_{l,i,j} = \theta_{l,i,j}\Gamma(a+i+j-1-l)\Gamma(b-i-j+1). \tag{E.3}$$

Consequently, the multiplied factor is

$$\begin{aligned} c_{l,i} &= -\frac{\theta_{l-1,i,l}(a+i-1)}{\theta_{l-1,i-1,l}(b-i-l+1)}, \\ w_{l,i,j} &= w_{l-1,i,j} + c_{l,i}\, w_{l-1,i-1,j} \\ &= \Bigg[\theta_{l-1,i,j}(a+i+j-l-1) \\ &\quad - \frac{\theta_{l-1,i-1,j}\,\theta_{l-1,i,l}(a+i-1)(b-i-j+1)}{\theta_{l-1,i-1,l}(b-i-l+1)}\Bigg] \\ &\quad \times \Gamma(a+i+j-l-1)\Gamma(b-i-j+1). \end{aligned} \tag{E.4}$$

Comparing the RHS of the above expression to (E.3), we get

$$\begin{aligned} \theta_{l,i,j} &= \theta_{l-1,i,j}(a+i+j-l-1) - \theta_{l-1,i-1,j} \\ &\quad \times \frac{\theta_{l-1,i,l}\,(a+i-1)(b-i-j+1)}{\theta_{l-1,i-1,l}(b-i-l+1)}. \end{aligned} \tag{E.5}$$

Before doing any operation on $\mathbf{W}$, $\theta_{0,i,j} = 1$. Then, by (E.5), we obtain

$$\begin{aligned} \theta_{1,i,j} &= \frac{(j-1)(a+b-1)}{(b-i)}, \\ \theta_{2,i,j} &= \frac{(j-1)(j-2)(a+b-1)(a+b-2)}{(b-i)(b-i-1)}. \end{aligned} \tag{E.6}$$

Supposing

$$\theta_{l,i,j} = \prod_{k=1}^{l} \frac{(j-k)(a+b-k)}{(b-i-l+k)}, \tag{E.7}$$

then by (E.5) we have

$$\theta_{l+1,i,j} = \prod_{k=1}^{l+1} \frac{(j-k)(a+b-k)}{(b-i-l+k)}. \tag{E.8}$$

Therefore, our conjecture is right. Hence, for $i \geq 2$, the ith diagonal entry of the equivalent upper triangular $\mathbf{L}$,

$$\begin{aligned} w_{i-1,i,i} &= \Gamma(a+b)\Gamma(i)\Gamma(a+i) \\ &\quad \times \frac{\Gamma(b-2i+2)\Gamma(b-2i+1)}{\Gamma(a+b-i+1)\Gamma(b-i+1)}. \end{aligned} \tag{E.9}$$

Consequently,

$$\begin{aligned} |\mathbf{W}| &= \Gamma(a+1)\Gamma(b-1)\Gamma^{m-1}(a+b) \\ &\quad \times \prod_{k=2}^{m}\Gamma(k)\Gamma(a+k)\frac{\Gamma(b-2k+2)\Gamma(b-2k+1)}{\Gamma(a+b-k+1)\Gamma(b-k+1)}, \end{aligned} \tag{E.10}$$

which concludes this proof.

F. Proof of Lemma 5

The derivation of Lemma 5 is analogous to Appendix D. However, for deriving Lemma 5, we use Gaussian elimination by column operations from the right to the left, instead of row operations from the bottom to the top in Appendix D. After the Gaussian elimination, the left upper-diagonal triangle-matrix becomes a zero triangle-matrix. Consequently, the determinant of $\mathbf{W}$ is

$$|\mathbf{W}| = (-1)^{m(m-1)/2}\prod_{k=1}^{m}\Gamma(k)\Gamma(a+k-m). \tag{F.1}$$

G. Proof of Lemma 6

$f(n)$ can be written as

$$f(n) = \frac{\Gamma(n-a)}{\Gamma(n)} \cdots \frac{\Gamma(n-m+1-a)}{\Gamma(n-m+1)}. \tag{G.1}$$

We thus have

$$f(n+1) - f(n) = \left(\frac{n-a}{n} \cdots \frac{n-m+1-a}{n-m+1} - 1\right) f(n). \tag{G.2}$$

It is seen that $(n-a)/n \cdots (n-m+1-a)/(n-m+1) < 1$ and $f(n) > 0$. Hence, $f(n+1) - f(n) < 0$; that is, $f(n)$ is monotonically decreasing.

For $g(n)$,

$$\begin{aligned} &g(n+1) - g(n) \\ &\quad = \left((n+1)^{am}\frac{n-a}{n} \cdots \frac{n-m+1-a}{n-m+1} - n^{am}\right) f(n) \\ &\quad \leq \left[(n+1)^{am}\left(\frac{n-a}{n}\right)^{m} - n^{am}\right] f(n). \end{aligned} \tag{G.3}$$

If

$$(n+1)^{a} \cdot \frac{n-a}{n} < n^{a}, \tag{G.4}$$

then we have $g(n+1) - g(n) < 0$.

Define a function $h(x)$:

$$h(x) = (x-a)(x+1)^a - x^{a+1} = (x+1)^{a+1} - x^{a+1} - (a+1)(x+1)^a, \quad x > a. \tag{G.5}$$

In terms of mean value theory [34], for $\phi(x) = x^{a+1}$, there exists ξ which lets

$$\phi'(\xi) = (x+1)^{a+1} - x^{a+1}, \quad x < \xi < x+1. \tag{G.6}$$

where $\phi'(\xi)$ is the first derivative.

As

$$\phi''(x) = a(a+1)x^{a-1} > 0, \tag{G.7}$$

$\phi'(x)$ is monotonically increasing and thus

$$\phi'(\xi) < \phi'(x+1). \tag{G.8}$$

So, $h(x) < 0$.

Then, we have

$$\frac{x-a}{x} < \left(\frac{x}{x+1}\right)^a. \tag{G.9}$$

When $x = n$,

$$(n+1)^a \frac{n-a}{n} < n^a. \tag{G.10}$$

Consequently, $g(n+1) - g(n) < 0$, that is, $g(n)$ is monotonically decreasing.

H. Proof of Lemma 7

In terms of Euler's reflection formula

$$\Gamma(1-x)\Gamma(x) = \frac{\pi}{\sin(\pi x)},$$

$$\Gamma(a+n+1)\Gamma(-a-n) = \frac{\pi}{\sin(\pi(a+n+1))}, \tag{H.1}$$

$$\Gamma(a+1)\Gamma(-a) = \frac{\pi}{\sin(\pi(a+1))}.$$

Straightforwardly,

$$\frac{\Gamma(a+n+1)}{\Gamma(a+1)} = (-1)^n \frac{\Gamma(-a)}{\Gamma(-a-n)}, \tag{H.2}$$

that is,

$$(a+1)_n = (-1)^n(-a-n)_n. \tag{H.3}$$

I. Proof of Theorem 3

From the proof of Theorem 2, we see that

$$\mu^*_{\text{unc}}(\eta) = \frac{P_s|\mathbf{E}(\eta)|}{\prod_{k=1}^{N_{\min}} \Gamma(N_{\max}-k+1)\Gamma(N_{\min}-k+1)}, \tag{I.1}$$

where $\mathbf{E}(\eta)$ is an $N_{\min} \times N_{\min}$ matrix of $e_{ij}(\eta)$'s.

(1) When $2/\eta \in (0, |N_t - N_r| + 1)$, given by (24) and Table 1, we have

$$e_{ij}(\eta) = N_t^{2/\eta}\Gamma\left(d_{ij} - \frac{2}{\eta}\right). \tag{I.2}$$

By Lemma 3,

$$|\mathbf{E}(\eta)| = N_t^{\Delta^*_{\text{unc}}}\kappa_h\left(\frac{2}{\eta}, N_{\min}, N_{\min}, N_{\max}\right). \tag{I.3}$$

In this case, $\Delta^*_{\text{unc}}(\eta) = 2N_{\min}/\eta$. Substituting (I.3) into (I.1), we obtain the optimum distortion factor in this case in the closed form

$$\mu^*_{\text{unc}}(\eta) = P_s N_t^{\Delta^*_{\text{unc}}} \frac{\kappa_h(2/\eta, N_{\min}, N_{\min}, N_{\max})}{\prod_{k=1}^{N_{\min}} \Gamma(N_{\max}-k+1)\Gamma(N_{\min}-k+1)}. \tag{I.4}$$

In the light of Lemma 6, it monotonically decreases with $N_{\max}$.

(2) When $2/\eta \in (N_t + N_r - 1, \infty)$, in terms of (24) and Table 1, we have

$$e_{ij}(\eta) = N_t^{d_{ij}}\Gamma\left(d_{ij}\right)\frac{\Gamma\left(2/\eta - d_{ij}\right)}{\Gamma((2/\eta))}. \tag{I.5}$$

In terms of Lemmas 2 and 4, the determinant of $\mathbf{E}(\eta)$ is

$$|\mathbf{E}(\eta)| = N_t^{\Delta^*_{\text{unc}}}\kappa_l\left(\frac{2}{\eta}, N_{\min}, N_{\min}, N_{\max}\right). \tag{I.6}$$

In this case, $\Delta^*_{\text{unc}}(\eta) = N_t N_r$. Substituting (I.6) into (I.1), we obtain the optimum distortion factor in this case in the form

$$\mu^*_{\text{unc}} = P_s N_t^{\Delta^*_{\text{unc}}} \frac{\kappa_l(2/\eta, N_{\min}, N_{\min}, N_{\max})}{\prod_{k=1}^{N_{\min}} \Gamma(N_{\max}-k+1)\Gamma(N_{\min}-k+1)}. \tag{I.7}$$

(3) When $2/\eta \in [|N_t - N_r| + 1, N_t + N_r - 1]$, the analysis is relatively complex. Define a partition number

$$l = \left\lfloor \frac{2/\eta + 1 - |N_t - N_r|}{2} \right\rfloor \tag{I.8}$$

and partition the Hankel matrix $\mathbf{E}(\eta)$ in (23) as

$$\mathbf{E}(\eta) = \begin{pmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B}^T & \mathbf{C} \end{pmatrix}, \tag{I.9}$$

where $\mathbf{A}$ is the $l \times l$ submatrix and $\mathbf{C}$ is the $(N_{\min} - l) \times (N_{\min} - l)$ submatrix.

At high SNR, in terms of Table 1, when mod $(2/\eta + 1 - |N_t - N_r|, 2) \neq 0$, the entries of $\mathbf{A}$ and $\mathbf{C}$ approximate

$$\tilde{a}_{ij} = N_t^{d_{ij}}\Gamma\left(d_{ij}\right)\frac{\Gamma\left(2/\eta - d_{ij}\right)}{\Gamma(2/\eta)}\rho^{-d_{ij}}, \tag{I.10}$$

$$\tilde{c}_{ij} = N_t^{2/\eta}\Gamma\left(d_{ij} - \frac{2}{\eta}\right)\rho^{-2/\eta}; \tag{I.11}$$

when mod $(2/\eta+1-|N_t-N_r|,2)=0$, the form of $\tilde{c}_{ij}$ is the same as (I.11) whereas the form of $\tilde{a}_{ij}$ becomes

$$\tilde{a}_{ij}=\begin{cases} N_t^{d_{ij}}\Gamma\left(d_{ij}\right)\dfrac{\Gamma\left(2/\eta-d_{ij}\right)}{\Gamma(2/\eta)}\rho^{-d_{ij}}, & (i,j)\neq(l,l);\\ N_t^{2/\eta}\log\rho\rho^{-2/\eta}, & (i,j)=(l,l).\end{cases} \tag{I.12}$$

In terms of Schur determinant formula [31],

$$|\mathbf{E}(\eta)|=|\mathbf{A}|\,|\mathbf{C}-\mathbf{A}^*|, \tag{I.13}$$

where $\mathbf{A}^*=\mathbf{B}^T\mathbf{A}^{-1}\mathbf{B}$. By the method analogous to the derivation in Appendix B, we know that for high SNR

$$\mathbf{C}-\mathbf{A}^*\sim\tilde{\mathbf{C}}, \tag{I.14}$$

where $\tilde{\mathbf{C}}$ is composed of $\tilde{c}_{ij}$'s. Consequently,

$$|\mathbf{E}(\eta)|\sim\left|\tilde{\mathbf{A}}\right|\left|\tilde{\mathbf{C}}\right|. \tag{I.15}$$

Given the preceding derivation for high and low SCBR regimes, we have

$$\left|\tilde{\mathbf{A}}\right|=\begin{cases}\kappa_l\left(\dfrac{2}{\eta},l,N_{\min},N_{\max}\right)\mathfrak{C}, & \mathfrak{B}\neq 0;\\ \kappa_l\left(\dfrac{2}{\eta},l-1,N_{\min},N_{\max}\right)\log\rho\,\mathfrak{C}, & \mathfrak{B}=0,\end{cases} \tag{I.16}$$

$$\left|\tilde{\mathbf{C}}\right|=\kappa_h\left(\frac{2}{\eta}-2l,N_{\min}-l,N_{\min},N_{\max}\right)\mathfrak{D},$$

where

$$\mathfrak{B}=\mathrm{mod}\left\{\frac{2}{\eta}+1-|N_t-N_r|,2\right\},$$
$$\mathfrak{C}=\left(\frac{N_t}{\rho}\right)^{l(l+N_{\max}-N_{\min})}, \tag{I.17}$$
$$\mathfrak{D}=\left(\frac{N_t}{\rho}\right)^{2(N_{\min}-l)/\eta}.$$

Therefore, in this case,

$$\mu^*_{\mathrm{unc}}(\eta)=\begin{cases}\kappa_l\left(\dfrac{2}{\eta},l,N_{\min},N_{\max}\right)\mathfrak{A}, & \mathfrak{B}\neq 0,\\ \kappa_l\left(\dfrac{2}{\eta},l-1,N_{\min},N_{\max}\right)\log\rho\,\mathfrak{A}, & \mathfrak{B}=0\end{cases} \tag{I.18}$$

where

$$\mathfrak{A}=\frac{P_sN_t^{\Delta^*_{\mathrm{unc}}}\kappa_h(2/\eta-2l,N_{\min}-l,N_{\min},N_{\max})}{\prod_{k=1}^{N_{\min}}\Gamma(N_{\max}-k+1)\Gamma(N_{\min}-k+1)} \tag{I.19}$$

and the optimum distortion exponent is

$$\Delta^*_{\mathrm{unc}}(\eta)=l(l+|N_t-N_r|)+\frac{2(N_{\min}-l)}{\eta}. \tag{I.20}$$

This concludes the proof of this theorem.

J. Proof of Theorem 5

Let $\tilde{\mathbf{G}}$ denote the asymptotic form of $\mathbf{G}$ for high SNR. Since g_{ij} is a polynomial in ρ^{-1} given by (48) and the preliminaries in Section 3, in terms of Table 1, $|\tilde{\mathbf{G}}|$ can be written as $\sum_{m=1}^{M}|\tilde{\mathbf{G}}_m|$ where

$$\left|\tilde{\mathbf{G}}_m\right|=u_m\rho^{-\Delta^*_{\mathrm{cor}}}, \tag{J.1}$$

that is, they have the same degree over ρ^{-1}. Each entry of $\tilde{\mathbf{G}}_m$ is a monomial of ρ^{-1} denoted by $\tilde{g}_{m,ij}$. In terms of Table 1 and the preliminaries in Section 3, we learn that $\tilde{g}_{m,ij}$'s form is one of $\sigma_i^{-r_{m,j}}a(j,r_{m,j})\rho^{-(d_j+r_{m,j})}$ (Form 1) and $\sigma_i^{d_j-2/\eta}c_j\log^{\epsilon}\rho\rho^{-2/\eta}$ (Form 2), where $r_{m,j}$ is a nonnegative integer, $\epsilon=0,1$, and

$$a\left(j,r_{m,j}\right)=N_t^{d_j+r_{m,j}}\frac{\Gamma\left(2/\eta-d_j\right)\Gamma\left(d_j+r_{m,j}\right)}{\Gamma(2/\eta)\Gamma\left(r_{m,j}+1\right)\left(d_j+1-2/\eta\right)_{r_{m,j}}}, \tag{J.2}$$

$$c_j=N_t^{2/\eta}\Gamma\left(d_j-\frac{2}{\eta}\right). \tag{J.3}$$

If the entries of first l columns of $\tilde{\mathbf{G}}_m$ are of Form 1 and other entries are of Form 2, $\tilde{\mathbf{G}}_m$ can be partitioned as

$$\tilde{\mathbf{G}}_m=\left(\tilde{\mathbf{G}}_{m,1}\;\;\tilde{\mathbf{G}}_{m,2}\right), \tag{J.4}$$

where $\tilde{\mathbf{G}}_{m,1}$ is of size $N_{\min}\times l$ and $\tilde{\mathbf{G}}_{m,2}$ is of size $N_{\min}\times(N_{\min}-l)$. Since $\tilde{\mathbf{G}}_m$ is a full-rank matrix, $\tilde{\mathbf{G}}_{m,1}$ and $\tilde{\mathbf{G}}_{m,2}$ ought to be full rank as well. Apparently, $\tilde{\mathbf{G}}_{m,2}$ is a full-rank matrix; whereas, for $\tilde{\mathbf{G}}_{m,1}$, if there exist $r_{m,j_1}=r_{m,j_2}$ for $j_1\neq j_2$, $\tilde{\mathbf{G}}_{m,1}$ would not be full rank, because in that case, its submatrix constructed by the two columns with individual indices j_1 and j_2 would be rank-one. Thus, each $r_{m,j}$ must be distinct.

Now let us figure out l. Define a distortion exponent function as

$$\gamma(n)=\begin{cases}\displaystyle\sum_{k=1}^{n}d_k+\sum_{k=0}^{n-1}k+\frac{2(N_{\min}-n)}{\eta}, & n\in\mathbb{Z}\cap(0,N_{\min}];\\ \dfrac{2N_{\min}}{\eta}, & n=0.\end{cases} \tag{J.5}$$

Apparently, $\gamma(n)$ is on the curve of the two-order function $f(x)$,

$$f(x)=x^2+\left(|N_t-N_r|-\frac{2}{\eta}\right)x+\frac{2N_{\min}}{\eta}, \tag{J.6}$$

which is a symmetric convex function and whose minimum value is given by $x=(2/\eta-|N_t-N_r|)/2$.

Since $n=l$ gives the minimum $\gamma(n)$, when $2/\eta\in(0,|N_t-N_r|+1)$, $l=0$, $\Delta_{\mathrm{cor}}(\eta)=\gamma(0)=2N_{\min}/\eta$; when $2/\eta\in(N_t+N_r-1,+\infty)$, $l=N_{\min}$, $\Delta_{\mathrm{cor}}(\eta)=\gamma(N_{\min})=N_tN_r$.

When $\eta\in[|N_t-N_r|+1,N_t+N_r-1]$, we should have

$$\begin{aligned}\gamma(l)&\leq\gamma(l-1),\\ \gamma(l)&\leq\gamma(l+1),\end{aligned} \tag{J.7}$$

which gives

$$\frac{2}{\eta} - 1 - |N_t - N_r| \leq 2l \leq \frac{2}{\eta} + 1 - |N_t - N_r|. \quad \text{(J.8)}$$

Hence, for $\eta \in [|N_t - N_r| + 1, N_t + N_r - 1]$,

$$\begin{aligned} l &= \left\lfloor \frac{2/\eta + 1 - |N_t - N_r|}{2} \right\rfloor \\ &\quad \text{or} \left\lceil \frac{2/\eta - 1 - |N_t - N_r|}{2} \right\rceil, \\ \Delta^*_{\text{cor}}(\eta) &= \gamma(l) \\ &= l(l + |N_r - N_t|) + \frac{2(N_{\min} - l)}{\eta} \\ &= \sum_{k=1}^{N_{\min}} \min\left\{\frac{2}{\eta}, 2k - 1 + |N_t - N_r|\right\}. \end{aligned} \quad \text{(J.9)}$$

Note that $\gamma(\lfloor (2/\eta + 1 - |N_t - N_r|)/2 \rfloor) = \gamma(\lceil (2/\eta - 1 - |N_t - N_r|)/2 \rceil)$.

This concludes the proof of this theorem.

K. Proof of Theorem 6

From the proofs of Theorems 4 and 5, we have

$$\mu^*_{\text{cor}} = \frac{P_s |\mathbf{\Sigma}|^{-N_{\max}} \sum_{m=1}^{M} u_m}{\prod_{k=1}^{N_{\min}} \Gamma(N_{\max} - k + 1) |\mathbf{V}_2(\boldsymbol{\sigma})|}, \quad \text{(K.1)}$$

where u_m is defined in (J.1).

(1) Consider the case of $2/\eta \in (0, |N_t - N_r| + 1)$. We have $M = 1$ and

$$\widetilde{g}_{1,ij} = \sigma_i^{d_j - 2/\eta} c_j \rho^{-2/\eta}, \quad i = 1, \ldots, N_{\min}, \ j = 1, \ldots, N_{\min} \quad \text{(K.2)}$$

where d_j is defined in Theorem 4 and u_j is defined in (J.3). Thereby,

$$u_1 = N_t^{2N_{\min}/\eta} |\mathbf{V}_1(\boldsymbol{\sigma})| \prod_{j=1}^{N_{\min}} \Gamma\left(d_j - \frac{2}{\eta}\right) \prod_{i=1}^{N_{\min}} \sigma_i^{|N_t - N_r| + 1 - 2/\eta}. \quad \text{(K.3)}$$

So, in this case,

$$\begin{aligned} \mu^*_{\text{cor}}(\eta) &= \frac{|\mathbf{\Sigma}|^{-N_{\max}} |\mathbf{V}_1(\boldsymbol{\sigma})| \prod_{i=1}^{N_{\min}} \sigma_i^{|N_t - N_r| + 1 - (2/\eta)}}{|\mathbf{V}_2(\boldsymbol{\sigma})|} \\ &\quad \times \frac{P_s N_t^{2N_{\min}/\eta} \prod_{j=1}^{N_{\min}} \Gamma\left(d_j - 2/\eta\right)}{\prod_{k=1}^{N_{\min}} \Gamma(N_{\max} - k + 1)} \\ &= \prod_{k=1}^{N_{\min}} \sigma_k^{-2/\eta} \mu^*_{\text{unc}}(\eta). \end{aligned} \quad \text{(K.4)}$$

Note that $\mathbf{V}_1(\boldsymbol{\sigma})$ and $\mathbf{V}_2(\boldsymbol{\sigma})$ are Vandermonde matrices defined by (53) and (52), respectively, in the proof of Theorem 4.

(2) Consider the case of $2/\eta \in (N_t + N_r - 1, +\infty)$. We have $M = N_{\min}!$ and

$$\begin{gathered} \widetilde{g}_{m,ij} = \sigma_i^{-r_{m,j}} a\left(j, r_{m,j}\right) \rho^{-d_j - r_{m,j}}, \\ m = 1, \ldots, M, \quad i = 1, \ldots, N_{\min}, \quad j = 1, \ldots, N_{\min}, \end{gathered} \quad \text{(K.5)}$$

where

$$\begin{aligned} a\left(j, r_{m,j}\right) &= N_t^{d_j + r_{m,j}} \frac{\Gamma\left(d_j\right) \Gamma\left(2/\eta - d_j\right) \left(d_j\right)_{r_{m,j}}}{\Gamma(2/\eta) \Gamma\left(r_{m,j} + 1\right) \left(d_j + 1 - 2/\eta\right)_{r_{m,j}}} \\ &= N_t^{d_j + r_{m,j}} \frac{\Gamma\left(2/\eta - d_j\right) \Gamma\left(d_j + r_{m,j}\right)}{\Gamma(2/\eta) \Gamma\left(r_{m,j} + 1\right) \left(d_j + 1 - 2/\eta\right)_{r_{m,j}}}. \end{aligned} \quad \text{(K.6)}$$

By Lemma 5,

$$\left(d_j + 1 - \frac{2}{\eta}\right)_{r_{m,j}} = (-1)^{r_{m,j}} \left(\frac{2}{\eta} - d_j - r_{m,j}\right)_{r_{m,j}}. \quad \text{(K.7)}$$

Substituting (K.7) to (K.6), we have

$$\begin{aligned} a\left(j, r_{m,j}\right) &= (-1)^{r_{m,j}} N_t^{d_j + r_{m,j}} \\ &\quad \times \frac{\Gamma\left(d_j + r_{m,j}\right) \Gamma\left(2/\eta - d_j - r_{m,j}\right)}{\Gamma(2/\eta) \Gamma\left(r_{m,j} + 1\right)}. \end{aligned} \quad \text{(K.8)}$$

Hence,

$$\begin{aligned} u_m &= (-1)^{\sum_j r_{m,j}} \operatorname{sgn}(\mathbf{r}_m) |\mathbf{V}_2(\boldsymbol{\sigma})| \prod_{j=1}^{N_{\min}} a\left(j, r_{m,j}\right) \\ &= \operatorname{sgn}(\mathbf{r}_m) |\mathbf{V}_2(\boldsymbol{\sigma})| \\ &\quad \times \prod_{j=1}^{N_{\min}} N_t^{d_j + r_{m,j}} \frac{\Gamma\left(d_j + r_{m,j}\right) \Gamma\left(2/\eta - d_j - r_{m,j}\right)}{\Gamma(2/\eta) \Gamma\left(r_{m,j} + 1\right)}. \end{aligned} \quad \text{(K.9)}$$

Note that $\mathbf{r}_m$ is a permutation of $\{0, 1, \ldots, N_{\min} - 1\}$ and $\operatorname{sgn}(\mathbf{r}_m)$ denotes the signature of the permutation $\mathbf{r}_m$: $+1$ if $\mathbf{r}_m$ is an even permutation and -1 if $\mathbf{r}_m$ is an odd permutation.

Consequently, in the light of Leibniz formula [31],

$$\sum_{m=1}^{M} u_m = \frac{|\mathbf{V}_2(\sigma)|}{\prod_{k=1}^{N_{\min}} \Gamma(k)} \quad |\mathbf{Q}|, \quad \text{(K.10)}$$

where each entry of $\mathbf{Q}$ is

$$q_{ij} = N_t^{d_{ij}} \Gamma\left(d_{ij}\right) \frac{\Gamma\left(2/\eta - d_{ij}\right)}{\Gamma(2/\eta)}. \quad \text{(K.11)}$$

Note that d_{ij} is defined in the description of Theorem 1. Comparing (K.11) to (I.5), we find that q_{ij} and e_{ij} are identical. Therefore,

$$\mu^*_{\text{cor}}(\eta) = \prod_{k=1}^{N_{\min}} \sigma_k^{-N_{\max}} \mu^*_{\text{unc}}(\eta). \quad \text{(K.12)}$$

(3) Consider the case of $2/\eta \in [|N_t - N_r| - 1, N_t + N_r + 1]$. In terms of the proof of Theorem 5 and the preliminaries in Section 3, when $\bmod\{2/\eta + 1 - |N_t - N_r|, 2\} \neq 0$, $M = l!$,

$$\widetilde{g}_{m,ij} = \begin{cases} \sigma_i^{-r_{m,j}} a\left(j, r_{m,j}\right)\rho^{-d_j - r_{m,j}}, & j \leq l; \\ \sigma_i^{d_j - 2/\eta} c_j \rho^{-2/\eta}, & j \geq l+1; \end{cases} \tag{K.13}$$

when $\bmod\{2/\eta + 1 - |N_t - N_r|, 2\} = 0$, $M = (l-1)!$,

$$\widetilde{g}_{m,ij} = \begin{cases} \sigma_i^{-r_{m,j}} a\left(j, r_{m,j}\right)\rho^{-d_j - r_{m,j}}, & j \leq l-1; \\ \sigma_i^{-l+1}(-1)^{l-1}\dfrac{N_t^{2/\eta}}{\Gamma(l)}\log\rho\rho^{-2/\eta}, & j = l; \\ \sigma_i^{d_j - 2/\eta} c_j \rho^{-2/\eta}, & j \geq l+1. \end{cases} \tag{K.14}$$

Note that $a(j, r_{m,j})$ and c_j are given by (J.2) and (J.3), respectively; when $\bmod\{2/\eta + 1 - |N_t - N_r|, 2\} \neq 0$, $\mathbf{r}_m$ is a permutation of $\{0, 1, \ldots, l-1\}$; when $\bmod\{2/\eta + 1 - |N_t - N_r|, 2\} = 0$, $\mathbf{r}_m$ is a permutation of $\{0, 1, \ldots, l-2\}$. Thus,

$$u_m = \begin{cases} \operatorname{sgn}(\mathbf{r}_m)|\mathbf{V}_3(\boldsymbol{\sigma})|\displaystyle\prod_{j=1}^{l} a\left(j, r_{m,j}\right)\prod_{j=l+1}^{N_{\min}} N_t^{2/\eta}\Gamma\left(d_j - \frac{2}{\eta}\right), \\ \qquad \bmod\{2/\eta + 1 - |N_t - N_r|, 2\} \neq 0; \\ \operatorname{sgn}(\mathbf{r}_m)|\mathbf{V}_3(\boldsymbol{\sigma})|(-1)^{l-1}N_t^{2(N_{\min}-l+1)/\eta}\log\rho \\ \times\displaystyle\prod_{j=1}^{l-1} a\left(j, r_{m,j}\right)\prod_{j=l+1}^{N_{\min}}\Gamma\left(d_j - \frac{2}{\eta}\right), \\ \qquad \bmod\left\{\dfrac{2}{\eta} + 1 - |N_t - N_r|, 2\right\} = 0, \end{cases} \tag{K.15}$$

where each entry of $\mathbf{V}_3(\boldsymbol{\sigma})$,

$$v_{3,ij} = \sigma_i^{-\min\{j-1, 2/\eta - d_j\}}. \tag{K.16}$$

Comparing to the proof of Theorem 3 for the same case of η, we have

$$\begin{aligned} \mu_{\text{cor}}^*(\eta) &= \frac{(-1)^{l(l-1)/2}|\mathbf{V}_3(\boldsymbol{\sigma})|}{\prod_{k=1}^{N_{\min}}\sigma_k^{|N_t - N_r|+1}\prod_{1\leq m<n\leq N_{\min}}(\sigma_n - \sigma_m)} \\ &\quad \times \prod_{k=1}^{N_{\min}-l}\frac{(k)_l}{\left(|N_t - N_r| - (2/\eta) + l + k\right)_l}\mu_{\text{unc}}^*(\eta). \end{aligned} \tag{K.17}$$

This concludes the proof.

L. Proof of Theorem 7

When $2/\eta \in (0, |N_t - N_r| + 1)$ or $2/\eta \in (N_t + N_r - 1, +\infty)$, in terms of Theorem 6, straightforwardly, $\lim_{\boldsymbol{\Sigma}\to\mathbf{I}}\mu_{\text{cor}}^*(\eta) = \mu_{\text{unc}}^*(\eta)$.

Consider the case of $2/\eta \in [|N_t - N_r| - 1, N_t + N_r + 1]$. By Taylor expansion and Lemma 5, the entries of $\mathbf{V}_3(\boldsymbol{\sigma})$ are

$$\begin{aligned} v_{3,ij} &= \sum_{n=0}^{\infty}\frac{\left(-p_j - n + 1\right)_n}{n!}(\sigma_i - 1)^n \\ &= \sum_{n=0}^{\infty}\frac{(-1)^n\left(p_j\right)_n}{n!}(\sigma_i - 1)^n, \end{aligned} \tag{L.1}$$

where $p_j = \min\{j-1, 2/\eta - d_j\}$.

Thereby, when $\boldsymbol{\sigma}$ approaches a vector of ones,

$$|\mathbf{V}_3(\boldsymbol{\sigma})| = \sum_{m=1}^{(N_{\min}-1)!}\left|\mathbf{V}_{3,m}(\boldsymbol{\sigma})\right|, \tag{L.2}$$

where the entries of $\mathbf{V}_{3,m}(\boldsymbol{\sigma})$

$$v_{3,m,ij} = \begin{cases} 1, & j = 1; \\ \dfrac{(-1)^{s_{m,j}}\left(p_j\right)_{s_{m,j}}}{s_{m,j}!}(\sigma_i - 1)^{s_{m,j}}, & j \geq 1. \end{cases} \tag{L.3}$$

Note that $\mathbf{s}_m = \{s_{m,2}, \ldots, s_{m,N_{\min}}\}$ is a permutation of $\{1, 2, \ldots, N_{\min} - 1\}$.

The determinant of $\mathbf{V}_{3,m}(\boldsymbol{\sigma})$ is

$$\begin{aligned} \left|\mathbf{V}_{3,m}(\boldsymbol{\sigma})\right| &= (-1)^{n_1}|\mathbf{V}_1(\boldsymbol{\sigma} - \mathbf{1})|\operatorname{sgn}(\mathbf{s}_m) \\ &\quad \times\prod_{k=2}^{N_{\min}}\frac{1}{\Gamma(p_k)\Gamma(k)}\prod_{j=2}^{N_{\min}}\Gamma\left(s_{m,j} + p_j\right), \end{aligned} \tag{L.4}$$

where $n_1 = N_{\min}(N_{\min} - 1)/2$. In the light of Leibniz formula [31] and

$$|\mathbf{V}_1(\boldsymbol{\sigma} - \mathbf{a})| = |\mathbf{V}_1(\boldsymbol{\sigma})|, \quad \mathbf{a} = \{a, \ldots, a\}, \tag{L.5}$$

$|\mathbf{V}_3(\boldsymbol{\sigma})|$ can be written in the form

$$|\mathbf{V}_3(\boldsymbol{\sigma})| = (-1)^{N_{\min}(N_{\min}-1)/2}|\mathbf{V}_1(\boldsymbol{\sigma})||\mathbf{W}|\prod_{k=2}^{N_{\min}}\frac{1}{\Gamma(p_k)\Gamma(k)}, \tag{L.6}$$

where $\mathbf{W}$ is an $(N_{\min} - 1) \times (N_{\min} - 1)$ matrix with entries

$$\begin{aligned} w_{ij} &= \Gamma\left(i + p_{j+1}\right) \\ &= \begin{cases} \Gamma(i + j), & j \leq l - 1, \\ \Gamma\left(\dfrac{2}{\eta} - |N_t - N_r| - 1 + i - j\right), & j \geq l. \end{cases} \end{aligned} \tag{L.7}$$

By partial Gaussian elimination, $\mathbf{W}$ can be transformed to $\mathbf{W}'$ with a $(N_{\min} - l)\times(l-1)$ left-lower submatrix of zeros. Partition $\mathbf{W}'$ as

$$\mathbf{W}' = \begin{pmatrix} \mathbf{W}_1' & \mathbf{W}_2' \\ \mathbf{W}_3' & \mathbf{W}_4' \end{pmatrix}, \tag{L.8}$$

where $\mathbf{W}_3'$ is the submatrix of zeros, the entries of $\mathbf{W}_1'$ are

$$w_{1,ij}' = \Gamma(i+j-1), \quad 1 \le i, j \le l-1, \tag{L.9}$$

and the entries of $\mathbf{W}_4'$ are

$$\begin{aligned} w_{4,ij}' &= \left(\frac{2}{\eta} - |N_t - N_r| - j - l\right)_{l-1} \\ &\quad \times \Gamma\left(\frac{2}{\eta} - |N_t - N_r| - l + i - j\right), \\ &\qquad l \le i, j \le N_{\min} - 1. \end{aligned} \tag{L.10}$$

By Lemma 3,

$$|\mathbf{W}_1'| = \prod_{k=1}^{l-1} \Gamma(k)\Gamma(k+1). \tag{L.11}$$

By Lemma 5,

$$\begin{aligned} |\mathbf{W}_4'| &= (-1)^{n_2} \prod_{j=l}^{N_{\min}-1} \left(\frac{2}{\eta} - |N_t - N_r| - j - l\right)_{l-1} \\ &\quad \times \prod_{k=1}^{N_{\min}-l} \Gamma(k)\Gamma\left(\frac{2}{\eta} - N_{\max} + k\right), \end{aligned} \tag{L.12}$$

where $n_2 = (N_{\min} - l)(N_{\min} - l - 1)/2$.

Consequently, in terms of Theorem 6,

$$\begin{aligned} \lim_{\boldsymbol{\Sigma} \to \mathbf{I}} \mu_{\text{cor}}^* &= (-1)^{n_1+n_2+n_3} \\ &\quad \times \prod_{k=1}^{N_{\min}-l} \frac{\Gamma(2/\eta - N_{\max} + k)}{\Gamma(2/\eta - |N_t - N_r| - k - 2l + 1)} \\ &\quad \times \frac{\Gamma(|N_t - N_r| - (2/\eta) + l + k)}{\Gamma(|N_t - N_r| - 2/\eta + 2l + k)} \mu_{\text{unc}}^*, \end{aligned} \tag{L.13}$$

where $n_3 = l(l-1)/2$. Since for any function $f(x)$,

$$\prod_{k=1}^{N_{\min}-l} f(a + N_{\min} - k - l + 1) = \prod_{k'=1}^{N_{\min}-l} f(a + k'), \tag{L.14}$$

where $k' = N_{\min} - k - l + 1$,

$$\begin{aligned} &\lim_{\boldsymbol{\Sigma} \to \mathbf{I}} \mu_{\text{cor}}^*(\eta) \\ &\quad = (-1)^{n_1+n_2+n_3} \prod_{k=1}^{N_{\min}-l} \frac{((2/\eta) - N_{\max} + k - l)_l}{(N_{\max} - (2/\eta) - k + 1)_l} \mu_{\text{unc}}^*(\eta). \end{aligned} \tag{L.15}$$

By Lemma 5,

$$\left(\frac{2}{\eta} - N_{\max} + k - l\right)_l = (-1)^l \left(N_{\max} - \frac{2}{\eta} - k + 1\right)_l. \tag{L.16}$$

Thus,

$$\lim_{\boldsymbol{\Sigma} \to \mathbf{I}} \mu_{\text{cor}}^*(\eta) = (-1)^{n_1+n_2+n_3+n_4} \mu_{\text{unc}}^*(\eta), \tag{L.17}$$

where $n_4 = l(N_{\min} - l + 1)$. As

$$(-1)^{n_1+n_2+n_3+n_4} = (-1)^{n_1-n_2+n_3+n_4} = 1, \tag{L.18}$$

we have

$$\lim_{\boldsymbol{\Sigma} \to \mathbf{I}} \mu_{\text{cor}}^*(\eta) = \mu_{\text{unc}}^*(\eta). \tag{L.19}$$

This concludes the proof.

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