On the MIMO Interference Channel

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***Abstract*—We propose an iterative algorithm to design optimal linear transmitters and receivers in a K-user frequency-flat MIMO Interference Channel (MIMO IFC) with full channel state information (CSI). The transmitters and receivers are optimized to maximize the weighted sum rate (WSR) of the MIMO IFC. Maximization of WSR is desirable since it allows the system to cover all the rate tuples on the rate region boundary for a given MIMO IFC. This algorithm is rooted in a recent result showing a correspondence between local optima of the minimum weighted sum mean squared error (MWSMSE) and maximum weighted sum rate (MWSR) objective functions for the MIMO Broadcast Channel (BC). This connection between the MWSR and MWSMSE is shown hold true in the MIMO IFC and is exploited to design an alternating minimization algorithm for MIMO IFC that maximizes the weighted sum rate. An interesting by product of this paper is the result that for the BC, the overall iterative process proposed in [1] for the MIMO BC is identical to the extension of the iterative algorithm proposed in [2] to the MIMO BC. Since the former adopts the transmitter optimization procedure proposed in [3] we are able to establishing optimality of this seemingly ad-hoc technique for the MIMO BC.**

***Index Terms*—MIMO, MMSE, Interference Channel**

I. Introduction

A. The K-user MIMO interference channel

Multiple input multiple output (MIMO) systems have been shown to have tremendous potential in increasing the average throughput in cellular wireless communication systems. The performance gain in channel capacity, reliability and spectral efficiency in single user (point-to-point) MIMO (SU-MIMO) systems has spurred the inclusion of SU-MIMO in various cellular and wireless communication standards such as 3GPP high-speed packet access (HSPA) and long term evolution (LTE) where it has successfully demonstrated its ability to enhance the performance of wireless networks. In cellular systems where spectrum scarcity/cost is a major concern, frequency reuse factor 1 is desirable. Such systems however, have to deal with the additional problem of inter-cell interference which does not exist in simple point-to-point systems. Interference is being increasingly accepted as the major bottleneck limiting the throughput in wireless communication networks. Traditionally, the problem of interference has been dealt with through careful planning and (mostly static) radio resource management. With the widespread popularity of wireless devices following different wireless communication standards, the efficacy of such interference avoidance solutions is fairly limited. Indeed, major standardization bodies are now including explicit interference coordination strategies in next generation cellular communication standards. A systematic study of the performance of cellular communication systems where each cell communicates multiple streams to its users while enduring/causing interference from/to neighboring cells due to transmission over a common shared resource comes under the purview of MIMO interference channels (MIMO IFC). A K-user MIMO-IFC models a network of K transmit-receive pairs where each transmitter communicates multiple data streams to its respective receiver. In doing so, it generates interference at all other receivers.

B. Rate maximization: Beyond interference alignment

While the interference channel has been the focus of intense research over the past few decades, starting from the celebrated paper by Carleial [4], the capacity of this channel in general remains an open problem and is not well understood even for the 2-user case [5]. However, recent research [6] has shown that interference does not fundamentally limit the channel capacity and that at least in the high signal to noise ratio (SNR) regime, the per-user capacity of an interference channel with arbitrary number of users, scales at half the rate of each user's interference-free capacity. Such a scaling was obtained in [6] using the concept of interference alignment (IA) which has been shown to maximize the capacity pre-log factor in a K-user IFC. The key idea behind interference alignment is to process the transmit signal (data streams) at each transmitter so as to align all the undesired signals at each receiver in a subspace of suitable dimension. Alignment allows each receiver to suppress more interfering streams than it could otherwise cancel. The focus of this paper is on the K-user frequency-flat MIMO IFC. In contrast to the multi-user MIMO broadcast channel (MIMO BC) or multiple access channel (MIMO MAC) where it is reasonable to assume that the number of antennas at the transmitter or receiver exceed the total number of data streams that constitute the transmit (respectively receive) signal, in a frequency-flat MIMO IFC, the total number of streams contributing to the input signal at each receiver are, in general, greater than the number of antennas available at the receiver. This would lead one to believe that, at least in the high-SNR regime, the network (comprising of K user pairs) performance can be maximized (i.e, the sum-rate can be maximized) using IA since aligning the streams at the transmitter will now allow each receiver to cancel more steams than the number of "spare antennas" at its disposal. A distributed algorithm that exploits the reciprocity

of the MIMO IFC to obtain the transmit and receiver filters in a K-user MIMO IFC was proposed in [7]. It is was shown here that that IA is a suboptimal strategy at finite SNRs. In the same paper, the authors propose a signal-to-interference-noise-ratio (SINR) maximizing algorithm which outperforms the IA in finite SNRs and converges to the IA solution in the high SNR regime. However, this approach can be shown to be suboptimal for multiple stream transmission since it allocates equal power to all streams. Moreover, the convergence of this iterative algorithm has not be proved. Thus an optimal solution for MIMO IFC in finite SNRs remains an open problem.
Some early work on the MIMO IFC was reported in [8] by Ye and Blum for the asymptotic cases when interference to noise ratio (INR) is extremely small or extremely large. It was shown here that a ”greedy approach” where each transmitter attempts to maximize its individual rate regardless of its effect on other un-intended receivers is provably suboptimal. It was also noted here that the network capacity in general is neither a convex nor concave function of transmit covariance matrices thus making it difficult to find an analytical solution to the optimization problem. The MIMO IFC was studied in a game theoretic framework in [9] where such a greedy approach was modeled as a non-cooperative game and shown to have a unique Nash-equilibrium point subject to mild conditions on the channel matrices. The maximum weighted sum rate (MWSR) problem is well studied in the context of MIMO broadcast channel (MIMO-BC) in [10] (and references therein) and in [1] where the authors elegantly exploit the connection between the MWSR problem and the weighted minimum mean squared error (WMMSE) problem to obtain locally optimum solutions for the (non-convex) MWSR problem. There have been some early attempts to port the solution concepts of the MIMO BC and MIMO MAC to the MIMO IFC.T For instance, the problem of joint transmitter and receiver design to minimize the sum-MSE of a multiuser MIMO uplink was considered in [11] where iterative algorithms that jointly optimize precoders and receivers were proposed. Subsequently [12] applied this algorithm to the MIMO IFC where each user transmits a single stream and a similar iterative algorithm to maximize the sum rate was proposed in [13].

II. Signal Model

Fig. 1 depicts a K-user MIMO interference channel with K transmitter-receiver pairs. The k-th transmitter and its corresponding receiver are equipped with M_k and N_k antennas respectively. The k-th transmitter generates interference at all $l \neq k$ receivers. Assuming the communication channel to be frequency-flat, the $\mathbb{C}^{N_k \times 1}$ received signal $\mathbf{y}_k$ at the k-th receiver, can be represented as

$$\mathbf{y}_k = \mathbf{H}_{kk}\mathbf{x}_k + \sum_{\substack{l=1 \\ l \neq k}}^{K} \mathbf{H}_{kl}\mathbf{x}_l + \mathbf{n}_k \quad (1)$$

where $\mathbf{H}_{kl} \in \mathbb{C}^{N_k \times M_l}$ represents the channel matrix between the l-th transmitter and k-th receiver, $\mathbf{x}_k$ is the $\mathbb{C}^{M_k \times 1}$ transmit signal vector of the k-th transmitter and the $\mathbb{C}^{N_k \times 1}$ vector $\mathbf{n}_k$

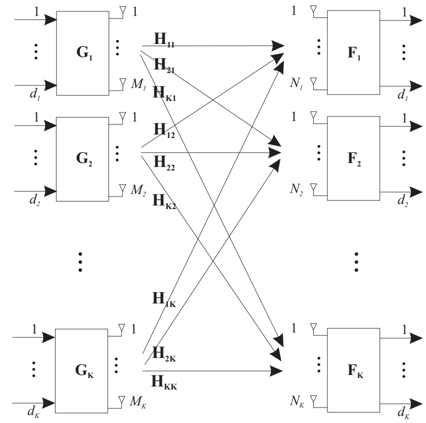

Fig. 1. MIMO Interference Channel

represents AWGN with zero mean and covariance matrix $\mathbf{R}_{nn}$. Each entry of the channel matrix is a complex random variable drawn from a continuous distribution. It is assumed that each transmitter has complete knowledge of all channel matrices corresponding to its direct link and all the other cross-links in addition to the transmitter power constraints and the receiver noise variances.

We denote by $\mathbf{G}_k$, the $\mathbb{C}^{M_k \times d_k}$ precoding matrix of the k-th transmitter. Thus $\mathbf{x}_k = \mathbf{G}_k\mathbf{s}_k$, where $\mathbf{s}_k$ is a $d_k \times 1$ vector representing the d_k independent streams of communicated between the k-th user pair. We assume $\mathbf{s}_k$ to have a Gaussian distribution with zero mean and unit variance, $\mathcal{N}(0, \mathbf{I}_k)$. The k-th receiver applies $\mathbf{F}_k \in \mathbb{C}^{d_k \times N_k}$ to suppress interference and retrieve its d_k desired streams. The output of such a receive filter is then given by

$$\mathbf{r}_k = \mathbf{F}_k\mathbf{H}_{kk}\mathbf{G}_k\mathbf{s}_k + \sum_{\substack{l=1 \\ l \neq k}}^{K} \mathbf{F}_k\mathbf{H}_{kl}\mathbf{G}_l\mathbf{s}_l + \mathbf{F}_k\mathbf{n}_k$$

III. Weighted sum rate maximization for MIMO IFC

The stated objective of our investigation is the maximization of the WSR of MIMO IFC. For a given MIMO IFC, the maximization of the weighted sum rate (WSR) allows to cover all the rate tuples on the rate region boundary. It is for this reason that, in this paper we consider the weighted sum rate maximization problem for a K-user frequency-flat MIMO IFC and propose an iterative algorithm for linear precoder/receiver design. While it is expected that the capacity-optimal transmission strategy may involve sophisticated non-linear techniques, from the practical standpoint, linear precoding (beamforming coupled with an appropriated power allocation strategy) in conjunction with linear processing at the receiver presents an attractive alternative due to its simplicity. For the MIMO IFC, one approach to linear transmit precoder design is joint design of precoding matrices to be applied at each transmitter based on channel state information (CSI) of all users. Such a *centralized* approach [8] requires information exchange among transmitters. Nevertheless, studying such systems can provide valuable insights on the limits of the so called *distributed* algorithms [14] [15] that do not require any information transfer among transmitters.

The WSR maximization problem can be mathematically expressed as follows.

$$\{\mathbf{G}_k^\star, \mathbf{F}_k^\star\} = \arg \min_{\{\mathbf{G}_k, \mathbf{F}_k\}} \mathcal{R} \quad \text{s.t} \ \operatorname{Tr}(\mathbf{G}_k^H \mathbf{G}_k) = P_k \ \ \forall k \quad (2)$$

where

$$\mathcal{R} = \sum_k -u_k R_k.$$

with u_k denoting the weight assigned to the k-th user's rate and P_k it's transmit power constraint. We use the notation $\{\mathbf{G}_k, \ \mathbf{F}_k\}$ to compactly represent the candidate set of transmitters $\mathbf{G}_k$ and receivers $\mathbf{F}_k \ \forall k \in \{1, \ldots, K\}$ and the corresponding set of optimum transmitters and receivers is represented by $\{\mathbf{G}_k^\star, \mathbf{F}_k^\star\}$. Assuming Gaussian signaling, the k-th user's achievable rate is given by

$$R_k = \log |\mathbf{I}_k + \mathbf{F}_k \mathbf{H}_{kk} \mathbf{G}_k (\mathbf{F}_k \mathbf{H}_{kk} \mathbf{G}_k)^H (\mathbf{F}_k \mathbf{R}_{\bar{k}} \mathbf{F}_k^H)^{-1}|. \quad (3)$$

we use here the standard notation $|.|$ to denote the determinant of a matrix.

The MIMO IFC rate region is known to be non-convex. The presence of multiple local optima complicates the computation of optimum precoding matrices to be applied at the transmitter in order to maximize the weighted sum rate. What is known however, is that, for a given set of precoders, MMSE receivers are optimal in terms of interference suppression.

A. Optimality of LMMSE interference suppression filters

We discuss here the optimality of linear minimum mean squared error (LMMSE) interference suppressors (in terms of maximizing weighted sum-rate in the finite SNR regime and maximizing achievable DoF in the high SNR regime) for a given set of linear precoders applied at the transmitters. In general, for fixed $\mathbf{G}_k$s, the received signal can be expressed as

$$\mathbf{y}_k = \mathbf{H}_{kk} \mathbf{x}_k + \mathbf{v}_k = \mathbf{H}_{kk} \mathbf{G}_k \mathbf{s}_k + \mathbf{v}_k \quad (4)$$

$\mathbf{v}_k = \sum_{l=1; l \neq k}^{K} \mathbf{H}_{kl} \mathbf{x}_l + \mathbf{n}_k$ accounts for the total interference and noise contribution in $\mathbf{y}_k$. The achievable rate at each receiver can now be expressed as

$$R_k = \log |\mathbf{I}_k + \mathbf{R}_{\bar{k}}^{-1} \mathbf{H}_{kk} \mathbf{G}_k \mathbf{G}_k^H \mathbf{H}_{kk}^H| \quad (5)$$

$$\mathbf{R}_{\bar{k}} = \mathbf{R}_{nn} + \sum_{l \neq k} \mathbf{H}_{kl} \mathbf{G}_l \mathbf{G}_l^H \mathbf{H}_{kl}^H.$$

$\mathbf{R}_{\bar{k}}$ represents the interference plus noise covariance matrix at the k-th receiver. The LMMSE receiver for the k-th user is then given by

$$\mathbf{F}_k^{LMMSE} = \mathbf{G}_k^H \mathbf{H}_{kk}^H (\mathbf{R}_{\bar{k}} + \mathbf{H}_{kk} \mathbf{G}_k \mathbf{G}_k^H \mathbf{H}_{kk}^H)^{-1} \quad (6)$$

It can be shown that by substituting $\mathbf{F}_k^{LMMSE}$ in (3), the resulting expression for R_k^{LMMSE} is exactly the same as (5). The implication is that, for a given set of linear beamforming filters applied at the transmitters, the LMMSE interference-suppressing filter applied at the receiver does not lose any information of the desired signal in the process of reducing the N_k dimensional $\mathbf{y}_k$ to a d_k dimensional vector $\mathbf{r}_k$. This is of course under the assumption that all interfering signals can be treated as Gaussian noise. In other words, the linear MMSE interference suppressor filter is information lossless and is thus optimal in terms of maximizing the WSR.

B. Gradient of weighted sum rate for the MIMO IFC

Consider the WSR maximization problem in (2). Let

$$\tilde{\mathbf{E}}_k = (\mathbf{I}_k + \mathbf{F}_k \mathbf{H}_{kk} \mathbf{G}_k (\mathbf{F}_k \mathbf{H}_{kk} \mathbf{G}_k)^H (\mathbf{F}_k \mathbf{R}_{\bar{k}} \mathbf{F}_k^H)^{-1})^{-1}. \quad (7)$$

Expressing the WSR in terms of $\tilde{\mathbf{E}}_k$, we have

$$\mathcal{R} = \sum_{k=1}^{K} -u_k \log |\tilde{\mathbf{E}}_k^{-1}|$$

The Karush-Kuhn-Tucker (KKT) conditions for this optimization problem obtained by setting the gradient of the WSR w.r.t $\mathbf{F}_k$ proves difficult to solve. Therefore we consider the an optimization problem where MMSE processing at the receiver is implicitly assumed. The rationale for this assumption will become clear as we proceed in this section. For now, we simply state that this assumption allows us to leverage a connection between the weighted sum MSE minimization problem and the WSR maximization problem that is the focus of our investigations. The alternative optimization problem that we consider is expressed as

$$\{\mathbf{G}_k^\star\} = \arg \min_{\{\mathbf{G}_k\}} \sum_{k=1}^{K} -u_k \log |\mathbf{E}_k^{-1}| \ \ \text{s.t} \ \operatorname{Tr}(\mathbf{G}_k^H \mathbf{G}_k) = P_k \ \forall k \quad (8)$$

where $\mathbf{E}_k$ is given by

$$\mathbf{E}_k = (\mathbf{I} + \mathbf{G}_k^H \mathbf{H}_{kk}^H \mathbf{R}_{\bar{k}}^{-1} \mathbf{H}_{kk} \mathbf{G}_k)^{-1}. \quad (9)$$

In order to obtain the stationary points for the optimization problem (8), we solve the Lagrangian:

$$J(\{\mathbf{G}_k, \lambda_k\}) = \sum_{k=1}^{K} -u_k \log |\mathbf{E}_k^{-1}| + \lambda_k (\operatorname{Tr}\{\mathbf{G}_k^H \mathbf{G}_k\} - P_k)$$

Now setting the gradient of the Lagrangian w.r.t. the transmit filter $\mathbf{G}_k$ to zero, we have:

$$\begin{gathered} \frac{\partial J(\{\mathbf{G}_k, \lambda_k\})}{\partial \mathbf{G}_k^*} = 0 \\ \sum_{l \neq k} u_l \mathbf{H}_{lk}^H \mathbf{R}_{\bar{l}}^{-1} \mathbf{H}_{ll} \mathbf{G}_l \mathbf{E}_l \mathbf{G}_l^H \mathbf{H}_{ll}^H \mathbf{R}_{\bar{l}}^{-1} \mathbf{H}_{lk} \mathbf{G}_k \\ -u_k \mathbf{H}_{kk}^H \mathbf{R}_{\bar{k}}^{-1} \mathbf{H}_{kk} \mathbf{G}_k \mathbf{E}_k + \lambda_k \mathbf{G}_k = 0 \end{gathered} \quad (10)$$

Notice that it is now possible to derive the gradient of the WSR expression w.r.t $\mathbf{G}_k$ for fixed $\mathbf{F}_i$ and $\mathbf{E}_i \ \ \forall i \neq k$. However, direct computation of λ_k that satisfies the KKT conditions now becomes complex. For single antenna receivers in a broadcast channel, a solution for transmit filter design that minimizes the MSE at the receiver was proposed in [3]. The key idea was to allow for scalars to compensate for transmit power constraints. Our approach to the design of the WSR maximizing transmit filters for the MIMO IFC is inspired by this idea. Before we explain the computation of λ_k any further, we digress in order to highlight an important connection between the WSR maximization and the weighted sum mean squared error (WSMSE) minimization problem that we exploit in our iterative algorithm.

Consider the problem where it is desired to optimize the transmit filters so as to minimize the WMMSE across all users (i.e., assuming MMSE receivers). Denote by $\mathbf{W}_k$ the weight matrix of the k-th user. Then this problem can be expressed as

$$\arg\min_{\{\mathbf{G}_k\}} \sum_{k=1}^{K} \mathrm{Tr}\{\mathbf{W}_k\mathbf{E}_k\} \text{ s.t. } \mathrm{Tr}\{\mathbf{G}_k^H\mathbf{G}_k\} = P_k \quad \forall k$$

and the corresponding Lagrangian reads

$$L(\{\mathbf{G}_k,\lambda_k\}) = \sum_{k=1}^{K} \mathrm{Tr}\{\mathbf{W}_k\mathbf{E}_k\} + \lambda_k(\mathrm{Tr}\{\mathbf{G}_k^H\mathbf{G}_k\} - P_k)$$

Deriving $L(\{\mathbf{G}_k,\lambda_k\})$ respect to $\mathbf{G}_k$ we have

$$\begin{array}{c}\frac{\partial \mathrm{L}(\{\mathbf{G}_k,\lambda_k\})}{\partial \mathbf{G}_k^*} = 0\\ \sum_{l\neq k}\mathbf{H}_{lk}^H\mathbf{R}_{\bar{l}}^{-1}\mathbf{H}_{ll}\mathbf{G}_l\mathbf{E}_l\mathbf{W}_l\mathbf{E}_l\mathbf{G}_l^H\mathbf{H}_{ll}^H\mathbf{R}_{\bar{l}}^{-1}\mathbf{H}_{lk}\mathbf{G}_k\\ -\mathbf{H}_{kk}^H\mathbf{R}_{\bar{k}}^{-1}\mathbf{H}_{kk}\mathbf{G}_k\mathbf{E}_k\mathbf{W}_k\mathbf{E}_k + \lambda_k\mathbf{G}_k = 0\end{array} \tag{11}$$

Comparing the gradient expressions for the two Lagrangians (10) and (11) we see that they can be made equal if

$$\mathbf{W}_k = u_k\mathbf{E}_k^{-1} \tag{12}$$

In other words, with a proper choice of the weighting matrices, a stationary point for the weighted sum minimum mean square error objective function is also a stationary point for the maximum WSR problem. This is the extension of [1] to the MIMO IFC. We exploit this relationship to henceforth compute the $\mathbf{G}_k$ that minimizes the WSMSE when $\mathbf{W}_k = u_k\mathbf{E}_k^{-1}$ instead of directly maximizing the WSR. We are now ready to extend the solution in [3] to MIMO IFC problem at hand. Since we are interested in minimizing the WSMSE, we have

$$\min \sum_{k=1}^{K} \mathrm{Tr}\{\mathbf{W}_k\mathbb{E}[(\mathbf{d} - \alpha_k^{-1}\mathbf{F}_k\mathbf{y}_k)(\mathbf{d} - \alpha_k^{-1}\mathbf{F}_k\mathbf{y}_k)^H]\}$$
$$\text{s.t. } \mathrm{Tr}\{\mathbf{G}_k^H\mathbf{G}_k\} = P_k$$

where the α_k allows to compensate for the (scalar) transmit-filter power constraint. Assuming $\mathbb{E}\{\mathbf{dd}^H\} = \mathbf{I}_k$, the MSE covariance matrix becomes:

$$\begin{aligned}\boldsymbol{\mathcal{E}}_k = &\ \mathbb{E}[(\mathbf{d} - \alpha_k^{-1}\mathbf{F}_k\mathbf{y}_k)(\mathbf{d} - \alpha_k^{-1}\mathbf{F}_k\mathbf{y}_k)^H]\}\\ = &\ \mathbf{I} - \alpha_k^{-1}\mathbf{G}_k^H\mathbf{H}_{kk}^H\mathbf{F}_k^H - \alpha_k^{-1}\mathbf{F}_k\mathbf{H}_{kk}\mathbf{G}_k\\ &+ \alpha_k^{-2}\mathbf{F}_k\mathbf{H}_{kk}\mathbf{G}_k\mathbf{G}_k^H\mathbf{H}_{kk}^H\mathbf{F}_k^H\\ &+ \alpha_k^{-2}\textstyle\sum_{l\neq k}\mathbf{F}_k\mathbf{H}_{kl}\mathbf{G}_l\mathbf{G}_l^H\mathbf{H}_{kl}^H\mathbf{F}_k^H + \alpha_k^{-2}\mathbf{F}_k\mathbf{R}_{nn}\mathbf{F}_k^H\end{aligned} \tag{13}$$

The corresponding Lagrangian can be written as:

$$J(\{\mathbf{G}_k,\alpha_k,\lambda_k\}) = \sum_{k=1}^{K} \mathrm{Tr}\{\mathbf{W}_k\boldsymbol{\mathcal{E}}_k\} - \lambda_k(\mathrm{Tr}\{\mathbf{G}_k^H\mathbf{G}_k\} - P_k) \tag{14}$$

Optimizing for α_k we get:

$$\begin{aligned}\frac{\partial J(\{\mathbf{G}_k,\alpha_k,\lambda_k\})}{\partial \alpha_k} = &\ \alpha_k^{-2}\mathrm{Tr}\{\mathbf{W}_k\mathbf{G}_k^H\mathbf{H}_{kk}^H\mathbf{F}_k^H\}\\ &+ \alpha_k^{-2}\mathrm{Tr}\{\mathbf{W}_k\mathbf{F}_k\mathbf{H}_{kk}\mathbf{G}_k\}\\ &-2\alpha_k^{-3}\mathrm{Tr}\{\mathbf{W}_k\mathbf{F}_k\mathbf{H}_{kk}\mathbf{G}_k\mathbf{G}_k^H\mathbf{H}_{kk}^H\mathbf{F}_k^H\}\\ &-2\alpha_k^{-3}\textstyle\sum_{l\neq k}\mathrm{Tr}\{\mathbf{W}_k\mathbf{F}_k\mathbf{H}_{kl}\mathbf{G}_l\mathbf{G}_l^H\mathbf{H}_{kl}^H\mathbf{F}_k^H\}\\ &-2\alpha_k^{-3}\mathrm{Tr}\{\mathbf{W}_k\mathbf{F}_k\mathbf{R}_{nn}\mathbf{F}_k^H\}\\ = &\ 0\end{aligned}$$

solving the expression w.r.t. α_k

$$\alpha_k = 2\frac{\mathrm{Tr}\{\sum_{l=1}^{K}\mathbf{W}_k\mathbf{F}_k\mathbf{H}_{kl}\mathbf{G}_l\mathbf{G}_l^H\mathbf{H}_{kl}^H\mathbf{F}_k^H + \mathbf{W}_k\mathbf{F}_k\mathbf{R}_{nn}\mathbf{F}_k^H\}}{\mathrm{Tr}\{\mathbf{W}_k\mathbf{G}_k^H\mathbf{H}_{kk}^H\mathbf{F}_k^H\} + \mathrm{Tr}\{\mathbf{W}_k\mathbf{F}_k\mathbf{H}_{kk}\mathbf{G}_k\}} \tag{15}$$

For the Lagrange multiplier λ_k, we set

$$\mathrm{Tr}\{\mathbf{G}_k^H\frac{\partial J(\{\mathbf{G}_k,\alpha_k,\lambda_k\})}{\partial \mathbf{G}_k^*}\} = 0$$

and obtain

$$\lambda_k = \frac{1}{P_k}(\sum_{l=1}^{K}\alpha_l^{-2}\mathrm{Tr}\{\mathbf{G}_k^H\mathbf{H}_{lk}^H\mathbf{F}_l^H\mathbf{W}_l\mathbf{F}_l\mathbf{H}_{lk}\mathbf{G}_k\} - \alpha_k^{-1}\mathrm{Tr}\{\mathbf{G}_k^H\mathbf{H}_{kk}^H\mathbf{F}_k^H\mathbf{W}_k\})$$

It is interesting to note that fixing the receivers to be MMSE leads to the simplified expression $\alpha_k = 1 \quad \forall k$. Therefore, assuming MMSE receives, the above expression simplifies to

$$\begin{aligned}\lambda_k = &\ \tfrac{1}{P_k}\left(\sum_{l\neq k}\mathrm{Tr}\{\mathbf{W}_l\mathbf{F}_l\mathbf{H}_{lk}\mathbf{G}_k(\mathbf{F}_l\mathbf{H}_{lk}\mathbf{G}_k)^H\}\right)\\ &-\tfrac{1}{P_k}\left(\sum_{l\neq k}\mathrm{Tr}\{\mathbf{W}_k\mathbf{F}_k\mathbf{H}_{kl}\mathbf{G}_l(\mathbf{F}_k\mathbf{H}_{kl}\mathbf{G}_l)^H\}\right)\\ &-\tfrac{1}{P_k}\left(\mathrm{Tr}\{\mathbf{W}_k\mathbf{F}_k\mathbf{R}_{nn}\mathbf{F}_k^H\}\right)\end{aligned} \tag{16}$$

Thus, assuming MMSE receivers, from (13) (14) and (16) we have the expression (17) for the transmit filter $\mathbf{G}_k$. We thus have the following two-step iterative algorithm to compute the precoders that maximize the weighted sum rate for a given MIMO IFC (c.f Table **Algorithm 1**)

Algorithm 1 MWSR Algorithm for MIMO IFC

Fix an arbitrary initial set of precoding matrices $\mathbf{G}_k$, $\forall \in k = \{1, 2 \ldots K\}$
set $n = 0$
repeat
  $n = n + 1$
  Given $\mathbf{G}_k^{(n-1)}$, compute $\mathbf{F}_k^n$ and $\mathbf{W}_k^n$ from (6) and (12) respectively $\forall k$
  Given $\mathbf{F}_k^n$ and $\mathbf{W}_k^n$, compute $\mathbf{G}_k^n$ $\forall k$
until convergence

$$\mathbf{G}_k = \left(\sum_{l=1}^{K} \mathbf{H}_{lk}^H \mathbf{F}_l^H \mathbf{W}_l \mathbf{F}_l \mathbf{H}_{kl} - \frac{1}{P_k}\left(\left(\sum_{l\neq k} \mathrm{Tr}\{\mathbf{W}_l \mathbf{J}_l^{(k)}\} - \mathrm{Tr}\{\mathbf{W}_k \mathbf{J}_k^{(l)}\}\right) - \mathrm{Tr}\{\mathbf{W}_k \mathbf{N}_k\}\right)\mathbf{I}\right)^{-1} \mathbf{H}_{kk}^H \mathbf{F}_k^H \mathbf{W}_k \tag{17}$$

$$\begin{aligned} \mathbf{J}_l^{(k)} &= \mathbf{F}_l \mathbf{H}_{lk} \mathbf{G}_k \mathbf{G}_k^H \mathbf{H}_{lk}^H \mathbf{F}_l^H \\ \mathbf{J}_k^{(l)} &= \mathbf{F}_k \mathbf{H}_{kl} \mathbf{G}_l \mathbf{G}_l^H \mathbf{H}_{kl}^H \mathbf{F}_k^H \\ \mathbf{N}_k &= \mathbf{F}_k \mathbf{R}_{nn} \mathbf{F}_k^H \end{aligned}$$

C. Convergence analysis

As mentioned earlier, the non-convexity of the MIMO IFC rate region precludes the possibility of a rigorous proof of convergence of our iterative algorithm to a global optimum. However, convergence to a local optimum is guaranteed and we devote this section to prove this. In order to prove the monotonic convergence of our iterative algorithm we use a more general optimization problem that requires optimization of receivers as well as MSE weight matrices in addition to the precoders applied at the transmitters.

Our new optimization problem is mathematically expressed as

$$\begin{aligned} \{\mathbf{G}_k^\star\} = \quad & \arg\min_{\{\mathbf{G}_k,\, \mathbf{F}_k,\, \mathbf{W}_k\}} \sum_k \mathrm{Tr}(\mathbf{W}_k \boldsymbol{\mathcal{E}}_k) \\ & -u_k \left(\log|\tfrac{1}{u_k}\mathbf{W}_k| + d_k^{max}\right) \\ & \mathrm{s.\,t} \sum_k \mathrm{Tr}(\mathbf{G}_k \mathbf{G}_k^H) = P_k. \end{aligned} \tag{18}$$

where $d_k^{max} = \min\{N_k, M_k\}$ represents the maximum number of independent data streams that can be transmitted to user k. Notice that the cost function for this new optimization problem is now a function of the MSE weights in addition to the transmit precoders and receivers. Let

$$f_k\left(\mathbf{W}_k, \mathbf{G}_k, \{\mathbf{F}_k\}\right) \triangleq \mathrm{Tr}(\mathbf{W}_k \boldsymbol{\mathcal{E}}_k) - u_k(\log|\frac{1}{u_k}\mathbf{W}_k| + d_k^{max}). \tag{19}$$

The optimization problem considered above is amenable to an alternating minimization solution. For a fixed set of precoding and weight matrices, the optimum receiver for the k-th user turns out to be $\mathbf{F}_k^{LMMSE}$ and requires the knowledge of only the set of precoding matrices $\{\mathbf{G}_k\}$. Plugging in the optimal receivers in the cost function and optimizing for the weight matrices $\mathbf{W}_k$ which is a function of $\mathbf{F}_k$ results in $\mathbf{W}_k^\star = u_k \mathbf{E}_k^{-1}$ where $\mathbf{E}_k$ is precisely the expression in (9). Finally, substituting $\mathbf{W}_k^\star$ and $\{\mathbf{F}_k^{LMMSE}\}$ in (19) it is immediately seen that the optimization problem in (18) corresponds to the original MSWR optimization problem. Thus the more general optimization problem reduces to the MWSR problem when optimized solely w.r.t the precoding matrices $\mathbf{G}_k$.

The alternating minimization solution explained above (first fixing $\{\mathbf{G}_k\}$ to find $\{\mathbf{F}_k\}$, $\{\mathbf{W}_k\}$ then computing $\{\mathbf{G}_k\}$ for fixed $\{\mathbf{F}_k\}$, $\{\mathbf{W}_k\}$) monotonically reduces the cost function in (18) at each step of the iteration. This, together with the fact that the cost function itself is lower bounded for a fixed power constraint on each transmitter proves convergence of the MWSR algorithm to a local optimum.

D. Asymptotic behavior of MWSR algorithm

As $SNR \to 0$, the interference due to other users is overwhelmed by the noise power seen at the receivers. The sum rate maximizing beamformers in this regime are simply the dominant right singular vectors obtained from the singular value decomposition of the direct link $\mathbf{H}_{kk}$ of the k-th user. The power allocation strategy reduces to that of single-user MIMO scenario. i.e., water filling. As $SNR \to \infty$, the solution offered by the algorithm can be interpreted as an "optimized interference alignment" solution. The optimality here is on similar lines of the optimality of MMSE-ZF solution versus ZF solution. Among all the possible IA solutions, the MWSR algorithm results in the solution that maximizes the weighted sum rate.

E. Some connections to MIMO BC

An alternative approach is the extension of [2] to the MIMO IFC and involves normalizing the transmit filter so as to satisfy the power constraint. i.e.,

$$\bar{\mathbf{G}}_k = \sqrt{\frac{P_k}{\mathrm{Tr}\{\mathbf{G}_k^H \mathbf{G}_k\}}}\mathbf{G}_k = \sqrt{P}_k \beta_k \mathbf{G}_k$$

This converts the constrained optimization problem considered in the previous section to an unconstrained optimization problem thereby doing away with the computation of the Lagrange multipliers. The sum rate expression with the beamformer normalization can be written as

$$\mathcal{R} = \sum_{k=1}^{K} u_k \log|\mathbf{I}_k + P_k \beta_k^2 \mathbf{H}_{kk} \mathbf{G}_k (\mathbf{H}_{kk}\mathbf{G}_k)^H \mathbf{R}_{\bar{k}}^{-1}|$$

where $\mathbf{R}_{\bar{k}}$ is now given by

$$\mathbf{R}_{\bar{k}} = \mathbf{R}_{nn} + \sum_{l\neq k} P_l \beta_l^2 \mathbf{H}_{kl} \mathbf{G}_l \mathbf{G}_l^H \mathbf{H}_{kl}^H.$$

To find the optimal transmit filter we derive the WSR expression first w.r.t. $\mathbf{G}_k$, and absorb the scalar contribution $P_k\beta_k$ of the resulting equation in $\bar{\mathbf{G}}_k$.

$$\frac{\partial \mathcal{R}(\mathbf{G}_k)}{\partial \mathbf{G}_k^*} = 0$$

$$\begin{gathered} -u_k \tfrac{1}{P_k} \bar{\mathbf{G}}_k \mathrm{Tr}\{\mathbf{E}_k \bar{\mathbf{G}}_k^H \mathbf{H}_{kk}^H \mathbf{R}_{\bar{k}}^{-1} \mathbf{H}_{kk} \bar{\mathbf{G}}_k\} + u_k \mathbf{H}_{kk}^H \mathbf{R}_{\bar{k}}^{-1} \mathbf{H}_{kk} \bar{\mathbf{G}}_k \mathbf{E}_k \\ + \textstyle\sum_{l\neq k} u_l \tfrac{1}{P_k} \bar{\mathbf{G}}_k \mathrm{Tr}\{\mathbf{E}_l \bar{\mathbf{G}}_l^H \mathbf{H}_{ll}^H \mathbf{R}_{\bar{l}}^{-1} \mathbf{H}_{lk} \bar{\mathbf{G}}_k \bar{\mathbf{G}}_k^H \mathbf{H}_{lk}^H \mathbf{R}_{\bar{l}}^{-1} \mathbf{H}_{ll}^H \bar{\mathbf{G}}_l\} \\ - \textstyle\sum_{l\neq k} u_l \mathbf{H}_{lk}^H \mathbf{R}_{\bar{l}}^{-1} \mathbf{H}_{ll} \bar{\mathbf{G}}_l \mathbf{E}_l \bar{\mathbf{G}}_l \tilde{\mathbf{H}}_{ll}^H \mathbf{R}_{\bar{l}}^{-1} \mathbf{H}_{lk} \bar{\mathbf{G}}_k = 0 \end{gathered} \tag{20}$$

In contrast to a MISO system, solving the above expression for $\bar{\mathbf{G}}_k$ is not straightforward for a general MIMO IFC. In a

MISO system, simply extending [2] makes it possible to fix all scalar quantities involved in the expression and thereby allowing us to find the the beamformer by iterating between the beamformer vectors and the fixed scalars. However, in moving from the MISO IFC to the MIMO IFC, the scalars now become matrices ($\mathbf{E}_k$ and $\mathbf{F}_k$) and hence a more structured reasoning is required.To this end, we derive the expression for the WSR, now with the receiver matrix in place as

$$\mathcal{R} = \sum_{k=1}^{K} u_k \log |\mathbf{I}_k + P_k \beta_k^2 \mathbf{F}_k \mathbf{H}_{kk} \mathbf{G}_k (\mathbf{F}_k \mathbf{H}_{kk} \mathbf{G}_k)^H (\mathbf{F}_k \mathbf{R}_{\bar{k}} \mathbf{F}_k^H)^{-1}|$$

Denoting the cascade of the receive filter and the channel matrix as $\tilde{\mathbf{H}}_{kl} = \mathbf{F}_k \mathbf{H}_{kl}$ and the noise covariance matrix after the receive filter as $\tilde{\mathbf{R}}_{\bar{k}}$, the WSR expression is rewritten as

$$\begin{aligned} \mathcal{R} &= \sum_{k=1}^{K} u_k \log |\mathbf{I}_k + P_k \beta_k^2 \mathbf{G}_k^H \tilde{\mathbf{H}}_{kk}^H \tilde{\mathbf{R}}_{\bar{k}}^{-1} \tilde{\mathbf{H}}_{kk} \mathbf{G}_k| \\ &= \sum_{k=1}^{K} u_k \log |\hat{\mathbf{E}}_k| \end{aligned}$$

Proceeding as before, we find the optimal transmit filter by first deriving this WSR expression w.r.t. $\mathbf{G}_k$, absorbing the scalar contribution $P_k \beta_k$ of the resulting equation in $\bar{\mathbf{G}}_k$ and finally, solving for $\bar{\mathbf{G}}_k$, assuming all $d_k \times d_k$ matrices to be constant to get (21). It can be shown that the expression for $\bar{\mathbf{G}}_k$ in (21) and $\mathbf{G}_k$ in (17) are identical to within some algebraic manipulations. Thus the extension of [2] to the MIMO IFC as well as the extension of [1] to the MIMO IFC yield exactly the same solution. Interestingly, it was observed that extending the approach in [2] to the MIMO BC leads to the same solution as that of [1] thus proving the optimality of integrating the [3] solution in the approach proposed in [1] (i.e., iterating between transmit filters and receive filters with corresponding weights). Indeed, it can be shown that the KKT condition $\mathbf{G}_k$ is satisfied when the solution for $\mathbf{G}_k$ and λ_k are substituted thereby proving optimality of using the [3] approach both for the MIMO BC and MIMO IFC.

IV. Simulation Results

We provide here some simulation results to compare the performance of the proposed max-WSR algorithm. i.i.d Gaussian channels (direct and cross links) are generated for each user. For a fixed channel realization transmit and receiver filters are computed based on IA algorithm and max-WSR algorithm over multiple SNR points. The non convexity of the problem may lead the algorithm to converge to a stationary point that represents a local optimum instead of the global one which we are interested in. To increase the probability of reaching the optimum a common strategy in non convex problem is to to choose multiple random initial beamforming matrices and adopting the solution of the algorithm that determines the best WSR. Using these filters individual rates are computed. The resulting rate-sum is averaged over several hundred Monte-Carlo runs. The average rate-sum plots are used to compare the performance of the proposed algorithm.

In Fig. 2, we plot the results for a 3-user MIMO IFC. The antenna distribution at the receive and transmit side is $M_k = N_k = 2\ \forall k$. The max-WSR algorithm results in a DoF allocation of $d_1 = 1\ d_2 = 1\ d_3 = 1$ with $u_k = 1\ \forall k$ In Fig. 3, we plot the results for a 3-user MIMO

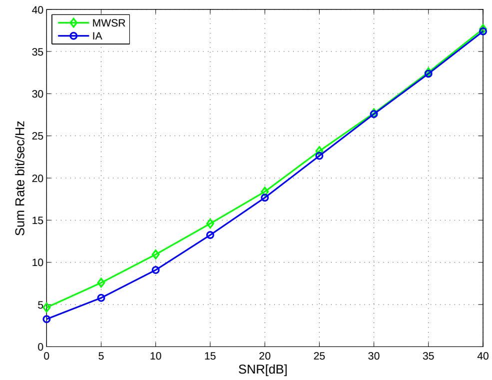

Fig. 2. 3-user MIMO IFC with $M_k = N_k = 2\ \forall k$.

IFC with $M_k = N_k = 3\ \forall k$. The resulting DoF allocation is $d_1 = 2\ d_2 = 1\ d_3 = 1$ with $u_k = 1\ \forall k$ In Fig. 4, the

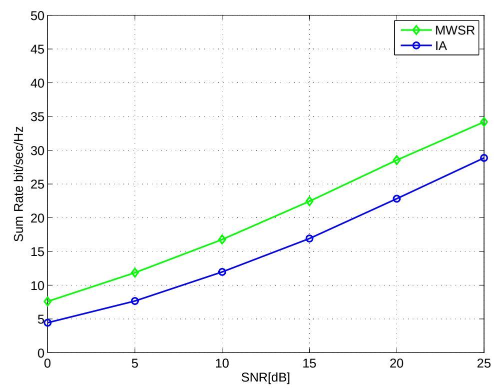

Fig. 3. 3-user MIMO IFC with $M_k = N_k = 3\ \forall k$

performance of a 3-user MIMO IFC with $M_k = N_k = 4\ \forall k$ is shown. With $u_k = 1\ \forall k$, the max-WSR algorithm allocates 2 streams for all users in this case. Finally, Fig. 5 shows the convergence behavior of our algorithm for the same 3-user MIMO IFC with $M_k = N_k = 4\ \forall k$ in a given SNR point, SNR=5dB

V. Conclusions

We addressed maximization of the weighted sum rate for the MIMO IFC. We introduced an iterative algorithm to solve this optimization problem. In the high-SNR regime, this algorithm leads to an optimized Interference Alignment (IA) solution Optimality here is on similar lines of the MMSE-ZF solution w.r.t general ZF solutions. In the finite SNR regime the performance of this algorithm is superior to that of IA and all known algorithms since it maximizes the WSR as opposed to

$$\bar{\mathbf{G}}_k = (\sum_{l=1}^{K} u_l \tilde{\mathbf{H}}_{lk}^H \tilde{\mathbf{R}}_{\bar{l}}^{-1} \tilde{\mathbf{H}}_{ll} \bar{\mathbf{G}}_l \hat{\mathbf{E}}_l \bar{\mathbf{G}}_l \tilde{\mathbf{H}}_{ll}^H \tilde{\mathbf{R}}_{\bar{l}}^{-1} \tilde{\mathbf{H}}_{lk} + \frac{1}{P_k}(u_k \mathrm{Tr}\{\hat{\mathbf{E}}_k \bar{\mathbf{G}}_k^H \tilde{\mathbf{H}}_{kk}^H \tilde{\mathbf{R}}_{\bar{k}}^{-1} \tilde{\mathbf{H}}_{kk} \bar{\mathbf{G}}_k\} - \sum_{l \neq k} u_l \mathrm{Tr}\{\hat{\mathbf{E}}_l \bar{\mathbf{G}}_l^H \tilde{\mathbf{H}}_{ll}^H \tilde{\mathbf{R}}_{\bar{l}}^{-1} \tilde{\mathbf{H}}_{lk} \bar{\mathbf{G}}_k \bar{\mathbf{G}}_k^H \tilde{\mathbf{H}}_{lk}^H \tilde{\mathbf{R}}_{\bar{l}}^{-1} \tilde{\mathbf{H}}_{ll}^H \bar{\mathbf{G}}_l\})\mathbf{I})^{-1} \tilde{\mathbf{H}}_{kk}^H \tilde{\mathbf{R}}_{\bar{k}}^{-1} \tilde{\mathbf{H}}_{kk} \bar{\mathbf{G}}_k u_k \quad (21)$$

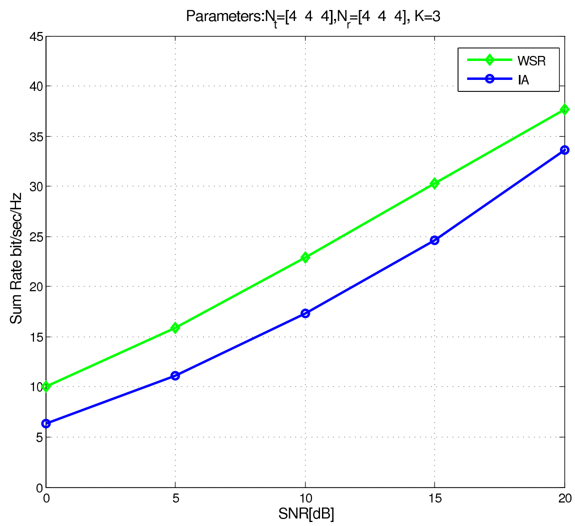

Fig. 4. 3-user MIMO IFC with $M_k = N_k = 4 \; \forall k$

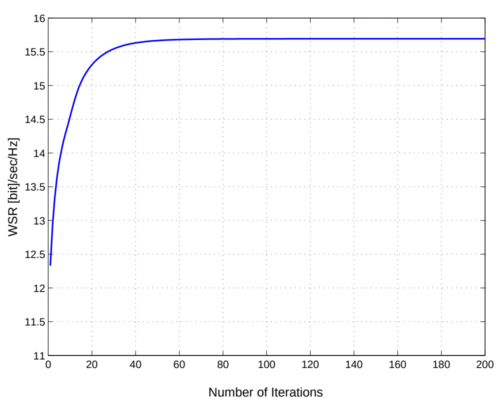

Fig. 5. Convergence behavior for a 3-user MIMO IFC with $M_k = N_k = 4 \; \forall k$ at SNR=5dB

previous attempts that maximize the sum rate. Convergence to a local optimum was also shown experimentally. Convergence to local optima is known and is related to the non-convexity of the MIMO IFC rate region. As an interesting by product of this paper we are able to show that the overall iterative process proposed in [1] for the MIMO BC is identical to the extension of the iterative algorithm proposed in [2] to the MIMO BC.

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