Capacity of multi-antenna block-fading channels

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Abstract

In this paper we determine the performance limits of a multiple transmit and receive antenna system over a fading channel. We assume a frequency-flat block-fading additive white Gaussian noise channel with delay constraint, transmit-power constraint, and perfect channel-state information available at both transmitter and receiver. The delay constraint is met by assuming that a code word spans a finite number of channel realizations. The relevant performance limits are the information outage probability and the "delay-limited" (or "non-ergodic") capacity. We show that the optimum coding scheme is obtained as the concatenation of an optimal code for the unfaded AWGN channel with an optimal beamformer. Numerical results show that very high rates are achievable without the need of deep interleaving.

I. Introduction

We consider a radio system consisting of a transmitter with K antennas and a receiver with L antennas. We assume a frequency-flat block-fading additive white Gaussian noise (BF-AWGN) channel with transmit-power constraint and perfect channel-state information (CSI) available both at the transmitter and at the receiver (see [1] for a list of references).

The BF-AWGN channel applies in many cases (as, for example, indoor wireless data networks) where the random channel gain varies slowly in time (see also [2]–[7]). Following [8;10;12], we assume that a code word spans a number M of fading blocks each carrying N channel symbols. The number of fading blocks per code word, M, is a measure of the interleaving delay so that a delay constraint can be translated into an upper bound on M itself.

We assume that CSI is available at the transmitter before sending a code word over the BF-AWGN channel. Fading blocks can be thought of as separated in time, frequency, or both. The transmitter can obtain CSI either by a dedicated feedback channel or by time-division duplex [13].

Recently, multiple antenna systems have received considerable attention [5;14;4;2;3;6;7]. Our goal is to minimize the outage probability at a given *fixed* code rate under a *long-term* (i.e., spanning a large number of code words) power constraint. We show that the optimal scheme consists of a Gaussian code for the AWGN channel (i.e., a code whose components' empirical distribution approaches the Gaussian distribution), followed by suitable beamforming matrices derived from the channel fading gains. The decoupling of coding and beamforming optimization stands in contrast to the case of no CSI at the transmitter, where particular space-code constructions prove to be useful [5]. System performance is measured in terms of *delay-limited capacity*, defined as the maximum rate for which the minimum outage probability is zero, for a given power constraint [9;10;12].

The paper is organized as follows. Sections 2 and 3 describe the channel model and power constraint details. Section 4 contains the derivation of the optimal transmission scheme. Section 5 contains the results on delay-limited capacity. Finally, Section 6 is devoted to a catalog of numerical results highlighting the performance of multi-antenna systems and elucidating the theoretical results.

II. Channel model

Assuming perfect symbol synchronization, the n-th signal sample output by the ℓ-th receive antenna during the m-th block can be written as

$$y_\ell[mN+n] = \sum_{k=1}^{K} a_{\ell,k}^{(m)} x_k[mN+n] + z_\ell[mN+n] \quad (1)$$

for $\ell = 1,\dots,L$, $m = 0\dots,M-1$, and $n = 0,\dots,N-1$. Here, $x_k[mN+n]$ is the n-th symbol of the m-th block transmitted by the k-th antenna. $a_{\ell,k}^{(m)}$ is the complex fading coefficient characterizing the transfer from the k-th transmit antenna to the ℓ-th receive antenna during the m-th block. Notice that these coefficients are independent of n since the channel is constant along each block. $z_\ell[mN+n]$ is a sample of an additive white complex Gaussian noise: $z_\ell[mN+n] \sim \mathcal{N}_c(0,1)$[1] The sequence of transmitted and received samples are represented by column vectors as follows:

- $\mathbf{y}_m[n] \triangleq (y_1[mN+n],\dots,y_L[mN+n])^T$, the received vector;
- $\mathbf{x}_m[n] \triangleq (x_1[mN+n],\dots,x_K[mN+n])^T$, the transmitted vector;
- $\mathbf{z}_m[n] \triangleq (x_1[mN+n],\dots,x_L[mN+n])^T$, the additive noise vector, $\mathbf{z}_m[n] \sim \mathcal{N}_c(\mathbf{0},\mathbf{I}_L)$.[2]

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[1] $\mathcal{N}_c(\mu,\sigma^2)$ denotes the distribution of a circular complex Gaussian random variable Z with mean $\mu = \mathrm{E}[Z]$, variance $\sigma^2 = 0.5E[|Z-\mu|^2]$, and independent real and imaginary parts.

[2] $\mathcal{N}_c(\boldsymbol{\mu},\boldsymbol{\Sigma})$ denotes the distribution of a circular complex Gaussian random vector $\mathbf{z}$ with mean $\boldsymbol{\mu} = \mathrm{E}[\mathbf{z}]$, real covariance matrix $\boldsymbol{\Sigma} = 0.5\,\mathrm{E}[\mathbf{z}\mathbf{z}^\dagger]$, and independent real and imaginary parts.

Denoting $\mathbf{A}_m \triangleq [a_{\ell,k}^{(m)}]_{\ell=1}^{L}{}_{k=1}^{K}$ the $L \times K$ matrix of the fading gains during the m block, we can write the received signal vector at the n-th sampling instant of the m-th block as

$$\mathbf{y}_m[n] = \mathbf{A}_m \mathbf{x}_m[n] + \mathbf{z}_m[n], n = 0, \ldots, N-1, \quad m = 0, \ldots, M-1. \tag{2}$$

III. Outage probability and delay-limited capacity

The M-block $K \times L$ BF-AWGN channel defined by (2), is characterized by the *instantaneous mutual information* (expressed in bit per complex symbol)

$$I(\underline{\mathbf{A}}) \triangleq \frac{1}{MN} \sum_{m=0}^{M-1} I(\underline{\mathbf{x}_m}; \underline{\mathbf{y}_m} \mid \underline{\mathbf{A}} = \underline{\mathbf{A}}) \tag{3}$$

where, with a slight abuse of notation, $I(\underline{\mathbf{x}_m}; \underline{\mathbf{y}_m} \mid \underline{\mathbf{A}} = \underline{\mathbf{A}})$ denotes the mutual information between $\underline{\mathbf{x}_m}$ and $\underline{\mathbf{y}_m}$ for a given realization of the channel matrix sequence $\underline{\mathbf{A}} \triangleq (\mathbf{A}_0, \ldots, \mathbf{A}_{M-1})$, and where we let $\underline{\mathbf{x}_m} \triangleq (\mathbf{x}_m[0], \ldots, \mathbf{x}_m[N-1])$ and
$\underline{\mathbf{y}_m} \triangleq (\mathbf{y}_m[0], \ldots, \mathbf{y}_m[N-1])$.

The information outage probability [8] of the M-block $K \times L$ BF-AWGN is defined by

$$P_{\text{out}} \triangleq P(I(\underline{\mathbf{A}}) < R). \tag{4}$$

In this paper we assume that, for each frame of M blocks, the transmitter has perfect knowledge of $\underline{\mathbf{A}}$ (see the discussion in Section I). In particular, we look for the optimal transmission scheme minimizing P_{out} under a transmit-power constraint.

The delay-limited capacity [9], is the maximum rate R at which the minimum outage probability is zero, for a given transmit-power constraint. In this paper, we consider a *long-term* power constraint specified by the following equation:

$$\frac{1}{MN} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} E_{\underline{\mathbf{A}}}[\text{Tr}(\mathbf{\Sigma}_m[n])] \leq \Gamma \tag{5}$$

where $E_{\underline{\mathbf{A}}}[\cdot]$ denotes expectation is with respect to $\underline{\mathbf{A}}$ and $\mathbf{\Sigma}_m[n] \triangleq 0.5\,\mathrm{E}[\mathbf{x}_m[n]\mathbf{x}_m[n]^\dagger | \underline{\mathbf{A}}]$, the covariance matrix of $\mathbf{x}_m[n]$, is a function of $\underline{\mathbf{A}}$.

IV. The optimal transmission scheme

First, we prove that there is no loss of optimality if we restrict to the case where the $\mathbf{x}_m[n]$'s are independent Gaussian special random vectors [3] with covariance matrix constant over each block. We have the following Proposition, whose proof will be omitted here (see [11] for a proof and further details).

[3] Following [4], a zero-mean complex random vector $\mathbf{x}$ is called *special* if

$$\begin{aligned} 2\,\mathrm{Re}(\mathbf{\Sigma}_m[n]) &= \mathrm{E}[\mathrm{Re}(\mathbf{x})\mathrm{Re}(\mathbf{x})^T] = \mathrm{E}[\mathrm{Im}(\mathbf{x})\mathrm{Im}(\mathbf{x})^T] \\ 2\,\mathrm{Im}(\mathbf{\Sigma}_m[n]) &= \mathrm{E}[\mathrm{Im}(\mathbf{x})\mathrm{Re}(\mathbf{x})^T] = -\,\mathrm{E}[\mathrm{Re}(\mathbf{x})\mathrm{Im}(\mathbf{x})^T] \end{aligned}$$

Proposition 1

The mutual information $I(\underline{\mathbf{A}})$ defined in (3) is maximum under the transmit-power constraint (5) when the channel-input vectors $\mathbf{x}_m[n]$, $n = 0, \ldots, N-1$ are independent zero-mean special complex Gaussian with covariance matrix $\mathbf{\Sigma}_m$ independent of n. In this case,

$$I(\underline{\mathbf{A}}) = \frac{1}{M} \sum_{m=1}^{M} \log\det(\mathbf{I}_L + \mathbf{A}_m \mathbf{\Sigma}_m \mathbf{A}_m)$$

For covariances $\mathbf{\Sigma}_m[n]$ independent of n, the constraint (5) reduces to

$$\frac{1}{M} \sum_{m=0}^{M-1} E_{\underline{\mathbf{A}}}[\text{Tr}(\mathbf{\Sigma}_m)] \leq \Gamma \tag{6}$$

Our approach is to reduce the vector problem to a scalar one and apply the results of [10;12]. To this purpose, as in [4;6;17], transform the M-block $K \times L$ BF-AWGN channel into a bank of equivalent parallel scalar channels. Let the singular value decomposition (SVD) of $\mathbf{A}_m$ be [16] $\mathbf{A}_m = \mathbf{U}_m \mathbf{S}_m \mathbf{V}_m^\dagger$ where $\mathbf{U}_m$ and $\mathbf{V}_m$ are $L \times L$ and $K \times K$ unitary matrices, respectively, and $\mathbf{S}_m$ is the diagonal matrix of the singular values of $\mathbf{A}_m$, i.e., the non-negative square-roots of the eigenvalues of $\mathbf{A}_m^\dagger \mathbf{A}_m$. The new channel obtained by pre-multiplying the input and post-multiplying the output of the original channel during the m-th block by $\mathbf{V}_m$ and by $\mathbf{U}_m^\dagger$, respectively, is described by

$$\widetilde{\mathbf{y}}_m[n] = \mathbf{S}_m \widetilde{\mathbf{x}}_m[n] + \widetilde{\mathbf{z}}_m[n] \tag{7}$$

for $m = 0, \ldots, M-1$ and $n = 0, \ldots, N-1$, where $\widetilde{\mathbf{x}}_m[n] = \mathbf{V}_m^\dagger \mathbf{x}_m[n]$, $\widetilde{\mathbf{y}}_m[n] = \mathbf{U}_m^\dagger \mathbf{y}_m[n]$, and $\widetilde{\mathbf{z}}_m[n] = \mathbf{U}_m^\dagger \mathbf{z}_m[n]$. Since $\mathbf{V}_m$ and $\mathbf{U}_m$ are invertible and $\widetilde{\mathbf{z}}_m[n]$ is distributed as $\mathbf{z}_m[n]$, the channels defined by (7) and by (2) are equivalent.

For a set of covariance matrices $\mathbf{\Gamma}_m = \mathrm{E}[\widetilde{\mathbf{x}}_m[n]\widetilde{\mathbf{x}}_m[n]^\dagger | \underline{\mathbf{A}}]$ with given diagonal elements, the instantaneous mutual information of channel (7) is maximum when $\widetilde{\mathbf{x}}_m[n]$ are independent complex Gaussian with i.i.d. real and imaginary parts [4]. Let $(\alpha_{mK+k})_{k=1}^{K}$ be the eigenvalues of $\mathbf{A}_m^\dagger \mathbf{A}_m$, and let $(\gamma_{mK+k})_{k=1}^{K}$ be the diagonal elements of $\mathbf{\Gamma}_m$, then the resulting instantaneous mutual information is given by

$$I(\underline{\mathbf{A}}) = \frac{1}{M} \sum_{m=0}^{M-1} \sum_{k=1}^{K} \log_2(1 + \alpha_{mK+k}\gamma_{mK+k}) \tag{8}$$

The channel given by (7) and the input-power constraint (5) are formally equivalent to the channel and constraint for a scalar MK-block BF-AWGN channel with maximum average SNR Γ/K. The only difference is that the instantaneous mutual information given by (8) is not divided by K. This fact suggests that multiple-antenna systems provide very high delay-limited capacity, as we will show in the following.

Given the above equivalence, results from [10;12] can be applied so as to obtain the assignment of the γ_{mK+k}'s as functions of the α_{mK+k}'s minimizing the outage probability subject to the long-term constraint. The resulting optimal transmission scheme is the concatenation with an optimal beamformer of a standard Gaussian code $\mathcal{C}$ (such that the empirical distribution of the code words components approaches the Gaussian distribution $\mathcal{N}_c(0,1)$) of length NMK and rate R. In each frame, the encoder selects a code word $\mathbf{c} \in \mathcal{C}$ partitioned into MN vectors $\mathbf{c}_m[n]$ of length K. The beamformer calculates $\mathbf{V}_m$, $(\alpha_{mK+k})_{k=1}^{K}$ from the SVD of $\mathbf{A}_m$, and the optimal $\mathbf{\Gamma}_m = \text{diag}(\gamma_{mK+1}, \ldots, \gamma_{mK+K})$ according to [10;12]. Finally, the transmitted vectors are

$$\mathbf{x}_m[n] = \mathbf{W}_m \mathbf{c}_m[n] \tag{9}$$

where $\mathbf{W}_m \triangleq \mathbf{V}_m \mathbf{\Gamma}_m^{1/2}$ is the beamforming matrix (for $m = 0, \ldots, M-1$ and $n = 0, \ldots, N-1$).

On the receiving side, $\mathbf{y}_m[n]$ is processed by a bank of "spatially-matched" filters described by the rows of $\mathbf{U}_m^\dagger$. The resulting received signal vector after spatial matched filtering is $\widetilde{\mathbf{y}}_m[n] = \mathbf{S}_m \mathbf{\Gamma}_m^{1/2} \mathbf{c}_m[n] + \widetilde{\mathbf{z}}_m[n]$.[4]

REMARK 1

Note that $\mathbf{V}_m$ depends only on $\mathbf{A}_m$ while $\mathbf{\Gamma}_m$ depends on all $\mathbf{A}_m$ for $m = 0, \ldots, M-1$. Also, notice that the coding and the beamforming operations are *decoupled*. Therefore, we conclude that when perfect CSI is available at the transmitter, no special space-time coding design [5] is required, and conventional optimal Gaussian codes suffice to achieve the minimum outage probability and the delay-limited capacity.

V. DELAY-LIMITED CAPACITY

The delay-limited capacity of the M-block $K \times L$ BF-AWGN channel with long-term power constraint Γ is obtained by equating to zero the outage probability P_{out} [10;12]. For the class of *regular* fading processes (see definition below), P_{out} is zero if the power-on region extends to the whole MK-dimensional non-negative orthant $\mathbb{R}_+^{MK}$, i.e., when $s^* \to \infty$ [10]. Thus, the delay-limited capacity can be obtained by solving for R the following equation

$$\Gamma = \mathrm{E}[\overline{\gamma}^{\,\text{lt}}(\overline{\boldsymbol{\alpha}}; R)] \tag{10}$$

where the expectation is taken over the random vector $\overline{\boldsymbol{\alpha}}$, and the dependence of $\overline{\gamma}^{\,\text{lt}}$ on R is indicated explicitly.

We are interested in the asymptotic behavior of the delay-limited capacity for high rates (i.e., as $R \to \infty$ or, equivalently, as $\Gamma \to \infty$). In order to state and prove a fairly general result, we focus on the class of regular fading processes satisfying the following condition:

DEFINITION 1

A $K \times L$ block fading process is defined regular *if: i) $\mathbf{A}_m$ has full rank $\kappa = \min(K, L)$ with probability 1; ii) the joint pdf of $\boldsymbol{\alpha} \triangleq (\alpha_1, \ldots, \alpha_{MK})^T$ is a continuous symmetric function; iii) $\mathrm{E}[1/\alpha_i]$ is finite for all $\alpha_i \neq 0$. Moreover, the joint pdf of the non-zero elements of $\boldsymbol{\alpha}$ is non-zero for all inner points of $\mathbb{R}_+^{M\kappa}$.*

[4] A scheme based on the same principle, aimed at maximizing mutual information rather than minimizing outage probability is proposed in [6] for a frequency-selective $K \times L$ channel with block transmission, and named Spatio-Temporal Vector Coding (STVC).

As an example, we notice that the independent Rayleigh BF-AWGN channel, where the elements of $\mathbf{A}_m$ are i.i.d. $\sim \mathcal{N}_c(0,1)$, with $\max(K, L) > 1$ is regular. The following propositions (see [11] for a proof) hold for a *regular* BF-AWGN channel.

PROPOSITION 2

The delay-limited capacity of a $K \times L$ BF-AWGN channel behaves asymptotically as $O(\log(\Gamma))$ for $\Gamma \to \infty$. More precisely,

$$C_{\text{delay-limited}} = \kappa \log_2 \left[\frac{\Gamma}{\kappa\, \mathrm{E}[(\prod_{i=1}^{M\kappa} \overline{\alpha}_i)^{-1/(M\kappa)}]} \right] + O\left(\frac{1}{\Gamma}\right) \tag{11}$$

where $\kappa \triangleq \min(K, L)$ and $(\overline{\alpha}_i)_{i=1}^{\kappa}$ are the (sorted) non-zero eigenvalues of $\mathbf{A}^\dagger \mathbf{A}$.

For illustration, we evaluate explicitly the RHS of (11) for the independent Rayleigh BF-AWGN channel with $K = L = 2, 3, 4$, and $M = 1$. In this case, the pdf of non-zero eigenvalues of $\mathbf{A}^\dagger \mathbf{A}$ is given by the Wishart distribution [15] and we obtain the following results:

$$C_{\text{delay-limited}} \approx \begin{cases} 2\log_2(0.3183\,\Gamma) & [K = L = 2] \\ 3\log_2(0.3625\,\Gamma) & [K = L = 3] \\ 4\log_2(0.3744\,\Gamma) & [K = L = 4] \end{cases} \tag{12}$$

where $\mathbf{\Gamma}(x) \triangleq \int_0^\infty u^{x-1} e^{-u}\, du$.

PROPOSITION 3

The delay-limited capacity of a $K \times L$ BF-AWGN channel behaves asymptotically as $O(\kappa)$ for $\kappa \to \infty$ and $\Gamma < \infty$. More precisely,

$$C_{\text{delay-limited}} = \kappa \log_2(\Gamma) + O(1) \tag{13}$$

VI. NUMERICAL RESULTS

In this section we illustrate some applications of the theory outlined above.

A. One transmit antenna

With a single transmit antenna and L receive antennas, the matrices $\mathbf{A}_m$ are column L-vectors, so that $\alpha_1[m] = |\mathbf{A}_m|^2$ is the only non-zero eigenvalue of $\mathbf{A}_m^\dagger \mathbf{A}_m$, $\mathbf{V}_m = 1$ (scalar), and the optimal beamforming matrices $\mathbf{W}_m$ reduce to the scalars $\sqrt{\gamma_1[m]}$. The instantaneous mutual information is $I(\underline{\mathbf{A}}) = \frac{1}{M} \sum_{m=0}^{M-1} \log_2(1 + |\mathbf{A}_m|^2 \gamma_1[m])$. In the case of independent Rayleigh BF-AWGN channel, $|\mathbf{A}_m|^2$ is chi-square distributed with $2L$ degrees of freedom.

B. One receive antenna

With K transmit antennas and a single receive antenna, the matrices $\mathbf{A}_m$ are row K-vectors, so that $\alpha_1[m] =$

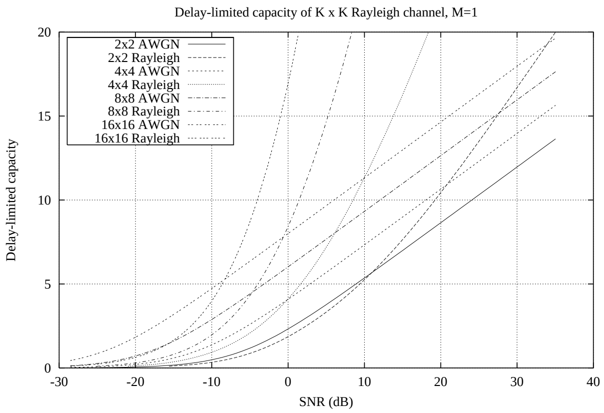

Fig. 1. Delay-limited capacity for the independent Rayleigh $K \times K$ BF-AWGN for $M = 1$ and $K = 2, 3, 4, 8$, and 16 obtained by Monte-Carlo integration. The capacity of the $K \times K$ AWGN channel is shown for comparison.

$|\mathbf{A}_m|^2$ is the only non-zero eigenvalue of $\mathbf{A}_m^\dagger \mathbf{A}_m$ and $\mathbf{V}_m$ is a unitary matrix whose first column is equal to $\mathbf{A}_m^\dagger / |\mathbf{A}_m|$. Again, the instantaneous mutual information is $I(\underline{\mathbf{A}}) = \frac{1}{M} \sum_{m=0}^{M-1} \log_2(1 + |\mathbf{A}_m|^2 \gamma_1[m])$ and in the case of independent Rayleigh BF-AWGN channel $|\mathbf{A}_m|^2$ is chi-square distributed with $2K$ degrees of freedom. As we can see, we have perfect reciprocity between transmitter-only and receiver-only diversity. This reciprocity, which holds for the AWGN channel, does not hold with the fading channel with no CSI at the transmitter [4]. Thus, we infer that reciprocity here is due to the availability of such CSI. For $M = 1$, the delay-limited capacity with optimal power allocation (corresponding to $\gamma_1[0] = (2^R - 1)/|\mathbf{A}_0|^2$) is $C_{\text{delay-limited}} = \log_2\left(1 + \Gamma / \,\mathrm{E}[|\mathbf{A}_0|^{-2}]\right)$. With independent Rayleigh BF-AWGN channel, $\mathrm{E}[|\mathbf{A}_0|^{-2}] = 1/(K-1)$, and hence $C_{\text{delay-limited}} = \log_2[1 + (K-1)\Gamma]$.

C. *More than one transmit/receive antennas*

Generalizing the results above to the case of more than one transmit/receive antennas, we consider the independent Rayleigh BF-AWGN channel (i.e., we assume that $\mathbf{A} \sim \mathcal{N}_c(\mathbf{0}, 0.5\mathbf{I})$) with $M = 1$ (i.e., every code word is affected by a constant fading value — no interleaving). Figures 1 and 2 show the delay-limited capacity versus SNR (Γ) for the $K \times K$ ($K = 2, 3, 4, 8$, and 16) and the $K \times 2$ (or $2 \times K$ for reciprocity, $K = 2$ to 8) BF-AWGN channel. Moreover, Fig. 1 reports, for comparison, the capacity of the $K \times K$ AWGN channel [4]

$$C_{\text{AWGN}} = \log_2(1 + K^2 \Gamma) \tag{14}$$

We note that delay-limited capacity with optimal power allocation exceeds the capacity of the $K \times K$ AWGN channel for all values of K above a certain value of SNR. This is a consequence of the fact that delay-limited capacity with optimal power allocation behaves asymptotically as $\kappa \log(\Gamma)$, while for the AWGN channel it behaves as $\log(\Gamma)$.

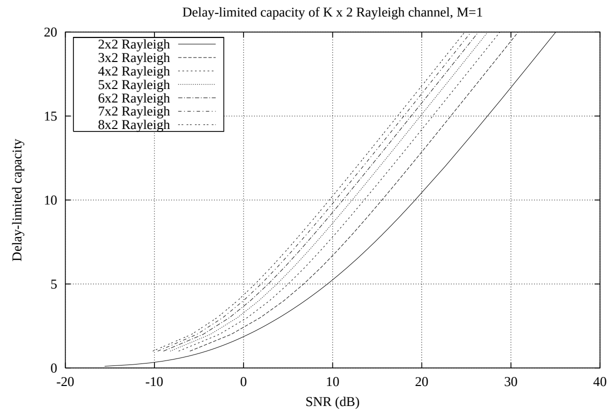

Fig. 2. Delay-limited capacity for the independent Rayleigh $K \times 2$ or $2 \times K$ BF-AWGN for $M = 1$ and $K = 2$ to 8 obtained by Monte-Carlo integration.

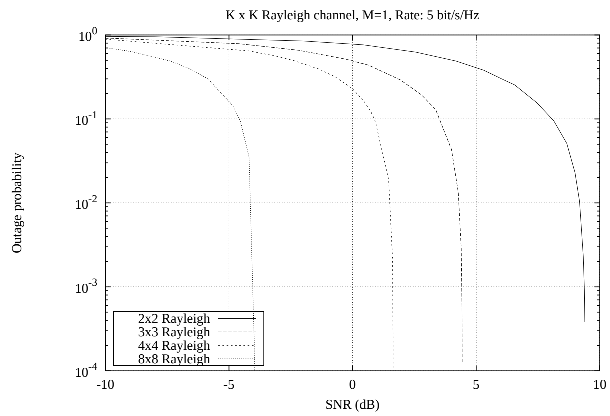

Fig. 3. Outage probability for the independent Rayleigh $K \times K$ BF-AWGN for $M = 1$ and $K = 2, 3, 4, 8$ obtained by Monte-Carlo integration.

D. *Outage Probability*

Figure 3 reports the outage probability corresponding to the $K \times K$ Rayleigh BF-AWGN channel with $K = 2$, 3, 4, and 8 and $M = 1$ versus the SNR Γ. The required transmission rate is $R = 5$ bit/s/Hz. It can be noticed that an outage probability level of 10^{-2} requires an SNR very close to that required at zero outage probability, i.e., the SNR required to have a delay-limited capacity equal to 5 bit/s/Hz.

E. *Delay-limited capacity versus antenna complexity*

Figs. 4 and 5 shows the delay-limited capacity versus the number of antennas (K) for the independent Rayleigh $K \times K$ BF-AWGN for $M = 1$ and 4, respectively. Besides the very high values of capacity achieved, it is interesting to note the linear growth of the capacity with K. Moreover, comparing the diagrams, we note that they are almost independent of M. Since M reflects the amount of interleaving allowed, this suggests that antenna diversity can be traded for interleaving (and hence interleaving delay), as observed in a different context in [18].

The results obtained for the independent Rayleigh $K \times 2$ (or $2 \times K$, by reciprocity) BF-AWGN channel with $M = 1$ are shown in Fig. 6. Again, increasing M yields little

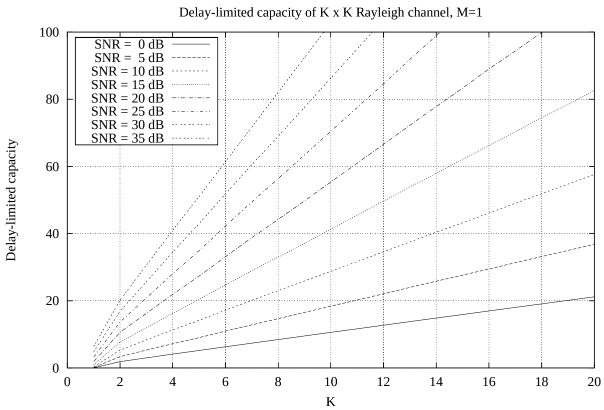

Fig. 4. Delay-limited capacity versus number of antennas (K) for the independent Rayleigh $K \times K$ BF-AWGN for $M = 1$ (results obtained by Monte-Carlo integration).

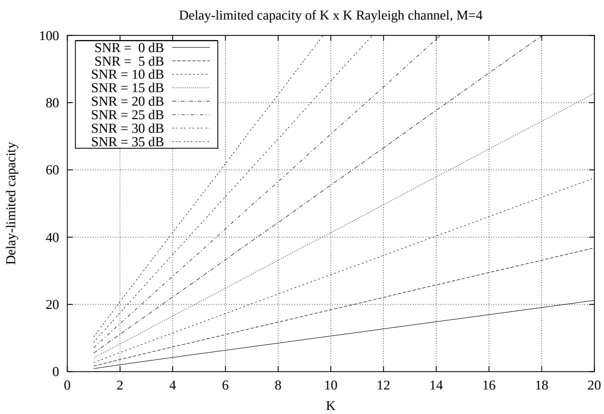

Fig. 5. Delay-limited capacity versus number of antennas (K) for the independent Rayleigh $K \times K$ BF-AWGN for $M = 4$ (results obtained by Monte-Carlo integration).

improvement in the delay-limited capacity.

VII. Conclusions

To understand the ultimate performance limits of a radio system consisting of a transmitter with K antennas and a receiver with L antennas, we have evaluated the minimum outage probability and the delay-limited capacity of a channel with transmit power constraint, independent flat fading between the transmit and receive antennas, Gaussian noise added independently at each receiver antenna, and channel state information available at the transmitter. Among other things, we have shown how the availability of channel-state information at the transmitter makes transmit-antenna diversity to be equivalent, in terms of capacity improvement, to receive-antenna diversity. Moreover, we have shown how antenna diversity can be a substitute for interleaving, in the sense that a target value of delay-limited capacity can be achieved by increasing diversity rather than interleaving depth (and hence interleaving delay).

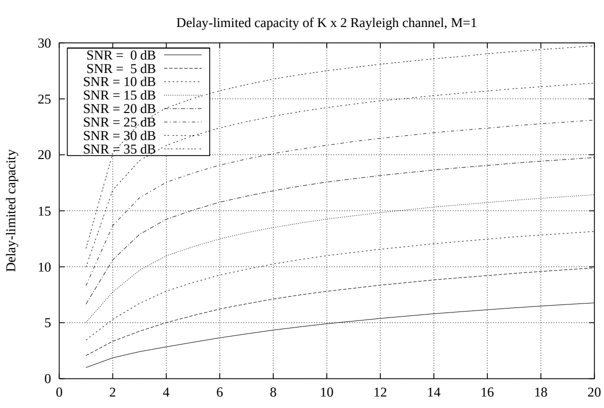

Fig. 6. Delay-limited capacity versus number of antennas (K) for the independent Rayleigh $K \times 2$ (or $2 \times K$) BF-AWGN for $M = 1$ (results obtained by Monte-Carlo integration).

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