Pilot-aided Adaptive MMSE Receivers for DS/CDMA

Giuseppe Caire, Urbashi Mitra

Abstract— **A weighted recursive least-squares algorithm for pilot-signal aided channel estimation in Direct-Sequence/Code-Division Multiple-Access (DS/CDMA) is proposed. Centralized and decentralized versions of the basic algorithm are considered. Since the algorithm tracks both the channel of the user of interest and the inverse covariance matrix of interference, it can be coupled with an adaptive linear MMSE receiver without additional complexity. The resulting receiver automatically performs the cancellation of the pilot-signals *before* data detection. Therefore, fairly significant power can be devoted to the pilot signals without affecting the overall interference level and without increasing the dimensionality of the desired signal space.**

Keywords— **CDMA, adaptive algorithms, multiuser receivers, frequency selective fading channels.**

I. Introduction

Multiuser detection has been a fruitful and rapidly growing research field for the last decade. Broadly speaking, this is motivated by the fact that the techniques developed for single-user communications, mostly devoted to combat Gaussian white noise, fail to give near-optimal performance if used in the presence of multiple-access interference (MAI). Under the common name of *multiuser detection* we find a wide range of receivers, which differ in complexity and performance (see [1] for a complete survey and a comprehensive list of references). We can distinguish between *centralized* and *decentralized* receivers. Centralized receivers make use of side information about all interfering users (spreading sequences, timing and propagation channels). These receivers are suited for base-station processing (uplink), where all this side information is either available or can be estimated consistently. Among centralized receivers we note the *optimal multiuser receiver* and receivers based on decision feedback or on parallel interference cancellation [1]. In contrast, decentralized receivers exploit knowledge of the spreading sequence, the timing and the propagation channel of the user of interest only. Remarkably, this is the same information necessary for a conventional matched filter that ignores the presence of MAI. Decentralized receivers are suited for mobile-station processing (downlink), where information relative to the other users is either difficult to obtain and/or forbidden, because of privacy reasons.

Some DS/CDMA systems [2], [3] make use of continuously transmitted pilot signals in order to perform channel estimation and enable coherent detection. In the downlink, a single pilot signal can be transmitted for each group of co-channel users (*i.e.*, users which go through the same beamformer). In the uplink, each user transmits the superposition of its data signal and its individual pilot signal. In both cases, pilot signals are to be seen as additional *virtual users* whose data sequence is known to the receiver. Conventional channel estimation is based on the assumption of discrete multipath: the channel is parameterized as a set of complex gains and delays, that are individually estimated by pilot-aided tracking loops, as in standard implementations of the *rake* receiver [4].

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In contrast, we start from a general discrete-time, finite-memory channel representation that does not necessarily assume discrete multipath. We propose a pilot-signal aided recursive channel estimation scheme based on weighted least-squares. Our basic algorithm can either work in a decentralized manner or be modified to work as a centralized algorithm that jointly estimates the channel of all interfering users. Since our algorithm simultaneously tracks both the channel for the user(s) of interest and the inverse covariance matrix of the interference, it can be coupled with an adaptive linear MMSE receiver without additional complexity. However, we hasten to say that our centralized algorithms have a much more general application, since virtually any (non-linear) centralized receiver needs the knowledge of the users' channels as side information [1].

It is well-known that linear receivers such as the MMSE and the decorrelator [1] suffer from the *dimensional crowding* problem: when the dimensionality of the signal space is larger than the length of the receiving filter there is no dimension left to null-out interference and the interference rejection property of the receiver is lost. This problem is particularly evident with pilot-signals, since each pilot-signal contributes to the overall signal space dimension. A nice feature of the proposed adaptive linear MMSE receiver is that it performs automatic active pilot-signal cancellation (APSC), *i.e.*, explicit cancellation of the pilots from the received signal, *before* data detection. Therefore, fairly significant power can be devoted to the pilot signals in order to achieve good channel estimation without affecting the interference level and without increasing the dimensionality of the signal space.

II. Discrete-time finite-memory signal model

Consider a system with K users. The k-th user's signal is given by

$$u_k(t) = \sum_m b_k[m] s_k(t - mT) \tag{1}$$

where $s_k(t)$ and $b_k[m]$ are the signature waveform and the m-th information symbol of user k, respectively. Users transmit individually and mutually uncorrelated sequences of unit-variance, zero-mean complex symbols. In

DS/CDMA, the signature waveforms are given by $s_k(t) = \sum_{\ell=0}^{L-1} s_{k,\ell}\psi(t-\ell T_c)$, where $\mathbf{s}_k = (s_{k,0},\ldots,s_{k,L-1})^T$ is the k-th user spreading sequence, $T_c = T/L$ is the chip interval, L is the processing gain and $\psi(t)$ is the chip pulse, assumed to be bandlimited in $[-W/2, W/2]$ and common to all users. We assume normalized waveforms $\int |s_k(t)|^2 dt = 1$.

User k transmits with delay τ_k through a channel with the baseband equivalent, time-varying, impulse response $c_k(t;\tau)$ [5]. We consider delays $\tau_k = q_k/W + \gamma_k$ where q_k is an integer in $[-LN_c/2, LN_c/2)$ and $0 \leq \gamma_k < 1/W$. The *fractional part*, γ_k, of the delay is modeled as an effect of the propagation channel, while the integer part q_k/W is modeled as an effect of the users' asynchronous transmission. User k signal contribution at the receiver is obtained by convolving $u_k(t-q_k/W)$ with $c_k(t;\tau-\gamma_k)$. For simplicity, we assume that the channel Doppler bandwidth B_d is much smaller than W, so that its output can be still considered as bandlimited in $[-W/2.W/2]$. [1] Then, from the sampling theorem we obtain

$$v_k(t) = \sum_i \left[\sum_j c_k[i;j] u_k((i-j-q_k)/W)\right] \mathrm{sinc}(Wt-i) \tag{2}$$

where $\mathrm{sinc}(t) = \sin(\pi t)/(\pi t)$ and where the coefficients of the time-varying discrete-time channel impulse response are given by

$$c_k[i;j] = \int c_k(i/W; j/W - \gamma_k - \tau)\mathrm{sinc}(W\tau)d\tau. \tag{3}$$

The overall received signal, given by the superposition of all users' signals plus background noise, is given by $y(t) = \sum_{k=1}^K v_k(t) + \nu(t)$, where $\nu(t)$ is a white circularly-symmetric complex Gaussian process with power spectral density N_0. The front-end baseband receiver is an ideal lowpass filter with bandwidth $[-W/2, W/2]$ and gain $1/\sqrt{W}$ followed by sampling at rate W with an arbitrary sampling epoch. We assume an integer number of samples per chip $N_c = WT_c$. Let $y[i]$ denote the sample of $y(t)$ at instant i/W *after lowpass filtering*. From (2) we obtain

$$y[i] = \frac{1}{\sqrt{W}} \sum_{k=1}^K \sum_j c_k[i;j] u_k[i-j-q_k] + \nu[i] \tag{4}$$

where $\{\nu[i]\}$ is an i.i.d. sequence with elements $\sim \mathcal{CN}(0, N_0)$. [2]

In order to obtain an approximated finite-memory signal model, we assume that the discrete-time impulse responses $c_k[i;j]$ and the sampled chip pulse $\psi(j/W)$ are negligible for $j \notin [0,P]$ (for all i) and for $j \notin [-Q,Q]$, respectively, where P and Q are suitable integers. Moreover, we assume that the receiver has a finite-length *processing window*, *i.e.*, for each symbol time n it processes a window of samples with indexes $i \in [nLN_c - M_1, nLN_c + M_2]$. The *processing window* size $\widetilde{L} = M_1 + M_2 + 1$ is left as a design parameter and it may span more than one symbol interval. Accordingly, we define the n-th channel output vector $\mathbf{y}[n]$ as the content of the receiver processing window at symbol time n, *i.e.*,

$$\mathbf{y}[n] = (y[nLN_c+M_2], y[nLN_c+M_2-1], \ldots, y[nLN_c-M_1])^T$$

and we let $\boldsymbol{\nu}[n]$ be the corresponding vector of noise samples.

Under the condition $B_d/W \ll 1$ (which holds in particular for wide-band CDMA signals), it is realistic to assume that the $c_k[i;j]$'s remain almost constant over the time interval spanned by the receiver processing window (of duration $(M_1+M_2+1)/W$). Hence, we can consider $c_k[nLN_c+i;j] = c_k[nLN_c;j]$ for all $i = -M_1,\ldots,M_2$ and represent the channel impulse response during the n-th symbol interval by the vector

$$\mathbf{c}_k[n] = (c_k[nLN_c;0],\ldots,c_k[nLN_c;P])^T \tag{5}$$

By inserting (1), (2) and (5) into (4), and after a little algebra, we obtain

$$\mathbf{y}[n] = \sum_{k=1}^K \sum_{m=-B_1}^{B_2} \mathbf{S}_k[m]\mathbf{c}_k[n] b_k[n-m] + \boldsymbol{\nu}[n] \tag{6}$$

where the matrices $\{\mathbf{S}_k[m] : m = -B_1,\ldots,B_2\}$, of size $(M_1+M_2+1)\times P$, are uniquely defined by q_k and $s_k(t)$, and have the (i,j)-th element given by

$$[\mathbf{S}_k[m]]_{i,j} = \frac{1}{\sqrt{W}} s_k((mLN_c - q_k + M_2 - i - j)/W) \tag{7}$$

for $i = 0,\ldots,\widetilde{L}-1$ and $j = 0,\ldots,P$. The summation limits B_1 and B_2 are obtained by noticing that $\mathbf{S}_k[m]$ is not identically zero over all possible $q_k \in [-LN_c/2, LN_c/2)$ if and only if $-B_1 \leq m \leq B_2$, where

$$\begin{aligned} B_1 &= \lfloor (M_2 + Q + LN_c/2)/(LN_c) \rfloor \\ B_2 &= \lfloor (M_1 + Q + P + 3LN_c/2 - N_c)/(LN_c) \rfloor \end{aligned} \tag{8}$$

Clearly, depending on the particular value of q_k, $\mathbf{S}_k[m]$ might be zero for some $m \in [-B_1, B_2]$. Then, each user contributes with *at most* $B_1 + B_2 + 1$ symbols to the vector $\mathbf{y}[n]$.

A. Pilot-signals

We assume that the users are partitioned into G groups of *co-channel users*. Users in the same group are synchronous and go through the same channel. Then, a single pilot-signal per group is sufficient in order to perform pilot-aided channel estimation for all users in the group. This assumption is fairly general and can model several situations in actual CDMA systems, for example:

i) In the uplink every user is characterized by a different channel ($G = K$ groups of size 1), so that an individual pilot per user is needed.

[1] In order to be rigorous, we should consider the bandwidth expansion due to the channel time-variation (Doppler spread). However, for $B_d/W \ll 1$, we can safely neglect the bandwidth expansion effect.

[2] $\mathcal{CN}(\boldsymbol{\mu}, \mathbf{R})$ denotes the circularly-symmetric complex multivariate Gaussian distribution with mean vector $\boldsymbol{\mu}$ and covariance matrix $\mathbf{R}$.

ii) In the case of multirate CDMA achieved by variable-length spreading or by multicode [6], a high-rate user can be decomposed into the superposition of several low-rate co-channel virtual users. Therefore, only one pilot per group of virtual users is needed.

iii) In the downlink, without beamforming or cell sectorization, all users from the same base-station are inherently synchronous and go through the same channel ($G = 1$ group of size K). Therefore a single pilot per cell is needed [7].

iv) In the downlink with sectorization or smarter beamforming, one pilot per beam is needed since users transmitted through different antenna patterns cannot be considered as co-channel.

In this this paper, we assume that pilot-signals have the same format as the data signals, *i.e.*, $p_g(t) = \sum_m d_g[m]s_g^{(p)}(t - mT)$, where $s_g^{(p)}(t)$ and $d_g[m]$ are the signature waveform and the m-th pilot-symbol of the g-th pilot-signal. Pilot signature waveforms are given by $s_g^{(p)}(t) = \sum_{\ell=0}^{L-1} s_{g,k}^{(p)}\psi(t - \ell T_c)$ where $\mathbf{s}_g^{(p)} = (s_{g,0}^{(p)}, \ldots, s_{g,L-1}^{(p)})^T$ is the g-th pilot spreading sequence. By repeating the derivation above, we obtain the vector channel model

$$\begin{aligned}\mathbf{y}[n] &= \sqrt{\gamma_1}\sum_{k=1}^{K}\sum_{m=-B_1}^{B_2}\mathbf{S}_k[m]\mathbf{c}_k[n]b_k[n-m] \\ &+\sqrt{\gamma_2}\sum_{g=1}^{G}\sum_{m=-B_1}^{B_2}\mathbf{S}_g^{(p)}[m]\mathbf{c}_g[n]d_g[n-m] \\ &+\boldsymbol{\nu}[n] \end{aligned} \quad (9)$$

where γ_1 and γ_2 define the power ratio between data and pilot signals, where the matrices $\mathbf{S}_g^{(p)}[m]$ are obtained from (7) by replacing $s_k(t)$ with $s_g^{(p)}(t)$ and where, by definition, $\mathbf{c}_k[n] = \mathbf{c}_g[n]$ if the k-th user belongs to the g-th group.

III. Pilot-aided adaptive channel estimation

We focus on the joint estimation of channels $\{\mathbf{c}_g[n] : g \in \mathcal{S}\}$, where $\mathcal{S} = \{g_1, \ldots, g_S\}$ is a subset of size S of $\{1, \ldots, G\}$. We assume that timing q_g, pilot spreading sequence $\mathbf{s}_g^{(p)}$ and pilot symbol sequence $\{d_g[n]\}$ are known for all $g \in \mathcal{S}$ and unknown for all $g \notin \mathcal{S}$. Notice that the receiver needs explicit knowledge of the coarse timing q_g only. The fractional part of the delay, γ_g, is implicitly handled by estimating the channel vector $\mathbf{c}_g[n]$.

For all n and all $g \in \mathcal{S}$, we define the matrices

$$\mathbf{H}_g[n] = \sqrt{\gamma_2}\sum_{m=-B_1}^{B_2}\mathbf{S}_g^{(p)}[m]d_g[n-m] \quad (10)$$

the block matrix $\mathbf{H}[n] = [\mathbf{H}_{g_1}[n]|\cdots|\mathbf{H}_{g_S}[n]]$, and the block vector

$$\mathbf{c}[n] = (\mathbf{c}_{g_1}[n]^T, \ldots, \mathbf{c}_{g_S}[n]^T)^T \quad (11)$$

of length $S(P+1)$. Then, (9) can be rewritten as

$$\mathbf{y}[n] = \mathbf{H}[n]\mathbf{c}[n] + \mathbf{w}[n] \quad (12)$$

where $\mathbf{w}[n]$ is uncorrelated with $\mathbf{H}[n]\mathbf{c}[n]$ and contains all user data signals, noise and all pilot signals not in the subset $\mathcal{S}$. With our assumptions, the sequence of matrices $\mathbf{H}[n]$ is known.

Then, consider the sequence of non-singular auxiliary matrices $\{\mathbf{M}[n]\}$ of size $\widetilde{L} \times \widetilde{L}$, and the Weighted Least-Squares (WLS) channel estimator minimizing the cost function

$$J(\mathbf{c}) = \sum_{i=1}^{n}\alpha^{n-i}(\mathbf{y}[i] - \mathbf{H}[i]\mathbf{c})^H\mathbf{M}[i](\mathbf{y} - \mathbf{H}[i]\mathbf{c}) \quad (13)$$

where $0 < \alpha \leq 1$ is an exponential forgetting factor. If $\widetilde{L} \geq S(P+1)$ and the matrices $\mathbf{H}[i]$ have full column-rank, the solution is easily obtained as

$$\widehat{\mathbf{c}}[n] = \left[\sum_{i=1}^{n}\alpha^{n-i}\mathbf{H}[i]^H\mathbf{M}[i]\mathbf{H}[i]\right]^{-1}\left[\sum_{i=1}^{n}\alpha^{n-i}\mathbf{H}[i]^H\mathbf{M}[i]\mathbf{y}[i]\right] \quad (14)$$

In this way, we simultaneously obtain the channel estimates for all users in the groups $g \in \mathcal{S}$. This problem falls into the class of Kalman filters with vector state and vector observations [8], for which recursive computation is possible. We still have to choose the sequence $\{\mathbf{M}[n]\}$. A simple choice is $\mathbf{M}[n] = \mathbf{I}$ for all n. Then, (13) becomes the classical exponentially-weighted Least-Squares cost function. A different sensible choice is $\mathbf{M}[n] = \mathbf{R}_w^{-1}$, where $\mathbf{R}_w = E[\mathbf{w}[n]\mathbf{w}[n]^H]$ is the “interference+noise” covariance in the the channel model (12) [9]. With this choice, (14) becomes an exponentially weighted version of the Best Linear Unbiased Estimator (BLUE) [8], that coincides with the maximum-likelihood estimator if $\mathbf{w}[n]$ were a Gaussian vector process. Unfortunately, the receiver has no knowledge of $\mathbf{R}_w$, therefore this must be estimated recursively too. We can write

$$\widetilde{\mathbf{R}}_w[n] = \sum_{i=1}^{n}\beta^{n-i}\widetilde{\mathbf{w}}[n]\widetilde{\mathbf{w}}[n]^H \quad (15)$$

where $0 < \beta \leq 1$ is an exponential forgetting factor (not necessarily equal to α) and where $\widetilde{\mathbf{w}}[n] = \mathbf{y}[n] - \mathbf{H}[n]\widehat{\mathbf{c}}[n]$. Then, with the choice $\mathbf{M}[n] = \widetilde{\mathbf{R}}_w[n-1]^{-1}$, we can approximate (14) by the following recursion (we omit the derivation for space limitations):

Recursive WLS channel estimator. Let $\widehat{\mathbf{c}}[0] = \mathbf{0}$, $\mathbf{M}[1] = \delta\mathbf{I}$ and $\boldsymbol{\Phi}[0] = \delta\mathbf{I}$, with $\delta > 0$. Then, for $n = 1, 2, \ldots$, let

$$\begin{aligned}\boldsymbol{\Phi}[n] &= \alpha\boldsymbol{\Phi}[n-1] + \mathbf{H}[n]^H\mathbf{M}[n]\mathbf{H}[n] \\ \widehat{\mathbf{c}}[n] &= \widehat{\mathbf{c}}[n-1] + \boldsymbol{\Phi}[n]^{-1}\mathbf{H}[n]^H\mathbf{M}[n] \\ &\quad\cdot(\mathbf{y}[n] - \mathbf{H}[n]\widehat{\mathbf{c}}[n-1]) \\ \widetilde{\mathbf{w}}[n] &= \mathbf{y}[n] - \mathbf{H}[n]\widehat{\mathbf{c}}[n] \\ \mathbf{M}[n+1] &= \frac{1}{\beta}\left[\mathbf{I} - \frac{\mathbf{M}[n]\widetilde{\mathbf{w}}[n]\widetilde{\mathbf{w}}[n]^H}{\beta + \widetilde{\mathbf{w}}[n]^H\mathbf{M}[n]\widetilde{\mathbf{w}}[n]}\right]\mathbf{M}[n] \end{aligned} \quad (16)$$

□

In the above recursion, the explicit computation of the inverse of $\boldsymbol{\Phi}[n]$ is needed. Unfortunately, this is an unavoidable feature of Kalman filters with vector observations [8].

$\mathbf{\Phi}[n]$ has dimension $S(P+1) \times S(P+1)$. Therefore, the number of groups S that can be estimated jointly also determines the algorithm computational complexity.

If $\widetilde{L} < S(P+1)$ or if the complexity of (16) is too large, we propose a suboptimal implementation of the channel estimator based on a parallel bank of individual estimators for all $g \in \mathcal{S}$. This can be obtained directly from (16) by constraining $\mathbf{\Phi}[n]$ to be in block-diagonal form, with S blocks $\mathbf{\Phi}_g[n]$ of size $(P+1) \times (P+1)$. The resulting algorithm is given by:

Parallel bank of Recursive WLS estimators. For all $g \in \mathcal{S}$ let $\widehat{\mathbf{c}}_g[0] = \mathbf{0}$, $\mathbf{M}[1] = \delta\mathbf{I}$ and $\mathbf{\Phi}_g[0] = \delta\mathbf{I}$, with $\delta > 0$ and let $\widehat{\mathbf{c}}[0] = (\widehat{\mathbf{c}}_{g_1}^T[0], \ldots, \widehat{\mathbf{c}}_{g_S}^T[0])^T$. Then, for $n = 1, 2, \ldots$:

1. For all $g \in \mathcal{S}$ let

$$\begin{aligned}
\mathbf{\Phi}_g[n] &= \alpha\mathbf{\Phi}_g[n-1] + \mathbf{H}_g[n]^H\mathbf{M}[n]\mathbf{H}_g[n] \\
\widehat{\mathbf{c}}_g[n] &= \widehat{\mathbf{c}}_g[n-1] + \mathbf{\Phi}_g[n]^{-1}\mathbf{H}_g[n]^H\mathbf{M}[n] \\
& \quad \cdot(\mathbf{y}[n] - \mathbf{H}[n]\widehat{\mathbf{c}}[n-1])
\end{aligned} \quad (17)$$

2. Let $\widehat{\mathbf{c}}[n] = (\widehat{\mathbf{c}}_{g_1}[n]^T, \ldots, \widehat{\mathbf{c}}_{g_S}[n]^T)^T$.
3. Update the inverse covariance matrix

$$\begin{aligned}
\widetilde{\mathbf{w}}[n] &= \mathbf{y}[n] - \mathbf{H}[n]\widehat{\mathbf{c}}[n] \\
\mathbf{M}[n+1] &= \frac{1}{\beta}\left[\mathbf{I} - \frac{\mathbf{M}[n]\widetilde{\mathbf{w}}[n]\widetilde{\mathbf{w}}[n]^H}{\beta + \widetilde{\mathbf{w}}[n]^H\mathbf{M}[n]\widetilde{\mathbf{w}}[n]}\right]\mathbf{M}[n]
\end{aligned}$$

□

With this receiver, S inverses of $(P+1) \times (P+1)$ matrices are needed. Moreover, normally the channel spread $P+1$ is much less than the processing window size $\widetilde{L}$, so that $\mathbf{\Phi}_g[n]$ is always invertible and S is not limited by P and $\widetilde{L}$, as in the case of (16).

IV. Adaptive MMSE receivers with APSC

Without loss of generality, we focus on the detection of user 1, assuming that it belongs to the user group 1 and that $1 \in \mathcal{S}$. We constrain the receiver to be a linear (time-varying) FIR filter with response $\mathbf{h}_1[n]$ of length $\widetilde{L}$ (*i.e.*, equal to the receiver processing window), followed by some (non-linear) detection algorithm based on the filter output sequence. Since all pilot-signals in $\mathcal{S}$ are known, they can be removed from the received signal vector without need of decision-feedback. The symbol-rate samples output by the receiver filter are given by

$$z_1[n] = \mathbf{h}_1[n]^H\widetilde{\mathbf{w}}[n] \quad (18)$$

where $\widetilde{\mathbf{w}}[n] = \mathbf{y}[n] - \mathbf{H}[n]\widehat{\mathbf{c}}[n]$ is already provided by the algorithms (16) and (17). We refer to this scheme as *Active Pilot-Signal Cancellation* (APSC). The signal-to-interference plus noise ratio (SINR) for the output $z_1[n]$ is given by

$$\text{SINR}[n] = \left[\frac{\mathbf{h}_1[n]^H\mathbf{R}_{\widetilde{w}}[n]\mathbf{h}_1[n]}{\gamma_1|\mathbf{h}_1[n]^H\mathbf{S}_1[0]\mathbf{c}_1[n]|^2} - 1\right]^{-1} \quad (19)$$

where $\mathbf{R}_{\widetilde{w}}[n]$ is the covariance matrix of $\widetilde{\mathbf{w}}[n]$ conditioned with respect to the channel vectors $\mathbf{c}_k[n]$ and their estimates $\widehat{\mathbf{c}}_k[n]$.

The baseline receiver is the single-user matched filter (SUMF) $\mathbf{h}_1[n] = \mathbf{S}_1[0]\mathbf{c}_1[n]$. This can be approximated by using the channel estimator (16), as

$$\mathbf{h}_1^{\text{sumf}}[n] = \mathbf{S}_1[0]\widehat{\mathbf{c}}_1[n] \quad (20)$$

where $\widehat{\mathbf{c}}_1[n]$ is the first subvector of length $P+1$ of $\widehat{\mathbf{c}}[n]$ provided by (16) or by (17). A more efficient choice for $\mathbf{h}_1[n]$ is the minimum mean-square error (MMSE) filter. Provided that APSC is perfect (*i.e.*, $\widetilde{\mathbf{w}}[n] = \mathbf{w}[n]$), this is given by $\mathbf{h}_1[n] = \mathbf{R}_w[n]^{-1}\mathbf{S}_1[0]\mathbf{c}_1[n]$, where $\mathbf{R}_w[n] = E[\mathbf{w}[n]\mathbf{w}[n]^H]$ is given by

$$\begin{aligned}
\mathbf{R}_w[n] &= \gamma_1\sum_{k=1}^{K}\sum_{m=-B_1}^{B_2}\mathbf{S}_k[m]\mathbf{c}_k[n]\mathbf{c}_k[n]^H\mathbf{S}_k[m]^H \\
& \quad +\gamma_2\sum_{g\notin\mathcal{S}}\sum_{m=-B_1}^{B_2}\mathbf{S}_g^{(p)}[m]\mathbf{c}_g[n]\mathbf{c}_g[n]^H\mathbf{S}_g^{(p)}[m]^H \\
& \quad +N_0\mathbf{I}
\end{aligned} \quad (21)$$

where again $\mathbf{c}_g[n] = \mathbf{c}_k[n]$ if user k belongs to group g. Algorithms (16) and (17) provides inherently a recursive estimate $\mathbf{M}[n+1]$ of $\mathbf{R}_w[n]^{-1}$. Then, we obtain easily an approximation of the MMSE filter as

$$\mathbf{h}_1^{\text{mmse}}[n] = \mathbf{M}[n+1]\mathbf{S}_1[0]\widehat{\mathbf{c}}_1[n] \quad (22)$$

In the case of a centralized receiver where $\mathcal{S} = \{1, \ldots, G\}$ (*i.e.*, all user groups are jointly estimated either by (16) or by (17)), the MMSE receiver can be calculated by computing explicitly the inverse of the *structured* covariance estimate by replacing $\mathbf{c}_k[n]$ with $\widehat{\mathbf{c}}_k[n]$ in (21). The resulting filter is

$$\mathbf{h}_1^{\text{mmse}}[n] = \widehat{\mathbf{R}}_w[n]^{-1}\mathbf{S}_1[0]\widehat{\mathbf{c}}_1[n] \quad (23)$$

The latter expression makes use of a good deal of additional information about the structure of the covariance matrix. Hopefully, this will improve the receiver performance. Efficient methods of computing (23) when $\widehat{\mathbf{R}}_w[n]$ is given by an identity matrix plus a sum of vector outer products are presented in [10].

V. Numerical results and Conclusions

We examine separately the uplink and the downlink of a simple DS/CDMA system.

A. Uplink

We consider a system with $K = 8$ users and processing gain $L = 16$. Each user transmits the superposition of data and pilot signals (this is the case of $G = K$). Spreading sequences are obtained by chip-wise multiplication of a Walsh-Hadamard (WH) sequence and a pseudo-noise (PN) sequence. Users have two distinct WH, one for data and the other for pilot, and one PN. Then, in the absence of multipath, the data and the pilot signal of the same user are mutually orthogonal. Each user is given a distinct PN sequence, but may use the same WH for other users. PN sequences are randomly generated with i.i.d. components over a 4PSK signal set (quaternary sequences) and also the modulation symbols for both the pilot and the data signals are 4PSK. The power-weighting coefficients are given by $\gamma_1 = 1/(1+\gamma_p)$ and $\gamma_2 = \gamma_p/(1+\gamma_p)$, where γ_p is the pilot-to-data power ratio. For simplicity, we assumed ideal Nyquist chip pulses $\psi(t) = \frac{1}{\sqrt{T_c}}\text{sinc}(t/T_c)$ and we chose receiver sampling rate $W = 1/T_c$, yielding $N_c = 1$ sample per chip. Without loss of generality, we let $q_1 = 0$ and we generated independently the delays q_k for $k = 2, \ldots, K$, uniformly distributed over the integers in $[-L/2, L/2)$, and γ_k for $k = 1, \ldots, K$, uniformly distributed over $[0, T_c)$. The channel vectors $\mathbf{c}_k[n]$ are obtained from (3), where the continuous-time channel responses $c_k(t;\tau)$ are derived from the multipath Rayleigh fading model $c_k(t;\tau) = \sum_{p=0}^{P'} g_p(t)\delta(\tau - \tau_p)$ where $g_p(t)$ are zero-mean mutually independent complex Gaussian WSS random processes with Jake's type power-spectral density (Doppler spectrum) $\sigma_p^2/(\pi\sqrt{B_d^2 - f^2})$ [11], for an exponentially decreasing *delay-intensity profile* [5] spanning 5 chips. The resulting channel vectors were scaled in order to achieve the desired user SNRs (in the time-varying case, SNRs are defined as time-averages over the whole simulation length, thus emulating a slow power control scheme). For each user, a snapshot of the random channel $\mathbf{c}_k[n]$ was generated for n ranging over the simulation length. The channels, as well as the delays q_k and the spreading sequences were fixed throughout all the simulations. Therefore, *we are not averaging over these parameters.* The receiver processing window is chosen to span three symbol intervals ($\widetilde{L} = 48$). In agreement with [12], extending the processing window over more than one symbol interval improves robustness to timing errors and performance in the presence of asynchronous transmission. We considered two SNR assignments: (a) all users have the same SNR= 10 dB (SNR is defined as the ratio of the total data+pilot symbol energy over N_0); (b) users $k = 1, \ldots, 4$ have SNR= 10 dB and users $k = 5, \ldots, 8$ have SNR= 20 dB. These situations are representative of perfect power-control and an uncompensated near-far effect.

Fig. 1 shows SINR$[n]$ vs. n for stationary channels (Doppler bandwidth $B_dT = 0$), perfect power control (a) and $\gamma_p = 0$ dB, for centralized adaptive receivers based on (22) where joint channel estimation is obtained by (16) (labeled "Joint") and by (17) (curves denoted by "Joint para."). Results for the receiver (23), based on the same joint channel estimation algorithms but with structured covariance matrix are denoted by "str.". For comparison, the results for ideal (non-adaptive) SUMF, MMSE with and without APSC are shown. Notice that the ideal MMSE without APSC achieves poor SINR, since with $\gamma_p = 0$ dB the system suffers from dimensional crowding (8 data plus 8 pilot signals with spreading gain 16). Fig. 2 shows the SINR curves for joint parallel channel estimation and receiver (22) for different values of γ_p. From these curves, it seems that $\gamma_p = -6$ dB yields a good trade-off between steady-state performance and convergence speed. Fig. 3 shows the SINR curves for time-varying channels with Doppler bandwidth $B_dT = 10^{-3}$, for $\gamma_p = -6$ dB and near-far case (b). From these results, we conclude that the proposed centralized receiver is able to track time-varying channels and remove the pilot-signals before detection, avoiding dimensional crowding. Moreover, it is clear that the pilot-to-data power ratio must be optimized carefully and that the receivers that exploit the structure of the interference covariance have close-to-optimal performance (a degradation of less than 2 dB with respect to the ideal MMSE with APSC in the time-varying case of Fig. 3).

B. Downlink

We considered a system with two cells, 5 user per cell and processing gain $L = 16$. Users from the same cell are co-channel and synchronous, therefore we are in the case of $K = 10$, $G = 2$ groups of 5 users each. One pilot-signal per cell is transmitted. The channel model is identical to the uplink, with the difference that only 2 channels (one per cell) and the relative delay between the two cells are to be generated. The user of interest (user 1) belongs to cell 1. We assumed a mobile terminal in between the two base-stations, receiving the same average power from both. The power-weight coefficients are $\gamma_1 = 1$ and γ_2 arbitrary, since we assume that the base station is not power-limited. The user SNR is defined as the ratio of the data symbol energy over N_0 and it is equal to 10 dB for all users. Spreading sequences are again obtained by chip-wise multiplication of WH sequences and quaternary PN sequences. In this case, all users from the same base-station have a distinct WH and a common PN. Two distinct PN are assigned to the two cells, but WH sequences may be reused inside each cell.

Fig. 4 shows the SINR curves for time-varying channels with Doppler bandwidth $B_dT = 10^{-3}$, for $\gamma_2 = 10$ dB. The receiver is decentralized and based on (22). We considered the cases of hard and soft handoff (HH and SH, respectively). In the case of HH, the mobile has knowledge of the pilot signal (and timing) of cell 1. Therefore, it can perform APSC only on this signal. In the case of SH, the mobile has knowledge of the pilot signal (and timing) of both cell 1 and 2. Therefore, it can estimate jointly the channels from the two base-stations and perform APSC on both pilot signals. Our plots report only the SINR at the output of the receiver for the signal coming from cell 1 (SINR per macrodiversity branch). However, we would like to point out that the proposed receiver scheme working in SH mode can be easily combined with any space-time coding scheme. For example, an encoded symbol sequence can be demultiplexed and transmitted in parallel from the two base-stations, achieving both coding and macrodiversity gains [13]. Interestingly, we notice that the performance of the proposed receivers and of the ideal (non-adaptive) MMSE are very similar for APSC on one or both pilots. In comparison, the ideal SUMF performs very differently. This is because the SUMF is not near-far resistant, and its performance is degraded by the interference of the strong pilot signal of cell 2, which is not actively canceled.

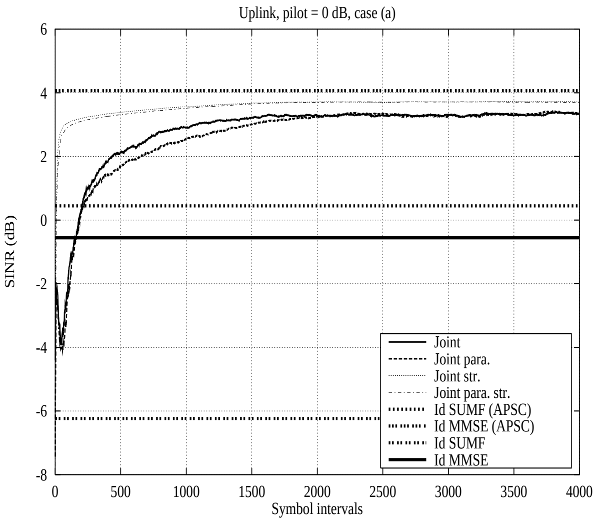

Fig. 1. SINR vs n for the uplink with $B_d T = 0$ and near-far case (a).

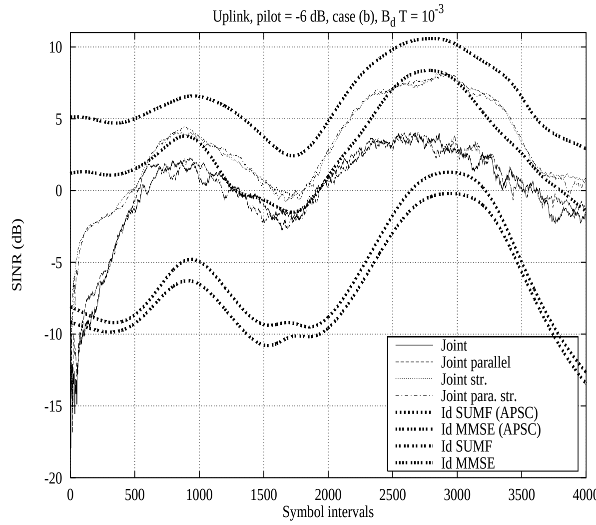

Fig. 3. SINR vs n for the uplink with $B_d T = 10^{-3}$ and near-far case (b), $\gamma_p = -6$ dB.

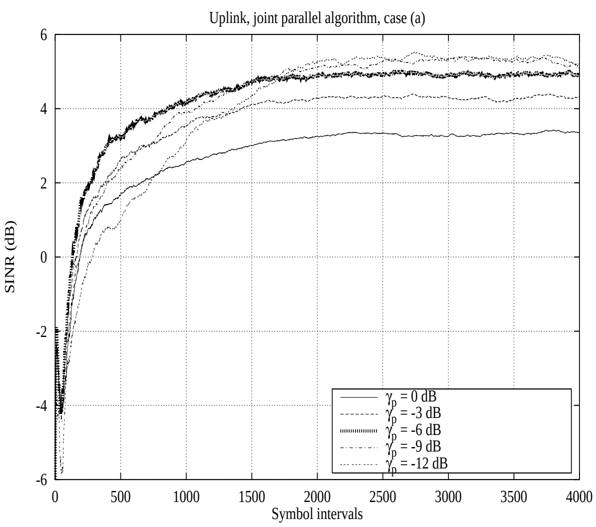

Fig. 2. SINR vs n for the uplink with $B_d T = 0$ and near-far case (a), for different values of γ_p.

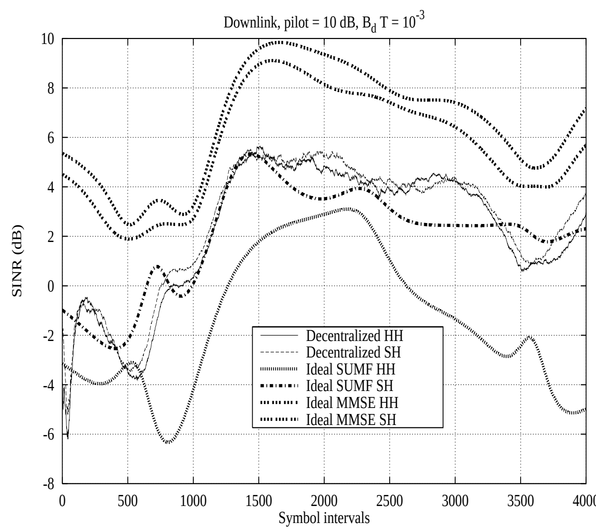

Fig. 4. SINR vs n for the downlink with $B_d T = 10^{-3}$ and pilot power 10 dB above data signals.

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