Performance Comparison for Low Complexity Blind Sensing Techniques in Cognitive Radio Systems

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***Abstract*—In this paper[1], we will provide a straightforward classification of some spectrum sensing strategies derived at Eurecom attempting to show the diversity and advantages of these spectrum sensing techniques. Specifically, two low complexity blind sensing algorithms were developed to detect spectrum holes in the primary user's bands: the distribution analysis detector (DAD) and the algebraic detector (AD), which are compared with the energy detector (ED) as reference algorithm. For performance evaluation we have chosen to thoroughly investigate the DVB-T primary user system. Simulation results show that the two proposed detectors offer high performances and detect primary users presence even at very low SNR with comparable complexity to ED.**

***Index Terms*—Cognitive radio, blind sensing, energy detector, distribution analysis detector, algebraic detector, low complexity.**

I. Introduction

As cognitive radio (CR) is based on the principle of exploiting any part of the radio spectrum not in use, whether licensed or not, it is crucial to sense the spectrum and find holes that can be exploited for a shorter or longer period of time [1]. For signal transmission which requires real time and small delay, this may become difficult depending on the intensity of exploitation of the spectrum [2]. For non-realtime applications the potential is great.

Spectrum sensing techniques are based on primary user modulation type, power and frequency. Primary users that use frequency hopping or spread spectrum signaling, where the power of the primary user signal is distributed over a wider frequency even though the actual information bandwidth is much narrower, are difficult to detect. Primary users can claim their frequency bands anytime while CR is operating at that band. In order to prevent interference to and from primary licence owners, a CR should be able to identify the presence of primary users as quickly as possible and should vacate the band immediately. Hence, sensing methods should be able to identify the presence of primary users within a certain duration. This requirement possesses a limit on the performance of sensing algorithms and creates a challenge for CRs. Some other challenges that need to be considered while designing effective spectrum sensing algorithms include implementation complexity, presence of multiple secondary users, coherence times, multipath and shadowing, cooperation, competition, robustness, heterogeneous propagation losses, and power consumption [3].

There are several spectrum sensing techniques that were proposed for CR [3]. A few completely blind sensing methods that do not consider any prior knowledge about the transmitted signal have been derived in the literature, but all of them suffer from the noise uncertainty and fading channel variations. One of the most popular is the energy detector (ED) [4]. Despite its easy implementation and low complexity, the ED does not perform well at a low signal-to-noise-ratio (SNR) and cannot differentiate between noise and signals. Moreover, this kind of detector is inconvenient when the level of noise is completely unknown. Two other blind techniques were proposed at EURECOM. The first technique analyzes the distribution of the primary user received signal to sense vacant frequency sub-bands over the spectrum band. Specifically, the distribution analysis detector (DAD) exploits model selection tools like the Akaike information criterion (AIC) to detect vacant holes in the spectrum band [5] [6]. For that, we assume that the noise of the radio spectrum band can still be adequately modeled using a Gaussian distribution. We then compute and analyze Akaike weights in order to decide if the distribution of the received signal fits the noise distribution or not. The second detector called algebraic detector (AD) exploits the change point detection [7]. Indeed, the detection of a used sub-band corresponds to the presence of a "spike-like" in the signal's spectrum which is represented with an N order local piecewise model. The two proposed detectors, in addition to their low complexity compared to the ED, offer high performances even at low SNR regime.

This paper presents part of the experimental results obtained during the European research project SENDORA (SEnsor Network for Dynamic and cOgnitive Radio Access) [8]. Among the objectives of the SENDORA project is the design of new robust spectrum sensing algorithms, whose detection power will be enhanced by processing data from several sources in order to perform distributed detection of the primary licensed users [8]. In this paper, we present part of some results obtained during this project, which deals with opportunistic spectrum sensing management. Specifically, we develop a common framework for the comparison of three low complexity blind sensing algorithms.

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The paper is organized as follows. After the presentation of the common framework study in Section II, the spectrum sensing techniques are discussed in Section III. In Section IV, the performance evaluation and advantages are described, and a comparison of the three detectors is given. Finally, Section V concludes the paper.

II. Framework Study

In this section, we describe the system model that will be used throughout this paper. For the radio channel measurement we have chosen to thoroughly investigate the DVB-T primary user system. In this system, the transmitted signal is convolved with a multi-path channel and a Gaussian noise is added. The received signal at time n, denoted by x_n, can be modeled as:

$$x_n = A_n s_n + e_n \tag{1}$$

where A_n being the transmission channel gain, s_n is the transmit signal sent from the primary user and e_n is a stationary, Gaussian noise with zero mean. The goal of spectrum sensing is to decide between the following two hypotheses:

$$x_n = \begin{cases} e_n & \mathrm{H}_0 \\ A_n s_n + e_n & \mathrm{H}_1 \end{cases} \tag{2}$$

We decide a spectrum band to be unoccupied if it contains only noise, as defined in H_0; on the other hand, once there exist primary user signals besides noise in a specific band, as defined in H_1, we say the band is occupied. Let P_F be the probability of false alarm given by:

$$P_F = P(\mathrm{H}_1 \mid \mathrm{H}_0) = P(x_n \text{ is present} \mid \mathrm{H}_0) \tag{3}$$

that is the probability of the spectrum detector having detected a signal under hypothesis H_0, and P_D the probability of detection expressed as:

$$\begin{aligned} P_D &= 1 - P_M = 1 - P(\mathrm{H}_0 \mid \mathrm{H}_1) \\ &= 1 - P(x_n \text{ is absent} \mid \mathrm{H}_1) \end{aligned} \tag{4}$$

the probability of the detector having detected a signal under hypothesis H_1, where P_M indicates the probability of missed detection.

We develop in this paper a common framework to make a comparison of three blind sensing algorithms. In order to decide on the nature of the received signal, we calculate a threshold for each detector. The decision threshold is determined using the required probability of false alarm P_{FA} given by (3). The threshold Th for a given false alarm probability is determined by solving the equation:

$$P_{FA} = P(T(x_n) > Th | H_0) \tag{5}$$

where $T(x_n)$ denotes the test static for the given detector. Note that, for each detector we compute a particular threshold Th that tests the decision statistic based on a fixed false alarm probability value.

III. Spectrum Sensing Techniques

A. Energy Detector (ED)

The ED is the most common method for spectrum sensing because of its non-coherency and low complexity. The energy detector measures the received energy during a finite time interval and compares it to a predetermined threshold. That is, the test statistic of the energy detector is:

$$\sum_{m=1}^{M} |x_m|^2 \tag{6}$$

where M is the number of samples of the received signal x_m. The computed energy is compared to a threshold value, that depends on the false alarm probability given by equation (5), to make a decision about the presence/absence of a primary user signal.

Conventional ED can be simply implemented like spectrum analyzer. It is universal in the sense that it does not require any knowledge about the signal to be detected. On the other hand, for the same reason it does not exploit any potentially available knowledge about the signal.

B. Distribution Analysis Detector (DAD)

The main idea of the blind DAD is to decide if the distribution of the observed signal x_k fits the candidate model. Let K be the number of independent observations $x_1, x_2, ..., x_K$. It is assumed that the samples of the received signal x_k are distributed according to an original probability density function f, called the operating model. The operating model is usually unknown, since only a finite number of observations is available. Therefore, an approximate probability model (i.e. candidate model) must be specified using the observed data, in order to estimate the operating model. The candidate model is denoted as g_θ, where the subscript θ indicates the U-dimensional parameter vector, which in turn specifies the probability density function. Akaike's proposal was to select the model which gives the minimum AIC [10] [11], defined by:

$$\mathrm{AIC} \quad = \quad -2 \sum_{k=1}^{K} \log g_{\hat{\theta}}(x_k) + 2U \tag{7}$$

The parameter vector θ for each family should be estimated using the minimum discrepancy estimator $\hat{\theta}$, which minimizes the empirical discrepancy.

The sensing technique selects the distribution that best fits the data. In fact, we consider that the norm of the Gaussian noise can be modeled using a Rayleigh distribution and the presence of a signal can be modeled using a Rice distribution. From the received signal, we estimate the parameters $\hat{\theta}$ for the Rayleigh and Rice distribution. Then, we compute the AIC for both distributions according to (7). The probability density function for the Rayleigh distribution is given by:

$$g_{Rayleigh}(x \mid \sigma) = \frac{x}{\sigma^2} \exp\left(\frac{-x^2}{2\sigma^2}\right) \tag{8}$$

which leads to a log-likelihood function:

$$L_{Rayleigh}(\sigma) = \sum_{i=1}^{M} \log x_i - M \log \sigma^2 - \frac{1}{2\sigma^2} \sum_{i=1}^{M} x_i^2 \quad (9)$$

where the parameter $\theta = (\sigma)$ and M is the number of observation samples. The MLE of the parameter σ is given by:

$$\hat{\sigma}^2 = \frac{1}{2M} \sum_{i=1}^{M} x_i^2 \quad (10)$$

The probability density function for the Rice distribution is given by:

$$g_{Rice}(x \mid v, \sigma) = \frac{x}{\sigma^2} \exp\left(\frac{-(x^2+v^2)}{2\sigma^2}\right) I_0\left(\frac{xv}{\sigma^2}\right) \quad (11)$$

where $I_0\left(\frac{xv}{\sigma^2}\right)$ is the modified Bessel function of the first kind with order zero. The approximated probability density function leads to the following log-likelihood function:

$$\begin{aligned} L_{Rice}(v,\sigma) &= \log\left(\frac{\prod_{i=1}^{p} x_i}{\sigma^{2p}} \exp\left(-\frac{\sum_{i=1}^{p}\left(x_i^2+v^2\right)}{2\sigma^2}\right)\right. \\ &\times \left.\prod_{i=1}^{p} I_0\left(\frac{x_i v}{\sigma^2}\right)\right) \end{aligned} \quad (12)$$

Parameters v and σ are a solution of the following set of equations [12]:

$$\begin{cases} v - \frac{1}{p}\sum_{i=1}^{p} x_i \frac{I_1\left(\frac{x_i v}{\sigma^2}\right)}{I_0\left(\frac{x_i v}{\sigma^2}\right)} = 0 \\ 2\sigma^2 + v^2 - \frac{1}{p}\sum_{i=1}^{p} x_i^2 = 0 \end{cases} \quad (13)$$

where $I_1\left(\frac{x_i v}{\sigma^2}\right) = -I_0\left(\frac{x_i v}{\sigma^2}\right) + \frac{\sigma^2}{2xv} I_0\left(\frac{x_i v}{\sigma^2}\right)$ is the modified Bessel function with order one.

In order to show the comparison results between the distributions in a clearly manner, we introduce the Akaike weights W_{Rice} and $W_{Rayleigh}$ derived from the AIC values [9]. Akaike weights for Rice can be expressed as:

$$W_{Rice} = \frac{exp\left(-\frac{1}{2}\Phi_{Rice}\right)}{exp\left(-\frac{1}{2}\Phi_{Rice}\right) + exp\left(-\frac{1}{2}\Phi_{Rayleigh}\right)} \quad (14)$$

where

$$\Phi_{Rice} = \text{AIC}_{Rice} - \min\left(\text{AIC}_{Rice}, \text{AIC}_{Rayleigh}\right) \quad (15)$$

Similarly, we can express Akaike weights for Rayleigh. AIC_{Rice} and $\text{AIC}_{Rayleigh}$ are computed using equation (7) taking $g_{\hat{\theta}} = g_{Rice}$ and $U = 2$ for a Rice distribution and $g_{\hat{\theta}} = g_{Rayleigh}$ and $U = 1$ for a Rayleigh distribution. The test statistic of the DAD detector is given by:

$$\begin{cases} W_{Rice} - W_{Rayleigh} < Th & \text{noise} \\ W_{Rice} - W_{Rayleigh} > Th & \text{signal} \end{cases} \quad (16)$$

According to the system requirement on P_{FA}, we calculate a proper threshold Th through simulations. If $W_{Rice} - W_{Rayleigh} > Th$, we declare that the primary user is present, otherwise, we declare the primary user is absent.

C. Algebraic Detector (AD)

The AD is a new approach based on the advances lead in the fields of differential algebra and operational calculus. In this method, the primary user's presence is rather casted as a change point detection in its transmission spectrum [7]. In this approach, the mathematical representation of the amplitude spectrum of the received signal X_n in frequency domain is assumed to be a piecewise N^{th} order polynomial signal expressed as follows:

$$X_n = \sum_{k=1}^{K} \chi_k[n_{k-1}, n_k] p_k(n - n_{k-1}) + E_n \quad (17)$$

where $\chi_k[n_{k-1}, n_k]$ is the characteristic function, $p_k(n)$ is an N^{th} order polynomials and E_n is the additive corrupting noise. K is the number of subsection, and n is the normalized frequency. For simulation results, $M/K = 1000$.

Let S_n be the clean version of the received signal given by:

$$S_n = \sum_{k=1}^{K} \chi_k[n_{k-1}, n_k] p_k(n - n_{k-1}) \quad (18)$$

And let b be a frequency bandwidth such that in each interval $I_b = [n_{k-1}, n_k] = [\nu, \nu + b]$, $\nu \geq 0$ one and only one change point occurs. Denoting $S_\nu(n) = S_{n+\nu}$, $n \in [0, b]$ for the restriction of the signal in the interval I_b and redefine the change point relatively to I_b say n_ν given by:

$$\begin{cases} n_\nu = 0 & \text{if } S_\nu \text{ is continuous} \\ 0 < n_\nu \leq b & \text{otherwise} \end{cases} \quad (19)$$

The primary user presence on a sensed sub-band is equivalent to finding $0 < n_\nu \leq b$ in this band. The AD gives the opportunity to build a whole family of detectors for spectrum sensing, depending on a given model order N. Depending on this model order, we can show that the performance of the AD increases as the order N increases.

The proposed algorithm is implemented as a filter bank composed of N filters mounted in a parallel way. The impulse response of each filter is:

$$h_{k+1,n} = \begin{cases} \frac{\left(n^l (b-n)^{N+k}\right)^{(k)}}{(l-1)!} & 0 < n < b \\ 0 & \text{otherwise} \end{cases} \quad (20)$$

where $k \in [0..N-1]$ and l is chosen such that $l > 2 \times N$. The proposed expression of $h_{k+1,n}$ was determined by modeling the spectrum by a piecewise regular signal in the frequency domain and casting the problem of spectrum sensing as a change point detection in the primary user transmission [7]. Finally, in each stage of the filter bank, we solve the following equation:

$$\varphi_{k+1} = \sum_{m=0}^{M/K} W_k h_{k+1,m} X_m \quad (21)$$

where W_m are the weights for numeric integration defined by:

$$\begin{array}{ll} W_k = 0.5 & \text{for } k = 0, M \\ W_k = 1 & \text{otherwise} \end{array} \quad (22)$$

In order to infer whether the primary user is present in its sub-band, a decision function is computed as follows:

$$Df = \prod_{k=0}^{N-1} |\varphi_{k+1}| \tag{23}$$

The decision is made by computing the threshold Th to the decision function over the sensed sub-band. The threshold Th is set through simulations.

IV. Performance Evaluation

The complexity of a sensing detector is a major concern in spectrum sensing. Using the implementation steps of the three detectors, we will study the complexity required for each detector to derive their sensing algorithm. As the ED is well known for its simplicity, the comparison is made with reference to it. The complexity of the algorithms is measured through the number of complex multiplications that the algorithms has to perform for the calculation of the test statistics. We summarize the number of multiplications required for each technique in Table I. Note that M refers to the number of samples of the received signal x_n and N is the model order of the AD. From these results, we find that the two proposed detectors have the same complexity for $N = 2$, with over 2 times the complexity as compared to the ED. While the ED and the AD have the same complexity for $N = 1$.

Sensing technique	Complexity
Energy detector	M
Distribution analysis detector	2M
Algebraic detector	NM

TABLE I
Complexity comparison of the three sensing techniques: energy detector (ED), distribution analysis detector (DAD) and algebraic detector (AD)

For simulation results, the choice of the DVB-T primary user system is justified by the fact that most of the primary user systems utilize the OFDM modulation format. This choice is done in the context of the European research project SENDORA [13]. The channel models implemented are AWGN, Rician and Rayleigh channels. The latter two correspond to the two different types of propagation that have to be handled in practice, namely line-of-sight (LOS) and Non-line-of-sight (NLOS). Slow fading is simulated by adding log-normal shadowing. The simulation scenarios are generated by using different combinations of parameters given in Table II. The evaluation framework for all simulations has been implemented in Matlab.

Fig. 1 depicts the detection comparison of three detectors. This figure shows the probability of detection versus SNR ranging between -15 dB and -2 dB at a constant false alarm rate ($P_{FA} = 0.05$). We used here a DVB-T primary user system with Rician channel. Threshold values are computed according to the common framework and depend only on the P_{FA} value. From the simulation results, we show that the DAD detector and the AD detector for $N = 3$ outperform the ED under the same SNR condition. For the AD detector, we remark that the detection rate goes higher as the polynomial order gets higher. We find also that the proposed detectors generally work well under low SNR condition, while it is not the case of ED detector due to the fact that it does not differentiate between primary user data and noise.

Bandwidth	8MHz
Mode	2K
Guard interval	1/4
Channel models	Rayleigh/Rician (K=1)
Maximum Doppler shift	100Hz
Frequency-flat	Single path
Sensing time	1.25ms
Location variability	10dB

TABLE II
The transmitted DVB-T primary user signal parameters

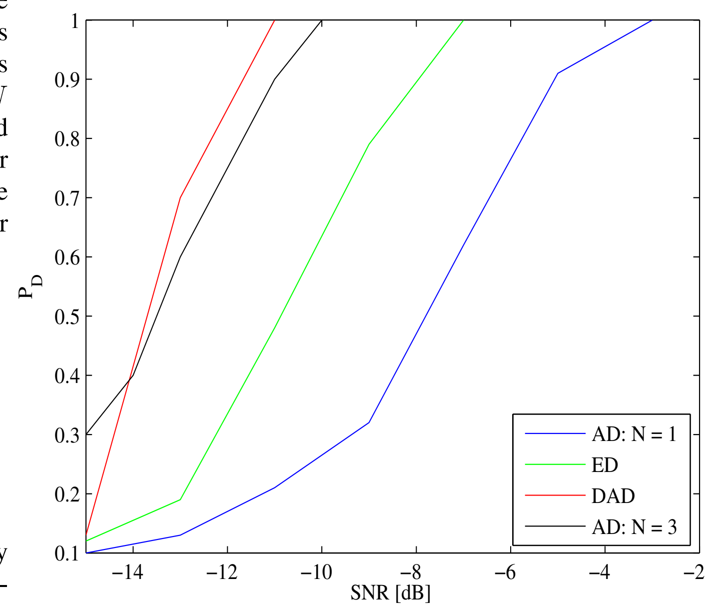

Fig. 1. Probability of detection vs. SNR for the three detectors: energy detector (ED), distribution analysis detector (DAD) and algebraic detector (AD) with $P_{FA} = 0.05$ and $N = (1, 3)$ for a DVB-T primary user system with Rician channel.

V. Conclusion

In this paper, we presented a performance comparison of three low complexity blind sensing techniques using a common simulation framework. We focuss on the performance of local sensing algorithms. We considered for simulation results a realistic network setting using a DVB-T primary user system. From this comparison, we show that the distribution analysis detector and the algebraic detector offer high performances and could also be used to make the estimation safer and faster. Specifically, primary users are detected even at very low SNR with a comparable complexity to ED.

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