Transmission Strategies and Sum Rate Maximization in Multi-User TDD Systems

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Abstract— **In a system comprising of a multiple antenna enabled base station (BS) and multiple single antenna users, very high data rates can be obtained if BS transmits to multiple users simultaneously. This simultaneous transmission to multiple users over the same bandwidth is realizable only if BS knows the forward channels linking its transmit antennas to these users.**

We study a time-division duplexed (TDD) broadcast channel with initial assumption of channel information neither at the BS nor at the users' side. We propose two simple transmission strategies, one where users feedback independent of their channel realizations (earlier proposed in [1]) and the other where users feedback based upon their channel realizations. We derive a lower bound of the sum rate which reflects the rate loss compared to a system with perfect channel knowledge and the corresponding approximate sum rate expressions are developed for both schemes. These expressions capture the benefits of channel feedback, multi-user diversity and inter-user interference cancellation, and the cost of exchange of information required, and hence can be optimized for the sum rate maximization.

I. Introduction

In broadcast channels, first introduced in [2], if a BS has M transmit antennas and the number of users in the system is K with $K \geq M$, this broadcast channel can support data rates M times larger than a single antenna BS, although all users may have single antenna each in both cases [3], [4]. Apart from the multiplexing gain of M, broadcast channels enjoy another gain due to surplus number of users by selecting good users. The term multi-user diversity was coined by Knopp and Humblet in [5]. It has been shown in [6] [7] that the sum capacity of the Gaussian broadcast channel has a scaling factor with the number of users as $M \log(\log(K))$, where K is the total number of users in the system whose channel information is available at the BS. The price to pay to achieve the multiplexing gain of M is that BS must know the forward channel to at least M users [8]. Now to further achieve the multi-user diversity gain factor $M \log(\log(K))$, BS should know the channel state information (CSI) of all of these K users where normally K will be much larger than M.

The study of achievable rates for multi-user MIMO downlink (DL) removing all the assumptions of CSI at receiver (CSIR) and at transmitter (CSIT) for frequency-division duplexed (FDD) systems was carried out in [9]. They compared achievable rates with analog and quantized feedback schemes under the assumption of infinitely large channel coherence lengths and by restricting the number of users (K) equal to M. Later in [10], training and feedback parameters were optimized as a function of channel coherence length and signal-to-noise-ratio (SNR), although the number of users was still restricted to M. In another work [11], the authors analyze the trade-off of multi-user diversity and accuracy of channel information at the BS, keeping the total number of feedback bits fixed. They conclude that with a fixed feedback load accurate channel information is more important than having multi-user diversity.

In [7], the authors had given a very innovative scheme coined as Random Beam Forming (RBF) where only a few bits of feedback are required from every user (with asymptotically large number of users) and the sum rate was shown to converge to the sum capacity, obtainable through dirty paper coding (DPC) [12]. Later in [13] it was shown that, with perfect CSIT and asymptotically large number of users, even linear zero-forcing (ZF) precoding achieves the full multiplexing gain M and the full multi-user diversity gain $M \log(\log(K))$ of the broadcast channel.

If only M users feedback to the BS, multi-user diversity gain is completely lost. If all users feedback, feedback load can be prohibitively large for large systems. So the characterization of optimal number of feeding back users is of prime practical importance. A very important aspect, which often gets overlooked in the analysis of multi-user systems, is the consideration of channel coherence time. The channels in practice have finite coherence times which should be carefully partitioned between obtaining feedback and data transmission to maximize the throughput.

In this work, we make no CSI assumption but we don't prevent any side (transmitter and receivers) to learn/feedback the channel and subsequently use this information for scheduling/precoding/decoding of data. To make the task tractable, we simplify the problem by selecting time-division duplex (TDD) broadcast channel. TDD channel with reciprocity assumption simplifies the acquisition of CSIT as uplink (UL) pilot transmission from users to the BS acts as channel feedback [14], [15]. We restrict the CSIT acquisition at the BS only through training and hence we use the terms training and feedback synonymously in the sequel. So we have a fixed bandwidth available which can be used for data and training/feedback, a BS having M transmit antennas and K single antenna users and the objective would be to maximize the DL sum rate by appropriately choosing the amount of feedback load. Two transmission strategies are given in this contribution, one where a subset of users feedback independent of their channel realizations (hence termed as **Oblivious Users**). In the second

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scheme, the users learn their channel information first and then only good users (in terms of channel norm) feedback (hence termed as **Informed Users**). The sum rate lower bound derived in this setup shows the rate loss w.r.t. a perfect CSI system and its maximization gives the optimal amount of feedback load.

The references [14] and [16] are related to our work as they treat the TDD broadcast channel without any CSI assumption. But there are major differences in the scope. They try to exploit the channel hardening effect [17] due to large number of BS antennas without users having trained about their effective channels which causes rates saturated in SNR. Our analysis is for the systems with larger number of users than BS transmit antennas because this setting is certainly much more practical than its opposite counterpart. Moreover, in both of our transmission strategies (presented in later sections), users are explicitly trained about their effective channels after precoding and hence rates are unbounded in SNR.

This paper is organized as follows. Section II describes system model. The transmission strategies are proposed in sections III and IV. Sum rate maximization analysis appears in section V, followed by conclusions in section VI.

Notation: $\mathbb{E}$ denotes statistical expectation. Lowercase letters represent scalars, boldface lowercase letters represent vectors, and boldface uppercase letters denote matrices. $\mathbf{A}^{\dagger}$ denotes the Hermitian of matrix $\mathbf{A}$.

II. System Model

The system we consider, consists of a BS having M transmit antennas and K single-antenna user terminals. In the DL, the signal received by k-th user can be expressed as

$$y_k = \mathbf{h_k^{\dagger}}\mathbf{x} + n_k, \qquad k = 1, 2, \ldots, K \tag{1}$$

where $\mathbf{h_1}, \mathbf{h_2}, \ldots, \mathbf{h_K}$ are the channel vectors of users 1 through user K with $\mathbf{h_k} \in \mathbb{C}^{M\times 1}$ ($\mathbb{C}^{M\times 1}$ denotes the M-dimensional complex space), $\mathbf{x} \in \mathbb{C}^{M\times 1}$ denotes the M-dimensional signal transmitted by the BS and $n_1, n_2, \ldots, n_K$ are independent complex Gaussian additive noise terms with zero mean and unit variances. We denote the concatenation of the channels by $\mathbf{H_F^{\dagger}} = [\mathbf{h_1}\mathbf{h_2}\cdots\mathbf{h_K}]$, so $\mathbf{H_F}$ is the $K\times M$ forward channel matrix with k-th row equal to the channel of the k-th user ($\mathbf{h_k^{\dagger}}$). The input must satisfy a transmit power constraint of P i.e., $\mathbb{E}[||\mathbf{x}||^2] \leq P$. P also denotes the DL SNR due to unit variance noise. All users have a peak per symbol power constraint of P_{pk}.

The channel is assumed to be block fading having coherence length of T symbol intervals [18]. The entries of the forward channel matrix $\mathbf{H_F}$ are independent and identically distributed (i.i.d.) complex Gaussian with zero mean and unit variance. Initially all users and BS transmitter are oblivious of the channel realization in each block.

III. Transmission Scheme with Oblivious Users

In this section, first we describe our transmission scheme and then we derive sum rate lower bound. This scheme divides the coherence length of T symbol intervals in three phases, 1) uplink training, 2) downlink training and 3) coherent data transmission.

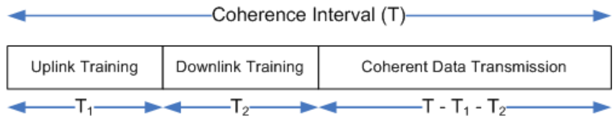

Fig. 1. Transmission Phases with Oblivious Users

The first phase is the UL training phase where K^{obl} of the K users present in the system transmit pilots to the BS. Because of TDD mode, this phase is equivalent to a feedback phase. As K^{obl} single antenna users transmit pilots, the length of this uplink training interval is $T_1 = \beta K^{obl}$ with $\beta \geq 1$. Users can use orthogonal codes to be able to transmit pilots simultaneously. For k-th user channel $\mathbf{h_k}$, the BS estimate and corresponding estimation error are denoted by $\mathbf{\hat{h}_k}$ and $\mathbf{\tilde{h}_k}$. The mean square error (MSE) in CSIT per channel coefficient, denoted by σ_h^2, is given by

$$\sigma_h^2 = \mathbb{E}[|\mathbf{H}_{ij} - \hat{\mathbf{H}}_{ij}|^2] = \frac{1}{P_{pk}T_1 + 1} \tag{2}$$

See [19] and [14] for details about this phase. This training interval length $T_1 = \beta K^{obl}$ is basically the price of obtaining CSIT at the BS through feedback which reduces the effective channel coherence time to $T - T_1$.

We adopt ZF precoding at the BS preceded by semi-orthogonal user selection (SUS) algorithm of [13]. In ZF precoding, unit-norm beamforming vector for k-th selected user (denoted as $\mathbf{\bar{v}_k}$), is selected such that it is orthogonal to the channel vectors of all other selected users. Hence with perfect CSIT, each user experiences no multi-user interference. For the case in hand with imperfect CSIT, there is some residual interference. If we represent ZF beamforming matrix by $\mathbf{\bar{V}} = [\mathbf{\bar{v}_1}\mathbf{\bar{v}_2}\cdots\mathbf{\bar{v}_M}]$, the transmitted signal $\mathbf{x}$ becomes $\mathbf{x} = \mathbf{\bar{V}}\mathbf{u}$ and the signal received by k-th selected user (1) can be expressed as

$$\begin{aligned} y_k &= \mathbf{h_k^{\dagger}}\mathbf{\bar{V}}\mathbf{u} + n_k \\ &= \mathbf{h_k^{\dagger}}\mathbf{\bar{v}_k}u_k + \sum_{j\neq k}\mathbf{h_k^{\dagger}}\mathbf{\bar{v}_j}u_j + n_k, \end{aligned} \tag{3}$$

where $\mathbf{u}$ is the data vector with u_k data intended for k-th selected user.

The second phase is the DL training phase where the BS transmits pilots so that the scheduled users estimate their corresponding effective channels. It was shown in [19] that only one symbol interval is sufficient to let the M selected users learn their effective scalar channels ($\mathbf{h_k^{\dagger}}\mathbf{\bar{v}_k}$ for user k). Moreover with the fact that BS is able to transmit with sufficient power reducing the estimation error, we assume that selected users are able to estimate their effective scalar channels perfectly and we ignore the overhead of this phase. When this second phase ends, both sides of the broadcast channel have necessary channel state information, although CSIT is imperfect. In the third data phase, we adopt independent data transmission with equal power allocation P/M to finally selected M users.

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A. Sum Rate Lower Bound

The signal received by k-th user from eq. (3) can be further written as

$$y_k = \hat{\mathbf{h}}_{\mathbf{k}}^{\dagger}\bar{\mathbf{v}}_{\mathbf{k}}u_k + \tilde{\mathbf{h}}_{\mathbf{k}}^{\dagger}\bar{\mathbf{v}}_{\mathbf{k}}u_k + \sum_{j\neq k}\tilde{\mathbf{h}}_{\mathbf{k}}^{\dagger}\bar{\mathbf{v}}_{\mathbf{j}}u_j + n_k. \tag{4}$$

This uses the fact that $\mathbf{h}_{\mathbf{k}}^{\dagger}\bar{\mathbf{v}}_{\mathbf{j}} = \tilde{\mathbf{h}}_{\mathbf{k}}^{\dagger}\bar{\mathbf{v}}_{\mathbf{j}}$ for $k \neq j$ due to ZF beamforming and by splitting the effective channel $\mathbf{h}_{\mathbf{k}}^{\dagger}\bar{\mathbf{v}}_{\mathbf{k}}$ in two parts, where $\hat{\mathbf{h}}_{\mathbf{k}}^{\dagger}\bar{\mathbf{v}}_{\mathbf{k}}$ is perfectly known at the BS. The above equation can be written as

$$y_k = \hat{\mathbf{h}}_{\mathbf{k}}^{\dagger}\bar{\mathbf{v}}_{\mathbf{k}}u_k + \sum_{j=1}^{M}\tilde{\mathbf{h}}_{\mathbf{k}}^{\dagger}\bar{\mathbf{v}}_{\mathbf{j}}u_j + n_k. \tag{5}$$

From the above equation, where we have relegated some signal part $\tilde{\mathbf{h}}_{\mathbf{k}}^{\dagger}\bar{\mathbf{v}}_{\mathbf{k}}u_k$ into interference and by treating all the interference terms which appear due to imperfect CSIT as an additional source of Gaussian noise as in [20] and [21], a lower bound of the SINR of k-th user can be written as

$$\mathrm{SINR}_k^{obl} = \frac{\frac{P}{M}|\hat{\mathbf{h}}_{\mathbf{k}}^{\dagger}\bar{\mathbf{v}}_{\mathbf{k}}|^2}{1+\frac{P}{M}\sum_{j=1}^{M}\mathbb{E}|\tilde{\mathbf{h}}_{\mathbf{k}}^{\dagger}\bar{\mathbf{v}}_{\mathbf{j}}|^2}. \tag{6}$$

The variance of each interference coefficient $(\tilde{\mathbf{h}}_{\mathbf{k}}^{\dagger}\bar{\mathbf{v}}_{\mathbf{j}})$ can be computed based upon the fact that BS does MMSE estimation which makes estimation error $\tilde{\mathbf{h}}_{\mathbf{k}}$ independent of any function of channel estimates $(\hat{\mathbf{h}}_{\mathbf{k}})$ of which beamforming vectors are one particular example.

$$\mathbb{E}|\tilde{\mathbf{h}}_{\mathbf{k}}^{\dagger}\bar{\mathbf{v}}_{\mathbf{j}}|^2 = \mathbb{E}\left[\bar{\mathbf{v}}_{\mathbf{j}}^{\dagger}\mathbb{E}\left(\tilde{\mathbf{h}}_{\mathbf{k}}\tilde{\mathbf{h}}_{\mathbf{k}}^{\dagger}\right)\bar{\mathbf{v}}_{\mathbf{j}}\right] = \sigma_h^2\mathbb{E}\left[\bar{\mathbf{v}}_{\mathbf{j}}^{\dagger}\bar{\mathbf{v}}_{\mathbf{j}}\right] = \sigma_h^2 \tag{7}$$

Furthermore by using $\hat{\mathbf{h}}_{\mathbf{k}} = \sqrt{1-\sigma_h^2}\mathbf{g}_{\mathbf{k}}$ where $\mathbf{g}_{\mathbf{k}} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_{\mathbf{M}})$ in the numerator, the SINR becomes

$$\mathrm{SINR}_k^{obl} = \frac{1-\sigma_h^2}{1+P\sigma_h^2}\frac{P}{M}|\mathbf{g}_{\mathbf{k}}^{\dagger}\bar{\mathbf{v}}_{\mathbf{k}}|^2, \tag{8}$$

where the first term represents the SINR loss factor w.r.t. a perfect CSI system as $\mathbf{g}_{\mathbf{k}}$'s (and hence $\bar{\mathbf{v}}_{\mathbf{k}}$'s) are perfectly known at the BS. During the data phase, the lower bound (LB) of the per symbol sum rate can be written as

$$\mathrm{LB} = \sum_{k=1}^{M}\mathbb{E}_{\mathbf{g}_{\mathbf{k}}}\log\left(1+\frac{1-\sigma_h^2}{1+P\sigma_h^2}\frac{P}{M}|\mathbf{g}_{\mathbf{k}}^{\dagger}\bar{\mathbf{v}}_{\mathbf{k}}|^2\right), \tag{9}$$

where the users being transmitted have been selected using SUS algorithm.

If one deals with the same system (K users and M antenna BS) with perfect CSI assumption ($\sigma_h^2 = 0, T_1 = 0$), the sum rate obtained through SUS and ZF beamforming would be

$$R_{\mathrm{ZF}}(K,M,P) = \sum_{k=1}^{M}\mathbb{E}_{\mathbf{g}_{\mathbf{k}}}\log\left(1+\frac{P}{M}|\mathbf{g}_{\mathbf{k}}^{\dagger}\bar{\mathbf{v}}_{\mathbf{k}}|^2\right). \tag{10}$$

And for large user regime, it was shown in [13] to be well-approximated by

$$R_{\mathrm{ZF}}(K,M,P) \approx M\log\left(1+\frac{P}{M}\log(K)\right). \tag{11}$$

So the lower bound of the sum rate can be written in terms of the sum rate of a perfect CSI system as

$$\mathrm{LB} = R_{\mathrm{ZF}}(K^{obl}, M, P_m) \tag{12}$$

where P_m is the reduced SNR given by

$$P_m = \frac{1-\sigma_h^2}{1+P\sigma_h^2}P. \tag{13}$$

By taking into account the loss of coherence interval T due to feedback (training) interval of length T_1, sum rate lower bound becomes

$$\mathrm{LB}^{obl} = \frac{T-\beta K^{obl}}{T}\sum_{k=1}^{M}\mathbb{E}_{\mathbf{g}_{\mathbf{k}}}\log\left(1+\frac{1-\sigma_h^2}{1+P\sigma_h^2}\frac{P}{M}|\mathbf{g}_{\mathbf{k}}^{\dagger}\bar{\mathbf{v}}_{\mathbf{k}}|^2\right). \tag{14}$$

The biggest virtue of this lower bound is that it gives the achievable sum rate in terms of the sum rate of a perfect CSI system (employing SUS and ZF precoding) with loss appearing as an SNR reduction factor and as reduced multiplexing gain due to feedback interval.

We can make the approximation of large user regime as in eq. (11), following the footsteps of [13] and putting the value of σ_h^2 from eq. (2), the above sum rate will become

$$\mathrm{SR}^{obl} = \frac{T-\beta K^{obl}}{T}M\log\left(1+\frac{\frac{P}{M}\frac{P_{pk}\beta K^{obl}}{P_{pk}\beta K^{obl}+1}\log(K^{obl})}{1+P\frac{1}{P_{pk}\beta K^{obl}+1}}\right). \tag{15}$$

Due to the approximation made at this final step, this sum rate expression is not necessarily a lower bound. We have verified through extensive simulations that it closely follows the lower bound and the true sum rate of the system even for moderate number of users, although we don't plot these simulation results due to space limitations.

IV. Transmission Scheme with Informed Users

This scheme also consists of transmission phases through which both the BS and all users get necessary channel information. The users who feedback are selected based upon their channel realizations. This scheme divides the coherence length of T symbol intervals in four phases, 1) initial downlink training, 2) uplink training, 3) downlink training and 4) coherent data transmission.

Fig. 2. Transmission Phases for Informed Users

In the first phase, termed as initial downlink training, BS transmits pilots to users based upon which all users estimate their channel vectors. As BS has M antennas, this training interval length is lower bounded by M, giving $T_{iDL} \geq M$. As T_{iDL} is independent of the number of users K and as the BS can transmit with sufficient power to provide good estimates,

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we don't take into account the estimation error during this phase but only subtract $T_{iDL} = M$ from the coherence length. Another point is that when the selected users transmit pilots so that BS obtains CSIT, imperfection in users' estimates don't propagate to CSIT.

Once the users have acquired the information regarding their respective channels, there could be plenty of criteria to prioritize some of the users for feedback depending upon their channel realizations but we restrict ourselves to the simple scheme where K^{inf} users with largest channel norms are selected for feedback. The next three transmission phases are exactly similar as those for transmission scheme with oblivious users. In the second phase of uplink training, K^{inf} users with the largest channel norms feedback their channel information to the BS by UL pilot transmission. Based upon this CSIT, BS uses SUS algorithm to further select M best users and computes corresponding ZF beamforming vectors. Then in the third phase of downlink training, BS transmits through these ZF beamforming vectors so that selected users estimate their effective scalar channels. The last phase is the coherent data transmission phase with equal power allocated to all M independent users' streams.

Important Remark. In this informed users scheme, we let only the strongest users train the BS about their channels. Strictly speaking, this is impractical as how can users know about being the strongest or not with only the local CSIR. But the underlying idea is to analyze the performance and determine the optimal feedback load if good users feedback. Then those many users can be made to feedback, on the average, by intelligent selection of a threshold with which users compare their channel strengths locally as detailed in [22].

A. Sum Rate Lower Bound

We'll be quite brief here as the treatment resembles a lot as for oblivious users' setting. If K^{inf} users transmit pilots in the UL direction, the feedback length would be $T_1 = \beta K^{inf}$ and the MSE of CSIT per channel coefficient is given by

$$\sigma_h^2 = \frac{1}{P_{pk}\beta K^{inf} + 1}. \quad (16)$$

Following the same steps as for oblivious users' case, we can write the rate expression for a single selected user. The length of the data phase in this scheme would be $T - M - \beta K^{inf}$, where additional M factor appears due to initial DL training and βK^{inf} denotes the length of the feedback phase. Thus the sum rate is given by

$$\mathrm{SR}^{inf} = \frac{T - M - \beta K^{inf}}{T} M \log\left(1 + \frac{\frac{P}{M}\frac{P_{pk}\beta K^{inf}}{P_{pk}\beta K^{inf}+1}\log(K)}{1 + P\frac{1}{P_{pk}\beta K^{inf}+1}}\right) \quad (17)$$

One striking difference in the sum rate of this scheme is the channel strength factor (multi-user diversity factor) of $\log(K)$ where K is the number of users in the system. This difference arises due to the fact that only the best users, with respect to the strength of channel norm, feedback in this scheme.

V. Sum Rate Maximization

We had introduced two parameters namely the number of users who feedback and the β factor. The sum rate expressions show that β always appears in product form with the number of users who feedback. So β can always be selected to be 1 without any loss of optimality of the sum rate and hence the amount of feedback load appears as the number of users who should feedback. Let's formulate the problem for the scheme with oblivious users. If the optimal number of feeding back users which maximizes the sum rate is denoted by K^{obl*}, the objective function can be written as

$$K^{obl*} = \arg\max_{K^{obl}} \frac{T - K^{obl}}{T} M \log\left(1 + \frac{\frac{P}{M}\frac{P_{pk}K^{obl}}{P_{pk}K^{obl}+1}\log(K^{obl})}{1 + P\frac{1}{P_{pk}K^{obl}+1}}\right). \quad (18)$$

The analytical solutions to this equation and the other (corresponding to the informed scheme) do not seem to have closed form expressions but the optimal value of feeding back users for both of these schemes can be found by trivially simple numerical optimization.

A. Optimal Users vs DL SNR

First we see how the optimal number of users (feeding back) should scale with SNR. We plot the graph of the optimal number of users versus SNR in Fig. 3 and also plot corresponding sum rate achieved by using that optimal number of users for each value of SNR in Fig. 4. We take T=1000 symbol intervals, there are 200 users in the system with per user peak power constraint of 5 dB and the BS is equipped with $M = 4$ antennas.

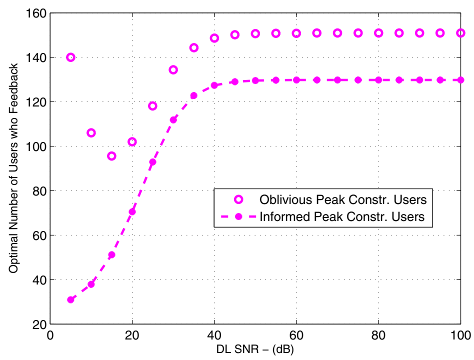

Fig. 3. Optimal Users versus SNR

The behavior of the curves of optimal number of users feeding back for two schemes versus SNR is not very straight forward. At high SNR, both schemes require very good quality CSIT and due to peak power constrained users, it translates to obtaining feedback from each user for longer intervals which comes out to be a lot of users transmitting feedback (users have orthogonal codes and hence can be separated).

At low SNR both curves show very different behavior. The reason is at low SNR, system is basically noise limited

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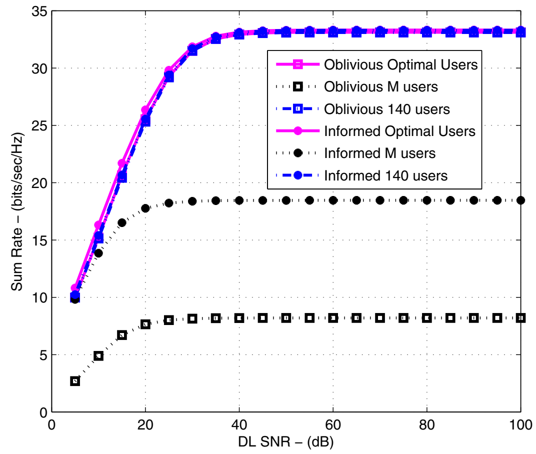

Fig. 4. Sum Rate with Optimal Users versus SNR

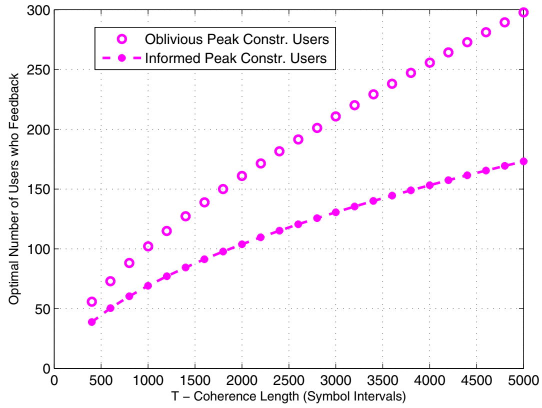

Fig. 5. Optimal Users versus Coherence Length

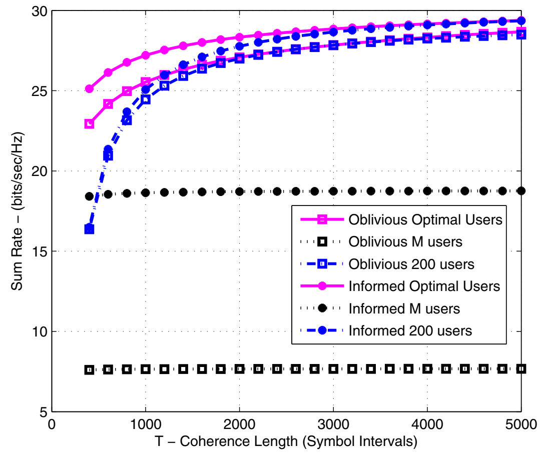

Fig. 6. Sum Rate with Optimal Users versus Coherence Length

and multi-user diversity factor is very important hence the users with very strong channels should be scheduled. In informed users scheme, only strong users feedback so it requires feedback from small number of users and as SNR increases and CSIT quality starts becoming more important, more users start feeding back (this is the way to improve CSIT quality for peak-power constrained users). The scheme with oblivious users (where users feedback independent of their channel realizations) requires feedback from a large number of users initially to enjoy multi-user diversity but that consumes a lot of coherence time in feedback so the number of users who feedback decreases further and later starts increasing again to provide high quality CSIT.

Although the optimal feeding back users in two schemes differ significantly from lower to medium SNR values, the corresponding sum rates overlap completely. At low to medium SNR values, informed user strategy gives slightly higher rates but this difference is minor.

We have also plotted the sum rate when 140 users (this is the number of users at high DL SNR) feedback in each coherence interval for both schemes. These curves also overlap fully the sum rates of two schemes with optimal feedback load (Fig. 4). It indicates that for a fixed channel coherence length, a fixed reasonable value of feeding back users (normally much larger than M) can achieve the cost-benefit trade-off of feedback significantly. In other words, the sum rate as a function of SNR is not very sensitive to the number of users who feedback.

B. Optimal Users vs Channel Coherence Time

We now analyze how the optimal number of users behaves with the change in channel coherence time. So we plot two graphs, one showing the optimal number of users versus coherence interval in Fig. 5 and the other showing the sum rate corresponding to the optimal number of users versus coherence interval in Fig. 6. Here BS has $M = 4$ antennas, its power constraint is 20 dB and there are 500 users in the system with each user restricted to a peak power constraint of 5 dB.

The curves of optimal number of users versus channel coherence time show almost linear increase. The number of users, who feedback, scales up with the increase in channel coherence time. The optimal number of users, who feedback, is always more for the scheme with oblivious users than in the case of informed users. This behavior can be anticipated from Fig. 3 which shows that from low to medium DL SNR values, the optimal number of users in the oblivious scheme is more than that in the informed scheme.

Sum rate curves for optimal users have been plotted at $\text{SNR} = 20$ dB so informed user scheme performs betters as can be guessed from Fig. 4. Sum rate curves have also been plotted for fixed number of users (200) feeding back but contrary to the optimal number of users versus SNR curves where a single suitable number of users feeding back (fixed feedback load) captures the gain of optimal feedback, here it is not possible to find one such number of users (feeding back) capturing the sum rate gains as of with optimal feedback load for a large range of channel coherence length. So the sum rate

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as a function of T is relatively sensitive to the number of users who feedback.

VI. Conclusions

Two transmission strategies were proposed which completely account for how the feedback is obtained and utilized. Novel lower bounds and simple analytical expressions for the sum rate of both schemes were developed which can be maximized by optimizing over the feedback load. Furthermore we studied how the optimal feedback load varies with DL SNR and channel coherence length for sum rate maximization.

Appendix A: Average Power Constrained Users

We treated the case with peak power constrained users in the system. If the users are average power constrained with P_{avg}, then feedback behavior will change as MSE of CSIT changes. If there are K users in the system with channel coherence length of T, the total UL energy available in each coherence block is $P_{avg}KT$. Now if K^{obl} users have to feedback, each one of these can transmit an energy of $P_{avg}KT/K^{obl}$. Here the use of orthogonal codes is not necessary because, due to weaker power constraint, users can transmit their available power in short intervals. As BS receives this energy for every channel coefficient, CSIT estimation error at the BS will be

$$\sigma_h^2 = \frac{1}{\frac{P_{avg}KT}{K^{obl}} + 1}. \tag{19}$$

Although here users, when feedback, will be able to transmit pilots with larger energy (if $K >> K^{obl}$), yet they will be transmitting only occasionally, the probability of which will reduce with more users in the system and hence long term average power constraint will be satisfied [23]. The sum rate expressions for two schemes when users are average power constrained can be obtained by plugging in the MSE of CSIT from eq. (19).

$$\mathrm{SR}^{obl} = \frac{T - \beta K^{obl}}{T} M \log\left(1 + \frac{\frac{P}{M}\frac{P_{avg}KT}{P_{avg}KT + K^{obl}}\log(K^{obl})}{1 + P\frac{1}{\frac{P_{avg}KT}{K^{obl}}+1}}\right)$$

$$\mathrm{SR}^{inf} = \frac{T - M - \beta K^{inf}}{T} M \log\left(1 + \frac{\frac{P}{M}\frac{P_{avg}KT}{P_{avg}KT + K^{inf}}\log(K)}{1 + P\frac{1}{\frac{P_{avg}KT}{K^{inf}}+1}}\right)$$

These expressions can be optimized to evaluate the amount of feedback to maximize the sum rate. The expression for informed strategy shows that increase in K^{inf} has only negative impact hence, for average power constrained users the optimal number of feeding back users should be minimal, $K^{inf^*} = M$.

Acknowledgments

Eurecom's research is partially supported by its industrial members: BMW Group, Bouygues Telecom, Cisco, Hitachi, ORANGE, SFR, Sharp, STMicroelectronics, Swisscom, Thales. The research work leading to this paper has also been partially supported by the European Commission through the FP7 ICT Network of Excellence NewCom++ and project WHERE, and by the French ANR project APOGEE.

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