On Full Diversity Equalization for Precoded Block Transmission Systems

Shakti Prasad Shenoy*, Irfan Ghauri*, Dirk T.M. Slock†
*Infineon Technologies France SAS, 06560 Sophia Antipolis Cedex, France
Email: shakti.shenoy@eurecom.fr, irfan.ghauri@infineon.com
†Mobile Communications Department, EURECOM, 06904 Sophia Antipolis Cedex, France
Email: dirk.slock@eurecom.fr

***Abstract*—Equalization techniques other than maximum likelihood equalization (MLE) that collect full diversity offered by doubly selective channels are investigated. With appropriate precoding at the transmitter, it is known the both linear equalizers (LE) and MLE collect full multiplicative multipath-Doppler diversity available in the channel. We close the gap between LE and MLE and show that decision feedback equalizers (DFE) also achieve full channel diversity in these channels. For a class of precoders proposed in [1] we propose a novel hybrid equalization scheme that also benefits from full channel diversity at a lower complexity compared to full blown MLE. Finally, we show that the amount of redundancy introduced in these precoders is in fact an over-kill for MLE and show that redundancy proportional to channel delay spread is largely enough to allow MLE to collect full channel diversity at a complexity that is exponential in channel delay spread. Simulation results are provided to substantiate all claims made in the paper.**

I. Introduction

Practical wireless communication channels are prone to signal fading due to the presence of multiple signal paths (time dispersive channel) , time-varying nature of the channel (frequency dispersive channel) or both (time-frequency dispersive or the so called, doubly selective channel). However, it is possible for the receiver to employ equalization techniques that optimally exploit the inherent diversity in these channels as a convenient counter-measure against fading. For instance, the frequency selectivity of time dispersive channels provide multipath diversity due to the presence of multiple independently fading components in the channel. In block transmission systems, when the channel coherence time is shorter than the transmit block length, temporal variations of the channel provides Doppler diversity [2] which can be exploited by the receiver. Doubly selective channels offer joint multipath-Doppler diversity that can be harnessed by suitable equalization techniques and proper precoding. In [1], the authors used the Complex-Exponential Basis Expansion Model (CE-BEM) [3] with $Q+1$ basis functions to model the doubly selective channel of memory L and introduced linear precoders that enable maximum likelihood equalization (MLE) to benefit from full $(Q+1)(L+1)$ channel diversity present in doubly selective channels. The cost paid to enable full diversity reception was a loss in bandwidth efficiency due to the redundancy introduced by the precoders. The issue of reducing this bandwidth loss was addressed to some extent by the authors in the same paper. However their investigations are limited to MLE. It is therefore natural to ask the following questions

1) Is it possible to benefit from full channel diversity using equalizers with lesser complexity?
2) What is the minimum loss of bandwidth incurred in order to enable full diversity reception?

The first question was answered positively in [4] for the case of frequency-selective only channels and in [5] and [6] for the case of time-selective only and doubly selective channels where it was shown that linear equalization (LE) benefits from full channel diversity with appropriate precoding. In this paper we address the first question first by filling the gap between LE and MLE to show that DFE also benefits from full channel diversity in doubly selective channels when enough redundancy is introduced using linear precoders. We then propose a hybrid equalization technique that enjoys full channel diversity at a complexity lower than full blown MLE for *square precoders* where LE is not able to benefit from full diversity. We then address the second question and show that the amount of redundancy required to enable full diversity reception with MLE of reasonable complexity is of the order of channel delay spread.

The precoders that we address in this paper are classified into three categories based on the redundancy they introduce in the time and frequency domain. Precoders that introduce redundancy in both time and frequency domain are called *tall-tall* precoders, those that introduce redundancy only in time domain (referred to as square precoders in [1]) are called *square-tall* precoders and those that do not introduce redundancy in either time or frequency domain are called *square-square* precoders. We will focus on full diversity equalization for all three types of precoders.

II. Signal Model

In Fig. 1 we show the block diagram of the transmission model. At the transmitter, complex data symbols $s[i]$ are first parsed into N-length blocks.The n-th symbol in the k-th block is given by $[\mathbf{s}[k]]_n = s[kN+n]$ with $n \in [0, 1, ..., N-1]$. Each block $\mathbf{s}[k]$ is precoded by a $M \times N$ matrix $\mathbf{\Theta}$ where $M \geq N$ and the resultant block $\mathbf{x}[k]$ is transmitted over the block fading channel. We consider a channel memory of order

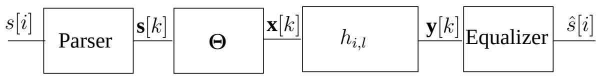

Fig. 1. Block diagram of transmission model.

L. It is well known that the temporal variation of the channel taps in doubly selective channels with a finite Doppler spread can be captured by finite Fourier bases. We therefore use CE-BEM [3] with $Q+1$ basis functions to model the time variation of each tap in a block duration. The basis coefficients remain constant for the block duration but are allowed to vary with every block. The time-varying channel for each block transmission is thus completely described by the $Q+1$ Fourier bases and $(Q+1)(L+1)$ coefficients. In general Q is chosen such that $Q = 2\lceil f_{max} M T_s \rceil$ where $1/T_s$ is the sampling frequency and f_{max} is the Doppler spread of the channel. The coefficients themselves are assumed to be zero-mean complex i.i.d Gaussian random variables. Using i as the discrete time (sample) index, we can represent the l-th tap of the channel in the k-th block

$$h_{i,l} = \sum_{q=0}^{Q} h_q(k,l) e^{j2\pi f_q i},$$

$l \in [0, L]$, $f_q = (q - Q/2)/M$. The corresponding receive signal is formed by collecting M samples at the receiver to form $\mathbf{y}[k] = [y(kM+0), y(kM+1), \ldots, y(kM+M-1)]^T$. When $M \geq L$, this block transmission system can be represented in matrix-vector notation as [1]

$$\mathbf{y}[k] = \mathbf{H}_{ds}[k;0]\boldsymbol{\Theta}\mathbf{s}[k] + \mathbf{H}_{ds}[k;1]\boldsymbol{\Theta}\mathbf{s}[k-1] + \mathbf{v}[k], \tag{1}$$

where $\mathbf{v}[k]$ is a AWGN vector whose entries have zero-mean and variance σ_v^2 and is defined in the same way as $\mathbf{y}[k]$. $\mathbf{H}_{ds}[k;0]$ and $\mathbf{H}_{ds}[k;1]$ are $M \times M$ matrices whose entries are given by $[\mathbf{H}_{ds}[k;t]]_{r,s} = h_{(kM+r, tM+r-s)}$ with $t \in [0,1], r, s \in [0, ..., M-1]$. Defining $\mathbf{D}[f_q]$ as a diagonal matrix whose diagonal entries are given by $[\mathbf{D}[f_q]]_{m,m} = e^{j2\pi f_q m}, m \in [0, 1, ..., M-1]$, and further defining $[\mathbf{H}_q[k;t]]_{r,s} = h_q(k, tM + r - s)$ as Toeplitz matrices formed of BEM coefficients, it is straightforward to represent Eq. (1) as

$$\mathbf{y}[k] = \sum_{t=0}^{1} \sum_{q=0}^{Q} \mathbf{D}[f_q]\mathbf{H}_q[k;t]\boldsymbol{\Theta}\mathbf{s}[k-t] + \mathbf{v}[k], \tag{2}$$

III. Tall-Tall precoders for doubly selective channels

The precoding matrix $\boldsymbol{\Theta}$ considered here is given by

$$\boldsymbol{\Theta} = \mathbf{F}_{P+Q}^{H}\mathbf{T}_1 \otimes \mathbf{T}_2.$$

where $\otimes$ represents the Kronecker product of matrices, $\mathbf{F}_{P+Q}$ is a $(P+Q)$-point DFT matrix, $\mathbf{T}_1 = [\mathbf{I}_P \;\; \mathbf{0}_{P\times Q}]^T$, $\mathbf{T}_2 = [\mathbf{I}_K \;\; \mathbf{0}_{K\times L}]^T$. P and K are chosen such that $M = (P+Q)(K+L)$ and $N = PK$. This precoder was proposed in [1] and was shown to enable diversity order of $(Q+1)(L+1)$ for ML receivers in doubly selective channels. When this precoder is applied at the transmitter, the received signal can be represented as

$$\mathbf{y}[k] = (\mathbf{F}_{P+Q}^{H} \otimes \mathbf{I}_{K+L})\mathbf{H}[k]\mathbf{s}[k] + \mathbf{v}[k], \tag{3}$$

where $\mathbf{H}[k]$ is given by (see [6] for derivation).

$$\mathbf{H}[k] = \sum_{q=0}^{Q} (\mathbf{J}_{P+Q}[q]\mathbf{T}_1) \otimes (\mathbf{D}_{K+L}[f_q]\widetilde{\mathbf{H}}_q[k;0]\mathbf{T}_2). \tag{4}$$

In the following we will drop the block index k in the interest of simplifying notations. For this scheme, it was shown in [6] that MMSE-ZF equalization collects full diversity offered by the channel. In other words, the slope of the outage probability curve of the MMSE-ZF receiver in the high-SNR regime is $(Q+1)(L+1)$.

A. Diversity Order of MMSE-DFE

In principle, the structure of DFE is very similar to that of MMSE-ZF equalizer [7] [8], this motivates us to analyze the diversity order of B-DFE in such channels. The goal is to

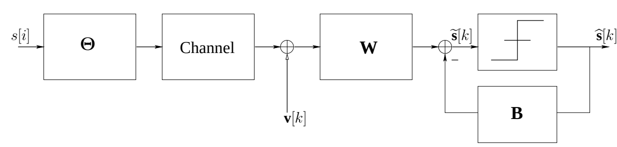

Fig. 2. Decision Feedback Equalization.

minimize $\mathrm{Tr}\{\mathbf{R}_{ee}\}$ where $\mathbf{e}[k] = \widetilde{\mathbf{s}}[k] - \mathbf{s}[k]$. In addition we impose the constraint that $\mathbf{B}$ is strictly upper triangular. The feedforward filter $\mathbf{W}$ and the feedback filter $\mathbf{B}$ are then given by

$$\mathbf{W} = (\mathbf{B} + \mathbf{I})\mathbf{H}_{ds}^{H}(\mathbf{R}_{vv} + \sigma_s^2 \mathbf{H}_{ds}\mathbf{H}_{ds}^{H})^{-1}, \tag{5}$$

where $\mathbf{B} = \mathbf{L}^H - \mathbf{I}$ and $\mathbf{L}$ is the result of LDL factorization of $(\sigma_s^{-2}\mathbf{I} + \mathbf{H}_{ds}^{H}\mathbf{R}_{vv}^{-1}\mathbf{H}_{ds})$. One can show that the mean squared error (MSE) of the linear MMSE equalizer and the MMSE-DFE are related as [9]

$$MSE_{LMMSE} = \underbrace{\sigma_n^2 \,\mathrm{Tr}\{\mathbf{D}^{-1}\}}_{MSE_{DFE}} + \sigma_n^2 \sum_{r=1}^{N}\sum_{s=r+1}^{N} [\mathbf{D}^{-1}]_{s,s} |[\mathbf{L}^{-1}]_{s,s}|^2. \tag{6}$$

This implies that $SINR_{DFE} \geq SINR_{LMMSE} \Rightarrow$ if LMMSE collects full diversity, so will the DFE *provided, the residual ISI component for the DFE (non-Gaussian distribution) does not impact the diversity order.* We show here that for MMSE designs (and by extension the MMSE-DFE), non-Gaussian ISI component in interference plus noise expression is bounded and therefore of noise becomes a non-issue as $SNR \rightarrow \infty$. Denote the MMSE equalizer by $\mathcal{G}$ and the output of the equalizer by $\widehat{\mathbf{y}}$. Then the n^{th} component of $\widehat{\mathbf{y}}$ and be expressed as

$$\widehat{y}_n = \underbrace{\sqrt{\rho}\mathbf{g}_n\mathbf{H}\theta_n}_{f_n} s_n + \sqrt{\rho}\mathbf{g}_n\mathbf{H}\boldsymbol{\Theta}\mathbf{s}_{-n} + \mathbf{g}_n\mathbf{v}, \tag{7}$$

	η	ML	ML-BLE	BDFE MMSE	BDFE MMSE-ZFE	BLE MMSE	BLE MMSE-ZFE
tall-tall	$\frac{PK}{(P+Q)(K+L)}$	[MG]	[TP]	[TP]	[TP]	[TP]	[SGS]
square-tall	$\frac{K}{K+L}$	[MG]	[TP]				
square-square	$\frac{PK}{(PK+L)}$	[TP]					

TABLE I
OVERVIEW OF FULL DIVERSITY COMBINATIONS AND BANDWIDTH EFFICIENCY η AS A FUNCTION OF PRECODER TYPE, AND FOR VARIOUS RECEIVERS.

LEGEND: [MG]:- [1], [SGS] :- [6], [TP]:- this paper
BDFE:- Block DFE, BLE:- Block Linear Equalizer, MMSE-ZF:- Minimum Mean Squared Error-Zero Forcing Equalizer

where θ_n is the n^{th} column of $\mathbf{\Theta}$ and $\mathbf{s}_{-n}$ is the transmit vector with the n^{th} symbol set to zero. We also use $\mathbf{\Theta}_{-n}$ to denote the precoding matrix sans its n^{th} column. Finally, $\mathbf{g}_n$ denotes the n^{th} row of $\boldsymbol{\mathcal{G}}$. Introduce a scaling factor for each $\widehat{y}_n$ to be

$$\gamma_n^2 = \mathbf{g}_n\mathbf{g}_n^H + \rho\mathbf{g}_n\mathbf{H}\mathbf{\Theta}_{-n}\mathbf{\Theta}_{-n}^H\mathbf{H}^H\mathbf{g}_n^H \quad 1 \leq n \leq N \tag{8}$$

The scaled vector $\bar{\mathbf{y}}$ then reads

$$\bar{\mathbf{y}} = \mathbf{\Gamma}\widehat{\mathbf{y}} = \mathbf{\Gamma}\boldsymbol{\mathcal{D}}\mathbf{s} + \widetilde{\mathbf{v}} \tag{9}$$

$\boldsymbol{\mathcal{D}} \triangleq \{f_1 \oplus f_2 \oplus \cdots \oplus f_N\}$, $\mathbf{\Gamma} = \{\gamma_1 \oplus \gamma_2 \oplus \cdots \oplus \gamma_N\}$ are diagonal matrices and the residual ISI and noise is collected in $\widetilde{\mathbf{v}}$. Following the treatment of noise is [10], we separate the contribution of residual non-Gaussian ISI and Gaussian noise in each component of $\widetilde{\mathbf{v}}$ as

$$\widetilde{v}_n = \frac{1}{\gamma_n}\left(\underbrace{\mathbf{g}_n\mathbf{v}}_{v_n^{(1)}} + \underbrace{\sqrt{\rho}\mathbf{g}_n\mathbf{H}\mathbf{\Theta}\mathbf{s}_{-n}}_{v_n^{(2)}}\right) \tag{10}$$

Since $E[v_n^{(1)}v_n^{(1)H}] + E[v_n^{(2)}v_n^{(2)H}] = 1$ and the constellation itself is of finite energy, one can show that $\|\widetilde{\mathbf{v}}^{(2)}\| \leq \beta$, $\beta > 0$ and is a constant independent of ρ. The contribution of the non-Gaussian component in the noise is therefore finite. Since an outage event gets situated in the exponentially receding Gaussian tail, the outage probability behaves asymptotically as if the noise was Gaussian

IV. SQUARE-TALL PRECODERS AND HYBRID EQUALIZATION

The tall-tall precoder succeeds in enabling full diversity reception with LE, DFE as well as MLE. However, this comes at a significant cost. This being the loss of bandwidth efficiency. In order to increase the bandwidth efficiency, the so-called square precoders were introduced in [1]. The basic idea is to embed a constellation-rotation precoder [11] in $\mathbf{\Theta}$. Such a class of precoders is given by $\mathbf{\Theta} = \mathbf{C}_P \otimes \mathbf{T}_2$ where $\mathbf{C}$ can be any square constellation-rotation precoder. Note that this precoder introduces redundancy of the order of channel delay spread in each *block* of the transmit signal vector (*super-block*). However, no redundancy is introduced in the Doppler domain. It is obvious that LE will not benefit from full channel diversity with this precoder. In [1], MLE was employed at the receiver to benefit from full channel diversity. However, there exists a possibility of combining the lower computational complexity of the LE with the full diversity benefits of MLE in a hybrid equalizer that limits ML processing only to extract Doppler diversity while employing a MMSE stage to benefit from multipath diversity. The idea of hybrid equalization (MMSE and MLE) is as follows. The received signal for the case of square-tall precoders is given by (dropping the block index k)

$$\mathbf{y} = \sum_{q=0}^{Q}\mathbf{D}[f_q]\mathbf{H}_q\mathbf{\Theta}\mathbf{s} + \mathbf{v} \tag{11}$$

$$= \boldsymbol{\mathcal{H}}(\mathbf{C}_P \otimes \mathbf{I}_K)\mathbf{s} + \mathbf{v} \tag{12}$$

$$\boldsymbol{\mathcal{H}} = \sum_{q=0}^{Q}(\mathbf{D}_P[f_q(K+L)] \otimes \mathbf{D}_{K+L}[f_q])(\mathbf{I}_P \otimes \widetilde{\mathbf{H}}_q\mathbf{T}_2) \tag{13}$$

where $\widetilde{\mathbf{H}}_q$ is a $K+L \times K+L$ Toeplitz matrix formed by the first $K+L$ rows and columns of $\mathbf{H}_q[k;0]$ described in Sec. II. $\boldsymbol{\mathcal{H}}$ is a block banded matrix given by $\boldsymbol{\mathcal{H}} = \{\mathcal{H}_0 \oplus \mathcal{H}_1 \oplus ... \oplus \mathcal{H}_{P-1}\}$ with each $\mathcal{H}_p \quad p \in \{0, P-1\}$ given by

$$\mathcal{H}_p = \sum_{q=0}^{Q}[\mathbf{D}_P[f_q(K+L)]]_{p,p}\mathbf{D}_{K+L}[f_q]\widetilde{\mathbf{H}}_q\mathbf{T}_2 \tag{14}$$

At the receiver, MMSE equalization is first performed separately for each of the P $(K+L)$-length blocks in the received super-block to yield P sets of K input estimates. Denote this MMSE equalizer by $\boldsymbol{\mathcal{F}} = \{\mathcal{F}_0 \oplus \mathcal{F}_1 \oplus ... \oplus \mathcal{F}_{P-1}\}$ and the *partially equalized* signal vector at the output of the MMSE equalization stage by $\widetilde{\mathbf{y}} = [\widetilde{y}_0, \widetilde{y}_1 ..., \widetilde{y}_{PK}]^T$. ML detection on $\widetilde{\mathbf{y}}$ to extract the symbol vector $\mathbf{s}$ will now benefit from full channel diversity. Note that, since the noise at the output of $\boldsymbol{\mathcal{F}}$ is no longer white, the covariance of the colored noise in $\widetilde{\mathbf{y}}$ should be taken into account in the ML metric. We do so by weighting the ML metric according to the weighted least squares (WLS) criterion. Denote this covariance matrix by $\mathbf{R}_{\mathcal{F}\mathcal{F}}$. The ML weight factor is then given by the diagonal matrix $\mathbf{\Phi}$ formed by the diagonal elements of $\mathbf{R}_{\mathcal{F}\mathcal{F}}^{-1}$. The symbol estimates are obtained by using the $\mathbf{\Phi}$ thus formed in the weighted MLE and the transmit symbols are the solution to

$$\arg\min_{\mathbf{s}\in\mathcal{S}}(\widetilde{\mathbf{y}} - \boldsymbol{\mathcal{F}}\,\boldsymbol{\mathcal{H}}\,\boldsymbol{\Theta}\,\mathbf{s})^H\boldsymbol{\Phi}(\widetilde{\mathbf{y}} - \boldsymbol{\mathcal{F}}\,\boldsymbol{\mathcal{H}}\,\boldsymbol{\Theta}\,\mathbf{s}), \tag{15}$$

This is accomplished by setting up K parallel Viterbi equalizers for the K $P\times P$ mixtures. to this end $\widetilde{\mathbf{y}}$ is re-ordered into K sets of P symbols which we denote here by $\mathbf{z}_i \quad i\in 0, K-1$ and $\mathbf{z}_i = [\widetilde{y}_i, \widetilde{y}_{i+K}, ..., \widetilde{y}_{i+(P-1)(K-1)}]^T$. Now, by appropriately re-ordering the channel matrix, ML detection is performed on the $P\times P$ mixture in each block. We comment here that for the detection of symbols in each of the $P\times P$ mixture $\mathbf{z}_i$, the noise covariance matrix is indeed diagonal, however there is non-zero correlation between each of the K $P\times P$ mixtures. In the interest of simplicity, we ignore the correlation across blocks in the weight matrix.

V. Square-Square Precoders and MLE

It turns out that for MLE, it is not required to introduce the order L redundancy in each block in the super-block as in the case of the square-tall precoders. A precoder given by $\boldsymbol{\Theta}_{min} = \mathbf{T}(\mathbf{C}_P\otimes\mathbf{I}_K)$ where $\mathbf{C}$ is a square constellation-rotation precoder and $\mathbf{T} = [\mathbf{I}_{PK},\ \mathbf{0}_{PK\times L}]^T$ $P\geq Q+1$ suffices to extract full channel diversity with MLE. The zero-padding matrix $\mathbf{T}$ ensures that the inter-super-block interference is nulled. At the receiver, Viterbi algorithm (VA) is employed to estimate the symbol sequence that was most likely transmitted. In order to do so, the received vector $\mathbf{y}$ is reordered as $\mathbf{y}_i = [y_i, y_{i+K}, ..., y_{i+(P-1)(K-1)}]^T$. $i\in\{1,...,K\}$ The branch metric for the VA is then given by

$$\|\mathbf{y}_i - \sum_{m=0}^{L} H_{i,m}\mathbf{C}\mathbf{s}_{i-m}\|^2$$

where $\mathbf{s}_i = [s_i^{(1)}, s_i^{(2)}, ..., s_i^{(P)}]^T$, $s_i^{(p)} = s_{(p-1)K+i}$ and $H_{i,m} = \{h_{i,m}\oplus h_{i+K,m}\oplus ...\oplus h_{i+(P-1)K,m}\}$. $h_{i,m}$ is the time domain channel coefficient corresponding to output time i (in each K length block) and channel delay offset m. The cost here is an increase in the complexity of MLE since at each time offset within a block one is required to track the optimal path which is one of the links from all possible initial states of that block to all states at that particular time offset. The payoff is increased bandwidth efficiency. For MLE the Viterbi algorithm can be applied with additional termination constraints. For instance, one has to deal with the consistencies between the states across block boundaries. That is, the optimal path is such that the initial state of the block indexed by $p\in\{1,..(P-1)\}$ should correspond to the end state of the block indexed by $(p-1)$.

A. Discussion

We discuss here the bandwidth efficiency of the precoder design and the associated equalization when full diversity reception is desired. We define the bandwidth efficiency $\boldsymbol{\eta}$ as the ratio of the length of the transmit signal vector and receive signal vector. II lists the bandwidth efficiency of the various precoders that we discuss in this paper. We therefore have

$$\boldsymbol{\eta}_{square-square} > \boldsymbol{\eta}_{square-tall} > \boldsymbol{\eta}_{tall-tall} \tag{16}$$

For all these cases using the Viterbi algorithm for MLE allows to reduce the complexity from being exponential in the block length K to exponential in the delay spread L. We also remark that MMSE-ZF becomes ZF in case the ZF and is unique (apart from ordering of the symbols). Therefore the proof that ISI in the error signal of a MMSE is negligible for diversity analysis as $SNR\to\infty$ is fairly general (for instance, it applies to both BDFE and BLE). We also note here the optimality of Vandermonde unitary constellation rotation precoder with unitary bases [11] for the square-tall and square-square precoders. When these precoders are used, the Matched Filter Bound (MFB) corresponds to a genie-aided analysis in which all (QAM) symbols apart from the one being detected are considered known. Since the MFB analysis corresponds to a Pairwise Error Probability (PEP) analysis in which the two input sequences differ only in the same (QAM) symbol, consider now a reduced genie-aided analysis in which all the ISI within the blocks is assumed known, this leaves us with a square mixture across the blocks. This corresponds exactly to the parallel channel scenario considered in [11] for which these precoder have been shown to be optimal in [11].

VI. Numerical results

We provide here simulation results to strengthen the arguments made in the previous section. The diversity order of a receiver can be estimated based on the slope of the BER curve at high SNR. multipath diversity of frequency selective channel. In Fig. 3 we plot the performance of both

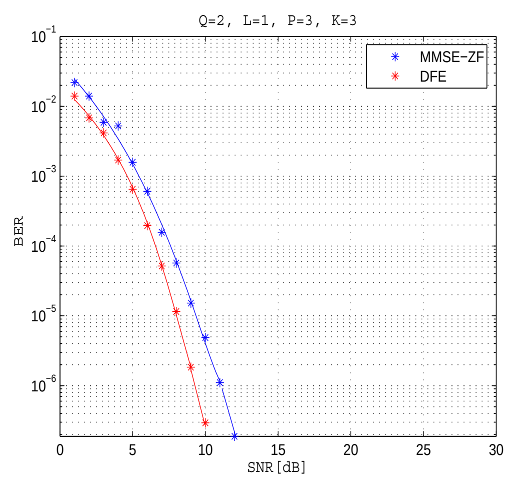

Fig. 3. Diversity order of LE and DFE.

the receivers for linearly precoded transmission in a channel parameterized by $Q=2, L=1, P=3, K=3$. The BER curve has a slope for both LE and DFE have a slope of $(Q+1)(L+1)$ which leads us to conclude that DFE also achieves full diversity in the channel when an appropriate diversity enabling precoder is used at the transmitter. As expected, the BER curve for the DFE has a better coding gain due to the fact that $SINR_{DFE} \geq SNR_{MMSE-ZF}$ In Fig. 4 we compare the diversity order with square-tall precoders with that of the square-square precoders when MLE is applied at the receiver. We see that both the precoders enable

MLE to exploit full channel diversity. However, the square-square precoders have a higher bandwidth efficiency while the square-tall precoders have better coding gain. Finally in

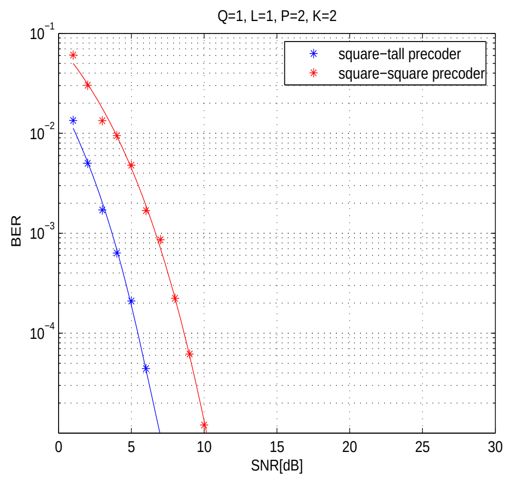

Fig. 4. Comparison of diversity order with square-tall and square-square precoders.

Fig. 5 we compare the performance of the hybrid equalizer for square-tall precoders. Note that MMSE-ZF receiver does not collect full diversity whereas the diversity order of the hybrid equalizer is the same as that of full blown MLE which has a much higher computational complexity compared to the hybrid equalizer.

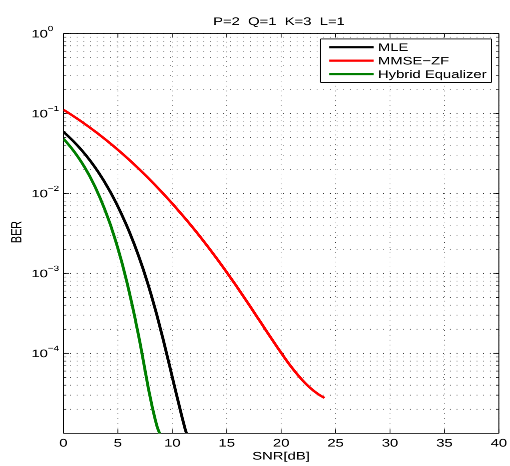

Fig. 5. Performance of hybrid equalizer with square-tall precoder.

VII. Concluding Remarks

In this contribution, we focus on full diversity equalization for precoded block transmission in doubly selective channels. We studied the diversity aspects of three classes of redundant precoders classified on the basis of redundancy in time-and-frequency (tall-tall), time-only (square-tall) and neither time nor frequency (square-square) domains. For the tall-tall case, we showed that the non-Gaussianity of the residual ISI in the DFE for MMSE designs is a non-issue with respect to diversity order of the equalizer, hence MMSE-DFEs benefit from full diversity in the channel with tall-tall precoders. For the square-tall precoders, we proposed a hybrid-equalizer that benefits from full channel diversity by requires ML processing only to extract Doppler diversity in the channel thereby reducing the complexity of the equalizer. Finally, we show that by using constellation rotation precoding, it suffices to introduce a redundancy of the order of channel delay spread in order to collect full channel diversity with MLE.

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