

# A Game Theoretic Framework for Decentralized Power Allocation in IDMA Systems

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**Abstract**—In this contribution we present a decentralized power allocation algorithm for the uplink interleaved division multiple access (IDMA) channel. Within the proposed optimal strategy for power allocation, each user aims at selfishly maximizing its own utility function. An iterative chip-by-chip (CBC) decoder at the receiver and a rational selfish behavior of all the users according to a classical game-theoretical framework are the underlying assumptions of this work. This approach leads to a channel inversion policy where the optimal power level is set locally at each terminal based on the knowledge of its own channel realization, the noise level at the receiver and the number of active users in the network.

## I. INTRODUCTION

Interleave division multiple access (IDMA) has been identified as a promising multiple access technique in the context of cellular networks [1] and self-organizing networks (e.g. *ad hoc networks*) [2]. In this domain, distributed power algorithms play a central practical and theoretical role. Nonetheless, the study of *distributed* power allocation algorithms for IDMA is still an unsolved problem. The centralized power allocation (CPA) has been tackled by several authors (see [3], [4], and references therein). In these works, iterative multiuser detection/channel decoding is assumed at the receiver and the base station determines the power to be transmitted by each user, according to a global optimum criterion and typically having complete channel state information (CSI). In this contribution, a novel framework to tackle the DPA problem in IDMA systems using tools from game theory is presented. The proposed approach assumes iterative multiuser detection/channel decoding at the receiver based on the chip-by-chip (CBC) algorithm [1], and relies on the signal to noise ratio (SINR) evolution technique [4] for non-heavily loaded systems.

In our approach, we assume that each user allocates the power by maximizing its own utility function and by assuming a competitive selfish and rational behavior of the other users. The proposed utility function for a given user is the ratio between a power of the users goodput (probability of an error-free detected frame)

and its own transmit power. Our decentralized approach requires only the knowledge of the noise power at the receiver, the channel gain of the user of interest, and the number of active users in the system. It yields a channel inversion policy for the power allocation. This policy is applied to an IDMA system with a repetition code of length  $N$  bits. Simulations show that the allocated power is substantially independent of the number of users in typical operation conditions for a practical system. Therefore, the knowledge of the number of active users in the system becomes irrelevant as observed in the centralized case in [3].

## II. SYSTEM MODEL

Consider the uplink of an IDMA system with  $K$  chip-synchronous active users. Each mobile and the base station are equipped with a single antenna. The base station performs CBC iterative multiuser decoding based on successive interference cancellation (SIC) or parallel interference cancellation (PIC) as proposed in [1]. Denote by  $b_k$  the information bits of the length- $M'$  message to be transmitted by the  $k^{th}$  user. An identical code with low rate  $R$  is applied to the messages of all the users. The coded information bits, referred to as chips, are permuted by an interleaver  $\pi_k$  of length  $M = \frac{M'}{R}$  chips. Each interleaver is unique in the network, i.e.  $\pi_k \neq \pi_i, \forall i \neq k$ . The base band signal  $r(j)$  sampled at the chip-rate at the receiver is

$$r(j) = \sum_{i=1}^K \sqrt{p_i} h_i x_i(j) + n(j), \quad j = 0 \dots M-1 \quad (1)$$

where  $j$  is the discrete time index for the chip interval and  $h_i$  and  $p_i$  represent the channel realization and the transmit power of the  $i^{th}$  user, respectively. Here,  $x_i(j)$  represents the chip transmitted by the  $i^{th}$  user at chip interval  $j$  and  $n(j)$  is the additive white Gaussian noise (AWGN) process with zero mean and variance  $\sigma^2$ . For the sake of simplicity, we introduce our results assuming an antipodal modulation, i.e.  $x_i(j) \in \{-1, 1\}$ , real valued channel realizations, and real noise samples. The extension to more complex modulation schemes is

straightforward. The receiver is made of an elementary multiuser signal estimator (ESE) based on the principle of maximum a posteriori (MAP) detection, a set of interleavers  $\pi_k$  and de-interleavers  $\pi_k^{-1}$ , and a set of  $K$  single user a posteriori probability decoders (DEC). The decoders feed back soft information to the ESE module in turbo configuration to iterate and improve the estimations. This structure is known as chip-by-chip (CBC) decoder and is described in [1]. The signal to interference and noise ratio (SINR) at the input of the decoder can be estimated at each iteration of the CBC detection by means of the SINR evolution technique [4]. It is shown in [3] that in the steady state, the SINR of user  $k$ , denoted as  $\gamma_k$ , converges to the solution to the following system of equations

$$\gamma_k^{ss} = \frac{p_k |h_k|^2}{\sum_{i=1, i \neq k}^K p_i |h_i|^2 f(\gamma_i^{ss}) + \sigma^2}, \quad \forall k \in [1, K], \quad (2)$$

where the super-index  $ss$  stands for steady state. The function  $f(x) \in [0, 1]$ ,  $\forall x \geq 0$  represents the amount of multiple access interference (MAI) that is eliminated at each iteration of the CBC detection [4]. This function depends on the coding scheme and can be obtained by Monte Carlo simulations as described in [1]. The system (2) might have several solutions. Nonetheless, it has been shown in [3] that in the case of  $|h_i|^2 p_i = |h_j|^2 p_j \forall i \neq j$ , the system has a unique solution given by the fix point equation

$$\gamma_k^{ss} = \frac{p_k |h_k|^2}{(K-1) |h_k|^2 p_k f(\gamma_k^{ss}) + \sigma^2} \quad \forall k \in [1, K] \quad (3)$$

and  $\gamma_k^{ss} = \gamma^{ss}$ , with  $1 \leq k \leq K$ .

Additionally, it has been shown that the SINR evolution technique predicts precisely the SINR under the constraint of non-overloaded systems [1], [4]. However, we have found in our simulations that this is not the case for heavily overloaded systems. Under this constraint, the SINR evolution technique does not match the SINR obtained at the output of the ESE module. In order to exclude this case, we restrict our study to non-overloaded systems, i.e. we consider that  $\frac{K}{N} \leq 1$  always holds.

### III. A NON-COOPERATIVE POWER ALLOCATION GAME

We define the power allocation problem as a strategic game denoted by the triplet  $\{S, \mathcal{P}, (u_k)_{k \in S}\}$ , where  $S = \{1, \dots, K\}$  is the set of players or active mobiles,  $\mathcal{P}$  represents the set of strategies, and  $u_k$  is the utility function of user  $k$ . In this work, the strategy of each user consists of all the possible transmittable power levels  $p_k \in \mathbb{R}^+$ ,  $\forall k \in S$ . We follow the same line as in [3], [5] and [6] and enforce constraints on the maximum transmit power later. Furthermore, the utility function  $u_k$  of user

$k$  is defined as the ratio between the positive power  $s_k$  of its goodput  $g(\gamma_k)$ , i.e. the probability of an error-free detected packet, and its transmit power<sup>1</sup>

$$u_k(\mathbf{p}, \mathbf{h}) = \frac{(g(\gamma_k))^{s_k}}{p_k} \quad \forall k \in S, \quad (4)$$

where  $\mathbf{p} = (p_1, p_2, \dots, p_K)$  and  $\mathbf{h} = (h_1, h_2, \dots, h_K)$  are the vectors of transmit power levels and channel realizations, respectively. Expression (4) generalizes the utility function presented in [5] and [6] when  $s_k = 1$ ,  $\forall k \in S$ . With respect to the reference utility function with  $s_k = 1$  the user  $k$  will choose  $s_k > 1$  if it is interested in better performance at the expenses of a higher transmitted power. On the contrary, if it needs to save power then it will set  $s_k < 1$ . In this study, we assume that all the users are aware of the fact that they are all interested on the same QoS profile, i.e.  $s_1 = \dots = s_K = s$ .

The objective of the player  $k$  is to determine the transmit power  $p_k^*$  that selfishly maximizes its utility function  $u_k(\mathbf{p}^*, \mathbf{h})$  under the assumption that a similar strategy is adopted by all the other users. Thus, the optimum power [7] is obtained as solution to the following system of equations:

$$\begin{aligned} \frac{\partial}{\partial p_k} u_k(\mathbf{p}^*, \mathbf{h}) &= \left( \frac{g(\gamma_k)^{s-1}}{(p_k)^2} \right) \left( sg'(\gamma_k) p_k \frac{\partial}{\partial p_k} \gamma_k - g(\gamma_k) \right) \\ &= 0 \quad \forall k \in S. \end{aligned} \quad (5)$$

Here,  $g(\gamma_k)'$  represents the first derivative of  $g(\gamma_k)$ . Due to the nature of the channel (non-error free channel), the term  $\frac{g(\gamma_k)^{s-1}}{(p_k)^2}$ ,  $\forall k \in S$  is always non-zero. Thus, the system in (5) could be simplified as

$$sg'(\gamma_k) p_k \frac{\partial}{\partial p_k} \gamma_k - g(\gamma_k) = 0 \quad \forall k \in S. \quad (6)$$

Furthermore, it has been shown in [7] that

$$\frac{\partial}{\partial p_k} \gamma_k = \frac{\gamma_k}{p_k} \left( \frac{1 + \gamma_k f'(\gamma_k)}{1 - \gamma_k^2 f'(\gamma_k)} \right) q_k(\gamma), \quad \forall k \in S \quad (7)$$

where  $\gamma = (\gamma_1, \dots, \gamma_K)$  and

$$q_k(\gamma) = \left( 1 - \frac{\gamma_k f(\gamma_k) + \gamma_k^2 f'(\gamma_k)}{(1 - \gamma_k^2 f'(\gamma_k)) (1 + \gamma_k f(\gamma_k)) (A + 1)} \right)$$

and

$$A = \sum_{i=1}^K \frac{\gamma_i^2 f'(\gamma_i)}{(1 - \gamma_i^2 f'(\gamma_i))}.$$

Replacing (7) in (6) we obtain the system of equations

$$sg'(\gamma_k) \gamma_k \left( \frac{1 + \gamma_k f'(\gamma_k)}{1 - \gamma_k^2 f'(\gamma_k)} \right) q_k(\gamma) - g(\gamma_k) = 0, \quad \forall k \in S \quad (8)$$

<sup>1</sup>Further discussions on the definition of this utility function in  $p_k = 0$  can be found in [5] and [6].

In (8), the variables are only  $\gamma_k, \forall k \in \mathcal{S}$ . Therefore, the system does not depend directly on the channel realizations and the transmit power of each user. This property was also observed in the centralized approach [3]. Moreover, in the system (8), all the equations are identical and the system is invariant to variable permutations. This implies that the solutions are also identical, i.e.  $\gamma_1 = \gamma_2 = \dots = \gamma_K = \gamma$ . Thus, the system (8) reduces to a single equation

$$sg'(\gamma)\gamma q(\gamma) - g(\gamma) = 0 \quad (9)$$

where

$$q(\gamma) = \frac{(1-\gamma^2 f'(\gamma)) + (K-1)(\gamma^2 f'(\gamma))(1+\gamma f(\gamma))}{(1-\gamma^2 f'(\gamma))(1+(K-1)\gamma^2 f'(\gamma))}.$$

We define  $z(\gamma) = sg'(\gamma)\gamma q(\gamma) - g(\gamma)$  and we name it *target function*. The zeros of the target function are candidates to be the optimal SINR  $\gamma^*$ . The optimal SINR corresponds to the SINR at which the utility function of each user is maximized. If the set of optimal SINR  $\gamma_i^* = \gamma^*, \forall i \in \mathcal{S}$  is known, it is possible to obtain an expression for the optimal transmit power level from expressions (24) and (25) in [3],

$$p_k^* = \frac{1}{|h_k|^2} \left( \frac{\sigma^2 \gamma^*}{1 - (K-1)\gamma^* f(\gamma^*)} \right) \quad \forall k. \quad (10)$$

Eqn. (10) holds under the constraint<sup>2</sup>

$$K < \left\lfloor \frac{1}{\gamma^* f(\gamma^*)} + 1 \right\rfloor \quad (11)$$

since only positive power levels are meaningful. Note that the expression (10) could be also obtained from the fix point equation (3) as unique solution.

Interestingly, the power allocation for a given user depends on the optimal SINR  $\gamma^*$ , its channel gain, and the number of active users. Therefore, the knowledge of all the other users' channel gains in the network is not required.

#### IV. A DECENTRALIZED POWER ALLOCATION ALGORITHM

In this section, we propose a power allocation algorithm based on the game investigated in Section III under the constraint of non-overloaded systems, i.e.  $\frac{K}{N} \leq 1$ . In general, the target function  $z(\gamma)$  is not linear and the solution to  $z(\gamma) = 0$  requires a numerical approach. The algorithm we present here is based on the secant method. The iterative search of the optimum SINR  $\gamma^*$  is initialized by choosing two values  $\gamma_1$  and  $\gamma_2$  such that

- both  $\gamma_1$  and  $\gamma_2$  are not lower than the minimum SINR required  $\gamma_{\min}$  for reliable communications at rate  $R$  without considering SIC or PIC. According

<sup>2</sup>The constraint is widely satisfied since the function  $f(\gamma)$  decreases rapidly with  $\gamma$  [1], [3]

to the Shannon capacity law  $C = \frac{1}{2} \log_2(1 + \gamma)$ .

Thus,  $\gamma_{\min} = 2^{2R} - 1$  and  $\gamma_{\min} \leq \gamma_1 \leq \gamma_2$ ;

- $z(\gamma_2) \leq z(\gamma_1)$ .

Furthermore, in practical systems, the users have power constraints. It might happen that the optimal power  $p_k^*$  for a given optimal SINR  $\gamma^*$  exceeds the maximum transmittable power  $p_{\max}$ . In this case, a user  $i, \forall i \in \mathcal{S}$  could either transmit at the maximum power  $p_i = p_{\max}$  or not transmit  $p_i = 0$  [3]. Let us denote as  $\gamma_{k,0}^{p_{\max}} = \frac{p_{\max}|h_k|^2}{(K-1)p_k|h_k|^2 + \sigma^2}$  the SINR achieved by the  $i^{\text{th}}$  user before the first iteration of the CBC algorithm when transmitting at the maximum power  $p_{\max}$ . Indeed, if  $\gamma_{k,0}^{p_{\max}}$  enables reliable decoding, i.e.  $\gamma_{k,0}^{p_{\max}} \geq \gamma_{\min}$ , then the user transmits at the maximum power level  $p_{\max}$ . In this case the user does not reach the optimal  $\gamma^*$ . In this case, its utility function is not maximized. However, a reliable decoding is always ensured.

If the condition  $\gamma_{k,0}^{p_{\max}} \geq \gamma_{\min}$  is not satisfied, the  $k^{\text{th}}$  user does not transmit and is considered in *outage*. Note that, the condition over the SINR  $\gamma_{k,0}^{p_{\max}}$  is a necessary but not sufficient condition for a user to be not decoded. In fact, certain users could attain an SINR  $\gamma_{k,0}^{p_{\max}}$  higher than  $\gamma_{\min}$  after decoding as a result of the iterative decoding. However, a user cannot evaluate this possibility. In fact, due to the incomplete knowledge available about other users, it can not determine if iterative decoding is able to sufficiently improve the initial SINR and enables a reliable decoding. Then, transmitting might result in a waste of energy and additional interference for all the other users.

Therefore, the power allocation rule is

$$p_k = \begin{cases} p_k^* & \text{if } p_k^* \leq p_{\max} \\ p_{\max} & \text{if } p_k^* > p_{\max} \text{ and } \gamma_{k,0}^{p_{\max}} \geq \gamma_{\min} \\ 0 & \text{otherwise} \end{cases} \quad (12)$$

Let us denote with  $\varepsilon > 0$  the desired accuracy to determine  $\gamma^*$ . The power allocation algorithm is summarized as follows

##### 1) Initialization of the secant method

$\gamma_1 = \gamma_{\min}$   
while ( $z(\gamma_1) \geq z(\gamma_{\min})$ ) then  
 $\gamma_1 = \gamma_1 + \Delta$  with  $\Delta \in (0, \frac{1}{2}]$   
end

$\gamma_2 = \gamma_1 + \Delta$  with  $\Delta \in (0, \frac{1}{2}]$

##### 2) Iterative step of the secant method

while  $|\gamma_{i+1} - \gamma_i| > \varepsilon$   
 $\gamma_{i+1} = \gamma_i - \frac{\gamma_i - \gamma_{i-1}}{z(\gamma_i) - z(\gamma_{i-1})} z(\gamma_i)$   
end

##### 3) End of the secant method

$\gamma^* = \gamma_{i+1}$ .

##### 4) Power allocation Determine the transmit power according to (12).

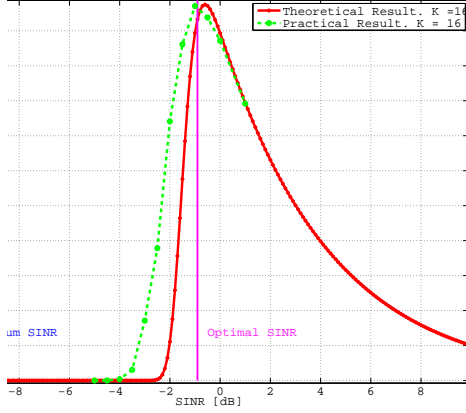


Figure 1. Average Utility Function versus the SINR in decibels for a standard IDMA system with  $K = 16$  users,  $M = 1000$  bits and coding rate  $R = \frac{1}{16}$ . Solid and dashed lines correspond to the SINR evolution and the system simulation, respectively.

## V. PERFORMANCE ASSESSMENT

The performance of the proposed power allocation scheme is assessed assuming antipodal modulation and a repetition code with different rates  $R = \frac{1}{N}$  and  $N = 16, 32, 64$ . The channel gains and the noise are real and Gaussian distributed with zero mean and unit variance.

We refer to the SINR at which the utility function (4) is maximized as the optimal operating point  $\gamma^*$ . An approximate expression for the bit error rate when a repetition code with rate  $R = \frac{1}{N}$  is used is

$$P_e \approx \sum_{i=0}^{\lfloor \frac{N}{2} \rfloor} \binom{N}{\lfloor \frac{N}{2} \rfloor + i} \left( Q(\sqrt{2\gamma_k}) \right)^{\lfloor \frac{N}{2} \rfloor + i} \left( 1 - Q(\sqrt{2\gamma_k}) \right)^{\lfloor \frac{N}{2} \rfloor - i} \quad (13)$$

In (13), the function  $Q(\cdot)$  is the complementary error function of a Gaussian random variable. It represents the probability of error of one chip in the case of antipodal modulation. Hence, the goodput can be written as:

$$g(\gamma_k) = (1 - P_e)^M. \quad (14)$$

For the coding scheme considered in this contribution, the function  $f(\cdot)$  is determined by Monte Carlo simulations as described in [1]. The obtained results match those presented in [3] and [8] perfectly.

In Figure 1, we plot the average utility function versus the SINR  $\gamma$  when  $s = 1$ . The solid line shows the theoretical performance determined by plugging (14) in (4), while the dashed lines are obtained with the actual BER at the output of the decoder. We noticed in all the non-overloaded cases that the utility has a unique maximum. The effects of the coding rate, the frame length, and the number of users on the optimum

operating point are assessed in Figures 2, 3, 4, and 5. In Figure 2, the target function (9) is plotted as a function of the SINR  $\gamma$  for three different coding rates while the frame length and the number of users are kept constant. Lower coding rates determine lower optimal SINRs. In Figure 3, the target function is plotted as a function of the SINR  $\gamma$  considering three different frame lengths, keeping the coding rate and the number of users constant. In this case, longer frames lead to higher optimal SINR values. In Figure 4 we plot the target function as a function of the SINR for several exponents  $s$  of the generalized utility function (4), while keeping the number of users, the frame length and the coding rate constant. When users are interested in better performance rather than saving power ( $s > 1$ ), the optimal SINR increases and vice versa. In Figure 5, the target function is plotted for 4, 8, and 16 active users as a function of the SINR when the coding rate and frame length are kept constant. The four lines overlap completely. Therefore, the variation of the number of active users  $K$  has a negligible effect on the optimal SINR. Then, the optimal operating point is practically independent of the number of users. This effect was also observed in the centralized case in [3].

In general, a convenient solution for a non-cooperative game is the Nash equilibrium (NE). By definition, the NE is a solution such that no player is interested on changing its strategy since no improvement could be obtained in its own utility while keeping the other users' utilities unchanged. In this case, even though the simulations results show that there is a unique solution to the game ( $\gamma^*$ ), we could not prove analytically the uniqueness of the NE since the functions  $f(\gamma)$  and  $g(\gamma)$  depend on the coding scheme. However, particular cases have been already studied. In the case where no interference cancellation is performed, i.e.  $f(\gamma) = 1$ , the proof is similar to the one presented in [5] with  $s = 1$  for the CDMA case.

## VI. CONCLUSIONS

We provided a novel framework for decentralized power allocation in IDMA systems based on a game-theoretic approach. In this context, each user aims at selfishly maximizing its own utility function assuming a similar behavior is adopted by all the other users. It leads to a channel inversion power allocation policy where the optimal power level could be set at each terminal based on the knowledge of its own channel realization, the noise energy at the receiver, and the number of active users in the network. Interestingly, we found that under practical operation conditions, the knowledge of the number of active users becomes irrelevant as observed in the centralized case in [3].

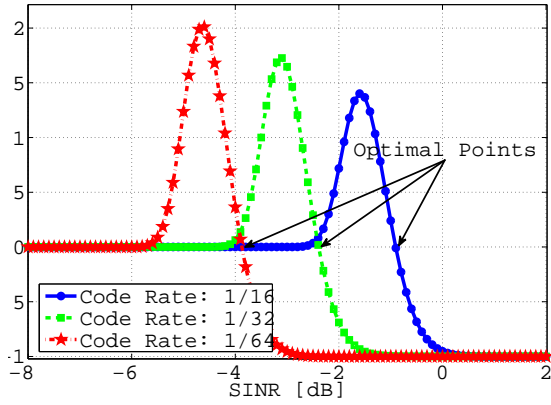


Figure 2. Target function  $z(\gamma)$  versus SINR in decibels for a standard IDMA system with parameters  $K = 16$  users,  $M = 1000$  bits. The solid, dot-dashed and dashed lines correspond to the coding rates  $R = \frac{1}{16}$ ,  $R = \frac{1}{32}$  and  $R = \frac{1}{64}$  respectively.

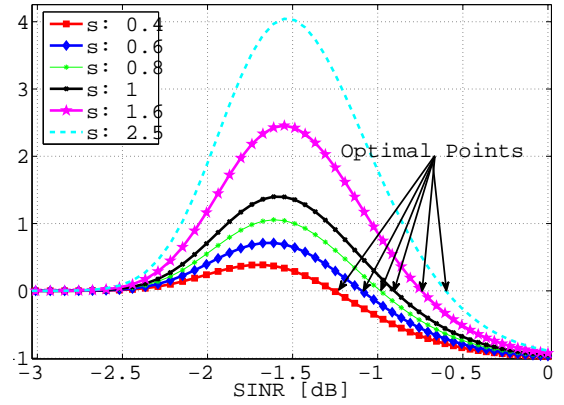


Figure 4. Target function  $z(\gamma)$  versus SINR in decibels for a standard IDMA system with coding rate  $R = \frac{1}{16}$ , frame length  $M = 1000$  bits,  $K = 16$  users and different values for the parameter  $s$

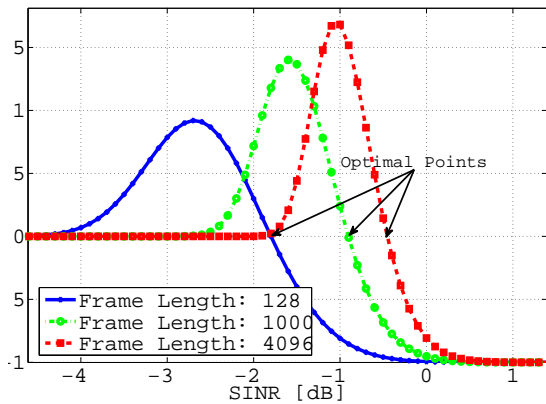


Figure 3. Target function  $z(\gamma)$  versus SINR in decibels for a standard IDMA system with parameters  $K = 16$  users, coding rate  $R = \frac{1}{16}$  bits. The solid, dot-dashed and dashed lines correspond to the frame lengths 128, 1000 and 4096 bits respectively.

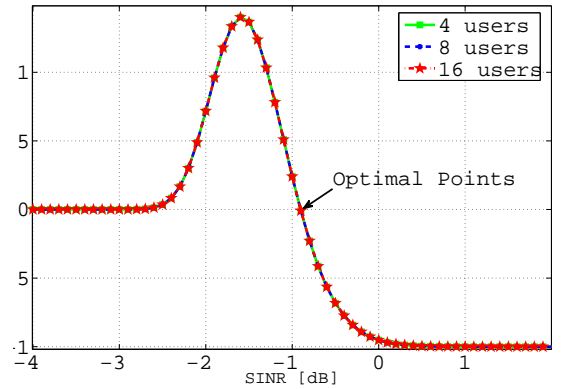


Figure 5. Target function  $z(\gamma)$  versus SINR in decibels for a standard IDMA system with coding rate  $R = \frac{1}{16}$ , frame length  $M = 1000$  bits,  $K = 4$ ,  $K = 8$  and  $K = 16$  users respectively.

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