SPATIO-TEMPORAL ARRAY PROCESSING FOR APERIODIC CDMA DOWNLINK TRANSMISSION

Giuseppe Montalbano, Irfan Ghauri, and Dirk T. M. Slock

Institut Eurécom
B.P. 193, 06904 Sophia–Antipolis CEDEX, France
E-Mail: {montalba, ghauri, slock}@eurecom.fr

ABSTRACT

We consider a DS-CDMA system employing aperiodic spreading sequences (APS) as signature sequences for different users. Multiple transmission antennas (MA) are employed for increasing the network capacity. It is assumed that partial or total knowledge of the downlink channel is available at the base-station due to either a time-division duplex (TDD) or some feedback structure in the network. Spatial and temporal dimensions can no longer be jointly exploited due to the aperiodic spreading. Hence, spatio-temporal pre-cancellation of downlink interference (zero-forcing) is no longer possible. However, beamforming can be applied to maximize the signal-to- noise plus interference (SINR) ratio on a user-to-user basis. We proceed with such an approach and provide closed-form relations for the signal and interference terms at the output of a mobile station RAKE receiver. We also show that using the spatial dimension at the base station enhances the system performance. The mobile receiver can later employ an equalizer to do away with the residual intersymbol interference, thus maximizing the matched filter bound (MFB) at its output.

1. INTRODUCTION

Third generation wireless communication systems envision the use of DS-CDMA employing aperiodic spreading sequences for the downlink, typically consisting of periodic Walsh-Hadamard sequences followed by masking by a symbol aperiodic base-station specific overlay sequence. Alternatively, even the scrambling sequence can be user dependent. In scenarios where high rate users are present in the downlink of a DS-CDMA based system operating in a multipath environment, a RAKE receiver with a relatively large number of fingers will be required to combine multipath signals. However the RAKE receiver is plagued by the near-far effect. Power control therefore arose to be a major concern in multicellular multiuser downlink transmission despite the presumed noise-like nature of multiple access interference (MAI).

The use of adaptive antenna arrays at the base station can increase the capacity of a mobile radio network by increasing transmit diversity and due to the capability of mitigating the interference. In the downlink however, the possibility of spatial diversity reception by multiple antennas (MA) is limited due to complexity and space limitations. The presence of the spatial dimension, eventually joined to oversampling of the transmitted signals and/or to other forms of diversity (e.g., polarization diversity, in-phase and quadrature modulations for mono-dimensional symbol constellations) allows the increase of the system loading fraction, actually limited by the multipath multi-user interference, by increasing diversity. When periodic spreading sequences are adopted, effective spatio-temporal processing can be carried out at the base station transmitter relying on symbol rate wide sense stationarity. Under these circumstances in [2] and [1] it was demonstrated that orthogonality between the spread signals can be restored at each receiver by properly filtering/spreading the symbols intended for different users, basing upon the information of the channel state associated with each user. Then a number of interfering users more than the processing gain may be located in the same cell, in particular accounting for the users in soft hand-off mode. The application of these techniques is not straightforward when the symbol rate cyclostationarity no longer exists due to the use of aperiodic overlay spreading sequences which spread/randomize the orthogonal user sequences. It has to be noted that, assuming the fading processes slow enough, in the structure of this downlink problem, the only entity fixed over the processing interval is the propagation channel. The actual channel as seen from the base station to a certain user will consists of the cascade of spreading, transmit filters, propagation channel, receive filters and RAKE receiver. Due to the aperiodicity of spreading sequences the previous cascade results in a time-variant filter from symbol to symbol. This precludes the possibility of performing feasible adaptive temporal pre-filtering at the base station, as in [2] and [1], because the pre-filters need to be up-dated every symbol period.

It has been observed that in outdoor propagation the most scattering phenomena occur in the proximity of the mobile user and not of the base station. This translates in a relatively small angle spread at the base station antenna. Due to cost reasons, the base station array consists of just a few antennas. This yields a small antenna aperture, namely a poor spatial resolution, so that in practice very few main nominal multipath directions can be resolved. As showed in the literature on channel modeling (e.g. [9]), for outdoor channels there exist one or two main distinct directions of multipath components called *clusters*. On the other hand, due to the spreading operation in CDMA systems, a very high multipath temporal-delay resolution can be achieved, and temporally sparse channels exist in outdoor propagation with distinct multipath delays. The spatio-temporal channel can therefore be modeled as a clusterized channel where, to each nominal direction of propagation, correspond several multipath components which are temporally resolvable. Such a channel model can be factorized in spatial and temporal channel components for each spatio-temporal channel path cluster. The above arguments lead to an approach considering only the transmit spatial processing (namely beamforming) at the base station, and to maintain the temporal processing as the one traditionally done when RAKE receivers are employed at the mobile. Some related work is also found in [13].

In the sequel we consider a scenario where d intra-cell users, each with a RAKE receiver, captures signals transmitted from a base station with m antennas. We shall consider both full and partial channel state information for each user, corresponding to TDD and FDD mode respectively. The goal consists of designing a proper set of beamforming weight vectors in order to maximize the minimum signal to interference plus noise ratio (SINR) at the d mobile receivers, under the constraint of a limited transmit power at the base station.

2. CHANNEL MODEL

Due to the high temporal resolution of CDMA systems we consider a specular path propagation channel model that consists of Q multipath components. The multipath channel as seen form the base station can be modeled in the continuous-time domain as follows

$$\boldsymbol{h}^T(\tau,\, t) = \sum_{q=1}^{Q} \alpha_q(t)\mathbf{a}^T(\theta_q)p(\tau - \tau_q) \tag{1}$$

where τ_q, θ_q, and $\alpha_q(t)$ denote the delay, the angle and the fading attenuation associated to the qth path, respectively, $p(t)$ denotes the chip pulse shaping filter, and $\mathbf{a}(\theta)$ represents the array response vector. If the angle delay-spread is small compared to the base station antenna array resolution, paths can be collected in clusters yielding the following model for the lth channel cluster[1]

$$\boldsymbol{h}_l^T(\tau,\, t) = \mathbf{a}^T(\theta_l) \sum_{q_l=1}^{Q_l} \alpha_{q_l}(t)p(\tau - \tau_{q_l}) \tag{2}$$

where Q_l, $\alpha_{q_l}(t)$ and τ_{q_l} are the number of multipaths components, the fading coefficients and the delays associated to the paths in the lth cluster, and θ_l is the corresponding direction of propagation. The whole channel can be modeled as the superposition of the single cluster channels, i.e.,

$$\boldsymbol{h}^T(\tau,\, t) = \sum_{q=l}^{L} \boldsymbol{h}_l^T(\tau,\, t) \tag{3}$$

In a TDD framework assuming a similar multipath channel model for the uplink, the mobile velocity sufficiently slow compared to the round-trip time and the transmitter and receiver properly calibrated, the uplink and downlink channels for a certain user can be assumed to be approximately the same. So, the uplink channel estimate can be assumed as downlink channel for the transmit filters design. On the contrary operating in the FDD mode the uplink and downlink channels are not the same. The parameters in the channel model which can be assumed approximately constant between the uplink and the downlink channels are the angles, the delays and the variances of the amplitudes. Since the difference in phase between up- and downlink is random it can be assumed uniformly distributed, whereas the magnitudes for both links are also random but can be assumed to have the same variance. The variances of the path amplitudes can be estimated by non-coherent averaging over a certain time interval. The angles can be estimated if the array manifold at the downlink carrier frequency is known. For particular array geometries and relatively small uplink–downlink frequency shifts, the array response can be transposed from the uplink to the corresponding response in the downlink via a linear transformation [7] without requiring explicit angle estimation. Another approach consists of performing a *beamspace* transformation (namely a spatial DFT) to estimate the beams in which the signal energy is located [12]. The downlink transmission then occurs through the same beams as the uplink reception.

[1]Path clusters can better be characterized by a nominal direction θ_l and an angle spread σ_l assuming a certain spatial distribution, although we shall neglect for simplicity this issue in this formulation.

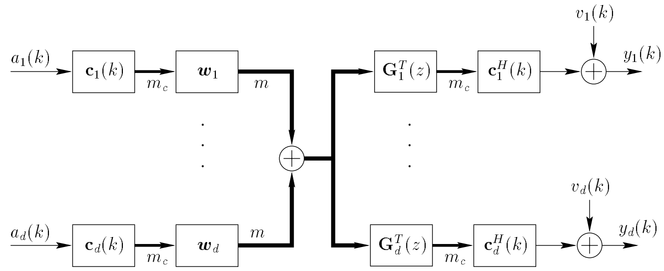

Figure 1: Transmission filters channels and RAKE receivers for d users

3. SIGNAL MODEL

We assume a CDMA based system employing aperiodic spreading sequences, $c_i(t;\, kT)$, for $i = 1,\, \ldots,\, d$, where T is the symbol period and k is an integer. The spreading factor is m_c and $T_c = T/m_c$ denotes the chip period. Due to the time-variant nature of the spreading sequences the cascade of the code filter, the transmit filter, the channel and the receive filter (a RAKE) results in a time-variant system. In the sequel the problem of the maximization of the minimum SINR of the d users is addressed, accounting for the presence of an equalizer following the RAKE receiver. In this case the ISI for the signal of interest will be considered as contributing to the signal and not to the interference energy. Notice that the actual temporal channel as seen from the base station is given by the autocorrelation sequence of the channel itself, where the RAKE is also accounting for the effect of the beamforming weight vector of the user of interest. This is straightforward if we commute the de-spreading and channel matched filtering operations in the RAKE receiver.
Assuming the channels, $\boldsymbol{h}_i$, time-invariant for the observation time, the ith user discrete-time received signal at the RAKE output, for $i = 1\, \ldots,\, d$, is

$$\begin{aligned} y_i(k) = {} & \mathbf{c}_i^H(k)\mathbf{G}_i^T(\zeta)\textstyle\sum_{j=1}^{d} \boldsymbol{w}_j \otimes \mathbf{c}_j(k)a_j(k) \\ & + v_i(k) \end{aligned} \tag{4}$$

where the $a_j(k)$ are the transmitted symbols intended for the jth user, ζ^{-1} is the unit sample delay operator (i.e., $\zeta^{-1}y_i(k) = y_i(k-1)$), $\mathbf{G}_i^T(z)$ is the channel-channel matched filter cascade transfer function between the base station and the ith user channel, $\mathbf{c}_i^H(k)$ is the ith user correlator, $\boldsymbol{w}_j$ is the beamforming weight vector for the transmitted chips $\mathbf{c}_j(k)a_j(k)$, that has to be optimized, $\otimes$ denotes Kronecker product, $\mathbf{c}_j(k)$ is the spreading code for the kth symbol of the jth user, and $v_i(k)$ is the additive noise at the output of the ith RAKE receiver. We remark that $v_i(k)$ will be a colored noise in general due to matched filtering to the channel. The channel $\mathbf{G}_i^T(z)$ is a $m_c \times m$ matrix, and $\mathbf{c}_i^H$ is a $1 \times m_c$ row vector. The product of $\boldsymbol{w}_j \otimes \mathbf{c}_j(k) = \boldsymbol{f}_j(k)$ generates a $m \times 1$ column vector, with $m = m_c m_a$ where m_a is the number of multiple antennas.

4. TEMPORAL CHANNEL STRUCTURE

Assume that the channel is of the form (3). For simplicity we consider that there is only one path cluster. Several clusters can be analyzed separately and their effect can be combined afterwards. In the single cluster case the channel can easily be factorized in spatial and temporal components. Hence we analyze the temporal channel component, and we evaluate the signal and interference energies transmitted through it averaged over the spreading codes statistics. The temporal channel for the ith, $h_i(t)$ is assumed to be an FIR filter of duration approximately equal to n_i chip periods. Let $\boldsymbol{h}_i = [h_i^H(0) \ldots h_i^H(n_i-1)]^H$ denote the discrete-time representation of the ith channel. Let N_i be the length of the channel in symbol periods. The autocorrelation sequence of the actual channel, written as $\boldsymbol{g}_i = [g_i^H(-n_i+1) \ldots g_i^H(n_i-1)]^H$ has duration $2n_i-1$ chip periods, or $2N_i-1$ symbol periods. The whole cascade spreading-channel autocorrelation-despreading will last at most $2N_i+1$ symbol periods, namely $m_c(2N_i+1)$ chip periods. Without loss of generality we may zero pad $\boldsymbol{g}_i$, in order to have $2N_i m_c+1$ coefficients in the channel autocorrelation sequence. Hence the overall energy in the spreader-channel-channel matched filter and correlator cascade for the ith user can be written as follows

$$S_i = \boldsymbol{g}_i^H \mathrm{E}\{\mathcal{C}_i(k)^H \boldsymbol{B}_i^H(k)\boldsymbol{B}_i(k)\mathcal{C}_i(k)\}\boldsymbol{g}_i \qquad (5)$$

where $\mathrm{E}\{\cdot\}$ denotes the expectation operator, $\mathcal{C}_i(k)$ is a $(2N_i+1)m_c \times (2N_i m_c+1)$ Toeplitz matrix with $[\boldsymbol{c}_i(k)\boldsymbol{0}_{2N_i m_c\times 1}]^T$ as first column and $[c_i(1,k)\boldsymbol{0}_{1\times 2N_i m_c}]$ as first row, $\boldsymbol{B}_i(k) = \mathrm{blockdiag}(\boldsymbol{c}_i^H(k-N_i), \cdots, \boldsymbol{c}_i^H(k-N_i))$ is a $(2N_i+1)\times(2N_i+1)m_c$ block diagonal matrix, and, hereinafter $\boldsymbol{g}_i$ denotes the zero padded version of the channel autocorrelation sequence.
For the interference from jth user signal to the ith user receiver a similar expression as (5) holds, i.e.,

$$I_{ji} = \boldsymbol{g}_i^H \mathrm{E}\{\mathcal{C}_j(k)^H \boldsymbol{B}_i^H(k)\boldsymbol{B}_i(k)\mathcal{C}_j(k)\}\boldsymbol{g}_i \qquad (6)$$

In the following we shall distinguish between the case of complex and real spreading sequences when taking the expectation in (5) and (6).

4.1. Complex Spreading Sequences

We consider complex circularly symmetric spreading sequences $\boldsymbol{c}_i(k)$ where, without loss of generality, we normalize the chip energy to one. Taking the expectation in (5) and (6) we obtain

$$\boldsymbol{S}_i = \mathrm{E}\{\mathcal{C}_i(k)^H \boldsymbol{B}_i^H(k)\boldsymbol{B}_i(k)\mathcal{C}_i(k)\} = m_c \mathrm{diag}([\boldsymbol{1}_{N_i m_c}\ m_c\ \boldsymbol{1}_{N_i m_c}])$$

and

$$\boldsymbol{I}_{ji} = \mathrm{E}\{\mathcal{C}_j(k)^H \boldsymbol{B}_i^H(k)\boldsymbol{B}_i(k)\mathcal{C}_j(k)\} = m_c\mathbf{I}$$

respectively. Then (5) and (6) reduces to

$$\begin{aligned} S_i^c &= m_c^2\|\boldsymbol{h}_i\|^4 + m_c(\|\boldsymbol{g}_i\|^2 - \|\boldsymbol{h}_i\|^4) \\ I_{ji}^c &= m_c\|\boldsymbol{g}_i\|^2 \end{aligned} \qquad (7)$$

where the superscript $(\cdot)^c$ denotes the use of complex spreading sequences. We remark that having normalized to one the energy per chip the energy per symbol is $\sigma_a^2 = m_c$.

4.2. Real Spreading Sequences

If we consider real spreading sequences $\boldsymbol{c}_i(k)$, taking the expectation in (5) yields an extra contribution with respect to the complex case, namely

$$\begin{aligned} &\boldsymbol{S}_i = m_c\mathrm{diag}([\boldsymbol{1}_{N_i m_c}\ m_c\ \boldsymbol{1}_{N_i m_c}]) + \\ &\mathrm{antidiag}([\boldsymbol{0}_{(N_i-1)m_c}, 1, 2, \ldots, \\ &\ldots(m_c-1), 0, (m_c-1), \ldots, 1, \boldsymbol{0}_{(N_i-1)m_c}]) \end{aligned} \qquad (8)$$

On the contrary the interference term $I_{ji}^c = I_{ji}^r = I_{ji}$ does not change. The signal energy consists of two contributions, namely

$$S_i^r = S_i^c + 2m_c\left(\sum_{n=-m_c+1}^{m_c-1} n(\mathbf{Re}\{g(n)\})^2\right) \qquad (9)$$

where S_i^c is given in (7). The additional contribution arising when using real spreading sequences can be negligible for large channel delay spreads.

4.3. Channel Covariance Matrices and Extension to Multi-Cluster Channels

Once the signal and the interference energy terms have been computed, the signal and interference spatial channel covariance matrices for the ith user are given by

$$\boldsymbol{R}_{ii} = \mathbf{a}(\theta_i)\mathbf{a}^H(\theta_i)S_i \qquad \boldsymbol{R}_{ji} = \mathbf{a}(\theta_i)\mathbf{a}^H(\theta_i)I_{ji} \quad \text{for } j \neq i. \qquad (10)$$

The approach above can be extended to treat multi-cluster channels, by simply computing each cluster contribution separately as shown previously, and summing up the corresponding covariance matrices.

5. TRANSMIT BEAMFORMING OPTIMIZATION

With the previous definitions for $\boldsymbol{R}_{ji}$ and $\boldsymbol{w}_j$ the SINR for the ith user is given by

$$\mathrm{SINR}_i = \frac{\boldsymbol{w}_i^H\boldsymbol{R}_{ii}\boldsymbol{w}_i}{\sum_{j=1,\,j\neq i}^d \boldsymbol{w}_j^H\boldsymbol{R}_{ji}\boldsymbol{w}_j + \nu_i} \qquad (11)$$

where $\nu_i = \sigma_{v_i}^2/\sigma_a^2$ where $\sigma_a^2 = \mathrm{E}\{|a_i(k)|^2\}$ for any i and $\sigma_{v_i}^2$ is the variance of the filtered noise $v_i(k)$. We denote $\mathrm{SINR}_i = \gamma_i$ for any i. Hence the general optimization problem is

$$\max_{\{\boldsymbol{w}_i\}} \min_i \{\gamma_i\} \qquad (12)$$

or

$$\min_{\{\boldsymbol{w}_i\}} \max_i \{\gamma_i^{-1}\} \qquad (13)$$

Then let $\boldsymbol{w}_i = \sqrt{p_i}\boldsymbol{u}_i$, with $\|\boldsymbol{u}_i\|_2 = 1$, the vector of the inverse SINR's $\boldsymbol{\gamma}^{-1} = [\gamma_1^{-1} \ldots \gamma_d^{-1}]^T$ and the vector of the transmit powers $\boldsymbol{p} = [p_1, \ldots, p_d]^T$. We also need to limit the maximum transmit power at the base station, i.e., $\|\boldsymbol{p}\|_1 \leq p_{\max}$. The criterion (13) can be reformulated as

$$\min_{\boldsymbol{p},\{\boldsymbol{u}_i\}} \|\boldsymbol{\gamma}^{-1}\|_\infty \quad \text{s.t. } \|\boldsymbol{p}\|_\infty \leq p_{\max}, \|\boldsymbol{u}_i\|_2 = 1\,\forall i \qquad (14)$$

Then we define the normalized power delivered by the jth base station to the ith user as

$$c_{ji} = \boldsymbol{u}_j^H\boldsymbol{R}_{ji}\boldsymbol{u}_j \quad .$$

For any i we have

$$\gamma_i^{-1}p_i c_{ii} = \sum_{j\neq i} p_j c_{ji} + \nu_i \quad . \qquad (15)$$

In order to account for all the users we introduce the matrix $\boldsymbol{D}_c = \operatorname{diag}(c_{11}, \ldots, c_{dd})$, the matrix $\boldsymbol{C}^T$ defined as

$$[\boldsymbol{C}^T]_{ij} = \begin{cases} c_{ji} & \text{for } j \neq i \\ 0 & \text{for } j = i \end{cases}$$

the vector $\boldsymbol{\nu} = [\nu_1 \ldots \nu_d]^T$ and the matrix $\boldsymbol{P} = \operatorname{diag}(\boldsymbol{p})$. Then we have the following equation

$$\boldsymbol{\gamma}^{-1} = \boldsymbol{D}_c^{-1}\boldsymbol{P}^{-1}[\boldsymbol{C}^T\boldsymbol{p} + \boldsymbol{\nu}] \ . \tag{16}$$

So the criterion (14) generally leads to a set of coupled problems which cannot be solved analytically. Furthermore it can be shown that once the vectors $\boldsymbol{u}_i$ are fixed the optimum power assignment vector $\boldsymbol{p}$ is unique. We consider in the following an analytical approach for the optimization of the minimum signal to interference ratio (SIR) for the normalized weight vectors design. Once the we have obtained the vectors $\boldsymbol{u}_i$'s we can plug them in the problem (14) and solve for $\boldsymbol{p}$.

5.1. SIR Optimization

The SIR for the ith user is defined as

$$\mathrm{SIR}_i = \frac{\boldsymbol{w}_i^H \boldsymbol{R}_{ii} \boldsymbol{w}_i}{\sum_{j=1,\, j\neq i}^{d} \boldsymbol{w}_j^H \boldsymbol{R}_{ji} \boldsymbol{w}_j} \tag{17}$$

The equation (16) in the absence of noise reduces to

$$\boldsymbol{\gamma}^{-1} = \boldsymbol{D}_c^{-1}\boldsymbol{P}^{-1}\boldsymbol{C}^T\boldsymbol{p} \tag{18}$$

where now $\gamma_i = \mathrm{SIR}_i$ for any i. Considering the criterion (14) and the definition (17) it is straightforward to see that the optimum is achieved when all the MAI is zero so that $\gamma_i^{-1} = 0$ for all i's. Then, if m_a is greater than the number of all the nominal propagation directions of all the users channels, the optimum approach in the absence of noise would lead to a zero-forcing (ZF) MAI solution. In practice this condition never arises so non-ZF approaches need to be considered. Note that since the optimum still involves $\gamma_i = \gamma$ for any i, the equation (18) reduces to

$$\gamma^{-1}\boldsymbol{p} = \boldsymbol{A}^T\boldsymbol{p} \tag{19}$$

where $\boldsymbol{A}^T = \boldsymbol{D}_c^{-1}\boldsymbol{C}^T$ is a non-negative matrix. Moreover $\boldsymbol{p}$ has to be a non-negative vector and γ^{-1} has to be non-negative as well. On the basis of the following theorems ([8],[6])

Theorem 1
For a non-negative matrix, the eigenvalue of the largest norm is positive, and its corresponding eigenvector can be chosen to be non-negative.

Theorem 2
For a non-negative matrix $\boldsymbol{A}^T$, the non-negative eigenvector corresponding to the eigenvalue of the largest norm is positive.

Theorem 3
Given the matrix $\boldsymbol{A}^T$ there exists only one solution to equation (19).

we can say that for a given set of unit norm vectors $\{\boldsymbol{u}_i\}$ then the optimum yields $\gamma^{-1} = \lambda_{\max}(\boldsymbol{A}^T)$ and $\boldsymbol{p} = V_{\max}(\boldsymbol{A}^T)$.
Having an estimate of $\boldsymbol{p}$, we can optimize $\{\boldsymbol{u}_i\}$. Indeed the optimization criterion is given by

$$\min_{\{\boldsymbol{u}_i\}} \lambda_{\max}(\boldsymbol{A}^T) \tag{20}$$

In order to simplify the problem formulation without loss of generality, we consider $\boldsymbol{u}_i^t$'s normalized such that $\boldsymbol{u}_i^H\boldsymbol{R}_{ii}\boldsymbol{u}_i = 1$, so that $\boldsymbol{D}_c = \mathbf{I}$ and $\boldsymbol{A}^T = \boldsymbol{C}^T$. Then the criterion (20) becomes

$$\min_{\{\boldsymbol{u}_i\}} \boldsymbol{q}^T\boldsymbol{A}^T\boldsymbol{p} \quad \text{s.t.} \quad \boldsymbol{u}_i^H\boldsymbol{R}_{ii}\boldsymbol{u}_i = 1 \tag{21}$$

where $\boldsymbol{q} = V_{\max}(\boldsymbol{A})$. The criterion (21) leads to a set of d decoupled problems whose solution is given by $\boldsymbol{u}_i = \frac{\boldsymbol{e}_i}{\sqrt{\boldsymbol{e}_i^H\boldsymbol{R}_{ii}\boldsymbol{e}_i}}$, where $\boldsymbol{e}_i = V_{\max}(\boldsymbol{R}_{ii}, \sum_{j\neq i}^{d} q_j\boldsymbol{R}_{ij})$ for any i. The new set of vectors $\{\boldsymbol{U}_i^t\}$ can be used to re-optimize the powers $\boldsymbol{p}$ according to (19).

5.1.1. $\|\boldsymbol{A}^T\|_1$ *minimization based solution*

As sub-optimal approach or initialization we might use the following criterion

$$\min_{\{\boldsymbol{u}\}} \|\boldsymbol{A}^T\|_1 \quad \text{s.t.} \quad \boldsymbol{u}_i^H\boldsymbol{R}_{ii}\boldsymbol{u}_i = 1 \tag{22}$$

This approach has the advantage of optimizing the direction vectors $\{\boldsymbol{u}_i\}$ independently from the powers $\boldsymbol{p}$. In that sense it is suitable to initialize an iterative procedure to find the global optimum. Indeed it leads to a set of d decoupled minimization problems whose solution is given by $\boldsymbol{u}_i = \frac{\boldsymbol{e}_i}{\sqrt{\boldsymbol{e}_i^H\boldsymbol{R}_{ii}\boldsymbol{e}_i}}$, where, in this case, $\boldsymbol{e}_i = V_{\max}(\boldsymbol{R}_{ii}, \sum_{j=1}^{d}\boldsymbol{R}_{ij})$ for any i.
Note that the criterion (22) corresponds to minimizing the power delivered to the undesired users while maximizing the power delivered to the desired user, by each spatial filter $\boldsymbol{u}_i$. A similar criterion was already proposed in [9, 10] to optimize the weight vectors for transmit beamforming.

5.1.2. $\lambda_{\max}(\boldsymbol{A}^T)$ *minimization based algorithm*

According to the previous arguments, we propose the iterative procedure summarized in Table 1 to find the global optimum in the absence of noise.

Table 1: $\lambda_{\max}(\boldsymbol{A}^T)$ minimization based algorithm

(i)	Initialize $\boldsymbol{u}_i$ using (22) for $i = 1, \ldots, d$;
(ii)	Compute $\boldsymbol{q} = V_{\max}(\boldsymbol{A})$;
(iii)	Compute $\boldsymbol{e}_i = V_{\max}(\boldsymbol{R}_{ii}, \sum_{j\neq i} q_j\boldsymbol{R}_{ij})$;
(iv)	Compute $\boldsymbol{u}_i = \frac{\boldsymbol{e}_i}{\sqrt{\boldsymbol{e}_i^H\boldsymbol{R}_{ii}\boldsymbol{e}_i}}$;
(v)	Go back to (ii) until convergence;
(vi)	Compute $\boldsymbol{p} = V_{\max}(\boldsymbol{A}^T)$;
(vii)	Compute $\boldsymbol{w}_i = \sqrt{p_i}\boldsymbol{u}_i$.

5.2. Power assignment optimization

Assuming a given set $\{\boldsymbol{u}_i\}$, since the optimum involves all the γ_i's to be the same, the expression (16) can be arranged in order to include the constraint on the transmitted power as follows

$$\boldsymbol{Q}\tilde{\boldsymbol{p}} = \gamma^{-1}\boldsymbol{S}\tilde{\boldsymbol{p}} \tag{23}$$

where $\tilde{\boldsymbol{p}} = [\boldsymbol{p}^T\, 1]^T$,

$$\boldsymbol{Q} = \begin{bmatrix} \boldsymbol{A}^T & \boldsymbol{\mu} \\ \boldsymbol{0}_{1\times d} & 0 \end{bmatrix} \qquad \boldsymbol{S} = \begin{bmatrix} \mathbf{I}_d & \boldsymbol{0}_{d\times 1} \\ \boldsymbol{s}^T & -p_{\max} \end{bmatrix}$$

where $\boldsymbol{\mu} = \boldsymbol{D}_c^{-1}\boldsymbol{\nu}$, $\boldsymbol{s} = [\|\boldsymbol{u}_1\|^2 \ \ldots \ \|\boldsymbol{u}_d\|^2]^T$ and $\boldsymbol{s}^T\boldsymbol{p} = p_{\max}$. Then similarly to [6] since $\boldsymbol{S}$ is invertible we have

$$\boldsymbol{E}\tilde{\boldsymbol{p}} = \gamma^{-1}\tilde{\boldsymbol{p}}, \qquad \boldsymbol{E} = \boldsymbol{S}^{-1}\boldsymbol{Q} = \begin{bmatrix} \boldsymbol{A}^T & \boldsymbol{\mu} \\ \frac{\boldsymbol{s}^T\boldsymbol{A}^T}{p_{\max}} & \frac{\boldsymbol{s}^T\boldsymbol{\mu}}{p_{\max}} \end{bmatrix} \tag{24}$$

which is a non-negative matrix. Relying on theorems 1–3 we can say that $\gamma^{-1} = \lambda_{\max}(\boldsymbol{E}^T)$ and $\tilde{\boldsymbol{p}} = V_{\max}(\boldsymbol{E})$. Further, note that we can always re-scale $\tilde{\boldsymbol{p}}$ in order to make its last element equal to one.

6. SIMULATIONS

In this section we consider a scenario with $d = 4$ users operating in CDMA system employing complex spreading sequences and spreading factor $m_c = 6$, corresponding to a loading fraction of 66%. Each user has a single cluster channel characterized by a nominal angle and a several delays. The users are in near-far conditions, namely the useful signal powers are proportional to 20 dB, -60 dB, 20 dB, and -20 dB. The angles are $\theta_1 = -45$, $\theta_2 = -25$, $\theta_3 = 5$, $\theta_4 = 30$ degrees respectively for the first the second, the third and the fourth user. An array of $m_a = 3$ antennas is used at the base station to transmit at the 4 users. In figure 6 it is shown the convergence of the proposed algorithm. The output SIR is 11.6 dB 2.8 dB with 3 and 1 antenna respectively (i.e., only RAKE temporal processing). Figure 6 shows the beam radiation pattern after the optimization. The arrows in the graph represent the angles associated with the different users while their amplitude is approximately proportional to the strength of the corresponding user. In general the beamformer of strong users attempts at putting nulls in correspondence of weak users while weak users aim at maximizing the energy in their own direction. Finally figure 6 shows the output SINR after the optimization versus the post-correlation SNR. The SINR improvement with respect to pure RAKE processing is larger as the SNR increases. This is due to the zero-forcing nature of the optimization algorithm.

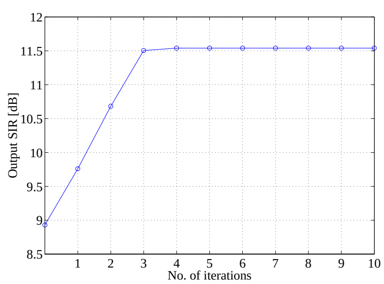

Figure 2: Optimum SIR (dB) convergence vs. no. of iterations

7. CONCLUSIONS

We addressed the problem of SINR maximization at the mobile stations by performing spatial filtering at the base-station and emplying a RAKE receiver at the mobile stations. It is shown that in cases where spatio-temporal processing cannot be employed to precancel the interference, significant performance gains can still be achieved by spatial filtering only. The case of complex spreading was treated along with that of real spreading sequences. The behavior of these two cases is slightly different, leading to an extra term for the signal part in the real codes' case. An algorithm for power allocation and spatial filter optimization was also presented.

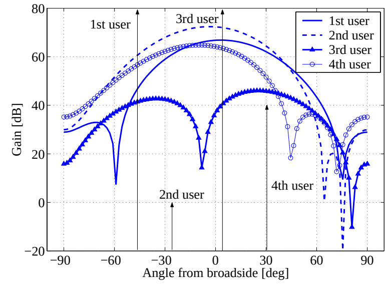

Figure 3: Radiation patterns after optimization

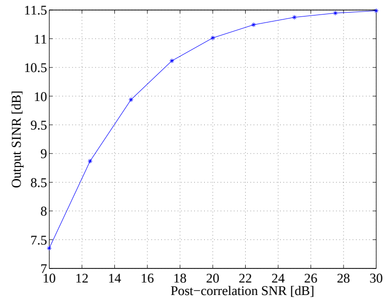

Figure 4: Optimum output SINR vs. input SNR

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