Blind Decentralized Projection Receiver for Asynchronous CDMA in Multipath Channels*

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Abstract

An asynchronous Direct Sequence Code Division Multiple Access (DS-CDMA) system employing periodic spreading sequences is considered to be operating in a frequency selective channel. The cyclostationary spread signal is received at multiple sensors and/or is sampled multiple times per chip (oversampling), leading to a stationary vector-valued received signal. Hence, such a model represents a very particular multi-input multi-output (MIMO) system with plentiful side information in terms of distinct spreading waveforms for the input signals. Depending upon the finite impulse response (FIR) length of the propagation channel, and the processing gain, the channel of a certain user spans a certain number of symbol periods, thus inducing memory or intersymbol interference (ISI) in the received signal in addition to the multiple-access interference (MAI) contributed by concurrent users. The desired user's multipath channel estimate is obtained by means of a new *blind* technique which exploits the spreading sequence of the user and the second-order statistics of the received signal. The blind *Minimum Mean Square Error-Zero Forcing* (MMSE-ZF) receiver or *projection* receiver is subsequently obtained. This receiver represents the proper generalization of the *anchored* MOE receiver [1] to the asynchronous case with delay spread. Classification of linear receivers obtained by various criteria is provided and the MMSE-ZF receiver is shown to be obtainable in a decentralized fashion by proper implementation of the unbiased minimum output energy (MOE) receiver, leading to the minimum variance distortionless response (MVDR) receiver for the signal of the desired user. This MVDR receiver is then adapted blindly by applying Capon's principle. A channel impulse response is obtained as a by-product. Lower bounds on the receiver filter length are derived, giving a measure of the ISI and MAI tolerable by the receiver and ensuring its identifiability.

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Récepteur Aveugle Décentralisé pour les Systèmes à Accès Multiple par Repartition de Codes (AMRC) Asynchrones pour des Canaux à Trajets Multiples

Résumé

Dans cette article, nous traitons du problème de l'annulation d'interférences dans un système à Accès Multiple par Répartition de Codes (AMRC) par séquences directes, pour des canaux sélectifs en fréquence et des conditions asynchrones. Le signal étalé cyclostationnaire est reçu par des antennes multiples ou est sur-échantillonné par rapport à la période chip. Il est donc converti en un signal vectoriel qui est stationnaire. Le sur-échantillonnage ou/et les antennes multiples donnent naissance à un système multi-entrées multi-sorties (MIMO) qui est abondant en connaissances *a priori*, dans la forme des séquences d'étalement distinctes pour les signaux d'entrée (utilisateurs). En fonction de la longueur du cananl proprement dit, qui est à réponse impulsionnelle finie (RIF), et au facteur d'étalement, l'ordre du canal total d'un utilisateur s'étale sur un certain nombre de symboles, engendrant la mémoire ou l'interférence entre symboles (ISI). L'interférence d'accès multiples (MAI) est ajoutée par les utilisateurs concurents. L'estimation du canal de l'utilisateur donné est obtenue par une nouvelle technique aveugle qui utilise la séquence d'étalement de l'utilisateur et les statistiques de l'ordre deux du signal reçu. En conséquence, le récepteur minimisant l'erreur quadratique moyenne-forçage à zéro (MMSE-ZF), est obtenu. Ce récepteur réprésente la généralisation au cas asynchrone et pour les canuax à trajets multiples, du récepteur ancré, celui-là même qui minimise l'énergie à la sortie [1].

Nous classifions les différents récepteurs linéaires obtenus par des critères diverses et montrons que le récepteur MMSE-ZF peut être déterminé d'une manière décentralisée en appliquant le critère MOE non-biaisé. Cela nous donne ainsi le récepteur à réponse sans distortion à minimum de variance (MVDR) pour le signal de l'utilisateur donné. Le récepteur MVDR est ensuite adapté en aveugle en appliquant le principe de Capon. Cela nous donne, par ailleurs, l'estimation de la réponse impulsionnelle du canal. Ainsi, nous dérivons les bornes inférieures sur la longueur du filtre de réception, obtenant alors, des mesures de l'ISI et du MAI tolérables par le récepteur.

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1 Introduction

Multiuser detection [2] has been the focus of intense academic research during the last decade, following the advent of DS-CDMA systems as potential candidates for wireless cellular networks. Capacity of wireless systems is mostly interference limited. For classical FDMA/TDMA systems, this is mostly intercell (cochannel) interference. For CDMA systems equipped with the conventionally used matched filter detector, the interference consists mainly of intracell cochannel interference. The goal of multiuser detection is to mitigate the influence of interference. Industrial interest in multiuser detection has also struck a strident note with the emergence of the major third generation proposals for cellular mobile communications based upon wideband DS-CDMA, some of which, e.g. [3] contain the necessary provisions for the application of multiuser algorithms.

For communications over an additive white Gaussian noise (AWGN) channel, and synchronous DS-CDMA users, transmission of mutually orthogonal waveforms for all users results in a perfectly orthogonal system. A conventional matched filter receiver, matched to the desired user's spreading sequence results in automatic interference rejection. This behavior of the matched filter persists irrespective of the powers of interferers due to the orthogonality of the modulation scheme. However, any diversion from this ideal system, e.g., choice of non-orthogonal spreading codes, deviation of the TX-RX filters from a Nyquist pulse, non-optimality of the RX timing, mutual asynchrony of users, multipath propagation, or a combination of these phenomena results in a non-zero interference term at the output of the matched filter. Now, the relative powers of interfering users have a significant impact on this term, thus giving rise to the much dreaded *near-far* problem [2]. When powers can be perfectly controlled [4], then, under asynchronous conditions in an AWGN channel, the matched filter receiver is still an optimal decentralized receiver from the average signal to interference plus noise (SINR) maximization point of view, if aperiodic (noise-like) spreading sequences spread successive symbols of users. This behavior of the matched filter is explained by the nature of PN interference from other users (cyclostationary with chip period, hence stationary after chip rate sampling) which essentially acts much the same way as uncorrelated channel noise. Consequently, the performance might still be acceptable yielding a reasonable bit-error rate if the number of users is much lesser than the processing gain (small loading fraction, yielding far lower capacity than an orthogonal system). In a multipath channel, the matched filter is the coherent RAKE receiver [4], which is matched to the cascade of the spreading sequence and the propagation channel, thus combining the delayed multipath signals coherently. The noise-like nature of the interfering users persists at the RAKE output, but now, the phenomenon of dimensional crowding creeps in, since each interferer's delayed multipath component contributes as an extra interference.

Most studies [5] (and references therein) show that the RAKE easily becomes interference limited for moderate loading fractions.

The search for near-far resistant multiuser detectors debuted with [6] where the optimum detector for asynchronous multiple-access Gaussian channels was presented. However, owing to the huge complexity (exponential in number of users) of the optimum detector, it was considered to be impractical in heavily loaded systems. Simpler, linear multiuser detectors (complexity linear in number of users) like the decorrelating detector and the minimum mean square error (MMSE) detector were proposed in [7] [8] and [9] respectively. The desirable feature of these detectors is that they are near-far resistant to varying degrees [2]. However, when these detectors are formulated as linear operators on the RAKE outputs for the multiple users (which are sufficient statistics if the rest of the signal is AWGN), then the *information complexity* of these detectors is fundamentally the same as for the optimum detector, i.e., knowledge of parameters like delays, spreading sequences and received powers (for the MMSE detector) is needed for the implementation of these algorithms. We shall classify these receivers as *joint multiuser detectors*, where one attempts to jointly detect the symbols of all users. In order to estimate the parameters like arrival delays, path amplitudes, and phases, some training information will necessarily be required for all users.

Another breakthrough step in multiuser detection, following developments in the field of *blind* channel identification and detection [10], was the introduction of the *blind adaptive multiuser detector* [1], where it was shown that the multiuser problem could be cast in a single user *decentralized* framework, thus enabling multiple access interference cancellation based on single user information (desired user delay and spreading sequence). In this framework, the linear receiver operates directly on the received signal. The receiver in [1] is the so-called *anchored* minimum-output energy (MOE) receiver. The anchored receiver is split into two components - one fixed, and proportional to the desired user's signature waveform (matched filter receiver), while the other, its orthogonal complement. The algorithm constrains the inner product of the received signal with the desired user spreading sequence to be fixed, thus restricting the optimization problem to within the constrained space. No effort is made to exploit the structure of the MAI except for the assumption of it being uncorrelated with the desired signal. A decentralized scheme of this nature can evidently be of considerable interest in some applications, like at the mobile terminal in a cellular network, where knowledge of interferer parameters is not readily available, or as a suboptimal/initialization approach at the base station. Blind adaptive multiuser detectors based on second-order statistics (non-decision directed/ data-aided) are developed for the case of short/periodic spreading sequences, leading to cyclostationarity at symbol period.

The problem addressed in [1] was that of DS-CDMA communications over a flat channel (no delay spread) [11]. A constrained optimization scheme was proposed in [12] for multipath channels by forcing to zero the receiver response to all but one of the multipath components. An immediate performance loss was noticeable resulting from the rejection of a major part of the desired signal energy contained in the other paths. This signal cancellation effect was alleviated in [13], where the receiver's output energy was minimized subject to a fixed response constraint for the desired signal. Connections with the *Capon* philosophy were drawn in that paper. The above mentioned receivers can be shown to converge asymptotically (SNR$\rightarrow \infty$) to the zero-forcing (ZF) or the decorrelating solution. It was shown in [14] that in order to accommodate a number of users approaching the code space dimension (spreading factor), longer receivers are required for the ZF solution to be achievable. Moreover, we presented in [14] the optimal MMSE receiver for multipath channels and asynchronous conditions, obtained by applying multichannel linear prediction to the received cyclostationary signal. Direct estimation of the MMSE receiver from spreading sequence properties and the noise subspace was introduced in [15] following the observation that the MMSE receiver vector dwells in the signal subspace. The ZF and the MMSE detectors in the case of high data-rate systems in dispersive channels inducing significant ISI, were investigated in [16]. The channel estimate in this work was obtained as a generalization to longer delay spreads of the subspace technique originally proposed in [17]. Both these schemes, however, evoke a high computational complexity since a subspace decomposition is required. It is worth mentioning that in the context of blind methods based on second-order statistics and spatio-temporal processing techniques [18], direct sequence CDMA systems allow quite robust channel estimation (compared to TDMA systems) due to the bandwidth expansion and integrated *a priori* knowledge and structure in terms of distinct spreading sequences that enables separation of user signals.

We propose, in this work, a new decentralized blind minimum mean-square error zero-forcing (MMSE-ZF) receiver for DS-CDMA systems in multipath channels. The receiver is MMSE-ZF in the sense that among all ZF receivers, it is the one that minimizes the mean-square error. The MMSE-ZF receiver is also called the projection receiver [19] or the decorrelating detector [7]. This blind receiver exploits spreading sequence properties in conjunction with the second-order statistics of the received signal to estimate the FIR channel for the desired user at a low cost. The delay spread is assumed to be possibly more than a symbol period, and can be different for different users.

The rest of the paper is organized as follows. We present the DS-CDMA signal model in the section 2. In section 3, the non-blind MMSE-ZF receiver is derived and its interpretation in terms of existing methods is provided. Section 4 lays the groundwork for the blind MMSE-ZF receiver by providing analogies between the interference cancellation problem and some related results from the array processing literature. Section 5 is dedicated to the derivation of the blind MMSE-ZF receiver through an alternate and simple method, namely the unbiased minimum output energy (MOE) criterion. Blind channel estimation via the blind MMSE-ZF algorithm is also discussed. An alternate interpretation of the MMSE-ZF receiver as a Generalized Sidelobe Canceler (GSC) is also discussed. Finally, simulation examples are presented in section 6 and concluding remarks in section 7.

2 Multiuser Data Model

Fig. 1 shows the baseband signal model. The K users are assumed to transmit linearly modulated signals over a linear multipath channel with additive Gaussian noise. It is assumed that the receiver employs M sensors to receive the mixture of signals from all users. The receiver front-end is an anti-aliasing low-pass filter. The continuous-time signal received at the mth sensor can be written in baseband notation

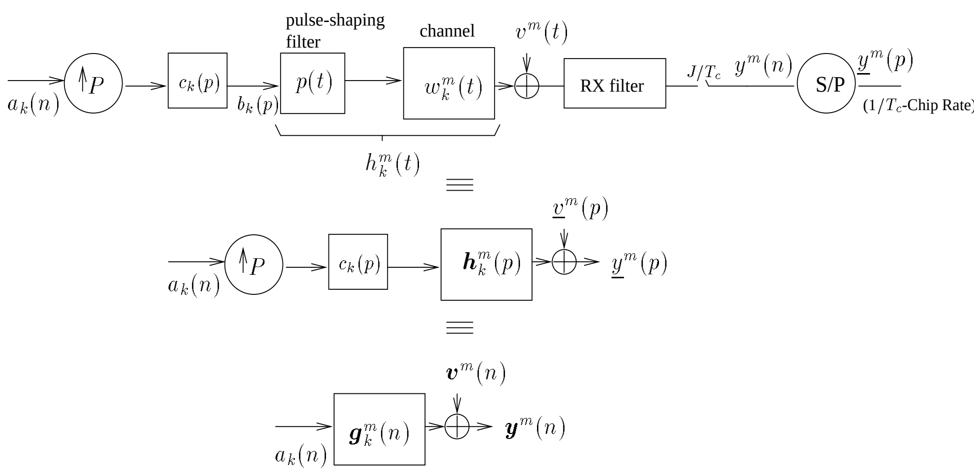

Figure 1: Signal model in continuous and discrete time, showing only the contribution from one user.

as

$$y^m(t) = \sum_{k=1}^{K} \sum_{n} a_k(n) g_k^m(t - nT) + v^m(t), \tag{1}$$

where the $a_k(n)$ are the transmitted symbols from user k, T is the common symbol period, $g_k^m(t)$ is the overall channel impulse response (including the spreading sequence, and the transmit and receive filters) for the kth user's signal at the mth sensor, and $\{v^m(t)\}$ is the complex circularly symmetric AWGN with power spectral density N_0. Assuming the $\{a_k(n)\}$ and $\{v^m(t)\}$ to be jointly wide-sense stationary, the process $\{y^m(t)\}$ is wide-sense cyclostationary with period T. The overall channel impulse response $g_k^m(t)$, is the convolution of the spreading code c_k and $h_k^m(t)$, itself the convolution of the chip pulse shape, the receiver filter, and the actual channel representing the multipath environment. This can be expressed as

$$g_k^m(t) = \sum_{p=0}^{P-1} c_k(p) h_k^m(t - pT_c), \tag{2}$$

where T_c is the chip duration. The symbol and chip periods are related through the processing gain/spreading factor P: $T = PT_c$. S/P in fig 1 denotes serial to parallel conversion (vectorization) with downsampling of a factor J. Sampling the received signal at J (oversampling factor) times the chip rate, we obtain the wide-sense stationary $PJ \times 1$ vector signal $\boldsymbol{y}^m(n)$ at the symbol rate. It is to be noted that the over-sampling aspect (with respect to the symbol rate) is inherent to DS-CDMA systems by their very nature, due to the large (extra) bandwidth and the need to acquire chip-level resolution. This aspect directly translates into temporal diversity and explains the interference cancellation capability of these systems.

We consider the channel delay spread between the kth user and all of the M sensors to be of length $l_k T_c$. Let $n_k \in \{0, 1, \cdots P-1\}$ be the chip-delay index for the kth user: $\boldsymbol{h}_k^m(n_k)$ is the first non-zero $J \times 1$ chip-rate sample of $\boldsymbol{h}_k^m(p)$. Let us denote by N_k, the FIR duration of $g_k^m(t)$ in symbol periods. It is a function of l_k, n_k, and P. We nominate the user 1 as the user of interest and assume that $n_1 = 0$ (synchronization to user 1). The symbol sequences for other users are relabelled (delayed or advanced), so that their relative delay with respect to user 1 falls in $[0, T)$.

Let $N = \sum_{k=1}^{K} N_k$. The vectorized oversampled signals at M sensors lead to a discrete-time $PMJ \times 1$ vector signal at the symbol rate that can be expressed as

$$\begin{aligned} \boldsymbol{y}(n) &= \sum_{k=1}^{K} \sum_{i=0}^{N_k-1} \boldsymbol{g}_k(i) a_k(n-i) + \boldsymbol{v}(n) \\ &= \sum_{k=1}^{K} \boldsymbol{G}_{k,N_k} A_{k,N_k}(n) + \boldsymbol{v}(n) = \boldsymbol{G}_N \boldsymbol{A}_N(n) + \boldsymbol{v}(n), \end{aligned} \tag{3}$$

$$\boldsymbol{y}(n) = \begin{bmatrix} \boldsymbol{y}_1(n) \\ \vdots \\ \boldsymbol{y}_P(n) \end{bmatrix}, \boldsymbol{y}_p(n) = \begin{bmatrix} \boldsymbol{y}_p^1(n) \\ \vdots \\ \boldsymbol{y}_p^M(n) \end{bmatrix}, \boldsymbol{y}_p^m(n) = \begin{bmatrix} y_{p,1}^m(n) \\ \vdots \\ y_{p,J}^m(n) \end{bmatrix}$$

$$\begin{aligned} \boldsymbol{G}_{k,N_k} &= [\boldsymbol{g}_k(N_k-1) \ldots \boldsymbol{g}_k(0)] \ , \boldsymbol{G}_N = [\boldsymbol{G}_{1,N_1} \ldots \boldsymbol{G}_{K,N_K}] \\ A_{k,N_k}(n) &= [a_k(n-N_k+1) \ldots a_k(n)]^T \ , \boldsymbol{A}_N(n) = \left[A_{1,N_1}^T(n) \ldots A_{K,N_K}^T(n)\right]^T, \end{aligned} \tag{4}$$

and the superscript ^{T} denotes transpose. For the user of interest (user 1), $\boldsymbol{g}_1(i) = (\boldsymbol{C}_1(i) \otimes I_{MJ})\, \boldsymbol{h}_1$, where, $\boldsymbol{h}_1$ is the $l_1 MJ \times 1$ propagation channel vector given by

$$\boldsymbol{h}_1 = \begin{bmatrix} \boldsymbol{h}_{1,1} \\ \vdots \\ \boldsymbol{h}_{1,l_1} \end{bmatrix}, \boldsymbol{h}_{1,l} = \begin{bmatrix} \boldsymbol{h}_{1,l}^1 \\ \vdots \\ \boldsymbol{h}_{1,l}^M \end{bmatrix}, \boldsymbol{h}_{1,l}^m = \begin{bmatrix} h_{1,l}^m(1) \\ \vdots \\ h_{1,l}^m(J) \end{bmatrix},$$

$\otimes$ denotes the Kronecker product, and the Toeplitz matrices $\boldsymbol{C}_1(i)$ are shown in fig. 2, where the band consists of the spreading code $(c_0 \cdots c_{P-1})^T$ shifted successively to the right and down by one position. For the interfering users, we have a similar setup except that owing to asynchrony, the band in fig. 2 is

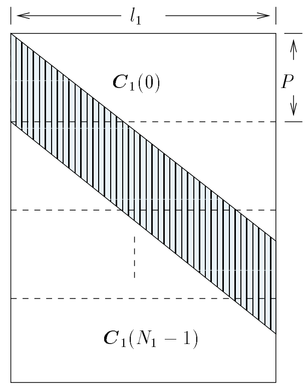

Figure 2: The Code Convolution Matrix $\boldsymbol{C}_1$.

shifted down n_k chip periods and is no longer coincident with the top left edge of the box. We denote by $\boldsymbol{C}_1$, the concatenation of the code matrices given above for user 1: $\boldsymbol{C}_1 = [\boldsymbol{C}_1^T(0) \cdots \boldsymbol{C}_1^T(N_1-1)]^T$.

It is clear that the signal model above addresses a multiuser setup suitable for joint interference cancellation provided the timing information and spreading codes of all sources are available. As we shall see in the following, it is possible to decompose the problem into single user ones, thus making the implementation suitable for decentralized applications such as at mobile terminals or as a suboptimal processing or initialization stage at the base station. To this end, let us stack L successive $\boldsymbol{y}(n)$ vectors in a super vector

$$\boldsymbol{Y}_L(n) = \mathcal{T}_L(\boldsymbol{G}_N)\boldsymbol{A}_{N+K(L-1)}(n) + \boldsymbol{V}_L(n), \tag{5}$$

where, $\mathcal{T}_L(\boldsymbol{G}_N) = [\mathcal{T}_L(\boldsymbol{G}_{1,N_1}) \ \cdots \ \mathcal{T}_L(\boldsymbol{G}_{K,N_K})]$ and $\mathcal{T}_L(\boldsymbol{x})$ is a banded block Toeplitz matrix with L block rows and $\begin{bmatrix}\boldsymbol{x} & \boldsymbol{0}_{p\times(L-1)}\end{bmatrix}$ as first block row (p is the number of rows in $\boldsymbol{x}$), and $\boldsymbol{A}_{N+K(L-1)}(n)$ is the concatenation of user data vectors ordered as $\left[A_{1,N_1+L-1}^T(n), A_{2,N_2+L-1}^T(n) \cdots A_{K,N_K+L-1}^T(n)\right]^T$. We shall refer to $\mathcal{T}_L(\boldsymbol{G}_{k,N_k})$ as the channel convolution matrix for the kth user. Consider the noiseless

received signal shown in fig. 3 for the contribution of user 1, from which the following observations can be made. Due to the limited delay spread, the effect of a particular symbol, $a_1(n-d)$, influences N_1 symbol periods, rendering the channel a moving average (MA) process of order $N_1 - 1$ [20]. We are interested in estimating the symbol $a_1(n-d)$ from the received data vector $\boldsymbol{Y}_L(n)$. One can notice that $a_1(n-d)$ appears in the portion $\boldsymbol{Y}_{N_1}$ of $\boldsymbol{Y}_L(n)$. The shaded triangles constitute the ISI, i.e., the effect of neighboring symbols on $\boldsymbol{Y}_{N_1}$. The contributions from the other (interfering) users to the received data

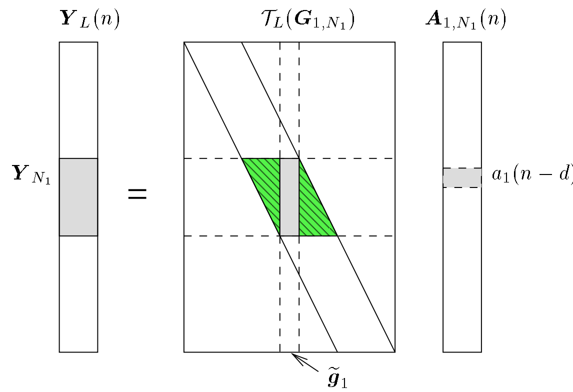

Figure 3: ISI for the desired user.

vector have a similar structure. Note that to handle ISI and MAI, it may be advantageous to consider the longer received data vector $\boldsymbol{Y}_L(n)$.

3 The MMSE-ZF/Projection Receiver

In the multiuser problem given in (5), there exists a multitude of possible zero-forcing constraints, ranging from zero MAI only, or zero ISI only, to zero forcing for both MAI and ISI, which we shall consider here. For the purpose of our problem, let us consider the ZF or the zero-distortion constraint, which can be written as,

$$\boldsymbol{F}^H \mathcal{T}(\boldsymbol{G}_N) = \boldsymbol{e}_d^T, \tag{6}$$

where, $\boldsymbol{e}_d^T = [0\cdots 0|\overbrace{0\cdots 0}^{d}\ 1\ 0\cdots 0|0\cdots 0]$, with d the "equalization" delay for the desired user.

Considering all user symbols $a_k(n)$ to be uncorrelated, the received signal covariance matrix can be written as $\boldsymbol{R}_{YY} = \sigma_a^2 \mathcal{T}\mathcal{T}^H + \sigma_v^2 \boldsymbol{I}$, where $\mathcal{T}$ replaces $\mathcal{T}(\boldsymbol{G}_N)$ to simplify the notation. The MMSE-ZF receiver is by definition the solution to the MMSE criterion under the ZF constraint, which can be written as

$$\begin{aligned} \min_{F:F^H\mathcal{T}=e_d^T} \boldsymbol{F}^H \boldsymbol{R}_{YY} \boldsymbol{F} &= \sigma_a^2 + \min_{F:F^H\mathcal{T}=e_d^T} \boldsymbol{F}^H \boldsymbol{R}_{VV} \boldsymbol{F} \\ &\Rightarrow \min_{F:F^H\mathcal{T}=e_d^T} \boldsymbol{F}^H \boldsymbol{F} \end{aligned} \tag{7}$$

Let us further express the receiver vector $\boldsymbol{F}$ as

$$\boldsymbol{F} = \mathcal{T}\boldsymbol{F}_1 + \mathcal{T}^\perp \boldsymbol{F}_2, \tag{8}$$

where, $\mathcal{T}^\perp$ spans the orthogonal complement of $\mathcal{T}$ and satisfies $P_{\mathcal{T}^\perp} = P_{\mathcal{T}}^\perp$, where $P_X = X(X^HX)^{-1}X^H$ is the projection operator onto the column space of the matrix X. From the ZF constraint, $\boldsymbol{F}^H\mathcal{T} = \boldsymbol{e}_d^T =$

$F_1^H \mathcal{T}^H \mathcal{T}$, and therefore,

$$F_1 = (\mathcal{T}^H \mathcal{T})^{-1} e_d. \tag{9}$$

Hence, $F = \mathcal{T}(\mathcal{T}^H \mathcal{T})^{-1} e_d + \mathcal{T}^{\perp} F_2$, where F_2 is the unconstrained part which becomes zero upon solving the minimization problem in (7). Thus $F = \mathcal{T}(\mathcal{T}^H \mathcal{T})^{-1} e_d$, and we can write the MMSE-ZF criterion as:

$$\min_{F:F^H \mathcal{T} = e_d^T} F^H R_{YY} F = \sigma_a^2 + \sigma_v^2 e_d^T \left(\mathcal{T}^H \mathcal{T}\right)^{-1} e_d. \tag{10}$$

The ZF solution in the noiseless case gives the distortionless response for the desired user's signal.

We can provide one more interpretation of the MMSE-ZF receiver in terms of a projection receiver as indicated in the following proposition.

Proposition 1: The MMSE-ZF receiver is equivalent to a projection receiver [19] that first projects the received data onto the orthogonal complement of the subspace spanned by ISI and MAI, and then projects the resulting vector onto a one-dimensional subspace that is matched to the signal part that remains in the data.

Proof: See appendix A.

The MMSE-ZF receiver derived above needs the knowledge of the channel convolution matrix (arrival delays and impulse responses of all user channels) for its implementation. However, as we shall see in the sequel, it is possible to determine this receiver blindly in a decentralized fashion, as a solution of Capon's method applied to the minimum variance distortionless response (MVDR) criterion.

4 Linearly Constrained Minimum Variance Beamforming

It is insightful to compare the problem of blind ISI and MAI rejection to that of beamforming and direction of arrival (DOA) estimation in the antenna array processing literature [21]. Let us look at a generic DOA estimation problem of a single narrowband source located at an angle θ_0 with respect to an antenna array. The observation or snapshot vector $Y(n)$ at the array output is

$$Y(n) = S(\theta_0) a(n) + V(n), \tag{11}$$

with $S(\theta_0)$ being the *array response* or *steering* vector associated with the look-direction θ_0, and $V(n)$ the complex additive (spatially) white Gaussian noise (AWGN) vector. $a(n)$ is the sampled source signal, with variance σ_a^2. In this problem there are two unknowns, namely the direction of arrival θ_0 (and the corresponding steering vector) and the source signal $a(n)$. In the first instance, we shall consider θ_0 known. A beamformer with the weight vector $\boldsymbol{F}$ is employed to obtain the estimate $\hat{a}(n) = \boldsymbol{F}^H \boldsymbol{Y}(n)$, where the superscript ^{H} stands for Hermitian transpose. Intuitively, any desirable beamformer should emphasize signals arriving from the direction θ_0, while the noise must be suppressed. We therefore impose the zero-distortion constraint, $\boldsymbol{F}^H \boldsymbol{S}(\theta_0) = 1$, on the beamformer, and minimize its output variance $E|\boldsymbol{F}^H \boldsymbol{Y}(n)|^2$ subject to this constraint. The weight vector of the linearly constrained minimum variance (LCMV) beamformer is the solution to the problem

$$\min_{F:F^H S(\theta_0)=1} E|\hat{a}_k|^2 \leftrightarrow \min_{F:F^H S(\theta_0)=1} \boldsymbol{F}^H \boldsymbol{R}_{YY} \boldsymbol{F} \ = \ \mathrm{MV}, \tag{12}$$

which results in

$$\boldsymbol{F} = \frac{1}{\boldsymbol{S}^H(\theta_0)\boldsymbol{R}_{YY}^{-1}\boldsymbol{S}(\theta_0)} \boldsymbol{R}_{YY}^{-1} \boldsymbol{S}(\theta_0), \ \ \mathrm{MV} = \left(\boldsymbol{S}^H(\theta_0)\boldsymbol{R}_{YY}^{-1}\boldsymbol{S}(\theta_0)\right)^{-1}. \tag{13}$$

At this point we realize that we do not yet know θ_0. However, we can obtain θ_0 by *Capon's* method [22] as the argument of the maximum of the minimum variance over all possible look directions. Thus,

$$\begin{aligned} \widehat{\theta} &= \arg\max_{\theta} \left(\boldsymbol{S}^H(\theta)\boldsymbol{R}_{YY}^{-1}\boldsymbol{S}(\theta)\right)^{-1} = \arg\min_{\theta} \boldsymbol{S}^H(\theta)\boldsymbol{R}_{YY}^{-1}\boldsymbol{S}(\theta) \\ &= \boldsymbol{S}^{-1}\left(V_{\max}(\boldsymbol{R}_{YY})\right) = \boldsymbol{S}^{-1}\left(\boldsymbol{S}(\theta_0)\right) = \theta_0, \end{aligned} \tag{14}$$

since $\boldsymbol{R}_{YY} = \sigma_a^2 \boldsymbol{S}(\theta_0)\boldsymbol{S}^H(\theta_0) + \sigma_v^2 \boldsymbol{I}$, and assuming a proper normalization of $\boldsymbol{S}(\theta)$. We denote by $V_{\max}(\boldsymbol{R}_{YY})$, the eigenvector of $\boldsymbol{R}_{YY}$ associated with the maximum eigenvalue.

Note that Capon's approach could be extended to the multisource case if the sources are uncorrelated and if they are treated jointly. Here, we shall stick to the decentralized single source formulation of the Capon's method. The rest of the developments in this paper are based upon the striking similarity between the purely spatial (beamforming) problem discussed above, and the ISI and MAI cancellation issue depicted in fig. 3. In particular, we show in the sequel that in the DS-CDMA problem, given certain conditions on the number of concurrent users and their channel orders, partial knowledge of the channel vectors, $\boldsymbol{g}_k(i)$'s in terms of distinct spreading code matrices, $\boldsymbol{C}_k(i)$'s, leads to an unambiguous estimate of the channel vector for the desired user.

5 Connections between Linear Receivers

We can classify the unbiased linear MOE[1] receiver in terms of the other optimization criteria as indicated in the following proposition.

[1] a derivative of the minimum variance distortionless response (MVDR) method, and a particular instance of the linearly constrained minimum-variance (LCMV) criterion

Proposition 2: The minimum mean-squared error (MMSE), and the minimum output energy (MOE) are interchangeable criteria under the unbiased constraint, and are equivalent to the maximization of the output SINR.

$$\arg \min_{F:F^H\tilde{g}_1=1} \mathrm{MSE}_{\mathrm{unbiased}} = \arg \min_{F:F^H\tilde{g}_1=1} \mathrm{OE} = \arg \max_{F} \ \mathrm{SINR}, \tag{15}$$

Proof: (i) Consider first, the MMSE criterion

$$\begin{aligned} \mathrm{MSE} &= E|a_{1,n-d} - \hat{a}_1|^2 = E|a_{1,n-d} - \boldsymbol{F}^H\boldsymbol{Y}|^2 \\ &= \sigma_a^2 - \sigma_a^2\boldsymbol{F}^H\tilde{\boldsymbol{g}}_1 - \sigma_a^2\tilde{\boldsymbol{g}}_1^H\boldsymbol{F} + \underbrace{\boldsymbol{F}^H\boldsymbol{R}_{YY}\boldsymbol{F}}_{\text{output energy}} \\ \Rightarrow \min_{F^H\tilde{g}_1=1} \mathrm{MSE} &= \text{unbiased MOE} \end{aligned}, \tag{16}$$

proving the first equality in (15).

(ii) The signal part in $\boldsymbol{Y}_L(n)$ is $\boldsymbol{Y}_{\mathrm{s}} = \tilde{\boldsymbol{g}}_1 a_{1,n-d}$, whereas the interference (MAI & ISI) plus noise is $\boldsymbol{Y}_{\mathrm{in}} = \overline{\mathcal{T}}_L\bar{\boldsymbol{A}} + \boldsymbol{V}_L$, where, $\overline{\mathcal{T}}_L$ is the same as $\mathcal{T}_L(G_N)$ with the column $\tilde{\boldsymbol{g}}_1$ removed. Then, for an arbitrary $\boldsymbol{F}$, assuming uncorrelated symbols, we obtain,

$$\mathrm{SINR} = \frac{\boldsymbol{F}^H\boldsymbol{R}_{\mathrm{s}}\boldsymbol{F}}{\boldsymbol{F}^H\boldsymbol{R}_{\mathrm{in}}\boldsymbol{F}} = \frac{\sigma_a^2\boldsymbol{F}^H\tilde{\boldsymbol{g}}_1\tilde{\boldsymbol{g}}_1^H\boldsymbol{F}}{\boldsymbol{F}^H\left(\boldsymbol{R}_{YY} - \sigma_a^2\tilde{\boldsymbol{g}}_1\tilde{\boldsymbol{g}}_1^H\right)\boldsymbol{F}}, \tag{17}$$

from where,

$$\begin{aligned} &\max_F \ \mathrm{SINR} \leftrightarrow \min_F \ \mathrm{SINR}^{-1} \leftrightarrow \ \min_F \ \frac{\boldsymbol{F}^H\boldsymbol{R}_{YY}\boldsymbol{F}}{\sigma_a^2|\boldsymbol{F}^H\tilde{\boldsymbol{g}}_1|^2} \\ &\Rightarrow \min_{F:F^H\tilde{g}_1=1} \boldsymbol{F}^H\boldsymbol{R}_{YY}\boldsymbol{F}, \end{aligned} \tag{18}$$

which is the unbiased MOE criterion of (19). □

5.1 Relationships between Various Constraints

At this juncture, we are able to identify the relationship between the unbiased linear MOE and the unbiased linear MMSE approaches which give the same receiver filter $\boldsymbol{F}$. Note that the unbiased MMSE yields the MMSE-ZF in the noiseless case and so does the unbiased MOE. A further observation is that the unbiasedness constraint is not the distortionless constraint (the ZF constraint) given by (6). It is only the ZF (distortionless) constraint which guarantees the minimum variance (σ_a^2) with a fixed response for the desired user signal, which is the desired goal in the original MVDR approach.

In [13], the authors interpreted zero distortion of the Capon's method as unbiasedness, and maximized the MOE to obtain the channel impulse response for the desired user. It was also shown that the distinct spreading sequences allowed identifiability of users' channel responses. However, unbiasedness is a weaker constraint as compared to the zero distortion constraint. Intuitively, with only the unbiasedness constraint, the other symbols (ISI) remain present in the estimator output and do not allow the application of the single user form of Capon's principle, which corresponds to the maximization of the MOE (minimum variance) under the zero-distortion constraint. However, unbiased MOE on noiseless data corresponds to the MMSE-ZF, which in turn is equivalent to zero forcing MOE on noiseless data. Hence, in conclusion, we can determine the MMSE-ZF receiver by applying the unbiased MOE on denoised data, leading to a simple treatment of the problem.

5.2 Blind Unbiased Linear MOE Receiver

Suppose that $\boldsymbol{F}$ is a linear FIR receiver applied to the received data, $\boldsymbol{Y}_L(n)$. The goal is to obtain a linear estimate of the transmitted symbol, $a_1(n-d)$ for the desired user symbol (with a possible delay of d symbols). Then, $\hat{a}_1(n-d) = \boldsymbol{F}^H\boldsymbol{Y}_L(n)$ is the linear estimate of the desired symbol. Finite alphabet information can later be applied to the this estimate to determine the symbol value. $\boldsymbol{F}$ is said to be *unbiased* if $\boldsymbol{F}^H\widetilde{\boldsymbol{g}}_1 = 1$, where, $\widetilde{\boldsymbol{g}}_1 = \boldsymbol{T}_1^H\boldsymbol{h}_1$ (see fig. 3), with $\boldsymbol{T}_1 = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{C}_1^H & \boldsymbol{0} \end{bmatrix} \otimes \boldsymbol{I}_{MJ}$ being the signature matrix for the desired user. $\widetilde{\boldsymbol{g}}_1 a_1(n-d)$ is the contribution of $a_1(n-d)$ to $\boldsymbol{Y}_L(n)$. The energy at the output of the receiver (noiseless case) can be written as $E|\boldsymbol{F}^H\boldsymbol{Y}_L(n)|^2 = \boldsymbol{F}^H\boldsymbol{R}^d_{YY}\boldsymbol{F}$, where the superscript ^{d} stands for noiseless or denoised data. The unbiased MOE criterion proposed in [13], which is a generalization of the instantaneous channel case of [1], is in principle a max/min problem solved in two steps with,

<u>step:1</u> *unbiased MOE*

$$\min_{F:F^H\widetilde{g}_1=1} \boldsymbol{F}^H\boldsymbol{R}^d_{YY}\boldsymbol{F} \Rightarrow \boldsymbol{F} = \frac{1}{\widetilde{\boldsymbol{g}}_1^H\boldsymbol{R}^{-d}_{YY}\widetilde{\boldsymbol{g}}_1}\boldsymbol{R}^{-d}_{YY}\widetilde{\boldsymbol{g}}_1, \tag{19}$$

with $\mathrm{MOE}(\hat{\boldsymbol{h}}_1) = \frac{1}{\widetilde{\boldsymbol{g}}_1^H\boldsymbol{R}^{-d}_{YY}\widetilde{\boldsymbol{g}}_1}$, followed by,

<u>step:2</u> *Capon's method*

$$\max_{\hat{\boldsymbol{h}}_1:||\hat{\boldsymbol{h}}_1||=1} \mathrm{MOE}(\hat{\boldsymbol{h}}_1) \Rightarrow \min_{\hat{\boldsymbol{h}}_1:||\hat{\boldsymbol{h}}_1||=1} \hat{\boldsymbol{h}}_1^H\left(\boldsymbol{T}_1\boldsymbol{R}^{-d}_{YY}\boldsymbol{T}_1^H\right)\hat{\boldsymbol{h}}_1, \tag{20}$$

from where, $\hat{\boldsymbol{h}}_1 = V_{min}(\boldsymbol{T}_1\boldsymbol{R}^{-d}_{YY}\boldsymbol{T}_1^H)$, which is the estimate (upto a scalar phase factor) of the desired user's FIR channel response.

5.3 Discussion

The minimum output energy (MOE) receiver, first proposed in [1] was developed for asynchronous users in the AWGN channel, where the channel is simply represented by a complex gain factor. The linear receiver $\boldsymbol{F}$ can be expressed as $\boldsymbol{F} = \boldsymbol{c}_1 + \boldsymbol{x}$, where, $\boldsymbol{c}_1$ is the fixed component, viz., the desired user's spreading sequence, and $\boldsymbol{x}$ is it orthogonal complement ($\boldsymbol{x}^H \boldsymbol{c}_1 = \boldsymbol{0}$), i.e., a blocking transformation for the desired signal. The fixed response, $\|c_1\| = 1$, (the *anchor*) constrains the desired signal's output variance to an arbitrary constant, which is determined by the channel gain, and the MOE criterion minimizes the output variance of the rest. The receiver is therefore determined blindly upto a scale factor. This receiver, upon scaling the output response to unity for the desired signal corresponds to the unbiased MMSE-ZF receiver of section 3, leading to a distortionless (ZF) response in the noiseless case. The extension to the multipath channels of this scheme is elaborated upon in [13]. However, in the later appraoch, the distortionless response (and thus the proper implementation of Capon's method) will only be guaranteed if denoised statistics were employed in the MOE cost function.

5.4 Unbiased MOE via the Generalized Sidelobe Canceler

The generalized sidelobe canceler (GSC) [21], is a particular implementation of the LCMV beamformer. Hence, the unbiased MOE criterion, which itself is a particular instance of the LCMV approach can be implemented in the GSC fashion as elucidated in the following. Let us denote by

$$\boldsymbol{T}_1 = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{C}_1^H & \boldsymbol{0} \end{bmatrix} \otimes \boldsymbol{I}_{MJ}, \text{ and } \quad \boldsymbol{T}_2 = \begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{C}_1^{\perp} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I} \end{bmatrix} \otimes \boldsymbol{I}_{MJ}, \tag{21}$$

the partial signature of the desired user and its orthogonal complement employed, respectively, in the upper and lower branches of the GSC, as shown in fig. 4. $\boldsymbol{C}_1^{\perp H}$ is the orthogonal complement of $\boldsymbol{C}_1$, the tall code matrix given in section 2 ($\boldsymbol{C}_1^{\perp}\boldsymbol{C}_1 = \boldsymbol{0}$). Then, $\boldsymbol{C}_1^H \boldsymbol{Y}_{N_1} = \boldsymbol{T}_1 \boldsymbol{Y}_L$ and the matrix $\boldsymbol{T}_2$ acts as a blocking transformation for all components of the signal of interest. Note that $P_{T_1^H} + P_{T_2^H} = \boldsymbol{I}$, where, P_X is the projection operator (projection on the column space of $\boldsymbol{X}$). Then the LCMV problem can be written as

$$\min_{F : F^H T_1^H = (h_1^H h_1)^{-1} h_1^H} \boldsymbol{F}^H \boldsymbol{R}_{YY}^d \boldsymbol{F} = \min_{\substack{F \,:\, F^H T_1^H h_1 = 1 \\ F^H T_1^H h_1^{\perp} = 0}} \boldsymbol{F}^H \boldsymbol{R}_{YY}^d \boldsymbol{F}, \tag{22}$$

where, $\begin{bmatrix} \boldsymbol{h}_1 & \boldsymbol{h}_1^{\perp} \end{bmatrix}$ is a square non-singular matrix, and $\boldsymbol{h}_1^H \boldsymbol{h}_1^{\perp} = \boldsymbol{0}$. Note that in the LCMV problem (GSC fomulation) there is a number of constraints to be satisfied. However, imposing the second set of constraints, namely $\boldsymbol{F}^H \boldsymbol{T}_1^H \boldsymbol{h}_1^\perp = 0$ has no consequence because the criterion automatically leads to their satisfaction once, $\text{span}\{\boldsymbol{R}_{YY}^d\} \cap \text{span}\{\boldsymbol{T}_1^H\} = \text{span}\{\boldsymbol{T}_1^H \boldsymbol{h}_1\}$, i.e., when the intersection of the signal subspace and the subspace spanned by the columns of $\boldsymbol{T}_1^H$ is one dimensional.

The matrix $\boldsymbol{T}_1$ is nothing but a bank of correlators matched to the l_1 delayed multipath components of user 1's code sequence. Note that the main branch in fig. 4 by itself gives an unbiased response for the desired symbol, $a_1(n-d)$, and corresponds to the (normalized) coherent RAKE receiver. For the rest, we have an estimation problem, which can be solved in the least squares sense, for some matrix $\boldsymbol{Q}$. This interpretation of the GSC corresponds to the pre-combining (or pathwise) interference (ISI and MAI) canceling approach (see [5] and references therein).

The vector of estimation errors is given by

$$\boldsymbol{Z}(n) = [\boldsymbol{T}_1 - \boldsymbol{Q}\boldsymbol{T}_2]\,\boldsymbol{Y}_L(n). \tag{23}$$

Since the goal is to minimize the estimation error variances, or in other words, estimate the interference term in the upper branch as closely as possible from $\boldsymbol{T}_2\boldsymbol{Y}_L(n)$, the interference cancellation problem settles down to minimization of the trace of the estimation error covariance matrix $\boldsymbol{R}_{ZZ}$ for a matrix filter $\boldsymbol{Q}$, which results in

$$\boldsymbol{Q} = \left(\boldsymbol{T}_1 \boldsymbol{R}^d \boldsymbol{T}_2^H\right)\left(\boldsymbol{T}_2 \boldsymbol{R}^d \boldsymbol{T}_2^H\right)^{-1}, \tag{24}$$

and where, $\boldsymbol{R}^d$ is the noiseless (denoised) data covariance matrix, $\boldsymbol{R}_{YY}$, with the subscript removed for convenience. The output $\boldsymbol{Z}(n)$ can directly be processed by a multichannel matched filter to get the symbol estimate, $\hat{a}_1(n-d)$, the data for the user 1.

$$\hat{a}_1(n-d) = \frac{1}{\tilde{\boldsymbol{g}}_1^H \tilde{\boldsymbol{g}}_1} \boldsymbol{F}^H \boldsymbol{Y}_L(n) = \frac{1}{\tilde{\boldsymbol{g}}_1^H \tilde{\boldsymbol{g}}_1} \boldsymbol{h}_1^H \left(\boldsymbol{T}_1 - \boldsymbol{Q}\boldsymbol{T}_2\right) \boldsymbol{Y}_L(n) \tag{25}$$

The covariance matrix of the prediction errors is then given by

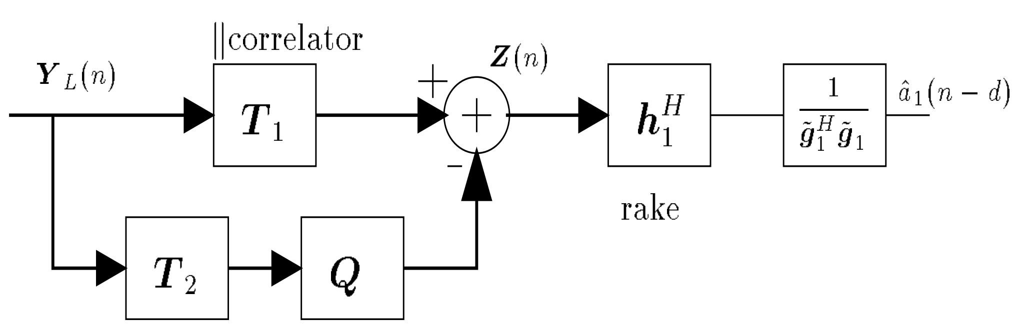

Figure 4: GSC implementation of the MMSE-ZF receiver.

$$\boldsymbol{R}_{ZZ}=\boldsymbol{T}_1\boldsymbol{R}^d\boldsymbol{T}_1^H-\boldsymbol{T}_1\boldsymbol{R}^d\boldsymbol{T}_2^H\left(\boldsymbol{T}_2\boldsymbol{R}^d\boldsymbol{T}_2^H\right)^{-1}\boldsymbol{T}_2\boldsymbol{R}^d\boldsymbol{T}_1^H, \tag{26}$$

From the above structure of the interference canceler, we observe that when $\boldsymbol{T}_1\left(\boldsymbol{Y}_L-\tilde{\boldsymbol{g}}_1 a_1(n)\right)$ can be perfectly estimated from $\boldsymbol{T}_2\boldsymbol{Y}_L$, the matrix $\boldsymbol{R}_{ZZ}$ is rank-1 in the noiseless case! Using this fact, the desired user channel can be obtained (upto a scale factor) as the maximum eigenvector of the matrix $\boldsymbol{R}_{ZZ}$, since $\boldsymbol{Z}(n)=(\boldsymbol{C}_1^H\boldsymbol{C}_1)\otimes\boldsymbol{I}_{MJ}\boldsymbol{h}_1\tilde{a}_1(n-d)$. It can further be shown easily that if $\boldsymbol{T}_2=\boldsymbol{T}_1^{\perp}$, then

$$\boldsymbol{T}_1\boldsymbol{R}_{YY}^{-1}\boldsymbol{T}_1^H=\left(\boldsymbol{T}_1\boldsymbol{T}_1^H\right)\boldsymbol{R}_{ZZ}^{-1}\left(\boldsymbol{T}_1\boldsymbol{T}_1^H\right), \tag{27}$$

where, $\boldsymbol{R}_{ZZ}$ is given by (26), and $\boldsymbol{Q}$, given by (24), is optimized to minimize the estimation error variance. $\boldsymbol{R}^d$ replaces $\boldsymbol{R}_{YY}$ in the above developments. From this, we can obtain the propagation channel estimate for the desired user, $\hat{\boldsymbol{h}}_1$ as $\hat{\boldsymbol{h}}_1=V_{max}\{\left(\boldsymbol{T}_1\boldsymbol{T}_1^H\right)^{-1}\boldsymbol{R}_{ZZ}\left(\boldsymbol{T}_1\boldsymbol{T}_1^H\right)^{-1}\}$. The above structure results in perfect interference cancellation (both ISI and MAI) in the noiseless case, the evidence of which is the rank-1 estimation error covariance matrix, and a consequent distortionless response for the desired user. In the noiseless case ($v(t)\equiv 0$), we have the following two cases of interest.

5.4.1 Uncorrelated symbols

In the absence of noise, with *i.i.d.* symbols, the stochastic estimation of $\boldsymbol{T}_1\boldsymbol{Y}$ from $\boldsymbol{T}_2\boldsymbol{Y}$ is the stochastic estimation of $\boldsymbol{T}_1\mathcal{T}_L(\boldsymbol{G}_N)\boldsymbol{A}$ from $\boldsymbol{T}_2\mathcal{T}_L(\boldsymbol{G}_N)\boldsymbol{A}$ with $\boldsymbol{R}_A=\sigma_a^2\boldsymbol{I}$. Hence, it is equivalent to the deterministic estimation of $\mathcal{T}_L^H(\boldsymbol{G}_N)\boldsymbol{T}_1^H$ from $\mathcal{T}_L^H(\boldsymbol{G}_N)\boldsymbol{T}_2^H$: $\|\mathcal{T}_L^H(\boldsymbol{G}_N)\boldsymbol{T}_1^H-\mathcal{T}_L^H(\boldsymbol{G}_N)\boldsymbol{T}_2^H\boldsymbol{Q}^H\|_2^2$. Then, given the condition

$$\begin{array}{l}\mathrm{span}\{\boldsymbol{T}_1^H\}\cap\mathrm{span}\{\mathcal{T}_L(\boldsymbol{G}_N)\}=\mathrm{span}\{\mathcal{T}_L(\boldsymbol{G}_N)\boldsymbol{e}_d^{'}\}\\ \Rightarrow\mathrm{span}\{\mathcal{T}_L(\boldsymbol{G}_N)\}\subset\mathrm{span}\{\boldsymbol{T}_2^H\}\oplus\mathrm{span}\{\tilde{\boldsymbol{g}}_1\}\\ \because\mathcal{T}_L(\boldsymbol{G}_N)\boldsymbol{e}_d^{'}=\mathcal{T}_L(\boldsymbol{G}_{1,N_1})\boldsymbol{e}_d=\tilde{\boldsymbol{g}}_1=\boldsymbol{T}_1\boldsymbol{h}_1,\end{array} \tag{28}$$

and where, $\boldsymbol{e}_d^{'}$ and $\boldsymbol{e}_d$ are vectors of appropriate dimensions with all zeros and one 1 selecting the desired column in $\mathcal{T}_L(\boldsymbol{G}_N)$ and $\mathcal{T}_L(\boldsymbol{G}_{1,N_1})$ respectively. We can write the channel convolution matrix $\mathcal{T}_L(\boldsymbol{G}_N)$ as

$$\mathcal{T}_L(\boldsymbol{G}_N)=\tilde{\boldsymbol{g}}_1\boldsymbol{e}_d^{'H}+\mathcal{T}_L(\boldsymbol{G}_N)P_{e_d^{'\perp}}=[\tilde{\boldsymbol{g}}_1\quad\boldsymbol{T}_2^H]\boldsymbol{B}, \tag{29}$$

for some $\boldsymbol{B}$. Then we can write,

$$\begin{array}{l}\mathcal{T}_L^H(\boldsymbol{G}_N)\left(\boldsymbol{T}_1^H-\boldsymbol{T}_2^H\boldsymbol{Q}^H\right)=\\ \boldsymbol{e}_d^{'}\boldsymbol{h}_1^H\boldsymbol{T}_1\boldsymbol{T}_1^H+\boldsymbol{B}^H\begin{bmatrix}\tilde{\boldsymbol{g}}_1\boldsymbol{T}_1^H\\ \boldsymbol{0}\end{bmatrix}-\boldsymbol{B}^H\begin{bmatrix}\boldsymbol{0}\\ \boldsymbol{T}_2\boldsymbol{T}_2^H\end{bmatrix}\boldsymbol{Q}^H\\ =\boldsymbol{e}_d^{'}\boldsymbol{h}_1^H\boldsymbol{T}_1\boldsymbol{T}_1^H+\boldsymbol{B}_1^H\tilde{\boldsymbol{g}}_1^H\boldsymbol{T}_1^H-\boldsymbol{B}_2^H\left(\boldsymbol{T}_2\boldsymbol{T}_2^H\right)\boldsymbol{Q}^H.\end{array} \tag{30}$$

Note that $\boldsymbol{e}_d^{'H}\boldsymbol{B}_i^H=0, i\in\{1,2\}$. This implies that the first term on the R.H.S. of (30) is not predictable from the third. Therefore, if the second term is perfectly predictable from the third, then the two terms cancel each other out and $\boldsymbol{R}_{ZZ}$ turns out to be rank-1, and $\hat{\boldsymbol{h}}_1=\left(\boldsymbol{T}_1\boldsymbol{T}_1^H\right)^{-1}V_{max}\left(\boldsymbol{R}_{ZZ}\right)$.

5.4.2 Correlated symbols

In the case of correlated symbols, with a finite amount of data, given the conditions in (28), it still holds that $\mathrm{span}\{\mathcal{T}_L^H(\boldsymbol{G}_N)\boldsymbol{T}_2^H\} = \mathrm{span}\{P_{e_d'^{\perp}}\mathcal{T}_L(\boldsymbol{G}_N)\}$. Now, we can write the received vector $\boldsymbol{Y}_L(n)$ as

$$\boldsymbol{Y}_L(n) = \mathcal{T}_L(\boldsymbol{G}_N)\boldsymbol{A} = \mathcal{T}_L(\boldsymbol{G}_N)\boldsymbol{e}_d^{'}a_1(n-d) + \overline{\mathcal{T}}_L\bar{\boldsymbol{A}}. \tag{31}$$

Now, the estimation of $\boldsymbol{T}_1\boldsymbol{Y}$ in terms of $\boldsymbol{T}_2\boldsymbol{Y} = \boldsymbol{T}_2\mathcal{T}_L(\boldsymbol{G}_N)\boldsymbol{A} = \boldsymbol{T}_2\overline{\mathcal{T}}_L\bar{\boldsymbol{A}}$ is equivalent to estimation in terms of $\bar{\boldsymbol{A}}$.

$$\begin{aligned}
\widetilde{\boldsymbol{T}_1\boldsymbol{Y}}|_{\boldsymbol{T}_2\boldsymbol{Y}} &= \boldsymbol{T}_1\boldsymbol{Y} - \widehat{\boldsymbol{T}_1\boldsymbol{Y}} \\
&= \boldsymbol{T}_1\boldsymbol{Y} - \left(\boldsymbol{T}_1\boldsymbol{R}_{YY}^d\boldsymbol{T}_2^H\right)\left(\boldsymbol{T}_2\boldsymbol{R}_{YY}^d\boldsymbol{T}_2^H\right)^{-1}\boldsymbol{T}_2\boldsymbol{Y} \\
\widetilde{\boldsymbol{T}_1\boldsymbol{Y}}|_{\bar{\boldsymbol{A}}} &= \boldsymbol{T}_1\mathcal{T}_L(\boldsymbol{G}_N)\boldsymbol{e}_d^{'}\tilde{a}_1(n-d) \\
&= \boldsymbol{T}_1\boldsymbol{T}_1^H\boldsymbol{h}_1\tilde{a}_1(n-d)|_{\bar{\boldsymbol{A}}}.
\end{aligned} \tag{32}$$

This results in,

$$\left(\boldsymbol{T}_1\boldsymbol{R}_{YY}^{-d}\boldsymbol{T}_1^H\right)^{-1} = \sigma^2_{\tilde{a}_1(n-d)|\bar{A}}\boldsymbol{h}_1\boldsymbol{h}_1^H, \tag{33}$$

The rank-1 results in a normalized estimate of the channel. It must however be noted that the estimation error variance of the desired symbol is now smaller ($\sigma^2_{\tilde{a}_1(n-d)} < \sigma_a^2$).

5.5 Identifiability Conditions for Blind MMSE-ZF Receiver

Continuing with the noiseless case, or with the denoised version of $\boldsymbol{R}_{YY}$, i.e., $\boldsymbol{R}_{YY}^d = \sigma_a^2\mathcal{T}_L(\boldsymbol{G}_N)\mathcal{T}_L^H(\boldsymbol{G}_N)$,

$$\min_{F:F^H\tilde{g}_1=1} \boldsymbol{F}^H\boldsymbol{R}_{YY}^d\boldsymbol{F} = \sigma_a^2, \quad \textit{iff} \quad \boldsymbol{F}^H\mathcal{T}_L(\boldsymbol{G}_N) = \boldsymbol{e}_d^{'H}, \tag{34}$$

i.e., the zero-forcing condition must be satisfied. Hence, the unbiased MOE criterion corresponds to ZF in the noiseless case. This implies that $\mathrm{MOE}(\hat{\tilde{\boldsymbol{g}}}_1) < \sigma_a^2 \;\; \textit{if} \;\; \hat{\tilde{\boldsymbol{g}}}_1 \not\sim \tilde{\boldsymbol{g}}_1$. We consider that:

(i). *FIR zero-forcing conditions are satisfied*, and
(ii). $\mathrm{span}\{\mathcal{T}_L(\boldsymbol{G}_N)\} \cap \mathrm{span}\{\boldsymbol{T}_1^H\} = \mathrm{span}\{\boldsymbol{T}_1^H\boldsymbol{h}_1\}$.

The two step max/min problem boils down to

$$\max_{\hat{h}_1:||\hat{h}_1||=1} \hat{\boldsymbol{h}}_1^H\left(\boldsymbol{T}_1\boldsymbol{T}_1^H\right)^{-1}\boldsymbol{T}_1\mathcal{T}_L P^{\perp}_{\mathcal{T}_L^H T_2^H}\mathcal{T}_L^H\boldsymbol{T}_1^H\left(\boldsymbol{T}_1\boldsymbol{T}_1^H\right)^{-1}\hat{\boldsymbol{h}}_1, \tag{35}$$

where, $P_X^\perp = \boldsymbol{I} - \boldsymbol{X}(\boldsymbol{X}^H\boldsymbol{X})^{-1}\boldsymbol{X}^H$. Then identifiability implies that $\mathcal{T}_L P^\perp_{\mathcal{T}_L^H T_2^H} \mathcal{T}_L^H = \boldsymbol{T}_1^H \boldsymbol{h}_1 \boldsymbol{h}_1^H \boldsymbol{T}_1 = \widetilde{\boldsymbol{g}}_1 \widetilde{\boldsymbol{g}}_1^H$, or

$$P^\perp_{\mathcal{T}_L^H T_2^H} \mathcal{T}_L^H(\boldsymbol{G}_N) = P_{e'_d} \mathcal{T}_L^H(\boldsymbol{G}_N), \tag{36}$$

Condition (i) above implies that $\boldsymbol{e}'_d \in \mathrm{span}\{\mathcal{T}_L^H(\boldsymbol{G}_N)\}$. From condition (ii), since $\boldsymbol{T}_1^H \boldsymbol{h}_1 = \mathcal{T}_L(\boldsymbol{G}_N)\boldsymbol{e}'_d$, we have

$$\begin{aligned} \mathrm{span}\{\mathcal{T}_L(\boldsymbol{G}_N)\boldsymbol{T}_2^H\} &= \mathrm{span}\{P^\perp_{e'_d} \mathcal{T}_L^H(\boldsymbol{G}_N)\} \\ \mathrm{span}\{\mathcal{T}_L^H(\boldsymbol{G}_N)\} &= \mathrm{span}\{\mathcal{T}_L^H(\boldsymbol{G}_N)\boldsymbol{T}_2^H\} \oplus \mathrm{span}\{\boldsymbol{e}'_d\} \end{aligned} \tag{37}$$

from which, $\mathcal{T}_L^H(\boldsymbol{G}_N) = P_{\mathcal{T}_L^H T_2^H} \mathcal{T}_L^H(\boldsymbol{G}_N) + P_{e'_d} \mathcal{T}_L^H(\boldsymbol{G}_N)$, which is the same as (36).

5.5.1 A Note on Sufficiency of Conditions

We consider first the conditions (**i**). Furthermore, in the following developments, we consider that $K < PMJ$, which is easily achievable with a small (e.g, 2) multiple sensor and/or oversampling factor. The effective number of channels is given by $(PMJ)_{\text{eff}} = \mathrm{rank}\{\boldsymbol{G}_N\}$, where $\boldsymbol{G}_N$ is given in (3). Let $\mathbf{G}_1(z) = \sum_{n=0}^{N_1-1} \boldsymbol{g}_1(n) z^{-n}$ be the channel transfer function for user 1, with $\mathbf{G}(z) = [\mathbf{G}_1(z) \cdots \mathbf{G}_K(z)]$. Then let us assume the following:

(**a**). $\mathbf{G}(z)$ *is irreducible,* i.e., $\quad \mathrm{rank}\{\mathbf{G}(z)\} = K, \forall z.$

(**b**). $\mathbf{G}(z)$ *is column reduced*: $\quad \mathrm{rank}\{[\boldsymbol{g}_1(N_1-1) \cdots \boldsymbol{g}_K(N_K-1)]\} = K.$

Given that the above two conditions hold, the channel convolution matrix $\mathcal{T}(\boldsymbol{G}_N)$ is full rank w.p. 1, and the FIR length L required is given by,

$$L \geq \overline{L} = \left\lceil \frac{N-K}{(PMJ)_{\text{eff}} - K} \right\rceil. \tag{38}$$

Note that condition (**a**) holds with probability 1 due to the quasi-orthogonality of spreading sequences. As for (**b**), it can be violated in certain limiting cases e.g., in the synchronous case where $\boldsymbol{g}_k(N_k-1)$'s contain very few non-zero elements. Under these circumstances, instantaneous (static) mixture of the sources can null out some of the $\boldsymbol{g}_k(N_k-1)$ (more specifically, at most $K-1$ of them). Then N gets reduced by at most $K-1$. However, even then, L given by (38) remains sufficient.

The condition (**ii**) can be restated as the following dimensional requirement:

$$\mathrm{rank}\{\mathcal{T}_L(\boldsymbol{G}_N)\} + \mathrm{rank}\{\boldsymbol{T}_1^H\} \leqslant \mathrm{row}\{\mathcal{T}_L(\boldsymbol{G}_N)\} + 1, \tag{39}$$

from where, under the irreducible channel and column reduced conditions,

$$L \geq \underline{L} = \left\lceil \frac{N-K+l_1MJ-1}{(PMJ)_{\text{eff}} - K} \right\rceil, \tag{40}$$

where, l_1 is the channel length for user 1 in chip periods. If (40) holds, then condition (**ii**) is fulfilled w.p. 1, regardless of the N_k's, i.e., the $\mathrm{span}\{\boldsymbol{T}_1^H\}$ does not intersect with all shifted versions of $\boldsymbol{g}_k$'s, $\forall k \neq 1$, which further means that no confusion is possible between the channel of the user of interest and those of other users, whether the mixing is static (same orders) or dynamic (different channel lengths), with lengths measured in symbol periods.

5.5.2 Violation of condition (*ii*)

If the channel length l_1 is over-estimated, such that N_1 gets over-estimated, then condition (**ii**) is violated w.p. 1. In that case, more than one shifted versions of $\boldsymbol{g}_1$ will fit in the column space of $\boldsymbol{T}_1^H$. The estimated channel in that case can be expressed as $\widehat{\mathbf{G}}_1(z) = \mathbf{G}_1(z)b(z)$, where, $b(z)$ is a scalar polynomial of the order that equals the amount by which the channel has been over- estimated. An adhoc but expensive solution to this would be to try all orders for N_1 and stop at the correct one. Once, the delay estimates have been obtained, however, overestimation of the channel order is highly unlikely in most DS-CDMA systems, where, the delay spread $< P$, and in which case, $N_k = 2$ for a synchronized user k.

5.6 Two-Sided Linear Prediction

We can give one more interpretation of the MMSE-ZF receiver in terms of two-sided linear prediction (TSLP) of the received signal. Let us consider the noiseless case ($v(t) \equiv 0$), and replace the $\boldsymbol{T}_1$ and $\boldsymbol{T}_2$ in (21) by,

$$\overline{\boldsymbol{T}}_1 = \begin{bmatrix} \mathbf{0} & \boldsymbol{I}_{PN_1} & \mathbf{0} \end{bmatrix} \otimes \boldsymbol{I}_{MJ}, \text{ and } \quad \overline{\boldsymbol{T}}_2 = \begin{bmatrix} \boldsymbol{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \boldsymbol{I} \end{bmatrix} \otimes \boldsymbol{I}_{MJ}, \tag{41}$$

This corresponds to the *least squares smoothing* approach of [23] in a single user case. We can proceed with a similar treatment as previously discussed in section 5.4 for the GSC implementation of the unbiased MOE algorithm. However, now,

$$\mathrm{rank}\{\boldsymbol{R}_{ZZ}\} \geq \overline{K}, \tag{42}$$

where $\overline{K}$ denotes the number of users with channel orders shorter than or equal to N_1 [24] [25]. A mixture (instantaneous when channel orders are the same) of different users' channels is now obtained. In the event of $\overline{K} = 1$ (the desired user), the composite channel vector $\widetilde{\boldsymbol{g}}_1$ can be obtained from the rank-1 $\boldsymbol{R}_{ZZ}$, although the stacking factor L required will be much longer than $\underline{L}$ given by (40).

From the above discussion it is obvious that the TLSP has some capability of multiuser interference cancellation in very special situations ($N_1 < N_k, \ \forall k \neq 1$). However, for the DS-CDMA system in question, the presence of $\boldsymbol{C}_1^{\perp}$ term in the blocking matrix $\boldsymbol{T}_2$ "cleans up" the contribution of interfering users without regard to their channel orders, and highlights the great degree of robustness of the system *vis-à-vis* the channel identification issue.

6 Numerical Examples

We consider $K = 5$ asynchronous users in the system with a spreading factor of $P = 16$. The channel for the jth user is modeled as a FIR channel of order l_k ranging from $8 - 21$ chip periods for different k's. The channel delay spread is therefore shorter than one symbol period for some users while longer for others. Near-far conditions prevail in that the interfering users are randomly (ranging from 8 to 10 dB.) stronger than the user of interest. Fig. 5 shows the bit error-rate performance of the blind MMSE-

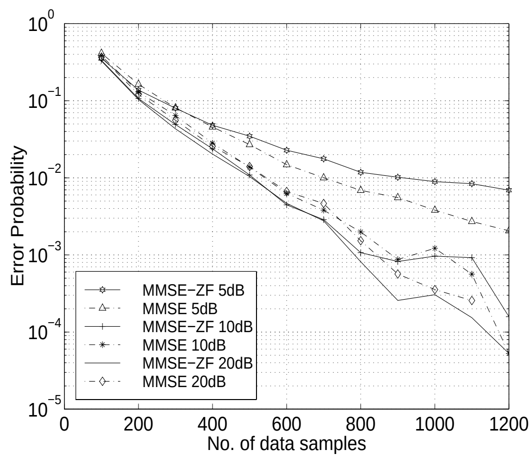

Figure 5: Error rate performance for spreading factor, P=16, and K=5 users.

ZF receiver and the MMSE receiver ($\sigma_a^2 \widehat{\boldsymbol{R}}_{YY}^{-1} \widetilde{\boldsymbol{g}}_1$). It can be seen that the performance depends on the quality of the correlation matrix estimate. Better results are therefore obtained if more data is available. This figure highlights the major drawback in the implementation of second-order statistics based linear receiver algorithms. Under power controlled conditions, with good choice of spreading sequences, and a small loading fraction, a simple RAKE receiver may outperform the linear receivers, unless a good estimate of $\widehat{\boldsymbol{R}}_{YY}$ is available. On the other hand, as seen in fig. 6, the channel is estimated fairly accurately (normalized mean squared error[2] (NMSE) of the order of -25 dB at 20 dB. SNR) with 70 symbols from the rank-1 $\boldsymbol{R}_{ZZ}$ (see section 5.4). Performance of the noise-subspace based algorithm

[2]NMSE= $E\frac{\|\boldsymbol{h}_1 - \hat{\boldsymbol{h}}_1\|^2}{\|\boldsymbol{h}_1\|^2} = \frac{1}{L}\sum_{i=1}^{L} \frac{\|\boldsymbol{h}_1 - \hat{\boldsymbol{h}}_1^{(i)}\|^2}{\|\boldsymbol{h}_1\|^2}$

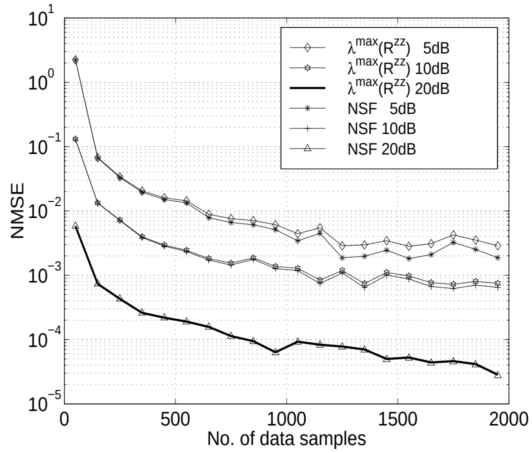

Figure 6: Channel estimation performance for spreading factor, P=16, and K=5 users.

[17] is also shown for several input SNR's. In fig. 7 and 8, we show the performance of blind MMSE-

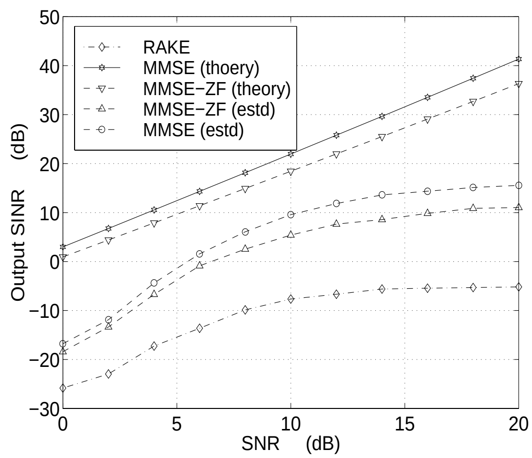

Figure 7: Output SINR performance of different receivers in near-far conditions for spreading factor, P=16, and K=5 users.

ZF receiver in near-far and power-controlled conditions, respectively, and compare it with that of the theoretical MMSE ($\boldsymbol{R}_{YY} = \sigma_a^2 \mathcal{T}_L(\boldsymbol{G}_N)\mathcal{T}_L(\boldsymbol{G}_N) + \sigma_v^2 \boldsymbol{I}$). A data record of 200 data samples is employed to estimate the receivers. It comes as no surprise that the optimal unbiased MMSE is not approached by any of the other receivers due to finite data effect. A theoretical curve for the MMSE-ZF is also provided. fig. 9 shows the quality of the channel estimates for the case when denoised statistics are employed in the unbiased MOE algorithm.

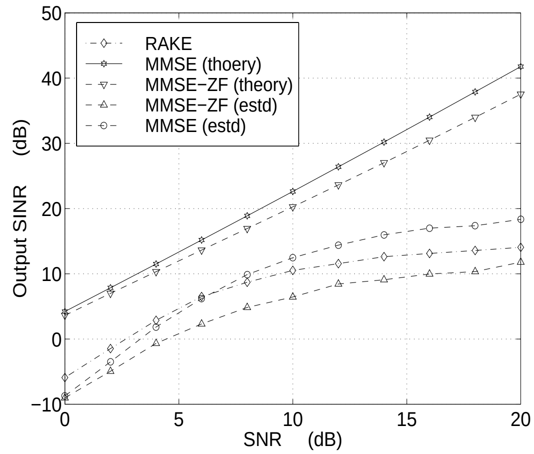

Figure 8: Output SINR performance of different receivers in power-controlled conditions for spreading factor, P=16, and K=5 users.

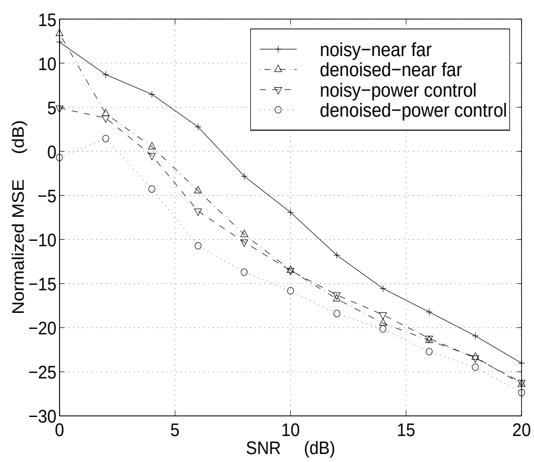

Figure 9: Normalized channel estimation MSE for the denoised and non-denoised $\boldsymbol{R}_{YY}$, for spreading factor, P=16, and K=5 users

7 Conclusions

The blind MMSE-ZF receiver for DS-CDMA was presented. The receiver was shown to be the proper extension of the anchored MOE receiver [1] to the general asynchronous case in multipath channels, leading to the distortionless response for the desired symbol of the desired user. It was also demonstrated, that the receiver is obtainable from the blind unbiased linear MOE criterion in a decentralized manner. A simpler implementation in the form of a Generalized Sidelobe Canceler (GSC) or the MVDR was also shown. In terms of its implementation, the blind algorithm, like the MMSE linear receiver, requires a large amount of data for the estimation of the channel covariance matrix thus making it rather impractical for rapidly changing environments (fast fading) and large numbers of users $(K \to P)$. Such algorithms can find their utility in indoor wireless LANs where channels change at relatively slow rates and a fair amount of data is available for the estimation of the covariance matrix. A possible implementation can be at the uplink, where, knowledge of spreading codes and timing of all users in the cell can be exploited to obtain a better $\hat{\boldsymbol{R}}_{YY}$.

Identifiability conditions of the blind MMSE-ZF receiver, for channels of arbitrary length (even longer than a symbol period) were given and it was shown that the channel is blindly identifiable w.p.1 (upto a scalar phase factor), unless it is overestimated.

A Appendix

The MMSE-ZF receiver was derived in section 3 as $\boldsymbol{F}^H = \boldsymbol{e}_d^T(\mathcal{T}^H\mathcal{T})^{-1}\mathcal{T}^H$. Where, we express a column permuted version of the channel convolution matrix $\mathcal{T}(\boldsymbol{G}_N)$ as $\mathcal{T} = \begin{bmatrix}\widetilde{\boldsymbol{g}}_1 & \bar{\mathcal{T}}\end{bmatrix}$. Then, $\widetilde{\boldsymbol{g}}_1 = \mathcal{T}\boldsymbol{e}_d$, and $\boldsymbol{e}_d = [1\ 0\ \cdots\ 0]$. Let us further define a square transformation matrix, $\boldsymbol{P}$, given by

$$\boldsymbol{P} = \begin{bmatrix} 1 & \boldsymbol{0} \\ \boldsymbol{X} & \boldsymbol{I} \end{bmatrix}, \tag{43}$$

so that

$$\mathcal{T}\boldsymbol{P} = [\widetilde{\boldsymbol{g}}_1 \quad \bar{\mathcal{T}}] \begin{bmatrix} 1 & \boldsymbol{0} \\ \boldsymbol{X} & \boldsymbol{I} \end{bmatrix} = \begin{bmatrix} P_{\bar{\mathcal{T}}}^{\perp}\widetilde{\boldsymbol{g}}_1 & \bar{\mathcal{T}} \end{bmatrix}, \tag{44}$$

and where, $\boldsymbol{X} = -(\bar{\mathcal{T}}^H\bar{\mathcal{T}})^{-1}\bar{\mathcal{T}}^H\widetilde{\boldsymbol{g}}_1$. Then, the MMSE-ZF receiver can be written as

$$\begin{aligned} \boldsymbol{F}^H &= \boldsymbol{e}_d^T\left[(\mathcal{T}\boldsymbol{P})^H(\mathcal{T}\boldsymbol{P})\right]^{-1}(\mathcal{T}\boldsymbol{P})^H \\ &= \boldsymbol{e}_d^T \begin{bmatrix} 1 & \boldsymbol{0} \\ \boldsymbol{X} & \boldsymbol{I} \end{bmatrix} \begin{bmatrix} \widetilde{\boldsymbol{g}}_1^H P_{\bar{\mathcal{T}}}^{\perp}\widetilde{\boldsymbol{g}}_1 & \boldsymbol{0} \\ \boldsymbol{0} & \bar{\mathcal{T}}^H\bar{\mathcal{T}} \end{bmatrix}^{-1} \begin{bmatrix} (P_{\bar{\mathcal{T}}}^{\perp}\widetilde{\boldsymbol{g}}_1)^H \\ \bar{\mathcal{T}}^H \end{bmatrix} \\ &= \frac{1}{\widetilde{\boldsymbol{g}}_1^H P_{\bar{\mathcal{T}}}^{\perp}\widetilde{\boldsymbol{g}}_1}\widetilde{\boldsymbol{g}}_1^H P_{\bar{\mathcal{T}}}^{\perp}, \end{aligned} \tag{45}$$

where $P^{\perp}_{\bar{\mathcal{T}}}$ is the projection operator that projects the received data vector $\boldsymbol{Y}_L(n)$ onto the low rank subspace defined by the orthogonal complement of the subspace spanned by the columns of $\bar{\mathcal{T}}$, and $\widetilde{\boldsymbol{g}}_1$ is the projection on the one-dimensional subspace matched to the desired signal. □

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BIOGRAPHIES

Irfan Ghauri received his bachelor degree in electrical engineering from the NWFP University of Engineering and Technology, Peshawar, Pakistan, in 1992, and the M.S. in Communication Systems and Computer Networks from the Ecole Nationale Supérieure d'Electrotechnique, d'Electronique, d'Informatique, et d'Hydraulique de Toulouse (ENSEEIHT), Toulouse, and Digital Communications from the Ecole Nationale Supérieure des Télécommunications (ENST), Paris, in 1995 and 1996 respectively. From 1992 to 1994, he worked as Communication Systems Engineer at Siemens Pakistan Engineering Co. Ltd. He is currently with Mobile Communications Department of the Eurécom Institute, Sophia Antiplois, France, where he is working towards his Ph.D. in signal processing techniques for advanced DS-CDMA receivers. His areas of interest include multiuser detection, spatio-temporal filtering, and blind and semi-blind techniques for interference cancellation in DS-CDMA systems.

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