Spatial Interference Cancellation and Pairwise Error Probability Analysis

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***Abstract*—Future wireless communication systems being characterized by tight frequency reuse, adaptive modulation and coding schemes and diversified data services will be interference limited by interfering signals of diverse rates and strengths. Keeping in view such a scenario, we propose in this paper the application of a recently proposed low complexity match filter (MF) based detector for spatial interference cancellation in the presence of one strong interferer. We derive an analytical upper bound for the coded pair wise error probability (PEP) for the proposed MF based detector using the *moment generating function* (MGF) based method and prove that this detector not only recuperates the diversity order lost by MMSE but also exhibits a coding gain as the interference gets stronger. We also study in this paper the effect of non Gaussian interference on coded PEP of MMSE linear detection and demonstrate the deficit of one order of diversity and a coding loss as the interference gets stronger. Our analysis provides insights to explain the relative performance of MMSE and proposed MF based detectors as a function of the strength of interference. Finally we demonstrate the strength of our new analytical PEP upper bounds by simulations.**

I. Introduction

To cope with the ever-increasing demands on higher spectral efficiency, a tight frequency reuse will be adopted for future wireless communication systems as 3GPP LTE [1] and IEEE 802.16m [2]. Adaptive modulation and coding schemes will be supported in the next generation wireless systems which combined with the diversified data services will lead to variable transmission rate streams. These system characteristics will overall lead to an interference-limited system. Most state-of-the-art wireless systems deal with the interference either by orthogonalizing the communication links in time or frequency [3] or allow the communication links to share the same degrees of freedom but model the interference as additive Gaussian random process [4]. Both of these approaches may be suboptimal as the first approach entails an *a priori* loss of degrees of freedom in both links, no matter how weak the potential interference is while the second approach treats interference as pure noise while it actually carries information and has the structure that can be potentially exploited in mitigating its effect.

Interference cancellation combined with interference coordination [3] is one of the effective means of improving system throughput as interference coordination will mitigate the number of interferers and interference cancellation strategies will reduce the effect of the remaining interferers. In this paper, we restrict our scope to one strong interferer. Different spatial interference cancellation techniques involving equalization and subtractive cancellation have been proposed in the literature [5] [6]. Amongst them, the MMSE linear detectors are being considered as likely candidates for 3GPP LTE [7]. Since the MMSE criterion is not directly related to the distribution of interference, it is of considerable interest to study the error probability performance of MMSE detectors in the presence of realistic non Gaussian interference. Gaussian assumption of the post detection interference (PDI) in MMSE is valid for asymptotic cases (large dimensional systems) [8] but the fidelity of this approximation in a lower dimensional system is questionable. With analysis of the MMSE detector available in the literature based on the assumption of Gaussianity [9], we extend it to analyzing the performance of MMSE taking into account the non Gaussian interference. We further investigate the effect of interference strength on the coding loss in MMSE linear detector.

Basing on the degraded performance of MMSE in the presence of one interferer, we propose in this paper the use of recently proposed low complexity match filter (MF) based detector [10] for spatial interference cancellation in single frequency reuse synchronized cellular networks. The MF based detector decodes the desired stream basing on the partial decoding of the interference stream [11] and therefore shows improved performance as the strength of interference increases. We study the effect of the strength of interference on coded pairwise error probability (PEP) of MF based detector and we derive an analytical upper bound using the *moment generating function* (MGF) based approach and show that this detector not only recuperates the diversity order lost by MMSE but also exhibits a coding gain (the horizontal shift of the BER curve) as the interference gets stronger. We demonstrate the strength of our new analytical PEP upper bounds by simulating the performance of a mobile station (MS) in the presence of one interferer of varying strength and alphabet size using the MMSE based approach and the MF based approach.

Regarding notations, we will use lowercase or uppercase letters for scalars and boldface letters for vectors and matrices. $\Re(.)$ and $\Im(.)$ indicate real and imaginary parts while $(.)^T$ and $(.)^\dagger$ indicate transpose and conjugate transpose. $|.|$ and $\|.\|$ indicate norm of scalar and vector respectively. The paper is divided into six sections. In section II we define the system model while section III discusses the MMSE and MF

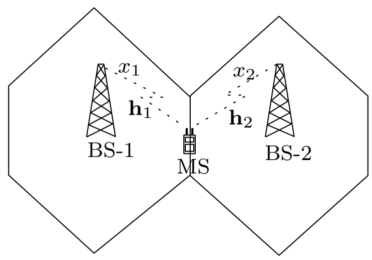

Fig. 1. Interference cancellation in single frequency reuse cellular network. x_1 is the desired signal while x_2 is the interference signal.

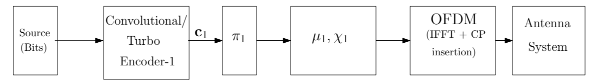

Fig. 2. Block diagram of the transmission chain of BICM OFDM system at BS-1. π_1 denotes the random interleaver, μ_1 the labeling map and χ_1 the signal set.

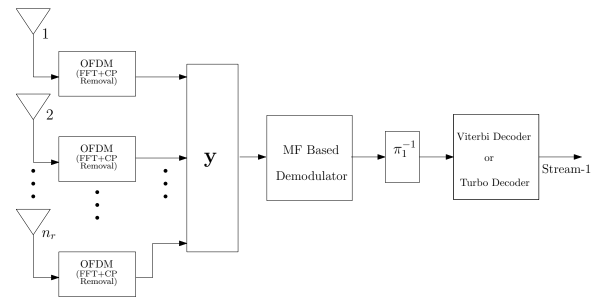

Fig. 3. Block diagram of the receiver at MS. π_1^{-1} denotes deinterleaver.

based detectors. Section IV contains the PEP analysis of MF based detector followed by MMSE linear detector analysis, simulation results and conclusions.

II. System Model

Consider a single frequency reuse cellular network as shown in Fig. 1. Keeping in view the upcoming wireless standards as LTE [1] and 802.16m [2], we assume that both the base stations (BSs) use bit interleaved coded modulation (BICM) based OFDM system for downlink transmission. Block diagram of the transmission chain at the BS and reception chain at the MS are shown in the Figs. 2 and 3 respectively. We assume receive diversity at the MS with n_r receive antennas. Let the two spatial streams arriving at the MS be $\mathbf{x}_1$ (desired stream) and $\mathbf{x}_2$ (interference stream). x_1 is the symbol of $\mathbf{x}_1$ over a signal set $\chi_1 \subseteq \mathcal{C}$ with a Gray labeling map $\mu_1 : \{0,1\}^{\log_2|\chi_1|} \rightarrow \chi_1$ and x_2 is the symbol of $\mathbf{x}_2$ over signal set χ_2 also with the Gray labeling. During the transmission at BS-1, code sequence $\mathbf{c}_1$ is interleaved by π_1 and then is mapped onto the signal sequence $\mathbf{x}_1 \in \chi_1$. Bit interleaver for the first stream can be modeled as $\pi_1 : k^{'} \rightarrow (k,i)$ where $k^{'}$ denotes the original ordering of the coded bits $c_{k'}$ of first stream, k denotes the time ordering of the signal $x_{1,k}$ and i indicates the position of the bit $c_{k'}$ in the symbol $x_{1,k}$.

We assume that the frequency reuse factor is one and cyclic prefix (CP) of appropriate length is added to the OFDM symbols at two base stations. Cascading IFFT at the transmitter and FFT at the receiver with CP extension, transmission at the k-th frequency tone can be expressed as::-

$$\mathbf{y}_k = \mathbf{h}_{1,k} x_{1,k} + \mathbf{h}_{2,k} x_{2,k} + \mathbf{z}_k, \qquad k = 1, 2, \cdots, K$$

where K is the total number of frequency tones. Each subcarrier corresponds to a symbol from a constellation map χ_1 for first stream and χ_2 for second stream. $\mathbf{y}_k, \mathbf{z}_k \in \mathbb{C}^{n_r}$ are the vectors of received symbols and circularly symmetric complex white Gaussian noise of double-sided power spectral density $N_0/2$ at the n_r receive antennas. $\mathbf{h}_{1,k} \in \mathbb{C}^{n_r}$ is the vector characterizing flat fading channel response from first transmitting antenna to n_r receive antennas at k-th subcarrier. This vector has complex-valued multivariate Gaussian distribution with $E\left[\mathbf{h}_{1,k}\right] = \mathbf{0}$ and $E\left[\mathbf{h}_{1,k}\mathbf{h}_{1,k}^{\dagger}\right] = \mathbf{I}$. The channels at different subcarriers are assumed to be independent. The complex symbols $x_{1,k}$ and $x_{2,k}$ of the two streams are also assumed to be independent with variances σ_1^2 and σ_2^2 respectively.

III. Detectors for Interference Cancellation

A. MMSE based Detection

The MMSE filter for $x_{1,k}$ is given as

$$\mathbf{h}_{1,k}^{MMSE} = \left(\mathbf{h}_{1,k}^{\dagger}\mathbf{R}_{2,k}^{-1}\mathbf{h}_{1,k} + \sigma_1^{-2}\right)^{-1} \mathbf{h}_{1,k}^{\dagger}\mathbf{R}_{2,k}^{-1} \qquad (1)$$

where $\mathbf{R}_{2,k} = \sigma_2^2 \mathbf{h}_{2,k}\mathbf{h}_{2,k}^{\dagger} + N_0\mathbf{I}$. After the application of MMSE filter on received vector $\mathbf{y}_k$, we get

$$y_k = \alpha_k x_{1,k} + \beta_k x_{2,k} + \mathbf{h}_1^{MMSE}\mathbf{z}_k \qquad (2)$$

$$= \alpha_k x_{1,k} + z_k \qquad (3)$$

where $\alpha_k = \mathbf{h}_1^{MMSE}\mathbf{h}_1$ and $\beta_k = \mathbf{h}_1^{MMSE}\mathbf{h}_2$. The bit metric for the $c_{k'}$ bit on first stream basing on (3) is given as

$$\lambda_1^i\left(\mathbf{y}_k, c_{k'}\right) \approx \min_{x_1 \in \chi_{1,c_{k'}}^i} \left[\frac{1}{N_k}\left|y_k - \alpha_k x_1\right|^2\right] \qquad (4)$$

where $N_k = \mathbf{h}_{1,k}^{MMSE}\mathbf{R}_{2,k}\mathbf{h}_{1,k}^{MMSE^{\dagger}}$ and $\chi_{1,c_{k'}}^i$ denotes the subset of the signal set $x_1 \in \chi_1$ whose labels have the value $c_{k'} \in \{0,1\}$ in the position i.

B. MF based Detector

The bit metric for bit $c_{k'}$ of the desired stream x_1 is given as [10]

$$\lambda_1^i\left(\mathbf{y}_k, c_{k'}\right) \approx \min_{x_1 \in \chi_{1,c_{k'}}^i} \left\{\frac{1}{N_0}\left|y_1 - \|\mathbf{h}_1\| x_1\right|^2 + \left|\varphi_{x_2}\right|^2 - \left|y_2^{'}\left(x_1\right)\right|^2\right\}$$

where

$$y_1 = \frac{\mathbf{h}_1^{\dagger}\mathbf{y}}{\|\mathbf{h}_1\|},\ y_2 = \frac{\mathbf{h}_2^{\dagger}\mathbf{y}}{\|\mathbf{h}_2\|},\ h_{21} = \frac{\mathbf{h}_2^{\dagger}\mathbf{h}_1}{\|\mathbf{h}_2\|},\ y_2^{'}\left(x_1\right) = y_2 - h_{21}x_1, \text{ and}$$

$$\left|\varphi_{x_2}\right|^2 = \min_{x_2 \in \chi_2} \Re^2\left(y_2^{'}\left(x_1\right) - \|\mathbf{h}_2\| x_2\right) + \Im^2\left(y_2^{'}\left(x_1\right) - \|\mathbf{h}_2\| x_2\right)$$

IV. PEP Analysis - MF Based Detector

The conditional PEP i.e. $P(\mathbf{c}_1 \to \hat{\mathbf{c}}_1|\mathbf{H}) = \mathcal{P}^{\hat{\mathbf{c}}_1}_{\mathbf{c}_1|\mathbf{H}}$ of MF based detector basing on the fact that it is a reduced complexity version of max log MAP detector is given as

$$\mathcal{P}^{\hat{\mathbf{c}}_1}_{\mathbf{c}_1|\mathbf{H}} = P\left(\sum_{k'} \min_{x_1\in\chi^i_{1,c_{k'}},x_2\in\chi_2} \frac{1}{N_0}\|\mathbf{y}_k - \mathbf{h}_{1,k}x_1 - \mathbf{h}_{2,k}x_2\|^2 \geq \sum_{k'} \min_{x_1\in\chi^i_{1,\hat{c}_{k'}},x_2\in\chi_2} \frac{1}{N_0}\|\mathbf{y}_k - \mathbf{h}_{1,k}x_1 - \mathbf{h}_{2,k}x_2\|^2\right) \quad (5)$$

where $\mathbf{H} = [\mathbf{H}_1 \cdots \mathbf{H}_K]$ and $\mathbf{H}_k = [\mathbf{h}_{1,k}\mathbf{h}_{2,k}]$ i.e. the channel at k-th frequency tone. For the worst case scenario once $d(\mathbf{c}_1 - \hat{\mathbf{c}}_1) = d_{free}$, the inequality on the right hand side of (5) shares the same terms on all but d_{free} summation points for which $\hat{c}_{k'} = \bar{c}_{k'}$ where $\bar{(.)}$ denotes the binary complement. Let

$$\tilde{x}_{1,k}, \tilde{x}_{2,k} = \arg\min_{x_1\in\chi^i_{1,c_{k'}},x_2\in\chi_2} \frac{1}{N_0}\|\mathbf{y}_k - \mathbf{h}_{1,k}x_1 - \mathbf{h}_{2,k}x_2\|^2$$
$$\hat{x}_{1,k}, \hat{x}_{2,k} = \arg\min_{x_1\in\chi^i_{1,\bar{c}_{k'}},x_2\in\chi_2} \frac{1}{N_0}\|\mathbf{y}_k - \mathbf{h}_{1,k}x_1 - \mathbf{h}_{2,k}x_2\|^2 \quad (6)$$

where $\|\mathbf{y}_k - \mathbf{h}_{1,k}x_{1,k} - \mathbf{h}_{2,k}x_{2,k}\|^2 \geq \|\mathbf{y}_k - \mathbf{h}_{1,k}\tilde{x}_{1,k} - \mathbf{h}_{2,k}\tilde{x}_{2,k}\|^2$. The conditional PEP is given as

$$\begin{aligned}\mathcal{P}^{\hat{\mathbf{c}}_1}_{\mathbf{c}_1|\mathbf{H}} &\leq P\left(\sum_{k,d_{free}} \frac{1}{N_0}\|\mathbf{y}_k - \mathbf{h}_{1,k}x_{1,k} - \mathbf{h}_{2,k}x_{2,k}\|^2 \geq \sum_{k,d_{free}} \frac{1}{N_0}\|\mathbf{y}_k - \mathbf{h}_{1,k}\hat{x}_{1,k} - \mathbf{h}_{2,k}\hat{x}_{2,k}\|^2\right) \\ &= Q\left(\sqrt{\sum_{k,d_{free}} \frac{1}{2N_0}\|\mathbf{H}_k(\hat{\mathbf{x}}_k - \mathbf{x}_k)\|^2}\right) \\ &= Q\left(\sqrt{\frac{1}{2N_0}\mathrm{vec}\left(\mathbf{H}^\dagger\right)^\dagger \boldsymbol{\Delta}\mathrm{vec}\left(\mathbf{H}^\dagger\right)}\right)\end{aligned} \quad (7)$$

where $\hat{\mathbf{x}}_k = [\hat{x}_{1,k}\ \hat{x}_{2,k}]^T$ and $\boldsymbol{\Delta} = \mathbf{I}_{n_r} \otimes \mathbf{D}\mathbf{D}^\dagger$ while $\mathbf{D} = \mathrm{diag}\left\{\hat{\mathbf{x}}_1 - \mathbf{x}_1, \hat{\mathbf{x}}_2 - \mathbf{x}_2, \cdots, \hat{\mathbf{x}}_{k,dfree} - \mathbf{x}_{k,d_{free}}\right\}$. Q is the Gaussian Q-function and vec indicates vectorization of a matrix. For a Hermitian quadratic form in complex Gaussian random variable $q = \mathbf{m}^\dagger\mathbf{A}\mathbf{m}$ where $\mathbf{A}$ is a Hermitian matrix and column vector $\mathbf{m}$ is a circularly symmetric complex Gaussian vector i.e. $\mathbf{m} \sim \mathcal{NC}(\boldsymbol{\mu}, \sum)$, the MGF is

$$E\left[\exp\left(-t\mathbf{m}^\dagger\mathbf{A}\mathbf{m}\right)\right] = \frac{\exp\left[-t\boldsymbol{\mu}^\dagger\mathbf{A}\left(\mathbf{I} + t\sum\mathbf{A}\right)^{-1}\boldsymbol{\mu}\right]}{\det\left(\mathbf{I} + t\sum\mathbf{A}\right)} \quad (8)$$

Using Chernoff bound $Q(x) \leq \frac{1}{2}\exp\left(\frac{-x^2}{2}\right)$ and the MGF, PEP is upper bounded as

$$\mathcal{P}^{\hat{\mathbf{c}}_1}_{\mathbf{c}_1} \leq \frac{1}{2\det\left(\mathbf{I} + \frac{1}{4N_0}\mathbf{I}\boldsymbol{\Delta}\right)} = \frac{1}{2\prod_{k=1}^{d_{free}}\left(1 + \frac{1}{4N_0}\|\hat{\mathbf{x}}_k - \mathbf{x}_k\|^2\right)^{n_r}} \quad (9)$$

$\|\hat{\mathbf{x}}_k - \mathbf{x}_k\|^2 \geq d^2_{1,\min} + d^2_{2,\min}$ if $\hat{x}_{2,k} \neq x_{2,k}$ and $\|\hat{\mathbf{x}}_k - \mathbf{x}_k\|^2 \geq d^2_{1,\min}$ if $\hat{x}_{2,k} = x_{2,k}$. There exists $2^{d_{free}}$ possible vectors of $\left[\hat{x}_{2,1}, \cdots, \hat{x}_{2,d_{free}}\right]^T$ basing on the binary criteria that $\hat{x}_{2,k}$ is equal or not equal to $x_{2,k}$. Taking into account all these cases combined with their corresponding probabilities, the PEP is upper bounded as

$$\mathcal{P}^{\hat{\mathbf{c}}_1}_{\mathbf{c}_1} \leq \frac{1}{2}\left(\frac{4N_0}{\sigma_1^2\breve{d}^2_{1,\min}}\right)^{n_r d_{free}} \times \left(\sum_{j=0}^{d_{free}} \mathrm{C}_j^{d_{free}} \frac{\left(P\left(\hat{x}_{2,k} \neq x_{2,k}\right)\right)^j\left(1 - P\left(\hat{x}_{2,k} \neq x_{2,k}\right)\right)^{d_{free}-j}}{\left(1 + \frac{\sigma_2^2\breve{d}^2_{2,\min}}{\sigma_1^2\breve{d}^2_{1,\min}}\right)^{jn_r}}\right) \quad (10)$$

where $d^2_{j,\min} = \sigma_j^2\breve{d}^2_{j,\min}$ with $\breve{d}^2_{j,\min}$ being the normalized minimum distance of the constellation χ_j for $j = \{1, 2\}$ and $\mathrm{C}_j^{d_{free}}$ is the binomial coefficient. It has been shown in Appendix that $P(\hat{x}_{2,k} \neq x_{2,k}) \to 0$ as $\sigma_2^2 \to \infty$ while $P(\hat{x}_{2,k} \neq x_{2,k})$ increases as σ_2^2 increases. (10) demonstrates a significant result of achieving full diversity by the MF based detector and converging to the performance of single stream using maximum ratio combining in case of very weak and strong interference. In the moderate region, as the strength of interference increases, $P(\hat{x}_{2,k} \neq x_{2,k})$ reduces and there is a coding gain for the detection of desired stream contrary to the case of MMSE where there is a coding loss as the interference gets stronger (shown in the next section).

V. PEP Analysis - MMSE Detector

A. Gaussian Assumption

Conditional PEP for MMSE basing on Gaussian assumption of PDI (4) is given as

$$\mathcal{P}^{\hat{\mathbf{c}}_1}_{\mathbf{c}_1|\mathbf{H}} = P\left(\sum_{k'} \min_{x_1\in\chi^i_{1,c_{k'}}} \frac{|y_k - \alpha_k x_1|^2}{N_k} \geq \sum_{k'} \min_{x_1\in\chi^i_{1,\hat{c}_{k'}}} \frac{|y_k - \alpha_k x_1|^2}{N_k}\right) \quad (11)$$

Let

$$\tilde{x}_{1,k} = \arg\min_{x_1\in\chi^i_{1,c_{k'}}} \frac{|y_k - \alpha_k x_1|^2}{N_k},\ \hat{x}_{1,k} = \arg\min_{x_1\in\chi^i_{1,\bar{c}_{k'}}} \frac{|y_k - \alpha_k x_1|^2}{N_k}$$

Considering the worst case scenario $d(\mathbf{c}_1 - \hat{\mathbf{c}}_1) = d_{free}$ and using the fact that $\frac{1}{N_k}|y_k - \alpha_k x_{1,k}|^2 \geq \frac{1}{N_k}|y_k - \alpha_k\tilde{x}_{1,k}|^2$, the conditional PEP is upper bounded as

$$\mathcal{P}^{\hat{\mathbf{c}}_1}_{\mathbf{c}_1|\mathbf{H}} \leq Q\left(\sqrt{\sum_{k,d_{free}} \frac{\alpha_k^2}{2N_k}|\hat{x}_{1,k} - x_{1,k}|^2}\right) \quad (12)$$

Bounding $|\hat{x}_{1,k} - x_{1,k}|^2 \geq d^2_{1,\min}$ and using the Chernoff bound

$$\mathcal{P}^{\hat{\mathbf{c}}_1}_{\mathbf{c}_1|\mathbf{H}} \leq \frac{1}{2}\exp\left(-\frac{d^2_{1,\min}}{4}\sum_{k,d_{free}} \mathbf{h}^\dagger_{1,k}\mathbf{R}^{-1}_{2,k}\mathbf{h}_{1,k}\right) \quad (13)$$

The eigenvalues of $\mathbf{R}_{2,k}^{-1}$ are

$$\lambda_l = \begin{cases} \left(\sigma_2^2 \left\|\mathbf{h}_{2,k}\right\|^2 + N_0\right)^{-1}, & l=1 \\ N_0^{-1}, & l=2,\cdots,n_r \end{cases} \tag{14}$$

Using the MGF (8), PEP conditioned on $\underline{\mathbf{h}}_2 = \left[\mathbf{h}_{2,1},\cdots,\mathbf{h}_{2,d_{free}}\right]$ is upper bounded as

$$\mathcal{P}_{\mathbf{c}_1|\underline{\mathbf{h}}_2}^{\hat{\mathbf{c}}_1} \leq \frac{1}{2}\left(\frac{4N_0}{d_{1,\min}^2}\right)^{d_{free}(n_r-1)}\left(\frac{4}{d_{1,\min}^2}\right)^{d_{free}}\prod_{l=1}^{d_{free}}\left(\sigma_2^2\|\mathbf{h}_{2,l}\|^2+N_0\right)$$

Channel independence at each subcarrier yields

$$\mathcal{P}_{\mathbf{c}_1}^{\hat{\mathbf{c}}_1} \leq \frac{1}{2}\left(\frac{4N_0}{\sigma_1^2\check{d}_{1,\min}^2}\right)^{d_{free}(n_r-1)}\left(\frac{4}{\sigma_1^2\check{d}_{1,\min}^2}\right)^{d_{free}}\left(n_r\sigma_2^2+N_0\right)^{d_{free}} \tag{15}$$

which not only demonstrates the well known result of the loss of one diversity order in MMSE in the presence of an interferer [9] but also exhibits a coding loss as interference gets stronger.

B. Non Gaussian Interference

Now we analyze MMSE in the presence of non Gaussian PDI i.e. $\beta_k x_{2,k}$. Using (11) and replacing y_k by (2), conditional PEP is upper bounded as

$$\mathcal{P}_{\mathbf{c}_1|\mathbf{H}}^{\hat{\mathbf{c}}_1} \leq Q\left(\sqrt{\sum_{k,d_{free}}\frac{\alpha_k^2}{2N_k}\left\{\left|x_{1,k}-\hat{x}_{1,k}\right|^2\right\}}+\frac{\sum_{k,d_{free}}\frac{2\alpha_k}{N_k}\Re\left(x_{2,k}^*\beta_k^*\left(x_{1,k}-\hat{x}_{1,k}\right)\right)}{\sqrt{\sum_{k,d_{free}}\frac{2\alpha_k^2}{N_k}\left|x_{1,k}-\hat{x}_{1,k}\right|^2}}\right) \tag{16}$$

Using the bounds $Q\left(a+b\right) \leq Q\left(a_{\min}-b_{\max}\right)$, $\Re\left(\mathbf{a}^\dagger\mathbf{b}\right) \leq \|\mathbf{a}\|\,\|\mathbf{b}\|$ and the Jensen's inequality i.e. $E\left(\sqrt{X}\right) \leq \sqrt{E\left(X\right)}$

$$\mathcal{P}_{\mathbf{c}_1|\mathbf{H}}^{\hat{\mathbf{c}}_1} \leq Q\left(\sqrt{\frac{d_{1,\min}^2}{2}\sum_{k,d_{free}}\mathbf{h}_{1,k}^\dagger\mathbf{R}_{2,k}^{-1}\mathbf{h}_{1,k}}-\frac{2d_{1,\max}d_{x_2,\max}\sqrt{\sum_{k,d_{free}}\mathbf{h}_{1,k}^\dagger\mathbf{R}_{2,k}^{-1}\mathbf{h}_{2,k}\mathbf{h}_{2,k}^\dagger\mathbf{R}_{2,k}^{-1}\mathbf{h}_{1,k}}}{\sqrt{2d_{1,\min}^2\sum_{k,d_{free}}\mathbf{h}_{1,k}^\dagger\mathbf{R}_{2,k}^{-1}\mathbf{h}_{1,k}}}\right)$$

where $d_{1,\max}$ and $d_{x_2,\max}$ is the maximum distance of constellation χ_1 and the maximum distance of a constellation point of χ_2 from the origin respectively. Using the Chernoff bound

$$\begin{aligned}\mathcal{P}_{\mathbf{c}_1|\mathbf{H}}^{\hat{\mathbf{c}}_1} \leq\ & \frac{1}{2}\exp\left(-\frac{d_{1,\min}^2}{4}\sum_{k,d_{free}}\mathbf{h}_{1,k}^\dagger\mathbf{R}_{2,k}^{-1}\mathbf{h}_{1,k}\right)\times \\ & \exp\left(-\frac{d_{1,\max}^2 d_{x_2,\max}^2\sum_{k,d_{free}}\mathbf{h}_{1,k}^\dagger\mathbf{R}_{2,k}^{-1}\mathbf{h}_{2,k}\mathbf{h}_{2,k}^\dagger\mathbf{R}_{2,k}^{-1}\mathbf{h}_{1,k}}{2d_{1,\min}^2\sum_{k,d_{free}}\mathbf{h}_{1,k}^\dagger\mathbf{R}_{2,k}^{-1}\mathbf{h}_{1,k}}\right)\times \\ & \exp\left(d_{1,\max}d_{x_2,\max}\sqrt{\sum_{k,d_{free}}\mathbf{h}_{1,k}^\dagger\mathbf{R}_{2,k}^{-1}\mathbf{h}_{2,k}\mathbf{h}_{2,k}^\dagger\mathbf{R}_{2,k}^{-1}\mathbf{h}_{1,k}}\right)\end{aligned} \tag{17}$$

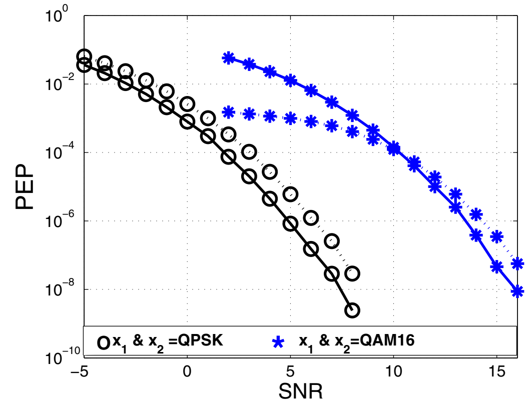

Fig. 4. PEP of x_1 for MMSE based detection in the presence of interference x_2 where $\sigma_1^2 = 1$ and $\sigma_2^2 = 0.5$. Continuous lines indicate approach of Gaussian assumption for PDI (eq.15) while dotted lines indicate actual QAM constellation for PDI (eq.17)

The expectation w.r.t $\mathbf{H}$ seemingly does not have closed from solution but its numerical evaluation and subsequent comparison with the PEP basing on the Gaussian assumption for PDI in Fig. 4 reveals that both have the same diversity thereby demonstrating the loss of one order of diversity in the case of MMSE based detection.

C. Simulation Results

We consider 2 BSs each using BICM OFDM system for downlink transmission using the *de facto* standard, 64 state $(133, 171)$ rate-$1/2$ convolutional encoder of 802.11n standard and the punctured rate $1/2$ turbo code[1] of 3GPP LTE [1]. The MS has two antennas. We consider an ideal OFDM based system (no ISI) and analyze the system in frequency domain. SIMO channel at each sub carrier from BS to MS has iid Gaussian matrix entries with unit variance. Perfect CSI is assumed at the receiver. Furthermore, all mappings of coded bits to QAM symbols use Gray encoding. We consider MMSE approach and the MF based approach. Fig. 5 shows the frame error rates of desired stream for the frame size of 1296 information bits. These simulation results show that the dependence of performance for MMSE detection is insignificant on the rate of the interference stream (constellation size) but its dependence on interference strength is substantial. The performance substantially degrades as the interferer gets stronger. For the proposed approach, a significant improvement is observed in the performance as the interference gets stronger which is in conformity with the PEP results of section IV. The improvement in performance is attributed to the partial decoding of interference [11] which improves as interference gets stronger. It is observed that for a given interference level, the performance is generally degraded as the rate of the interfering stream increases. The performance gap with respect to MMSE decreases as the desired and the interference streams

[1]The LTE turbo decoder design was performed using the coded modulation library www.iterativesolutions.com

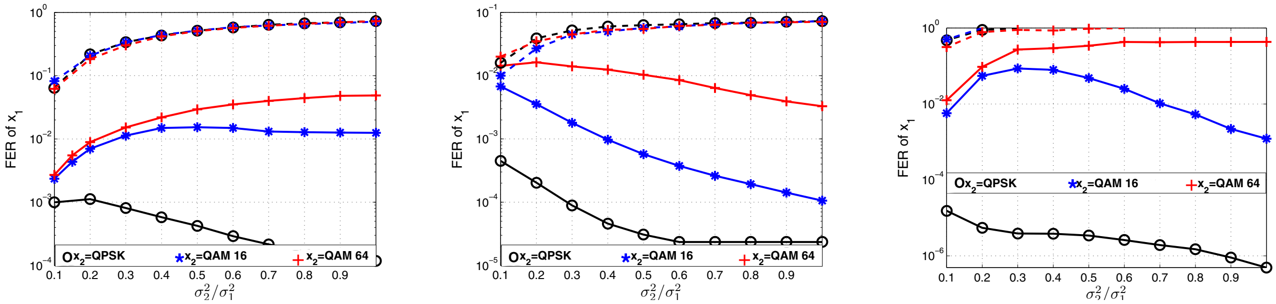

Fig. 5. Desired stream is x_1 while interference stream is x_2. Continuous lines indicate MF based detection while dashed lines indicate MMSE approach. For the left and center figures, x_1 is QAM16 and QAM64 with SNR of 11 and 20 dB respectively while $64-$state, rate $1/2$ convolutional code is used. For right figure x_1 is QAM 64, SNR is 13 dB while LTE turbo code is used with maximum of 5 decoding iterations using max log MAP decoder.

grow in constellation size which can be attributed to the proximity to Gaussian behavior of these larger constellations due to their high peak to average power ratio and to the optimality of MMSE for Gaussian alphabets.

VI. Conclusions

We have focused in this paper on spatial interference cancellation in single frequency cellular networks as 3GPP LTE. We have proposed the use of low complexity MF based detector which not only recuperates the degree of freedom lost by MMSE but also exhibits a coding gain as interference gets stronger which is contrary to the case of MMSE.

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Appendix

Considering (6), $P\left(\hat{x}_{2,k} \neq x_{2,k} | \mathbf{h}_{1,k}, \mathbf{h}_{2,k}, x_{1,k}\right) = \mathcal{P}^{\hat{x_2}}_{x_2|\mathbf{H}_k, x_1}$ is

$$= P\left(-2\Re\left(\left(\mathbf{h}_{1,k}\mathrm{x}_{1,k} + \mathbf{z}_k\right)^\dagger \mathbf{h}_{2,k}\mathrm{x}_{2,k}\right) \geq \left\|\mathbf{h}_{2,k}\mathrm{x}_{2,k}\right\|^2 | \mathbf{H}_k, x_{1,k}\right)$$

$$= Q\left(\sqrt{\frac{\left\|\mathbf{h}_{2,k}\mathrm{x}_{2,k}\right\|^2}{2N_0}} + \sqrt{\frac{2}{N_0}}\Re\left(\frac{\left(\mathbf{h}_{1,k}\mathrm{x}_{1,k}\right)^\dagger \mathbf{h}_{2,k}\mathrm{x}_{2,k}}{\sqrt{\left\|\mathbf{h}_{2,k}\mathrm{x}_{2,k}\right\|^2}}\right)\right)$$

where $\mathrm{x}_{j,k}$ denotes $(x_{j,k} - x_j)$. Using the relation $Q(a+b) \leq Q(a_{min} - |b_{max}|)$ and $\Re\left(\mathbf{a}^\dagger \hat{\mathbf{b}}\right) \leq \|\mathbf{a}\|$ where $\hat{\mathbf{b}}$ is the unit vector we get

$$\mathcal{P}^{\hat{x_2}}_{x_2|\mathbf{H}_k} \leq \frac{1}{2}\exp\left(-\frac{\left\|\mathbf{h}_{2,k}\right\|^2 d_{2,\min}^2}{4N_0} - \frac{\left\|\mathbf{h}_{1,k}\right\|^2 d_{1,\max}^2}{N_0} + \frac{\left\|\mathbf{h}_{2,k}\right\| \left\|\mathbf{h}_{1,k}\right\| d_{2,\min} d_{1,\max}}{N_0}\right) \quad (18)$$

Conditioned on the norm of $\mathbf{h}_{1,k}$ we make two non-overlapping regions as $\left(\left\|\mathbf{h}_{2,k}\right\| \geq \left\|\mathbf{h}_{1,k}\right\| \big| \mathbf{h}_{1,k}\right)$ and $\left(\left\|\mathbf{h}_{2,k}\right\| < \left\|\mathbf{h}_{1,k}\right\| \big| \mathbf{h}_{1,k}\right)$ with the corresponding probabilities as $\mathcal{P}^{<}_{\mathbf{h}_1}$ and $\mathcal{P}^{>}_{\mathbf{h}_1}$. Note that in the first region $\left\|\mathbf{h}_{2,k}\right\| \left\|\mathbf{h}_{1,k}\right\| \leq \left\|\mathbf{h}_{2,k}\right\|^2$ while for second region $\left\|\mathbf{h}_{2,k}\right\| \left\|\mathbf{h}_{1,k}\right\| < \left\|\mathbf{h}_{1,k}\right\|^2$. So

$$\begin{aligned}
\mathcal{P}^{\hat{x_2}}_{x_2} \leq & \frac{1}{2}E_{\mathbf{h}_1}\left[\left(\frac{4N_0}{d_{2,\min}^2 - 4d_{2,\min}d_{1,\max}}\right)^{n_r} \times \right. \\
& \exp\left(-\frac{\left\|\mathbf{h}_{1,k}\right\|^2 d_{1,\max}^2}{N_0}\right) E_{\mathbf{h}_2|\mathbf{h}_1}\left(\mathcal{P}^{<}_{\mathbf{h}_1}\right) + \left(\frac{4N_0}{d_{2,\min}^2}\right)^{n_r} \times \\
& \left. \exp\left(-\left\|\mathbf{h}_{1,k}\right\|^2 \frac{d_{1,\max}^2 - d_{2,\min}d_{1,\max}}{N_0}\right) E_{\mathbf{h}_2|\mathbf{h}_1}\left(\mathcal{P}^{>}_{\mathbf{h}_1}\right)\right] \\
\leq & \frac{1}{2}\left(\frac{4N_0}{\sigma_2^2 \check{d}_{2,\min}^2}\right)^{n_r}\left(\frac{N_0}{\sigma_1^2 \check{d}_{1,\max}^2}\right)^{n_r} \times \\
& \left(\frac{1}{\left(1 - \frac{4\sigma_1 \check{d}_{1,\max}}{\sigma_2 \check{d}_{2,\min}}\right)^{n_r}} + \frac{1}{\left(1 - \frac{\sigma_2 \check{d}_{2,\min}}{\sigma_1 \check{d}_{1,\max}}\right)^{n_r}}\right)
\end{aligned} \quad (19)$$

where we upper bound $E_{\mathbf{h}_2|\mathbf{h}_1}\left(\mathcal{P}^{<}_{\mathbf{h}_1}\right)$ and $E_{\mathbf{h}_2|\mathbf{h}_1}\left(\mathcal{P}^{>}_{\mathbf{h}_1}\right)$ by 1.

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