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Exploitation of Reciprocity in Measured MIMO Channels

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***Abstract*—In this work we study the feasibility of relative calibration to exploit channel reciprocity in a multiple-input multiple-output (MIMO) time division duplex (TDD) system. As an extension to the work of Guillaud et al. (2005), where the method was only applied to SISO and SIMO channels, real two-directional channel measurements were performed using the Eurecom MIMO Openair Sounder (EMOS). The setup considered here is more realistic, since a faster repetition rate, synchronization of transmitter and receiver over the air, and frequency offset, are accounted for. One of the major findings in the paper is a modification to the system model that correctly describes the non-negligible phase drift present in the measurements. We demonstrate that in a single-user MIMO channel and for low signal-to-noise (SNR) ratios, the relative calibration method can increase the capacity close to the theoretical limit.**

I. Introduction

In a wireless communication system, channel state information at the transmitter (CSIT) can greatly improve the capacity of the wireless link. This gain is significant in multiple-input multiple-output (MIMO) communication systems, where a gain in capacity of up to the number of transmit antennas is possible [1]. This gain becomes even more significant in multi-user MIMO systems, thanks to multi-user diversity, which crucially depends on the availability of CSIT.

CSIT can be acquired in several ways. In a frequency division duplex (FDD) system, a limited feedback channel is usually used to feed back a quantized version of the CSI [2]. In a time division duplex (TDD) system, the physical forward and the backward channel are reciprocal since they operate on the same carrier frequency [3]. In reality however, the communication channel does not only consist of the physical channel, but also the antennas, RF mixers, filters, A/D converters, etc., which are not necessarily identical for all devices. Therefore the system needs to be calibrated before channel reciprocity can be exploited.

Contrarily to absolute calibration [4] where external reference sources are used to measure and compensate for the imperfections of each RF chain independently, we focus here on approaches relying on relative calibration [5], [6], [7]. In this context, the calibration relies on the devices exchanging channel measurements, rather than on extra calibration hardware. We study the feasibility of relative calibration using real two-directional channel measurements, which were taken with the Eurecom MIMO Openair Sounder (EMOS). This work can be seen as a continuation of [6], where the method was only applied to SISO and SIMO channels. Furthermore the new measurements are more realistic, since they use a faster repetition rate and synchronization of transmitter and receiver is done over the air as in a real system.

Contributions. In this article, we present an overview of the numerical methods applicable to exploit channel reciprocity through relative calibration in realistic conditions, and demonstrate their application to channel measurements by evaluating the capacity increase due to channel knowledge at the transmitter. We introduce a new way to perform the estimation of the reciprocity parameters directly in the frequency domain, which makes the method more easily applicable to an OFDM system.

Organization. In Section II, we recall the reciprocity model introduced in [6], and propose an extension of this definition to the multi-user case. Section III is an overview of the numerical methods available to implement relative calibration within reasonable complexity. Experimental results based on bi-directional channel measurements are presented in Section IV-C.

II. Reciprocity model

The investigated reciprocity model is based on the technique introduced in [6], whereby the characteristics of the amplifiers at the transmitter and receiver are modelled with linear time-invariant filters. We recall it here.

Let us consider a point-to-point TDD communications system involving two devices denoted A and B. Denote the number of antennas at side A and B with N_A and N_B respectively. As depicted in Fig. 1, the channel as seen by transceivers in the digital domain, is comprised of the effective electromagnetic channel ($C(t)$), assumed identical in both directions, and filters modeling the imperfections of the power amplifiers (T_A, T_B) and low-noise amplifiers (R_A, R_B). In the case where antenna arrays are used, those are vector-input, vector-output filters.

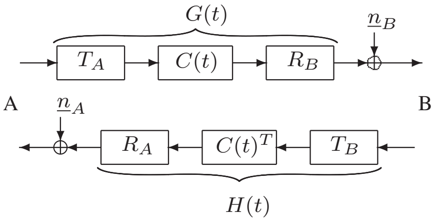

Fig. 1. Reciprocity model for the point-to-point case

The channel impulse response from node A to node B is denoted by $G(t)$, while in the reverse direction, the channel impulse response from node B to node A is denoted by $H(t)$.

In the ideal case, often considered in the literature, T_A, T_B, R_A and R_B are all identity filters. In that case, the channels are perfectly reciprocal ($H(t) = G(t)^T$) without requiring calibration. Conversely, we investigate practical methods applying to non-ideal cases.

As the notations imply, the filters modeling the amplifiers (T_A, T_B, R_A, R_B) are assumed to remain constant over the observed time horizon. For a given frequency f, the channel impulse response as measured by the digital signal processor is the cascade of the transmit filter, the electromagnetic channel, and the receive filter, i.e.

$$\begin{aligned} G(t,f) &= R_B(f)C(t,f)T_A(f), \quad (1) \\ H(t,f) &= R_A(f)C(t,f)^T T_B(f). \quad (2) \end{aligned}$$

In the time domain, a similar set of equations is obtained by replacing products by convolutions and matrices by linear filters in the equations above.

Departing from classical calibration techniques whereby T_A, T_B, R_A and R_B are estimated and compensated individually, [6] introduced the concept of *relative* calibration. It consists in introducing the filters $P_A = R_A^{-T} T_A$ and $P_B = T_B^T R_B^{-1}$. Eliminating C from eqs. (1) and (2), one obtains

$$G(t) = P_B^{-1} H(t)^T P_A. \quad (3)$$

In the considered point-to-point scenario, relative calibration consists in estimating directly P_A and P_B, using eq. (3) and the measured values of $G(t)$ and $H(t)$. Once these are known, the channel can be estimated through reciprocity using (3).

A. Design of reciprocity estimators for the point-to-point case

Let us consider a series of K bi-directional channel measurements, i.e. both $G(t)$ and $H(t)$ are assumed to be measured simultaneously (or with negligible time difference) at times t_i, $i = 1 \ldots K$. We wish to design an estimator for (P_A, P_B) based on the noisy channel measurements $(\hat{G}(t_i), \hat{H}(t_i))$, $i = 1 \ldots K$. Considering one single frequency, and dropping the index f for notational simplicity, the following estimator minimizes the objective function suggested by the reciprocity relationship $P_B G(t) - H(t)^T P_A = 0$. Since this relationship only applies to the true channels, we allow for compensation terms $\tilde{G}(t)$ and $\tilde{H}(t)$ to be added to $\hat{G}(t)$ and $\hat{H}(t)$, in the spirit of the Total Least-Squares (TLS) technique [8], in order to account for the uncertainty due to the estimation noise:

$$\begin{aligned} &(\hat{P}_A, \hat{P}_B) = \\ &\underset{\substack{(P_A, P_B, \tilde{G}_i, \tilde{H}_i), \\ \text{s.t.} ||P_A||_2^2 = 1}}{\operatorname{argmin}} \sum_{i=1}^{K} \left|\left| P_B \left(\hat{G}(t_i) + \tilde{G}_i \right) - \left(\hat{H}(t_i) + \tilde{H}_i \right)^T P_A \right|\right|_2^2 \\ &\qquad + \left|\left| \tilde{G}_i \right|\right|_2^2 + \left|\left| \tilde{H}_i \right|\right|_2^2 . \quad (4) \end{aligned}$$

It can be seen from eq. (4) that if the compensation terms $\tilde{G}$ and $\tilde{H}$ are exactly equal to the measurement noise, the first norm vanishes. The condition $||P_A||_2^2 = 1$ ensures that the trivial solution $(P_A, P_B) = (0, 0)$ is avoided. This condition is added without loss of generality since the set of parameters (P_A, P_B) is over-determined: it can be seen from eq (3) that the family of solutions where P_A and P_B are multiplied with the same scalar factor indeed represents a single solution to the problem at hand.

B. Extension to multiple users

The generalization of the above model to the case of multiple users is straightforward. Here we address the point-to-multipoint case, which has practical implications in cellular networks. Let us assume that device A is involved in multiple bidirectional TDD communications with N devices denoted $B_1 \ldots B_N$ (in the context of cellular communications, A can be understood as the base station, while B_n are the mobile nodes in the cell). Denoting by T_{B_n} and R_{B_n} the respective gains of transmit and receive RF subsystems of device n, the measured channels between A and B_n are

$$\begin{aligned} G_n(t) &= R_{B_n} C_n(t) T_A, \quad (5) \\ H_n(t) &= R_A C_n(t)^T T_{B_n}. \quad (6) \end{aligned}$$

Denoting $P_{B_n} = T_{B_n}^T R_{B_n}^{-1}$, we have as before $G_n(t) = P_{B_n}^{-1} H_n(t)^T P_A$. Note that T_A, R_A are common to all links, while T_{B_n}, R_{B_n} are specific to each node B_n. The estimator for the multi-user case is presented in eq. (7), as a multi-user generalization of (4).

$$(\hat{P}_A, \hat{P}_{B_1} \dots \hat{P}_{B_N}) = \underset{\substack{(P_A, P_{Bn}, \tilde{G}_{n,i}, \tilde{H}_{n,i}) \\ \text{s.t.} ||P_A||_2^2=1}}{\operatorname{argmin}} \sum_{\substack{n=1\dots N \\ i=1\dots K}} \left|\left| P_{Bi} \left(\hat{G}_n(t_i) + \tilde{G}_{n,i} \right) - \left(\hat{H}_n(t_i) + \tilde{H}_{n,i} \right)^T P_A \right|\right|_2^2 + \left|\left| \tilde{G}_{n,i} \right|\right|_2^2 + \left|\left| \tilde{H}_{n,i} \right|\right|_2^2 . \quad (7)$$

III. Approaches to solve the minimization problem

The quartic objective function defined in (4) makes the solution of the optimization problem non trivial. For relatively small problem sizes, this is solvable by standard non-convex optimization methods, although the complexity currently prevents any real-time exploitation. Another avenue to reduce the complexity of the considered estimation problem is to simplify the model above.

A. Frequency-flat SISO case

In the SISO case, the filters P_A and P_B are scalars and thus the products in (3) commute. Letting $P = P_B^{-1} P_A$ yields $G(t) = H(t)P$. Since both $G(t)$ and $H(t)$ are affected by estimation errors, the estimate of P can be estimated as the classical total least-squares solution: collecting K pairs of measurements in the vectors $\hat{\mathbf{g}} = [\hat{G}(t_1), \dots, \hat{G}(t_K)]^T$ and $\hat{\mathbf{h}} = [\hat{H}(t_1), \dots, \hat{H}(t_K)]^T$, $\hat{P}$ is estimated as

$$\underset{\tilde{\mathbf{h}}, \tilde{\mathbf{g}}, P}{\operatorname{argmin}} ||\tilde{\mathbf{h}}||_2^2 + ||\tilde{\mathbf{g}}||_2^2 \quad \text{s.t.} \quad (\hat{\mathbf{h}} + \tilde{\mathbf{h}})P = (\hat{\mathbf{g}} + \tilde{\mathbf{g}}). \quad (8)$$

This TLS problem can be easily solved using the classical solution based on the singular value decomposition (SVD) [9].

B. MIMO with Diagonal Reciprocity Matrices

The model of eqs. (1) and (2) incorporates cross-talk between all antenna pairs in an array. In reality, the effect of this phenomenon is likely to be negligible, making P_A and P_B diagonal. This decouples the MIMO problem (3) into $N_A N_B$ SISO problems

$$[G(t)]_{i,j} = [P_B^{-1}]_{i,i} [H(t)]_{j,i} [P_A]_{j,j}, \quad (9)$$

which are solved as in Section III-A.

C. Frequency-selective SISO case

The case of the frequency selective channel is not conceptually different from the flat-fading problem, except for the added complexity due to the increased dimensions. Two approaches can be envisioned:

- A per-subband approach, in which the reciprocity estimator is applied independently to each subband, i.e.,

  $$G(t, f) = H(t, f)P(f). \quad (10)$$

  The complexity of this approach scales linearly with the transmission bandwidth, however it fails to exploit the correlation across subbands between the reciprocity parameters. This correlation is expected to be high, since the impulse responses of P_A and P_B are expected to be extremely short in practice.
- Estimating the reciprocity parameters in the time domain by transforming (10) into the time domain:

  $$G(t, \tau) = H(t, \tau) * P(\tau), \quad (11)$$

  Under the assumption that $P(\tau) = P_B(\tau)^{-1} P_A(\tau)$ is a FIR filter, a solution to this problem is proposed in [6], based on the deconvolution algorithm of [10].

Those two approaches outlined above are compared in Section IV over real measured data.

IV. Experimental validation

A. Description of the Channel Measurements

The measurements were conducted with the Eurecom MIMO Openair Sounder (EMOS) [11]. The EMOS is a stripped-down version of the Eurecom OpenAirInterface.org platform that transmits additional pilot symbols instead of the scheduled access channels. It can be configured for multi-user and two-way channel sounding. The current EMOS hardware consists of laptop computers with Eurecom's dual-RF data acquisition cards and two clip-on 3G Panorama antennas. The cards operate at 1.900–1.920 GHz with 5 MHz channels[1]. For the evaluations we use a measurement taken in the Eurecom laboratory, were both nodes are stationary and in the same room.

The EMOS uses an orthogonal frequency division multiplexing (OFDM) modulated sounding sequence with 256 subcarriers (out of which 160 are non-zero) and a cyclic prefix length of 64. The subcarriers of the pilot symbols are multiplexed over the transmit antennas to ensure orthogonality in the spatial domain. One frame is 64 OFDM symbols (2.667 ms) long and is divided in a downlink (DL) transmission time interval (TTI) and an uplink (UL) TTI of equal length. The parameters are summarized in Table I.

For the two-way measurements, one node is configured as a base station (BS) and one node as a user terminal (UT). The UT estimates the DL while the BS estimates the UL. Both nodes store their estimates to disk. Since the BS also transmits a frame number using

[1]Eurecom has a frequency allocation for experimentation around its premises.

Parameter	Value
Center Frequency	1917.6 MHz
Sampling Rate	6.5 MHz
FFT size	256
Number of Subcarriers (Q)	160
Useful Bandwidth	4.0625 MHz
Max. Transmit Power	20 dBm
Number of Antennas (M)	2
Frame length (UL and DL)	2.667 ms

TABLE I
PARAMETERS OF THE EURECOM MIMO OPENAIR SOUNDER (EMOS).

its broadcast channel, the measurements can be aligned in a post-processing step.

Note that the synchronization of transmitter and receiver is done over the air. Also, due to the limited accuracy of the local oscillators on the cards, there are slight sampling offsets between transmitter and receiver. These sampling offsets manifest themselves as phase drifts in the frequency domain. These phase drifts are exactly antipodal in the uplink and the downlink and can be quite strong, such that neglecting them yields unusable results.

One way to overcome this problem is to resample one of the estimated channels to the respective other sample rate. However, this requires exact knowledge of the difference in sampling rates, which is not trivial to estimate. If the time-variation of the physical channel is much less than the frequency offsets of the cards, another approach is to model the uplink and downlink as conjugates of each other, i.e.,

$$G(t,f) = H(t,f)^*\, P(f). \tag{12}$$

In this paper we take the second approach, because we only consider stationary measurements.

B. Performance Metrics

The metric adopted here to evaluate the quality of the estimation of the reciprocal channel is the gain in channel capacity. Assume that we want to transmit from A to B over the channel G. We can distinguish three cases, depending on whether perfect, partial, or no channel state information is available at the transmitter (CSIT). For each case we evaluate the maximum achievable mutual information:

1) G is known only at B: the best is to choose the transmit covariance matrix $R_{\mathrm{Tx}} = I$. The capacity is classically $C_1 = \sum_i \log_2\left(1 + \frac{E_S}{N_A N_0}\lambda_i\right)$, where λ_i are the eigenvalues of GG^H and E_S/N_0 the SNR.
2) G is known to A and B: $C_2 = \sum_i \log_2\left(1 + \frac{E_S \gamma_i}{N_A N_0}\lambda_i\right)$, where the γ_i are obtained from the waterpouring algorithm [1].

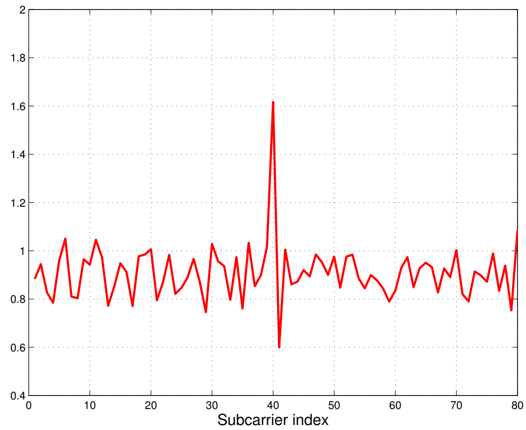

Fig. 2. Estimated frequency response of the reciprocity filter

3) A has only knowledge of G_{est} (estimated from the reciprocity matrices) and B knows G. We assume that we use the same transmission scheme as in case 2, but the transmit covariance $R_{\mathrm{Tx,est}}$ is calculated from G_{est} instead of G. $C_3 = \log_2 \det\left(I + \frac{E_S}{N_A N_0} G R_{\mathrm{Tx,est}} G^H\right)$.

C. Results

In this section we show feasibility of the reciprocity estimation using the channel measurements described in Section IV. We show results for wideband SISO channels as well as wideband MIMO channels with diagonal reciprocity matrices.

For the analysis in this paper, the measurements were split in blocks of approximately 200 frames each. The reciprocity matrices $P(f)$ were estimated using the first 20 consecutive frames of a block. Subsequently we use these estimates to calculate the downlink channel from the uplink channel for the rest of the frames

$$G_{\mathrm{est}}(t,f) = H(t,f)^* P(f). \tag{13}$$

1) SISO: By default we estimate the reciprocity filter $P(f)$ in the frequency domain using the SVD-based TLS solution from [9] per subcarrier (cf. Section III-A). One particular example of such an estimate is shown in Fig. 2. For comparison we transform this estimate to the time domain and compare it with the output of the time domain estimator [6] in Figs. 3 and 4. It can be seen that the two results are very similar, confirming the frequency domain algorithm proposed in this paper. Further it can be seen that the concatenation of the filters in nodes A and B deviates significantly from a Dirac impulse. This means that these filters cannot be neglected in a real system!

2) MIMO with diagonal reciprocity matrices: In the case of MIMO channel, where diagonal reciprocity ma-

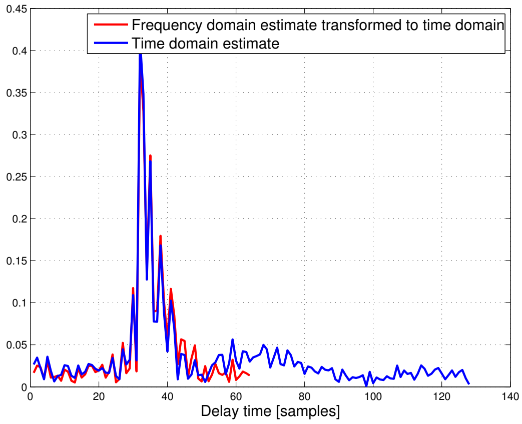

Fig. 3. Estimated impulse response of the reciprocity filter

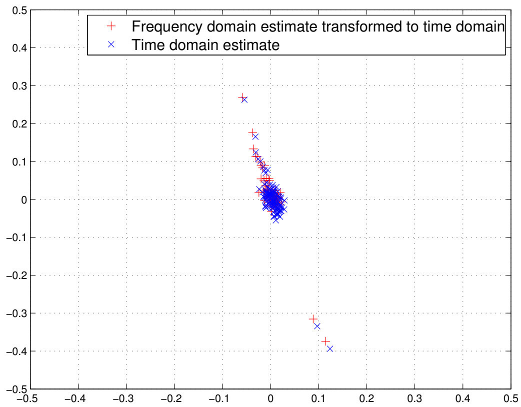

Fig. 4. IQ plot of the estimated impulse response of the reciprocity filter

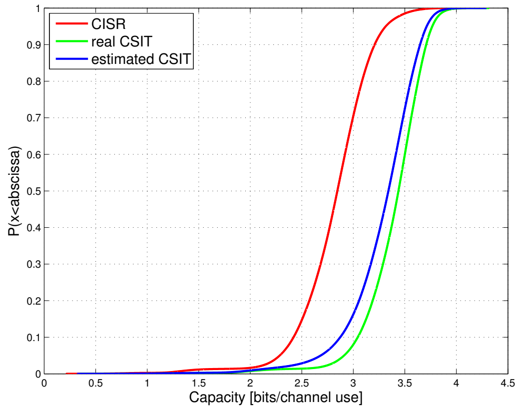

Fig. 5. CDF of the capacity of the MIMO channel at 10 dB SNR

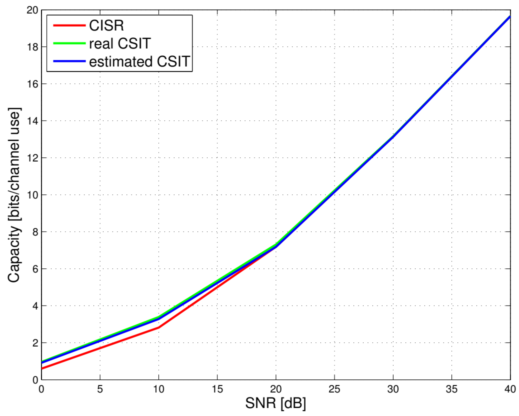

Fig. 6. Ergodic capacity of the MIMO channel

trices are assumed, the MIMO problem is decoupled in a series of SISO problems (cf. Section III-B).

In Fig. 6 we plot the ergodic (mean) capacity for different values of SNR, for the three cases outlined in Section IV-B. It can be seen that for high SNR the capacity of with and without CSIT become very similar, so obviously here hardly any performance improvements are possible. However, for low SNR, the channel knowledge obtained by the reciprocity estimation brings some gain. If we look closer at the low SNR case in Fig. 5, where we plot the CDF of C_1, C_2, and C_3 for an SNR of 10 dB using all the subcarriers and all the frames as samples, it can be seen that we almost reach the capacity curve with perfect CSIT. It can be concluded that for this case large performance gains are possible.

V. Conclusions

In this paper we have shown how to practically exploit channel reciprocity in a MIMO TDD system in order to obtain channel state information at the transmitter. We have verified the method using real two-way MIMO channel measurements that were conducted using the Eurecom MIMO Openair Sounder (EMOS). It was shown that the method is able to increase the capacity of a single-user MIMO system close to the theoretical limit.

Compared to previous work [6], the channel measurements in this paper are more realistic as they are synchronized over the air as in a real system. This results in residual frequency errors, which have to be compensated. In the case of stationary measurements this can easily be done by modeling the uplink and downlink as complex conjugates. In future work we will explore methods how to cope with these effects in non-stationary cases.

Future work will also include the extension to the multi-user MIMO case, where and even higher increase in capacity can be expected. Also, on the long term, we would also like to incorporate these methods in the real-time MODEM of Eurecom's OpenAirInterface.org platform.

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