

Hybrid TOA/AOD/Doppler-Shift Localization Algorithm for NLOS Environments

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Abstract—Accurate location estimation of the Mobile Terminal in a strictly Non-Line-of-Sight propagation environment is still a challenging problem. Existing techniques that attempt to tackle this problem, either perform poorly or are not very practical due to their high computational complexity. In this work we present a low-complexity 2-step approach for the joint estimation of the location and the speed in the presence of nuisance parameters. This hybrid method effectively extracts the location-related information contained in the Doppler Shift, without explicit knowledge of the AOA. It then combines this information with the AOD and the TOA and outputs a Least-Square estimate. Despite its simplicity, it can achieve an accuracy of 10cm in 75% of the cases for sufficiently high SNR¹.

I. INTRODUCTION

Traditional network-based geometrical localization techniques date back to the late sixties [1], [2]. They are based on the assumption of an existing Line-of-Sight (LOS) signal component in the wireless link between the transmitter and a few receivers - base stations (BS). However, in dense urban and indoor propagation environments, a LOS signal component rarely exists. The received signal usually consists of multipath components (MPC) created by reflection, diffraction and scattering of the transmitted signal on various objects, often and herein called scatterers. If the absence of a LOS signal component is ignored and traditional geometrical techniques are directly applied using one or more of the MPC, the performance is very poor. This is not surprising, since geometrical techniques are based on measured location and motion dependent parameters (LMDP), such as the angle of arrival (AOA), the angle of departure (AOD), the time of arrival (TOA) and the Doppler shift and in Non-Line-of-Sight (NLOS) environments, the LMDP of the MPC exhibit an error bias with respect to the true values that would correspond to a LOS path. This error bias results in large location estimation errors, except for very few scenarios, like e.g. when the scatterers are very closely co-located with the mobile terminal (MT).

The problem of localizing in NLOS environments has been addressed in more recent publications. Various techniques that

assume the existence of a LOS path between the MT and at least a few BS are based on identifying and removing the NLOS measurements [3], [4], weighting these measurements appropriately in order to minimize their impact [5] or enhancing performance assuming knowledge of the NLOS errors statistics [6]. On the other hand, techniques designed for localizing in strictly NLOS environments, include but are not limited to solving a constrained optimization problem where the error bias leads to inequalities instead of equalities [7], [8] or introducing a propagation model, creating a mapping between the LMDP of the MPC and the the MT coordinates and estimating the latter in the presence of nuisance parameters [9], [10].

In this contribution we adopt this last approach. We utilize the single-bounce model (SBM) and integrate it with a linear mobility model to describe accurately NLOS environments that change with time due to the movement of the MT. This model enables us to express four different subsets of LMDP, namely the AOA, AOD, TOA and Doppler Shifts as a function of the MT coordinates, the MT speed and the coordinates of the scatterers. However, in contrast to our previous work where we have proposed a maximum likelihood (ML) estimate based on knowledge of all of the above subsets of LMDP, herein we ignore the existence of the AOA (angle of the impinging wave at the MT, considering downlink transmission). The scenario is more realistic, considering that an AOA estimate might not be available or might be totally unreliable due to e.g. lack of calibration of the antenna array at the receiver or completely modified antenna pattern because of the way the user is holding his device. Moreover, although the Doppler shift for each MPC explicitly depends on its AOA, we show how to effectively extract useful - for localization purposes - information from this LMDP and combine it with the rest of the available information to form a least-square (LS) solution for the estimation of the coordinates of the scatterers in a first instance and then also the coordinates and the speed of the MT.

In a static SBM model, like the one the authors consider in [9], estimates of AOA, AOD and TOA are required for the MT location to be identifiable. Otherwise, the number of unknowns is always more than the number of available equations, no matter how rich the scattering environment might be. In a recent study for identifiability in NLOS SBM-based

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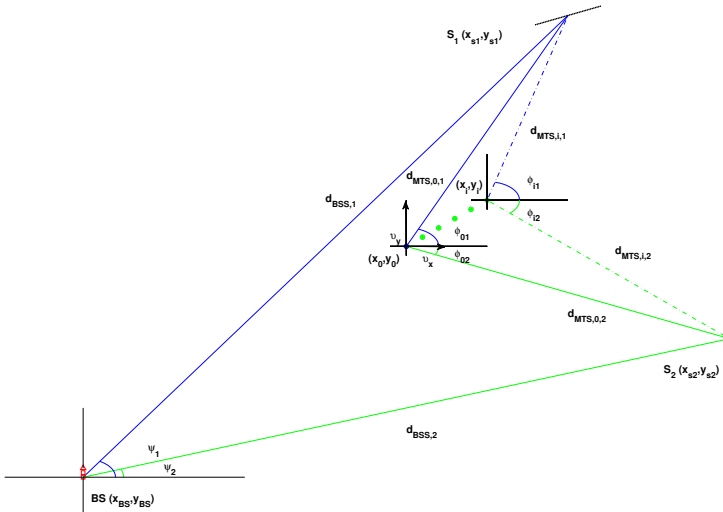


Fig. 1. Single Bounce model

localization [11], it was shown that for the dynamic channel model which is also used in this contribution, identifiability is feasible even in the absence of knowledge of the AOA. The reduced set of data however, results in a degradation in performance. Nevertheless the method performs accurately in approximately 75% of the cases for sufficiently high SNR, as will be shown in the simulation results section.

The rest of the paper is organized as follows: In the following section we present the channel model along with the scenario considered. In section III we find a closed-form solution for the coordinates of the scatterers. A closed-form solution for the coordinates and the speed of the MT is then given in section IV. Simulation results are presented in section V followed by conclusions and general remarks in section VI.

Notation: Throughout the paper, upper case and lower case boldface symbols will represent matrices and column vectors respectively. $(\cdot)^t$ will denote the transpose of any vector or matrix and $\|\cdot\|_2$ will denote its $L2$ norm. For time dependent scalars, vectors and matrices, the first subscript will denote time instant and the second subscript will denote the corresponding MPC while for constant scalars, vectors and matrices the subscript will denote the corresponding MPC. The difference between two values of a variable a in time instances i and i' will be denoted as $\delta a_{ii',j} = a_{i,j} - a_{i',j}$ while the difference between the values of variable a for two different MPC (at time instant i if time dependent) will be denoted as $\bar{a}_{i,j,j'} = a_{i,j} - a_{i,j'}$.

II. CHANNEL MODEL

In the scenario considered in this paper, a single BS is communicating with the MT through a multipath propagation channel with N_s scatterers that give rise to N_s distinct MPC at each time instant. In other words each MPC of the received signal is assumed to have bounced only once in one of the scatterers and thus there is a one-to-one correspondence between a scatterer and an MPC. Despite its simplicity, this

SBM can be widely applied, since in a physical propagation environment, the more bounces, the larger the attenuation will be, not only because the scatterer absorbs some of the signal's energy but also because more bounces usually implies a longer path length. The method and the results presented apply also to the case where the MT is communicating with multiple BS through channels that have 1 or more MPC.

The BS and the scatterers' location is fixed. The MT moves on a line with constant speed for the small period of time during which the estimation process takes place, so that its position vector \mathbf{p}_i at time i is given by:

$$\begin{bmatrix} x_i \\ y_i \end{bmatrix} = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} + \begin{bmatrix} v_x \\ v_y \end{bmatrix} \delta t_{i0}, \quad 0 \leq i < N_t \quad (1)$$

where $\delta t_{i0} = t_i - t_0$. The scatterers are uniformly distributed inside an ellipse with the BS and the (initial position of) MT placed at the foci.

A propagation channel with 2 MPC described by the SBM is depicted in figure 1. Let c denote the speed of light and f_c the carrier frequency. Also, with respect to figure 1, denote by $\phi_{i,j}$, ψ_j and $d_{BSS,j}$ and $d_{MTS,i,j}$, the AOA, AOD, distance between the BS and scatterer and distance between scatterer and the MT respectively, at time i and for MPC j . As mentioned in the introduction, only the AOD, ψ_j , the TOA $\tau_{i,j}$ or equivalently the total distance

$$d_{i,j} = \frac{1}{c} \tau_{i,j} = d_{BSS,j} + d_{MTS,i,j} \quad (2)$$

and the Doppler shift

$$f_{d,i,j} = \frac{f_c}{c} v \cos(\phi_{ij} - \alpha_v) \quad (3)$$

of each MPC is available. While the first two subsets of LMDP are shown in figure 1, Doppler shifts are not. Therefore, in order to be able to use trigonometric and geometric laws and derive a closed-form solution for the location estimation problem that uses all available data, a geometric interpretation of Doppler shift is required. Figure 2 depicts only the paths at time instances i and i' due to one of the two scatterers of figure 1, zoomed at the MT side. In this figure we have introduced two new kinds of line segments, z_i and u_i . Simply by observing the figure, it becomes obvious that

$$z_{i,j} = \frac{c}{f_c} f_{d,i,j} dt_{ii'}, \quad \forall j \quad (4)$$

i.e. $z_{i,j}$ is equal to the Doppler shift scaled by some known constant. Therefore estimates of line segments $z_{i,j}$ are available.

III. LS SOLUTION FOR THE NUISANCE PARAMETERS

Based on the previous observation we can formulate a LS solution for the nuisance parameters $\mathbf{p}_{nuis} = [d_{BSS,1}, \dots, d_{BSS,N_s}]^t$, i.e. since the AOD are known, we can solve for the location of the scatterers in polar coordinates. From figure 2, we have for all scatterers

$$z_{i,j}^2 + u_{i,j}^2 = d_{MTS,i',j}^2 - (d_{MTS,i,j} - z_{i,j})^2 = l_{i,i'}^2 \quad (5)$$

$$z_{i',j}^2 + u_{i',j}^2 = d_{MTS,i,j}^2 - (d_{MTS,i',j} + z_{i',j})^2 = l_{i',i}^2 \quad (6)$$

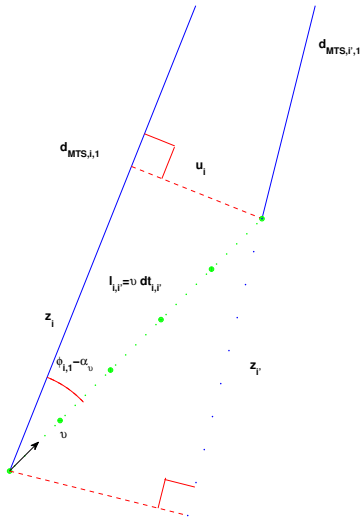


Fig. 2. Geometric Interpretation of Doppler Shift

Eliminating the common term $l_{i,i'}^2$ on the right hand side of the above equations, we get one constraint that must be satisfied by the distances $d_{MTS,i,j}$ and $d_{MTS,i,j'}$ of two scatterers with the MT:

$$d_{MTS,i',j}^2 - d_{MTS,i,j}^2 + 2z_{i,j}d_{MTS,i,j} = d_{MTS,i',j'}^2 - d_{MTS,i,j'}^2 + 2z_{i,j'}d_{MTS,i,j'} \quad (7)$$

and another one by replacing $z_{i,j}$ with $z_{i',j}$ and $z_{i,j'}$ with $z_{i',j'}$. Combining this two identical constraints and introducing the following differences of quantities between two time instances $\delta d_{i',j} = d_{i',j} - d_{i,j} = d_{MTS,i',j} - d_{MTS,i,j}$, $\delta z_{i',j} = z_{i',j} - z_{i,j}$ for ease of notation, we get the following linear constraint after some algebraic computations:

$$\delta z_{i',j}d_{MTS,i,j} - \delta z_{i',j'}d_{MTS,i,j'} = \delta d_{i',j}z_{i',j'} - \delta d_{i',j}z_{i,j} \quad (8)$$

Alternatively, to solve for the nuisance parameters, we can substitute $d_{MTS,i,j}$ using (2) and introduce $w_{i,j} = d_{i,j}z_{i,j}$ to get the following linear constraint:

$$\delta z_{i',j}d_{BSS,j} - \delta z_{i',j'}d_{BSS,j'} = \bar{w}_{i,j'j} - \bar{w}_{i,j'j} = \delta \bar{w}_{ii',j'j} \quad (9)$$

From 9 we have $N_s - 1$ independent equations for every pair of measurements. In total we can have $(N_s - 1) \times (N_t - 1)$ linear independent equations and thus we can formulate a LS problem. Define the matrix

$$\mathbf{Z}_i = \begin{bmatrix} z_{i,1} & -z_{i,2} & 0 & \cdots & 0 \\ 0 & z_{i,2} & -z_{i,3} & \vdots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & z_{i,N_s-1} & -z_{i,N_s-1} \end{bmatrix} \quad (10)$$

and the matrix containing differences $\Delta \mathbf{Z}_{ii'} = \mathbf{Z}_i - \mathbf{Z}_{i'}$. Furthermore define the vector

$$\bar{\mathbf{w}}_i = [\bar{w}_{i,12}, \dots, \bar{w}_{i,(N_s-1)N_s}]^t \quad (11)$$

and the corresponding vector containing differences $\delta \bar{\mathbf{w}}_{ii'} = \bar{\mathbf{w}}_i - \bar{\mathbf{w}}_{i'}$. Considering $N_t - 1$ time differences and stacking vertically all $\Delta \mathbf{Z}_{ii'}$ and $\delta \bar{\mathbf{w}}_{ii'}$ to create

$$\Delta \mathbf{Z} = [\Delta \mathbf{Z}_{12}^t, \dots, \Delta \mathbf{Z}_{(N_t-1)N_t}^t]^t \quad (12)$$

$$\delta \bar{\mathbf{w}} = [\delta \bar{\mathbf{w}}_{12}^t, \dots, \delta \bar{\mathbf{w}}_{(N_t-1)N_t}^t]^t \quad (13)$$

we get in matrix form the following set of linear equations:

$$\Delta \mathbf{Z} \mathbf{p}_{nuis} = \delta \bar{\mathbf{w}} \quad (14)$$

The LS estimate of \mathbf{p}_{nuis} is simply given by:

$$\hat{\mathbf{p}}_{nuis} = (\Delta \mathbf{Z}^t \Delta \mathbf{Z})^{-1} \Delta \mathbf{Z}^t \delta \bar{\mathbf{w}} \quad (15)$$

IV. LS SOLUTION FOR THE PARAMETERS OF INTEREST

Since the scatterers' locations and the distances between the scatterers and the MT have been estimated, the problem of estimating the location of the MT at time i is essentially equivalent to solving a TOA localization problem with N_s BS, all of which are in a LOS environment. One very appealing LS solution for this problem, is the so-called lines-of-position (LOP) approach that was presented in [12] for static channels. We can modify this method to estimate $\mathbf{p}_{int} = [x_0, y_0, v_x, v_y]^t$, i.e. to jointly estimate the initial position vector \mathbf{p}_0 and the speed vector v of the MT. The scatterers' cartesian coordinates are

$$x_{sj} = d_{BSS,j} \sin(\psi_j) \quad (16)$$

$$y_{sj} = d_{BSS,j} \cos(\psi_j) \quad (17)$$

where the estimate $d_{BSS,j}$ is the j th entry of $\hat{\mathbf{p}}_{nuis}$ and we have omitted the (\cdot) in notation for clarity. Define the following differences

$$\bar{d}_{MTS,i,jj'} = d_{MTS,i,j}^2 - d_{MTS,i,j'}^2 \quad (18)$$

$$\bar{x}_{s jj'} = x_{sj} - x_{sj'} \quad (19)$$

$$\bar{y}_{s jj'} = y_{sj} - y_{sj'} \quad (20)$$

$$\bar{x}_{s jj'}^2 = x_{sj}^2 - x_{sj'}^2 \quad (21)$$

$$\bar{y}_{s jj'}^2 = y_{sj}^2 - y_{sj'}^2 \quad (22)$$

$$q_{i,jj'} = \bar{d}_{MTS,i,jj'} - \bar{x}_{s jj'} - \bar{y}_{s jj'} \quad (23)$$

Then according to the LOP method, the coordinates of the MT at time i satisfy:

$$\bar{x}_{s jj'} x_i + \bar{y}_{s jj'} y_i = \frac{1}{2} q_{i,jj'} \quad (24)$$

Substituting (1) in (24) we get:

$$\bar{x}_{s jj'} x_0 + \bar{y}_{s jj'} y_0 + \bar{x}_{s jj'} \delta t_{i0} v_x + \bar{y}_{s jj'} \delta t_{i0} v_y = \frac{1}{2} q_{i,jj'} \quad (25)$$

By collecting $N_s - 1$ independent equations for time i we can compose the vectors

$$\mathbf{q}_i = [q_{i,12}, \dots, q_{i,(N_s-1)N_s}]^t \quad (26)$$

$$\bar{\mathbf{x}}_s = [\bar{x}_{s12}, \dots, \bar{x}_{s(N_s-1)N_s}]^t \quad (27)$$

$$\bar{\mathbf{y}}_s = [\bar{y}_{s12}, \dots, \bar{y}_{s(N_s-1)N_s}]^t \quad (28)$$

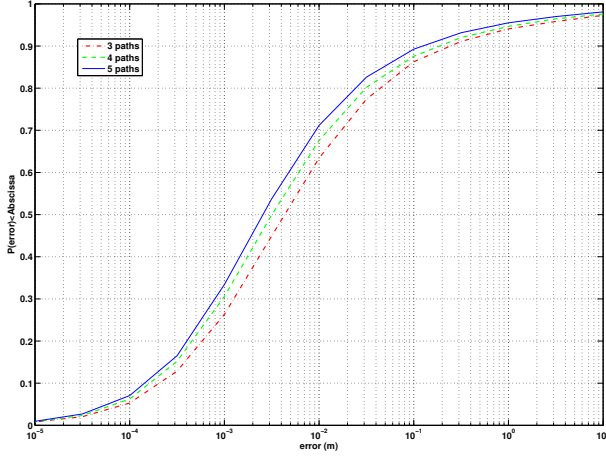


Fig. 3. CDF of $\|\tilde{\mathbf{p}}_{nuis}\|_2$

and from the last two vectors we can create the matrix

$$\mathbf{S}_i = \begin{bmatrix} \bar{\mathbf{x}}_s & \bar{\mathbf{y}}_s & \delta t_{i0} \bar{\mathbf{x}}_s & \delta t_{i0} \bar{\mathbf{y}}_s \end{bmatrix}. \quad (29)$$

Then by vertical concatenation of the above vectors and matrices for all time instances we can obtain

$$\mathbf{q} = [\mathbf{q}_0^t, \dots, \mathbf{q}_{N_t-1}^t]^t \quad (30)$$

$$\mathbf{S} = [\mathbf{S}_0^t, \dots, \mathbf{S}_{N_t-1}^t]^t. \quad (31)$$

Therefore we can again formulate a LS problem by stacking $(N_s - 1) \times N_t$ linear equations in the following matrix form

$$\mathbf{S} \mathbf{p}_{int} = \mathbf{q} \quad (32)$$

that we solve directly to obtain an estimate for \mathbf{p}_{int}

$$\hat{\mathbf{p}}_{int} = (\mathbf{S}^t \mathbf{S})^{-1} \mathbf{S}^t \mathbf{q} \quad (33)$$

V. SIMULATION RESULTS

As mentioned in the introduction, the results in [11] indicate that there is a noticeable degradation in the performance of ML location estimation in NLOS environments, when fewer than the four different subsets of LMDP are available in a NLOS SBM-based localization technique. This degradation is also expected for the method presented herein, which is a suboptimal 2-step LS solution. However due its low computational cost and its accuracy at high Signal-to-Noise Ratio (SNR), this method remains a very attractive solution. To demonstrate its performance we run $M = 10,000$ independent trials and derived the cumulative distribution function (CDF) of the location and speed estimation errors for propagation environments with various numbers of MPC. The propagation environment is equivalent to a pico-cell, with the BS located at the origin and the MT located at $\mathbf{p}_0 = [x_0, y_0] = [20, 30]$. To completely define the ellipse, in which the scatterers are located, for the simulations, we assume that the ratio of the maximum MPC length d_{max} over the length of the LOS path $d_{LOS} = d_{BSMT}$ is drawn from a uniform distribution with

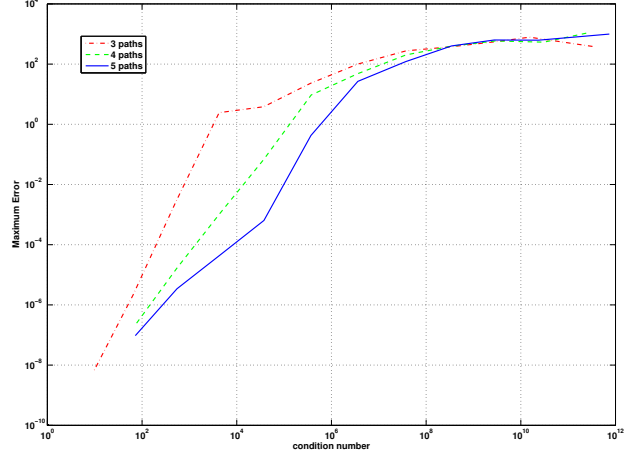


Fig. 4. $|\tilde{d}_{BSS,max}|$ versus condition number of $\Delta \mathbf{Z}$

support region $[1.5, 2]$. The method will yield similar results for any arbitrarily shaped and sized area. The magnitude of the MT speed v is drawn from a uniform distribution with support region $[1, 2]$ in m/sec , which corresponds to average walking speed and its direction α_v is drawn from a uniform distribution with support region $[0, 2\pi]$. The carrier frequency considered is $f_c = 1.9GHz$ and the total observation time was $t = 1sec$. Therefore we require the movement to be linear during only this small period of time, which is a very reasonable assumption. During the observation time $N_t = 20$ uniformly spaced ($\delta t_{(i+1)i} = 50msec \forall i$) measurements of the $3N_s$ LMDP were considered, and the $3N_s N_t$ noisy LMDP estimates are considered independent Gaussian random variables with mean their true value and standard deviation

$$\sigma_d = \sigma_{f_d} = \sigma_\psi = 10^{-6} \quad (34)$$

for all time instances and all scatterers. These values correspond to roughly $20dB$, according to the results in [13] where an ESPRIT-Based algorithm was implemented for the estimation of the LMDP. In figure 3 the CDF of the estimation error in the distances between the scatterers and the MT, which is equal to

$$\|\tilde{\mathbf{p}}_{nuis}\|_2 = \|\hat{\mathbf{p}}_{nuis} - \mathbf{p}_{nuis}\|_2 = \sqrt{\frac{1}{N_s} \sum_{j=1}^{N_s} (\hat{d}_{BSS,j} - d_{BSS,j})^2} \quad (35)$$

is plotted for the case of 3,4 and 5 MPC. Although the accuracy of the nuisance parameters estimation is of no importance, this figure not only serves as an indication of the performance of the first LS estimation but also gives a rough estimate of the percentage of the times when the method fails completely. Indeed, although the plot is truncated for clarity, there is a small percentage of approximately 2–3% for which the error's RMSE can be hundreds of meters. However, this only occurs when the matrix $\Delta \mathbf{Z}$ in (14) is extremely ill-conditioned. Thus, this situation can be predicted a priori by

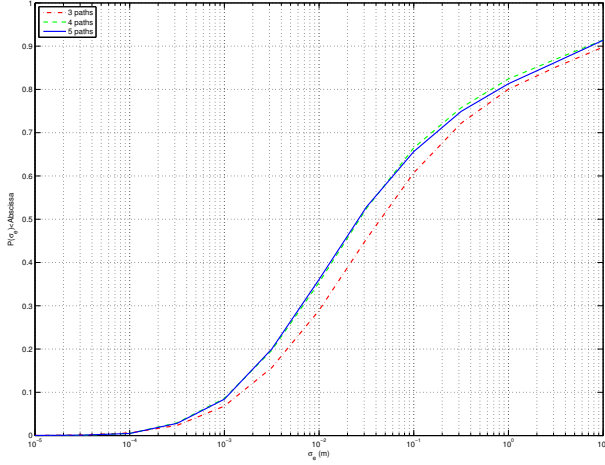


Fig. 5. CDF of $\|\hat{\mathbf{p}}_0\|_2$

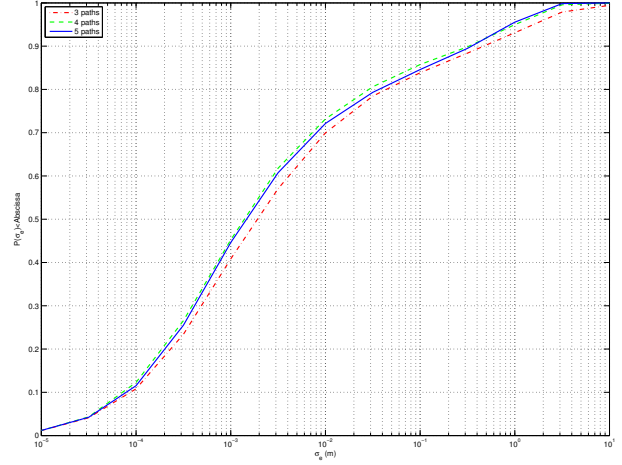


Fig. 6. CDF of $\|\hat{v}\|_2$

examining the condition number $c_{\Delta\mathbf{Z}}$ of $\Delta\mathbf{Z}$.

This becomes obvious from figure 4, where we have plotted the maximum absolute error among the estimated distances, $|\hat{d}_{BSS,max}|$, versus $c_{\Delta\mathbf{Z}}$, after some averaging to remove small-scale fluctuations so that the dependency becomes apparent. According to this figure, estimates that correspond to $c_{\Delta\mathbf{Z}}$ with order of magnitude 6 or higher, should be considered unreliable and thus should be discarded. In figure 5 the CDF of the estimation error in the location of the MT is plotted. This is given by

$$\|\hat{\mathbf{p}}_0\|_2 = \|\hat{\mathbf{p}}_0 - \mathbf{p}_0\|_2 = \sqrt{(\hat{x}_0 - x_0)^2 + (\hat{y}_0 - y_0)^2} \quad (36)$$

From this figure we can observe that the error is less than 10cm for 75% of the cases and less than 1m for 85% of the cases. As expected, an increase in the number of MPCs, i.e. a richest scattering environment results in better accuracy. The results, although at high SNR, are very impressive considering the sub-optimality of the method and the realistic but difficult-to-localize conditions under which the estimation is performed. Similarly in figure 6 the CDF of the speed estimation error

$$\|\hat{v}\|_2 = \|\hat{v} - v\|_2 = \sqrt{(\hat{v}_x - v_x)^2 + (\hat{v}_y - v_y)^2} \quad (37)$$

of the MT is plotted. The error is less than 0.1m/sec for 85% of the cases and thus the accurate speed estimate could be employed along with the position estimate in a filtering procedure (e.g. Kalman filtering) to further improve the latter.

VI. CONCLUSIONS

We have presented a high-accuracy low-complexity localization algorithm suitable for strictly NLOS propagation environments. It is based on the integration of a mobility model and the single-bounce model and utilizes effectively the available information in LMDP such as the TOA, the AOD and the Doppler Shift to output a simple Least-Square solution for both the parameters of interest and the nuisance parameters. It

performs accurately at high SNR for the majority of the cases and at the same time can predict the scenarios when it will fail. If higher localization accuracy is desired, this method's estimates can serve as an initial points in a higher-complexity Maximum Likelihood iterative algorithm.

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