APPROXIMATE FULL DIVERSITY LINEAR EQUALIZERS FOR DOUBLY SELECTIVE CHANNELS

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ABSTRACT

It is known that linear minimum mean square error zero-forcing (MMSE-ZF) equalization can achieve full joint multipath-Doppler diversity offered by doubly selective channels [1] when appropriate precoding is applied at the transmitter. However, brute force implementation MMSE-ZF equalizer involves a matrix inversion operation that results in significant computational burden. In order to alleviate this problem, first an iterative implementation of MMSE-ZF equalizer based on polynomial expansion (PE) approximation is proposed. Then, the structure of a matrix involved in this approximation is exploited to reduce the computational complexity of the PE approximation. Simulation results are provided to show that while this approach reduces the computational complexity compared to the brute-force implementation of the MMSE-ZF equalizer, it does not effect the diversity order.

1. INTRODUCTION

Practical wireless communication channels are prone to signal fading due to the presence of multiple signal paths (time dispersive channel) , time-varying nature of the channel (frequency dispersive channel) or both (time-frequency dispersive or the so called, doubly dispersive channel). However, its is possible for the receiver to employ equalization techniques that optimally exploit the inherent diversity in these channels as a convenient counter-measure against fading. For instance, the frequency selectivity of time dispersive channels provide multipath diversity due to the presence of multiple independently fading components in the channel. In block transmission systems, when the channel coherence time is shorter than the transmit block length, temporal variations of the channel provides Doppler diversity [2] which can be exploited by the receiver. Doubly selective channels offer joint multipath-Doppler diversity that can be harnessed by suitable equalization techniques and proper precoding. In [3], the authors used the Complex-Exponential Basis Expansion Model (CE-BEM) [4] with $Q+1$ basis functions to model the doubly selective channel of memory L and showed that by employing linear precoded block transmission, the maximum diversity in the channel is upper bounded by $(Q+1)(L+1)$ and can be achieved when maximum-likelihood equalization (MLE) is used at the receiver. However, MLE incurs a huge computational complexity therefore the diversity order achieved by linear equalization (LE) which is a low-complexity albeit sub-optimal alternative to optimal maximum-likelihood equalization (MLE) was investigated in [5] [6] [1]. It is now known that linear minimum mean squared error zero forcing (MMSE-ZF) receivers achieve maximal diversity offered by time/frequency and doubly selective channels. Recognizing the fact that the structure of MMSE-ZF receivers can be exploited in order to reduce the complexity of these full-diversity achieving receivers, low complexity linear equalizers were proposed in [7] and [8] for frequency selective channels. Since the computational complexity in the MMSE-ZF receiver is predominantly due to the matrix inversion involved in building the equalizer, we propose a reduced-complexity implementation of the MMSE-ZF receiver based on polynomial expansion (PE) approximation. PE approximation is not a new idea, it was first proposed in [9] where, in order to avoid explicit matrix inversion, the Cayley Hamilton theorem [10] was applied to express the matrix inverse as a finite sum of weighted matrix polynomials. The weights themselves were chosen to optimize a desired performance metric at the output of the equalizer. In this paper, we adopt an approach where the complexity reduction is achieved by first approximating the MMSE-ZF receiver using PE approximation an then exploiting the structure of a matrix involved in the PE approximation. In the sequel, we will explain our approach in more detail and show that a decrease in the computational effort is achieved with no loss on the diversity order of the resulting approximated equalizer.

2. SIGNAL MODEL

In Fig. 1 we show the block diagram of the transmission model. At the transmitter, complex data symbols $s[i]$ are

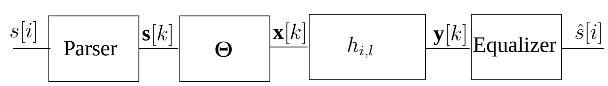

Figure 1: Block diagram of transmission model.

first parsed into N-length blocks.The n-th symbol in the k-th block is given by $[\mathbf{s}[k]]_n = s[kN+n]$ with $n \in [0, 1, ..., N-1]$. Each block $\mathbf{s}[k]$ is precoded by a $M \times N$ matrix $\boldsymbol{\Theta}$ where $M \geq N$ and the resultant block $\mathbf{x}[k]$ is transmitted over the block fading channel. We consider a channel memory of order L. It is well known that the temporal variation of the channel taps in doubly selective channels with a finite Doppler spread can be captured by finite Fourier bases. We therefore use CE-BEM [4] with $Q+1$ basis functions to model the time variation of each tap in a block duration. The basis coefficients remain constant for the block duration but are allowed to vary with every block. The time-varying channel for each block transmission is thus completely described by the $Q+1$ Fourier bases and $(Q+1)(L+1)$ coefficients.

In general Q is chosen such that $Q = 2\lceil f_{max}MT_s\rceil$ where $1/T_s$ is the sampling frequency and f_{max} is the Doppler spread of the channel. The coefficients themselves are assumed to be zero-mean complex i.i.d Gaussian random variables. Using i as the discrete time (sample) index, we can represent the l-th tap of the channel in the k-th block

$$h_{i,l} = \sum_{q=0}^{Q} h_q(k,l)e^{j2\pi f_q i},$$

$l \in [0, L]$, $f_q = (q - Q/2)/M$. The corresponding receive signal is formed by collecting M samples at the receiver to form $\mathbf{y}[k] = [y(kM+0), y(kM+1), \ldots, y(kM+M-1)]^T$. When $M \geq L$, this block transmission system can be represented in matrix-vector notation as [3]

$$\mathbf{y}[k] = \mathbf{H}_{ds}[k;0]\mathbf{\Theta}\mathbf{s}[k] + \mathbf{H}_{ds}[k;1]\mathbf{\Theta}\mathbf{s}[k-1] + \mathbf{v}[k], \quad (1)$$

where $\mathbf{v}[k]$ is a AWGN vector whose entries have zero-mean and variance σ_v^2 and is defined in the same way as $\mathbf{y}[k]$. $\mathbf{H}_{ds}[k;0]$ and $\mathbf{H}_{ds}[k;1]$ are $M \times M$ matrices whose entries are given by $[\mathbf{H}_{ds}[k;t]]_{r,s} = h_{(kM+r,tM+r-s)}$ with $t \in [0,1], r,s \in [0,...,M-1]$. Defining $\mathbf{D}[f_q]$ as a diagonal matrix whose diagonal entries are given by $[\mathbf{D}[f_q]]_{m,m} = e^{j2\pi f_q m}, m \in [0,1,...,M-1]$, and further defining $[\mathbf{H}_q[k;t]]_{r,s} = h_q(k, tM+r-s)$ as Toeplitz matrices formed of BEM coefficients, it is straightforward to represent Eq. (1) as

$$\mathbf{y}[k] = \sum_{t=0}^{1}\sum_{q=0}^{Q}\mathbf{D}[f_q]\mathbf{H}_q[k;t]\mathbf{\Theta}\mathbf{s}[k-t] + \mathbf{v}[k], \quad (2)$$

3. PRECODED TRANSMISSION IN DOUBLY SELECTIVE CHANNELS

The precoding matrix $\mathbf{\Theta}$ considered here is given by

$$\mathbf{\Theta} = \mathbf{F}_{P+Q}^H\mathbf{T}_1 \otimes \mathbf{T}_2.$$

where $\otimes$ represents the Kronecker product of matrices, $\mathbf{F}_{P+Q}$ is a $(P+Q)$-point DFT matrix, $\mathbf{T}_1 = [\mathbf{I}_P \ \mathbf{0}_{P\times Q}]^T$, $\mathbf{T}_2 = [\mathbf{I}_K \ \mathbf{0}_{K\times L}]^T$. P and K are chosen such that $M = (P+Q)(K+L)$ and $N = PK$. This precoder was proposed in [3] and was shown to enable diversity order of $(Q+1)(L+1)$ for ML receivers in doubly selective channels. When this precoder is applied at the transmitter, the received signal can be represented as

$$\mathbf{y}[k] = (\mathbf{F}_{P+Q}^H \otimes \mathbf{I}_{K+L})\mathbf{H}[k]\mathbf{s}[k] + \mathbf{v}[k], \quad (3)$$

where $\mathbf{H}[k]$ is given by (see Sec. 7 for derivation).

$$\mathbf{H}[k] = \sum_{q=0}^{Q}(\mathbf{J}_{P+Q}[q]\mathbf{T}_1) \otimes (\mathbf{D}_{K+L}[f_q]\widetilde{\mathbf{H}}_q[k;0]\mathbf{T}_2). \quad (4)$$

In the following we will drop the block index k in the interest of simplifying notations. For this scheme, it was shown in [1] that MMSE-ZF equalization collects full diversity offered by the channel. In other words, the slope of the outage probability curve of the MMSE-ZF receiver in the high-SNR regime is $(Q+1)(L+1)$. This being the case, it is of interest to explore the possibility of lowering the complexity of such a receiver since brute-force implementation of the MMSE-ZF receiver involves the computation of the matrix inverse $(\mathbf{H}^H\mathbf{H})^{-1}$ which is computationally expensive.

4. POLYNOMIAL EXPANSION APPROXIMATION FOR LE IN DOUBLY SELECTIVE CHANNELS

In order to reduce the above mentioned computational cost, we propose here a reduced complexity implementation of the MMSE-ZF equalizer for doubly selective channels. We start with an alternative representation of the received signal in Eq. (3)

$$\mathbf{y} = \mathbf{H}_{tv}\mathbf{\Theta}\mathbf{s} + \mathbf{v},$$

where $\mathbf{H}_{tv}$ represents the channel matrix in the time-domain and can in turn be represented as the sum of two matrices

$$\mathbf{H}_{tv} = \mathbf{H}_\kappa + \mathbf{H}_\nu,$$
$$\mathbf{H}_\kappa = \sum_{q=0}^{Q}(\mathbf{D}_{P+Q}[f_q(K+L)] \otimes e^{j\bar{\omega}_q}\widetilde{\mathbf{H}}_q),$$
$$\mathbf{H}_\nu = (\mathbf{D}_{P+Q}[f_q(K+L)] \otimes (\mathbf{D}_{K+L}[f_q] - e^{j\bar{\omega}_q}\mathbf{I}_{K+L})\widetilde{\mathbf{H}}_q).$$

$\bar{\omega}_q = \omega_q(K+L-1)/2$. Representing the received signal in this form allows us to iteratively estimate the transmit symbol vector $\mathbf{s}$. The symbol estimate after the m-th iteration is given by

$$\widehat{\mathbf{s}}^{(m)} = (\mathbf{H}_\kappa\mathbf{\Theta})^\dagger(\mathbf{y} - \mathbf{H}_\nu\mathbf{\Theta}\widehat{\mathbf{s}}^{(m-1)}). \quad (5)$$

where the superscript † represents the Moore-Penrose pseudo-inverse. From Eq. (5), we can derive the signal to interference noise ratio (SINR) expression for the n-th symbol of the symbol estimate $\widehat{\mathbf{s}}^{(m)}$ as

$$\text{SINR}_n = \frac{\rho[\mathbf{G}_s\mathbf{G}_s^H]_{n,n}}{\rho\bar{\mathbf{g}}\bar{\mathbf{g}}^H + [\mathbf{G}_v\mathbf{G}_v^H]_{n,n}}, \quad (6)$$

where ρ is the signal to noise ratio (SNR) and $\bar{\mathbf{g}}$ is the n-th row of $\mathbf{G}_s$ without the element $[\mathbf{G}]_{n,n}$ and

$$\mathbf{G}_s = \mathbf{I} + (-1)^m((\mathbf{H}_\kappa\mathbf{\Theta})^\dagger\mathbf{H}_\nu\mathbf{\Theta})^{m+1},$$
$$\mathbf{G}_v = (\sum_{k=0}^{m}(-1)^k((\mathbf{H}_\kappa\mathbf{\Theta})^\dagger\mathbf{H}_\nu\mathbf{\Theta})^k)(\mathbf{H}_\kappa\mathbf{\Theta})^\dagger.$$

Alternatively, it is possible to envisage a polynomial expansion approximation for the MMSE-ZF receiver that minimizes the mean squared error at the receiver. In this case, the symbol vector estimate after m iterations is given by

$$\widehat{\widehat{\mathbf{s}}}^{(m)} = \sum_{k=0}^{m}\mathbf{\Lambda}_k\mathbf{R}^k\mathbf{z}. \quad (7)$$

where

$$\mathbf{R} = -(\mathbf{H}_\kappa\mathbf{\Theta})^\dagger\mathbf{H}_\nu\mathbf{\Theta}, \qquad \mathbf{z} = (\mathbf{H}_\kappa\mathbf{\Theta})^\dagger\mathbf{y},$$

and the diagonal scale factor matrices $\mathbf{\Lambda}_k$ of order N are estimated by plugging in the expression for $\widehat{\widehat{\mathbf{s}}}^{(m)}$ in Eq. (7) in the LMMSE criterion

$$\mathbf{\Lambda}_k^{opt} = \arg\min_{\mathbf{\Lambda}_k : k\in\{0,1,m\}} E\|\mathbf{s} - \widehat{\widehat{\mathbf{s}}}^{(m)}\|^2. \quad (8)$$

Note that Eq. (5) corresponds to the special case of Eq. (8) where the diagonal elements of all $\mathbf{\Lambda}_k$ are unity. Another special case of Eq. (8) where the diagonal matrices $\mathbf{\Lambda}_k$ are reduced to scalar weighting coefficients λ_k are addressed before (for instance in [9]). Let $\boldsymbol{\lambda}_{n,k} = [\boldsymbol{\Lambda}_k]_{n,n}$ and $\boldsymbol{\lambda}_n = [\lambda_{n,0}, \cdots, \lambda_{n,m}]$ then Eq. (8) can be solved by finding the optimum $\boldsymbol{\lambda}_n^{opt}$ separately for each transmit symbol $n \in \{0, 1, \cdots, N-1\}$ in the symbol vector **s** as

$$\boldsymbol{\lambda}_n^{opt} = \arg\min_{\boldsymbol{\lambda}} E|s[n] - \boldsymbol{\lambda}_n \mathbf{q}[n]|^2. \qquad (9)$$

$\mathbf{q}[n] = [w_0[n] \ w_1[n] \ \cdots \ w_m[n]]^T$ and $w_m[n]$ are elements of $\mathbf{w}_m = \mathbf{R}^m \mathbf{z}$. Once the N vectors corresponding to $\boldsymbol{\lambda}_n^{opt}$ are obtained the diagonal matrices $\boldsymbol{\Lambda}_k$ are formed and substituted in Eq. (7) to get the symbol estimate. The SINR at the output of this equalizer is given by

$$\text{SINR}_n^{MMSE-PE} = \frac{\rho[\boldsymbol{\mathcal{G}}_s \boldsymbol{\mathcal{G}}_s^H]_{n,n}}{\rho \bar{\boldsymbol{g}} \bar{\boldsymbol{g}}^H + [\boldsymbol{\mathcal{G}}_v \boldsymbol{\mathcal{G}}_v^H]_{n,n}}, \qquad (10)$$

where $\bar{\boldsymbol{g}}$ is now the n-th row of $\boldsymbol{\mathcal{G}}_s$ without the element $[\boldsymbol{\mathcal{G}}]_{n,n}$ and

$$\begin{aligned} \boldsymbol{\mathcal{G}}_s &= \sum_{k=0}^{m} \boldsymbol{\Lambda}_k \mathbf{R}^k (\mathbf{I} + (\mathbf{H}_\kappa \boldsymbol{\Theta})^\dagger \mathbf{H}_\nu \boldsymbol{\Theta}), \\ \boldsymbol{\mathcal{G}}_v &= \sum_{k=0}^{m} \boldsymbol{\Lambda}_k \mathbf{R}^k (\mathbf{H}_\kappa \boldsymbol{\Theta})^\dagger. \end{aligned}$$

Since both Eq. (7) and Eq. (5) require the calculation of the pseudo-inverse $(\mathbf{H}_\kappa \boldsymbol{\Theta})^\dagger$ we now focus our attention to reducing the complexity of the matrix inversion that needs to be performed in order to obtain $(\mathbf{H}_\kappa \boldsymbol{\Theta})^\dagger$. Notice that $\mathbf{H}_\kappa \boldsymbol{\Theta}$ can be factored as shown in (12) where we replace the block-circulant-with-Toeplitz-blocks (BCTB) matrix in (11) with a block-circulant-with-circulant-blocks matrix (BCCB). i.e.,

$$\mathbf{H}_{BCCB} = \sum_{q=0}^{Q} (\mathbf{J}_{P+Q}[q] \otimes e^{j\bar{\omega}_q} \mathbf{H}_q^c).$$

where $\mathbf{H}_q^c$ is a circulant matrix whose first column is the same as the first column of $\widetilde{\mathbf{H}}_q$, This allows us to take advantage of the fact that $\mathbf{H}_{BCCB}$ is diagonalizable as

$$\boldsymbol{\mathcal{D}} = (\mathbf{F}_{P+Q}^H \otimes \mathbf{F}_{K+L}^H) \mathbf{H}_{BCCB} (\mathbf{F}_{P+Q} \otimes \mathbf{F}_{K+L}),$$

Now plugging this into (12) we have

$$\mathbf{H}_\kappa \boldsymbol{\Theta} = (\mathbf{I}_{P+Q} \otimes \mathbf{F}_{K+L}) \boldsymbol{\mathcal{D}} \boldsymbol{\Theta}_F.$$

where $\boldsymbol{\Theta}_F = (\mathbf{I}_{P+Q} \otimes \mathbf{F}_{K+L}^H) \boldsymbol{\Theta}$ which in turn leads us to

$$(\mathbf{H}_\kappa \boldsymbol{\Theta})^\dagger = (\boldsymbol{\mathcal{D}} \boldsymbol{\Theta}_F)^\dagger (\mathbf{I}_{P+Q} \otimes \mathbf{F}_{K+L})^H. \qquad (13)$$

The problem of computing $(\mathbf{H}_\kappa \boldsymbol{\Theta})^\dagger$ is thus reduced to the problem of computing $(\boldsymbol{\mathcal{D}} \boldsymbol{\Theta}_F)^\dagger$. This can be accomplished by formulating the problem of computing the pseudo-inverse as that of finding the $N \times (M-N)$ matrix Ξ that corresponds to the solution of the minimization problem [7]

$$\arg\min_{\Xi} \ \text{Tr}\{(\boldsymbol{\Theta}_F^\dagger \boldsymbol{\mathcal{D}}^{-1} + \Xi \boldsymbol{\Theta}_{\mathcal{N},F} \boldsymbol{\mathcal{D}}^{-1})^H (\boldsymbol{\Theta}_F^\dagger \boldsymbol{\mathcal{D}}^{-1} + \Xi \boldsymbol{\Theta}_{\mathcal{N},F} \boldsymbol{\mathcal{D}}^{-1})\} \qquad (14)$$

where $\boldsymbol{\Theta}_{\mathcal{N},F} = \boldsymbol{\mathcal{N}}(\boldsymbol{\Theta}^H)$, and $\boldsymbol{\mathcal{N}}(.)$ denotes the null space of a matrix. The solution to Eq. (14) allows us to compute $(\boldsymbol{\mathcal{D}} \boldsymbol{\Theta}_F)^\dagger$ as

$$\boldsymbol{\Theta}_F^\dagger \boldsymbol{\mathcal{D}}^{-1} [\mathbf{I} - \boldsymbol{\mathcal{D}}^{-H} \boldsymbol{\Theta}_{\mathcal{N},F} (\boldsymbol{\Theta}_{\mathcal{N},F}^H \boldsymbol{\mathcal{D}}^{-1} \boldsymbol{\mathcal{D}}^{-H} \boldsymbol{\Theta}_{\mathcal{N},F})^{-1} \boldsymbol{\Theta}_{\mathcal{N},F}^H \boldsymbol{\mathcal{D}}^{-1}] \qquad (15)$$

which involves inversion of a matrix of dimension $M-N$ in place of inversion of matrix of dimension M in the brute-force approach. Moreover, $\boldsymbol{\Theta}_F^\dagger$ is only dependent on the precoding matrix hence it can also be precomputed and used across blocks.

4.1 Optimization of transmission parameters

The reduction in computational complexity of $(\mathbf{H}_\kappa \boldsymbol{\Theta})^\dagger$ using Eq. (15) is related to the redundancy in the transmit block (i.e., $M-N$). One can therefore optimize the transmission parameters P, Q, K, L to reduce the computational complexity of overall PE approximation of the MMSE-ZF equalizer. Such an optimization has to take into account the interrelations of the transmission parameters. For a fixed f_{max} and T_s, we know that L is fixed but Q increases proportionally to M due to the fact that $\alpha = Q/M$ is fixed due to the relation $Q = 2\lceil f_{max} M T_s \rceil$). This also implies that the available diversity at the receiver increases with M

5. NUMERICAL RESULTS

In this section we provide simulation results to show that the approximation for the MMSE-ZF equalizer proposed in the paper does not entail any loss in terms of the diversity order collected by the receiver. It is well accepted that the slope of the outage probability curve in the high SNR regime provides an accurate estimate of the diversity order of a receiver. The outage probability curve was plotted by computing the post-equalization SINR (SNR for the case of brute for MMSE-ZF equalizer) for the symbol index n corresponding to $\lceil N/2 \rceil$ since it suffers the maximum inter-symbol-interference within the block. Monte-Carlo simulations were carried out for a fixed transmission rate for different SNR points. For the brute-force implementation of the MMSE-ZF receiver, the post-equalization SNR for an arbitrary symbol index n in the symbol block **s** is given by

$$\text{SNR}_n = \frac{\rho}{[(\mathbf{H}^H \mathbf{H})^{-1}]_{n,n}},$$

where **H** represents the equivalent doubly selective channel after precoding and ρ is the SNR. For the polynomial expansion equalizers, the post-equalization SINR was computed as given by Eq. (6) and Eq. (10). When the post-equalization SINR was below the SNR required to support the fixed transmission rate, the channel was declared to be in outage. The slope of the outage probability curve thus obtained provides an estimate of the diversity order. In addition to this, we compare the slope of the receiver to that of the matched filter bound (MFB) which is known to collect all the available diversity in the channel.

Fig. 2 illustrates the evolution of the diversity order slope achieved against the order of approximation in the polynomial expansion equalizer in Eq. (5). It is seen that the slope flattens out understandably at lower order approximations due to large approximation errors but starts to stabilize at about second order approximation of the equalizer.

$$\mathbf{H}_{\kappa}\mathbf{\Theta} = (\mathbf{F}_{P+Q}^{H} \otimes \mathbf{I}_{K+L}) \sum_{q=0}^{Q} (\mathbf{J}_{P+Q}[q] \otimes e^{j\bar{\omega}_q}\mathbf{H}_q)(\mathbf{T}_1 \otimes \mathbf{T}_2), \quad (11)$$

$$\mathbf{H}_{\kappa}\mathbf{\Theta} = (\mathbf{F}_{P+Q}^{H} \otimes \mathbf{I}_{K+L}) \sum_{q=0}^{Q} (\mathbf{J}_{P+Q}[q] \otimes e^{j\bar{\omega}_q}\mathbf{H}_q^{c})(\mathbf{T}_1 \otimes \mathbf{T}_2). \quad (12)$$

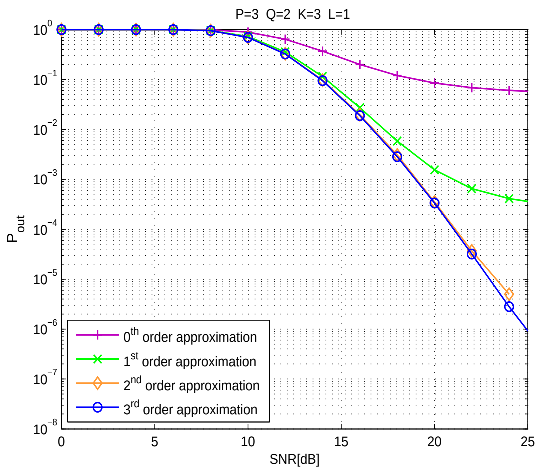

Figure 2: Evolution of diversity order for different iterations.

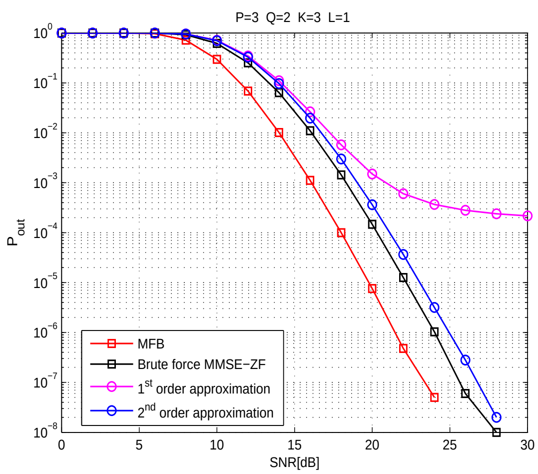

Figure 3: Diversity order of LE approximated by PE.

Fig. 3 shows the comparison of the diversity order of brute-force implementation of the MMSE-ZF equalizer for doubly selective channels for the case of $Q = 2$ and $L = 1$. Observe that the slope of the outage probability curves for both the implementations are the same. The polynomial expansion equalizer has an SNR offset when compared to the brute force implementation which is to be expected since the equalizer is an approximation of the MMSE-ZF receiver however, it succeeds in collecting full diversity offered by the doubly selective channel at relatively low order of approximation. The performance of PE approximation that

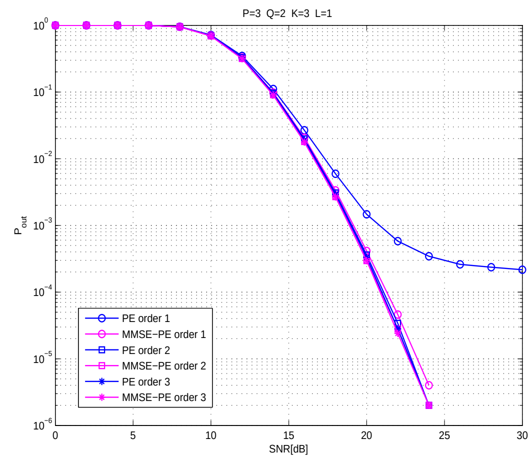

Figure 4: Comparison of performance of the two PE approximations.

minimizes the MSE at the receiver Eq. (7) is shown in Fig. 4. We see a significant enhancement in performance for the first order approximation when compared to the PE approximation in Eq. (5). However, for higher orders the the difference in their performance appears negligible.

6. CONCLUSIONS

In this contribution we propose two approximations for the MMSE-ZF equalizer for doubly selective channels. The proposed equalizers are based on polynomial expansion approximation and lead to a decrease in the computational complexity of the equalizer when compared with the brute-force implementation. Simulation results show that such an approximation does not incur a penalty in terms of the diversity order achieved by the approximated equalizers.

7. APPENDIX

Due to the presence of the zero-padding matrix $\mathbf{T}_2$, it can be easily shown that the inter-block-interference component in the received signal is zero, i.e., $\mathbf{H}_q[k;1]\mathbf{\Theta}\mathbf{s}[k-1] = 0$. As a result, the received block in Eq. (1) can now be represented as

$$\mathbf{y}[k] = \sum_{q=0}^{Q} \mathbf{D}[f_q]\mathbf{H}_q[k;0]\mathbf{\Theta}\mathbf{s}[k] + \mathbf{v}[k], \quad \text{(A1)}$$

Using standard Kronecker product identities, one can show that

$$\mathbf{H}_q[k;0]\mathbf{\Theta} = \mathbf{F}_{P+Q}^{H}\mathbf{T}_1 \otimes \widetilde{\mathbf{H}}_q[k;0]\mathbf{T}_2, \quad \text{(A2)}$$

where $\widetilde{\mathbf{H}}_q[k;0]$ is a $K+L \times K+L$ Toeplitz matrix formed by the first $K+L$ rows and columns of $\mathbf{H}_q[k;0]$. Eq. (A1) can then be re-written as

$$\mathbf{y}[k] = \sum_{q=0}^{Q} \mathbf{D}[f_q] \left(\mathbf{F}_{P+Q}^H \mathbf{T}_1 \otimes \widetilde{\mathbf{H}}_q[k;0]\mathbf{T}_2 \right) \mathbf{s}[k] + \mathbf{v}[k], \quad \text{(A3)}$$

Note that

$$\mathbf{D}[f_q] = \mathbf{D}_{P+Q}[f_q(K+L)] \otimes \mathbf{D}_{K+L}[f_q], \quad \text{(A4)}$$

The above equation represents $\mathbf{D}[f_q]$ as Kronecker product of time-variation over two scales. $\mathbf{D}_{P+Q}[f_q(K+L)]$ is a diagonal matrix of size $P+Q$ that represents time-variation at a coarse scale (complex-exponentials sampled at sub-sampling interval of $(K+L)T_s$ and $\mathbf{D}_{K+L}[f_q]$ is a diagonal matrix of size $K+L$ that represents the time-variation over a finer grid corresponding to the sampling period T_s. Using Eq. (A4) and standard matrix identities, we can decompose the received signal as in Eq. (A6) where $\mathbf{J}[q] = \mathbf{J}^{(q-Q/2)}$ and $\mathbf{J}$ is a circulant matrix with $[0, 1, \ \mathbf{0}_{1 \times P+Q-2}]^T$ as the first column. Since the matrix $(\mathbf{F}_{P+Q}^H \otimes \mathbf{I}_{K+L})$ has no effect on the diversity of the doubly selective channel, for the analysis of the diversity order of MMSE-ZF receiver, the effective channel matrix can be represented as

$$\mathbf{H}[k] = \sum_{q=0}^{Q} (\mathbf{J}_{P+Q}[q]\mathbf{T}_1) \otimes (\mathbf{D}_{K+L}[f_q]\widetilde{\mathbf{H}}_q[k;0]\mathbf{T}_2), \quad \text{(A7)}$$

Fig. 5 provides an illustration of the structure of the equivalent channel matrix due to precoding. Here $\bar{\mathbf{H}}_q$ represents the product matrix $\mathbf{D}_{K+L}[f_q]\widetilde{\mathbf{H}}_q[k;0]$ for ease of illustration. In particular, it is a block-Toeplitz matrix with constituent blocks which are in turn formed by the product of a diagonal matrix $\mathbf{D}_{K+L}[f_q]$ and a Toeplitz matrix formed by the corresponding BEM coefficients of the q-th basis function.

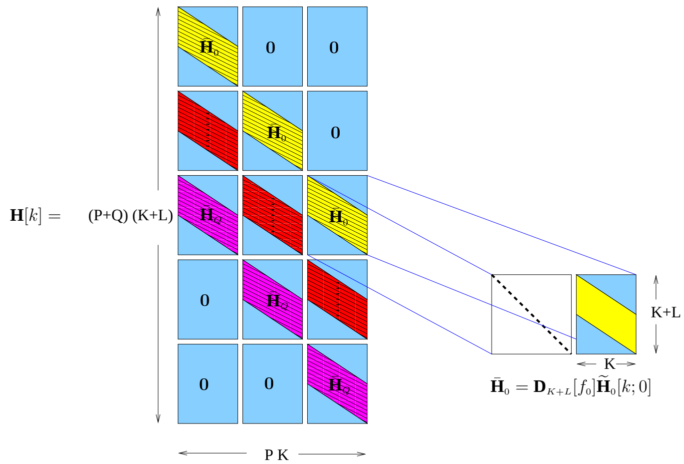

Figure 5: Equivalent channel matrix for doubly selective channel.

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$$\mathbf{y}[k] = \sum_{q=0}^{Q} \left((\mathbf{D}_{P+Q}[f_q(K+L)]\mathbf{F}_{P+Q}^H \mathbf{T}_1) \otimes (\mathbf{D}_{K+L}[f_q]\widetilde{\mathbf{H}}_q[k;0]\mathbf{T}_2) \right) \mathbf{s}[k] + \mathbf{v}[k], \quad \text{(A5)}$$

$$\mathbf{y}[k] = (\mathbf{F}_{P+Q}^H \otimes \mathbf{I}_{K+L}) \sum_{q=0}^{Q} \left((\mathbf{J}_{P+Q}[q]\mathbf{T}_1) \otimes (\mathbf{D}_{K+L}[f_q]\widetilde{\mathbf{H}}_q[k;0]\mathbf{T}_2) \right) \mathbf{s}[k] + \mathbf{v}[k], \quad \text{(A6)}$$