

# Enhanced Multiuser Random Beamforming: Dealing with the Not So Large Number of Users Case

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**Abstract**— We consider the downlink of a wireless system with an  $M$ -antenna base station and  $K$  single-antenna users. A limited feedback-based scheduling and precoding scenario is considered that builds on the multiuser random beamforming (RBF). Such a scheme was shown to yield the same capacity scaling, in terms of multiplexing and multiuser diversity gain, as the optimal full CSIT-based (channel state information at transmitter) precoding scheme, in the large number of users  $K$  regime. Unfortunately, for more practically relevant (low to moderate)  $K$  values, RBF yields degraded performance. In this work, we investigate solutions to this problem. We introduce a two-stage framework that decouples the scheduling and beamforming problems. In our scenario, RBF is exploited to identify good, spatially separable, users in a first stage. In the second stage, the initial random beams are refined based on the available feedback to offer improved performance toward the selected users. Specifically, we propose beam power control techniques that do not change the direction of the second-stage beams, offering feedback reduction and performance tradeoffs. The common feature of these schemes is to restore robustness of RBF with respect to sparse network settings (low  $K$ ), at the cost of moderate complexity increase.

**Index Terms**— MIMO systems, Random Beamforming, Power Control, Scheduling, Partial CSIT, Sparse networks.

## I. INTRODUCTION

In multiuser downlink multiple-input, multiple-output (MIMO) systems, the spatial multiplexing capability offered by multiple antennas can be advantageously exploited to boost the system capacity. A direct capacity gain proportional to the number of transmit antennas  $M$  can be achieved by serving multiple users in a space-division multiple access (SDMA) fashion. Recent information theoretic advances reveal that the capacity-achieving transmit strategy for the MIMO broadcast channel (i.e. channel from the transmitter to mobile users) is the so-called dirty paper coding (DPC) [1]–[3]. However, DPC involves high complexity and sensitivity to channel errors, making its implementation prohibitive in practical systems. In turn, several low-complexity strategies have recently been proposed to approach the capacity promised in multiuser MIMO systems. Among them, linear precoding based on zero-forcing beamforming (ZFBF) is shown to achieve a large fraction of DPC capacity while exhibiting reduced complexity [4], [5].

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Nevertheless, all these promising results unfortunately come at the critical assumption of good channel state information at transmitter (CSIT). Multiuser MIMO systems, unlike the point-to-point case, benefit substantially from CSIT, the lack of which may significantly reduce the system throughput. Providing CSIT at the base station (BS) poses serious challenges in practical settings where the channel information needs to be conveyed via a limited feedback channel in the uplink. The often impractical assumption of close-to-perfect CSIT, as well as the considerable capacity gap between perfect and no CSIT, have motivated research work on schemes employing partial CSIT. A tutorial on multiuser MIMO including how to deal with partial CSIT can be found in [6]. One popular approach to deal with incomplete channel information, often referred to as limited feedback, is to quantize the channel vector (or the precoder) based on a predetermined codebook known at both the BS and the terminals. The limited feedback model studied in point-to-point MIMO systems [7] has been extended for multiple antenna broadcast channels [8], [9]. In this framework, each user is allowed to feed back  $B$ -bit quantized information on its channel direction (CDI) through a finite rate uplink channel. When  $K \geq M$ , CDI is complemented with instantaneous channel quality information (CQI) and used in systems employing efficient user selection and ZFBF precoding [10]–[12], as a means to intelligently select  $M$  spatially separable users with large channel gains, approaching thus the capacity with full CSIT by means of multiuser diversity [13].

If we consider that each user is allowed to use only  $B = \log_2 M$  bits for CDI quantization, the optimal choice for a randomly generated codebook is one that contains orthonormal vectors. Therefore, the above channel vector quantization-based techniques can be viewed as extensions of an interesting, alternative low-rate feedback scheme, coined as multiuser random beamforming (RBF) and proposed in [14]. Therein,  $B = M$  random orthonormal beamforming vectors are generated and the best user on each beam is scheduled. The idea of [14] extends to the multiple beam case the concept of opportunistic beamforming, initially proposed in [15]. When the number of users  $K$  is large (dense networks), RBF is shown to yield the optimal capacity scaling of  $M \log \log K$  with only little feedback from the users, i.e. in the form of individual signal-to-interference-plus-noise ratio (SINR). The capacity growth of MIMO Gaussian broadcast channel with perfect CSIT is achieved by the RBF scheme. Although the beams are generated randomly and without a priori CSIT, for  $K \rightarrow \infty$  the selected group of users exhibits large channel gains as well as good spatial separability, and the probability that the random beam directions are nearly matched to certain

users is increased. On the other hand, a major drawback of this technique is that its performance is quickly degrading with decreasing  $K$ . Furthermore, this degradation is amplified when the number of transmit antennas increases. As the number of active users decreases and  $M$  increases, it becomes more and more unlikely that  $M$  randomly generated, *equipowered* beams will closely match the vector channels of any set of  $M$  users. This situation could easily be faced as traffic is normally bursty with frequent silent periods in data-access networks, thus the scheduler may not count on a large number of simultaneously active users at all times.

In this paper, we generalize our work in [16] and propose a new class of random unitary beamforming-inspired schemes that exhibits robustness in cells with - practically relevant - low to moderate number of users (sparse networks), while preserving the limited feedback and low-complexity advantages of RBF. One first key idea is based on splitting the design between the scheduling and the final beam computation (or "user serving") stages, thus taking profit from the fact the number of users to be served at each scheduling slot is much less than the number of active users (i.e.  $\mathcal{B} \leq M \ll K$ ). In the scheduling phase (stage 1), a coarse finite feedback rate user selection scheme is presented exploiting the concept of RBF [14]. We use the SINR reported by all users, which is measured upon the initial precoding matrix as a basis on which to further improve the design of the final beams that will be used to serve the selected users (stage 2). In general, the initial precoder can be designed based on any a priori channel knowledge; however here we assume that the first-stage beams are generated at random as in [14] since no a priori CSIT is assumed. Once the group of  $\mathcal{B}$  ( $1 \leq \mathcal{B} \leq M$ ) users is pre-selected using the SINR feedback on the random beams, additional CSIT may be requested to only the selected user group in order to design the final precoder. More specifically, we make the following proposals and contributions:

- The second-stage precoding matrix may require variable levels of additional CSIT feedback to be computed, depending on design targets, and the final beams will improve over the random beamforming used in [14]. In particular, while we expect little gain over [14] for large  $K$ , significant throughput gain appears for sparse networks in which the initial random beamformer may not provide satisfactory SINR for all  $M$  users.
- In this work, we restrict ourselves to the case that we do not change the initial beam direction and we then propose to adapt the power and the number of active beams to the number of users, the average signal-to-noise ratio (SNR) and the number of transmit antennas as a means to maximize the system throughput.
- In one variant of the proposed designs, we study the problem of power allocation across the  $\mathcal{B}$  (initially equipowered) random beams showing substantial capacity improvement over [14] for a wide range of values of  $K$ . The scheme requires  $\mathcal{B} \leq M$  real-valued scalar values to be fed back from each of the  $\mathcal{B}$  pre-selected users.
- For a 2-beam system, the global optimal beam power solution is provided in closed-form, whereas for the general

$\mathcal{B}$ -beam case, solutions based on iterative algorithms are proposed and numerically simulated.

- In another proposed robust variant of RBF, no additional CSIT feedback is required during the second stage. Instead, we exploit the SINR information obtained under the random beams in the first stage in order to not only perform scheduling but also to refine the number of active beams. An on/off beam power control is proposed as a low-complexity solution, yielding a dual-mode scheme switching from time-division multiple access (TDMA) transmission (only one beam is allocated non-zero power) to SDMA where all beams are active with equal power. The throughput gains over [14] are shown to be substantial for high SNR and low  $K$  values.

The remainder of this paper is organized as follows. The system model is described in Section II, and in Section III we review linear precoding and scheduling in MIMO broadcast channels. Section IV is devoted to the proposed two-stage scheduling and linear precoding framework. In Section V, power allocation techniques based on beam gain knowledge are proposed, while in Section VI an on/off beam power control scheme is investigated. We provide numerical results in Section VII, and conclude in Section VIII.

## II. SYSTEM MODEL

We consider a multi-antenna Gaussian broadcast channel in which a transmitter equipped with  $M$  antennas communicates with  $K \geq M$  single-antenna receivers. The received signal  $y_k(t)$  of the  $k$ -th user at time slot  $t$  is mathematically described as

$$y_k(t) = \mathbf{h}_k(t)\mathbf{x}(t) + n_k, \quad k = 1, \dots, K \quad (1)$$

where  $\mathbf{x}(t) \in \mathbb{C}^{M \times 1}$  is the vector of transmitted symbols at time slot  $t$ , and  $\mathbf{h}_k(t) \in \mathbb{C}^{1 \times M}$  is the channel vector of the  $k$ -th receiver, with components distributed as  $\sim \mathcal{CN}(0, 1)$  (Rayleigh fading). We assume that each of the receivers has perfect and instantaneous knowledge of its own channel  $\mathbf{h}_k$ , and that  $n_k$  is independent and identically distributed (i.i.d.) circularly symmetric additive complex Gaussian noise with zero mean and variance  $\sigma_k^2 = \sigma^2, \forall k$ . The transmitter is subject to a power constraint  $P$ , i.e.,  $\text{Tr}(\mathbf{x}\mathbf{x}^H) \leq P$ , where  $\text{Tr}(\cdot)$  is the trace operator, and all users are assumed to experience the same average SNR.

*Notation:* We use bold upper and lower case letters for matrices and vectors, respectively.  $\mathbb{E}(\cdot)$  denotes the expectation operator. The natural logarithm is referred to as  $\log(\cdot)$ , while the base 2 logarithm is denoted as  $\log_2(\cdot)$ .

## III. USER SELECTION AND LINEAR PRECODING FOR MIMO BROADCAST CHANNELS

Let  $\mathcal{S}(t) = \{k_i\}_{i=1}^{\mathcal{B}}$  be the set of selected users that are assigned non-zero rate at time slot  $t$ , with cardinality  $|\mathcal{S}(t)| = \mathcal{B}, 1 \leq \mathcal{B} \leq M$ . From now on, for simplicity, we drop the time index. Under linear beamforming, the transmitter multiplies the (normalized) data symbol for each user  $k$  by a vector  $\mathbf{w}_k$  so that the transmitted signal is  $\mathbf{x} = \sum_{k \in \mathcal{S}} \sqrt{P_k} \mathbf{w}_k s_k$  with

$\sqrt{P_k}$  denoting the  $k$ -th user transmit power scaling factor. The received signal for each user  $k \in \mathcal{S}$  is therefore

$$y_k = (\sqrt{P_k} \mathbf{h}_k \mathbf{w}_k)_{s_k} + \sum_{j \in \mathcal{S}, j \neq k} (\sqrt{P_j} \mathbf{h}_k \mathbf{w}_j)_{s_j} + n_k \quad (2)$$

where  $\sum_{k \in \mathcal{S}} P_k \|\mathbf{w}_k\|^2 \leq P$ . The instantaneous SINR at receiver  $k$  is

$$\text{SINR}_k = \frac{P_k |\mathbf{h}_k \mathbf{w}_k|^2}{\sum_{j \neq k} P_j |\mathbf{h}_k \mathbf{w}_j|^2 + \sigma^2} \quad k \in \mathcal{S} \quad (3)$$

Assuming user codes drawn from an i.i.d. Gaussian distribution, the supremum of the rates supported for user  $k$  is given by  $\mathcal{R}_k = \log_2(1 + \text{SINR}_k)$  and the average achievable sum rate  $\mathcal{R}$  is given by

$$\mathcal{R}(\mathcal{S}) = \sum_{k \in \mathcal{S}} \mathcal{R}_k = \sum_{k \in \mathcal{S}} \log_2(1 + \text{SINR}_k) \quad (4)$$

The user set  $\mathcal{S}$  is chosen to maximize the achievable sum rate, i.e.  $\mathcal{S}^* = \arg \max_{\mathcal{S} \subset \{1, \dots, K\}} \mathcal{R}(\mathcal{S})$ .

#### IV. TWO-STAGE SCHEDULING AND LINEAR PRECODING

In this section, we propose a MIMO downlink scheduling and beamforming framework in which the design is split into two stages. In the first stage, a coarse beamforming matrix is used (possibly even selected at random) and user group selection (of size  $\mathcal{B}$ ) is performed among all  $K$  active users. In the second stage, possibly additional channel quality information is collected for the selected user group, and an improved beamforming matrix is designed to serve them. The fact that  $\mathcal{B} \ll K$  is instrumental in reducing the total feedback requirement in this scenario. The two-stage framework can be described as follows:

##### Stage 1: User Selection

The transmitter generates a linear precoding matrix  $\mathbf{W}$  based on any a priori channel information we may have. In our case, since the channel conditions of the users are not known a priori, a unitary precoding matrix  $\mathbf{Q}$  is drawn randomly and equal power allocation is used ( $P_m = \frac{P}{\mathcal{B}}, \forall m$ ), as a means to reduce feedback burden and complexity requirements, i.e.  $\mathbf{W} = \mathbf{Q} = [\mathbf{q}_1 \dots \mathbf{q}_{\mathcal{B}}]$ . The  $\mathcal{B}$  columns  $\mathbf{q}_m \in \mathbb{C}^{M \times 1}$  of the precoder can be interpreted as random orthonormal beams, generated according to an isotropic distribution, as proposed in [14].

Each of the  $K$  users, say the  $k$ -th, calculates the SINRs over all equipowered beams, i.e.

$$\text{SINR}_{k,m} = \frac{|\mathbf{h}_k \mathbf{q}_m|^2}{\sum_{j \neq m} |\mathbf{h}_k \mathbf{q}_j|^2 + \mathcal{B} \sigma^2 / P} \quad m = 1, \dots, \mathcal{B} \quad (5)$$

finds the beam  $b_k$  that provides the maximum SINR, i.e.  $b_k = \arg \max_{m=1, \dots, \mathcal{B}} \text{SINR}_{k,m}$ , and feeds back  $\beta_k = \text{SINR}_{k,b_k}$  in addition to the beam index  $b_k$ . A simple and low-complexity user selection scheme is employed at the BS by selecting the users that have the highest SINR on each beam  $\mathbf{q}_m$ . The group of selected users is denoted as  $\mathcal{S}$ . In [14]  $\mathcal{B} = M$  beams are

activated. In the general case however, we could decide to activate the  $\mathcal{B} \leq M$  best beams only.

##### Stage 2: Final Precoding design

In our proposed framework, we follow up with a second stage where the  $\mathcal{B}$  users in  $\mathcal{S}$  may be allowed to report back to the BS additional limited feedback, denoted as  $\beta'_k, k \in \mathcal{S}$ . Based on the feedback information, the transmitter designs the final precoding matrix  $\mathbf{W}'(\mathcal{S}) = f(\beta'_k)$ , where  $f(\cdot)$  is some feedback-based beamforming design function. Note that in [14] there is no second stage, in other words  $\mathbf{W}'(\mathcal{S}) = \mathbf{Q}$ . The second-stage feedback can take on among others the following forms, depending on the system feedback rate constraint:

- *Strategy 1:*  $\beta'_k = \mathbf{h}_k$  (full CSIT)
- *Strategy 2:*  $\beta'_k = \hat{\mathbf{h}}_k$  (quantized channel vector)
- *Strategy 3:*  $\beta'_k = |\mathbf{h}_k \mathbf{q}_m|^2$  (BGI: beam gain information)
- *Strategy 4:*  $\beta'_k = \beta_k$  (no additional feedback)

Under strategy 3, the  $\mathcal{B}$  selected users feed back beam gain information for all  $\mathcal{B}$  active beams, i.e.  $\mathcal{B}$  real-valued scalars. Note that anyone of these two-stage schemes represents an efficient feedback reduction strategy considering the number of selected users  $\mathcal{B}$  is typically very small in comparison with  $K$ . For instance  $\mathcal{B} = 2$  or  $4$  in practical standardized systems while  $K$  could be a few tens even for moderately sparse networks. The optimal way of splitting the feedback load across the stage 1 (scheduling) and the stage 2 (beam design) is an interesting open problem, beyond the scope of this paper, although some design rules, where  $\beta'_k$  is given by a quantized version of the quantization error of the channel and ZFBF were already suggested [17].

The design of a two-stage feedback scheme will inevitably introduce a longer hand-shaking delay before the actual payload data can be sent to the mobile. For an efficient operation of feedback-based approach (whether single stage or two-stage), the total duration spent on feedback together with payload transmission must be significantly less than the coherence time of the channel  $T_{coh}$ . Therefore, for the 2-stage approach to be applicable, we envision a framing structure which encompasses the two stages of feedback, back to back, as an overall feedback preamble, prior to payload transmission. This preamble (minislot) of short duration  $\tau_m$ , during which users report their feedback messages is thus followed by a larger slot of duration  $\tau_s \gg \tau_m$ , which is dedicated to pilot and data transmission. The total framing interval duration should be kept less than the coherence time of the channel, i.e.  $\tau_s + \tau_m \leq T_{coh}$ . Note that the second stage of feedback collects fresh CSIT, so that the precoder design does not suffer from extra outdated degradation (compared with a single stage feedback). Nevertheless, the impact of employing two stages on the sum-rate performance and the resulting delays is assessed through simulations in Section VII by considering a time-varying channel.

When the number of active users  $K$  is large (dense networks), random beamforming can benefit from multiuser diversity by scheduling users with favorable channel conditions (highest SINR), improving thus the system capacity. The selected group of users exhibit large channel gains and good

spatial separability among them and the probability that the random beam direction is closely matched to certain users is increased with increasing  $K$ . For low to moderate number of users (sparse networks), the probability that all  $\mathcal{B}$  users enjoy a reasonable SINR is lower since the selected users may not be fully separable under a randomly generated unitary beamforming matrix  $\mathbf{Q}$ . Nevertheless, we point out that this user set, the user group selected by the scheduler under the initial random orthogonal beams, is likely to exhibit good separability conditions relative to the rest of the users, since it is at least the best user group for *one* orthogonal precoder  $\mathbf{Q}$ . Therefore, we argue that a design based on random  $\mathbf{Q}$  could be kept for the purpose of *scheduling*. In strategies 1-3, we propose to augment the random beamforming step (stage 1) with an additional yet low-rate CSIT feedback (stage 2), as a means to restore robustness and improve sum-rate performance. Note that the second stage only involves the  $\mathcal{B}$  pre-selected users. In this work we focus particularly on strategies 3 and 4 due to their low-rate feedback merits, and we propose power control techniques that do not refine the direction of the beamforming vectors.

## V. BEAM POWER CONTROL WITH BEAM GAIN INFORMATION

In this section, we consider that *strategy 3* is adopted during the second stage, thus the scheduler gains knowledge of  $\gamma_{k_m m} = |\mathbf{h}_{k_m} \mathbf{q}_m|^2$  for  $k_m \in \mathcal{S}$ . Without loss of generality (WLOG), we order users such that  $\gamma_{k_i i} \geq \gamma_{k_j j}, \forall i < j$  is assumed, and unless otherwise stated  $\mathcal{B} = M$ . If a moderate number of users exist, some of the random beams may not reach a target. This is measured at the BS in terms of the BGI  $\gamma_{k_m m}$ . In turn, the power control is used to reduce the resource allocated to the low-quality beams, to the benefit of the good-quality beams. As a result we propose to choose not to change the direction of the initial random beams. Based on this *beam gain information* (BGI)  $\gamma_{k_m m}$  we propose to design the beamforming matrix by applying a power allocation strategy across the beams of  $\{\mathbf{q}_m\}_{m=1}^M$ , i.e.  $\mathbf{w}_m = \sqrt{P_m} \mathbf{q}_m$ .

Define the vector of transmit powers  $\mathbf{P} = [P_1 \dots P_M]$  where  $P_m$  is the transmit power on beam  $m$ . The SINR of the selected user  $k_m \in \mathcal{S}$  over its preferred beam  $m$  can be expressed as:

$$\text{SINR}_{k_m, m}(\mathbf{P}) = \frac{P_m \gamma_{k_m m}}{\sigma^2 + \sum_{j \neq m} P_j \gamma_{k_m j}} \quad (6)$$

The beam power allocation problem for RBF in order to maximize the sum rate subject to a total power constraint can be formulated as:

$$\begin{aligned} \max_{\mathbf{P}} \mathcal{R}(\mathcal{S}, \mathbf{P}) &= \max_{\mathbf{P}} \sum_{m=1}^M \log_2(1 + \text{SINR}_{k_m, m}(\mathbf{P})) \\ \text{s.t.} \quad \sum_{m=1}^M P_m &\leq P, P_m \geq 0, m = 1, \dots, M \end{aligned} \quad (7)$$

We first remark that the power constraint is always satisfied with equality. This is easily verified by noting that any power

vector  $\mathbf{P}'$  with  $\sum_m P'_m < P$  cannot be the optimum power vector. For any  $\beta > 1$ , a power vector  $\mathbf{P}$  with  $P_m = \beta P'_m, m = 1, \dots, M$  such that  $\sum_m \beta P'_m = P$  increases the sum rate  $\mathcal{R}(\mathcal{S}, \mathbf{P})$ , since it increases all user rates.

In what follows we search for the optimal beam power allocation (power vector  $\mathbf{P}^*$ ) by finding

$$\mathbf{P}^* = \arg \max_{\mathbf{P} \in \mathcal{P}^M} \mathcal{R}(\mathcal{S}, \mathbf{P}) \quad (8)$$

where  $\mathcal{P}^M = \{\mathbf{P} | \sum_m P_m \leq P, P_m \geq 0, m = 1, \dots, M\}$  is the constraint set, which is a closed and bounded set. Although the sum rate function is concave in SINR, it is not strictly concave in power. Thus, the optimization problem is hard to solve due to non-convexity of the objective function, and no transformation into convex by relaxation seems doable. This problem is however typical of sum-rate maximizing power control [18]. In the following sections, we investigate a closed-form optimal solution for a 2-beam system and iterative solutions for the general case.

### A. Optimum Beam Power Allocation for $\mathcal{B} = 2$

For RBF system with  $\mathcal{B} = 2$  beams, the optimum beam power allocation policy under strategy 3 can be derived analytically. The sum rate for user set  $\mathcal{S} = \{k_1, k_2\}$  is given in terms of  $P_1 \in [0, P]$  by:

$$\begin{aligned} \mathcal{R}(\mathcal{S}, P_1) &= \sum_{m=1}^2 \log_2(1 + \text{SINR}_{k_m, m}) \\ &= \log_2 \left[ \left( 1 + \frac{P_1 \gamma_{k_1 1}}{\sigma^2 + (P - P_1) \gamma_{k_1 2}} \right) \left( 1 + \frac{(P - P_1) \gamma_{k_2 2}}{\sigma^2 + P_1 \gamma_{k_2 1}} \right) \right] \end{aligned} \quad (9)$$

Since the logarithm is a monotonically increasing function, we can consider the following objective function:

$$\mathcal{J}(P_1) = \left( 1 + \frac{P_1 \gamma_{k_1 1}}{\sigma^2 + (P - P_1) \gamma_{k_1 2}} \right) \left( 1 + \frac{(P - P_1) \gamma_{k_2 2}}{\sigma^2 + P_1 \gamma_{k_2 1}} \right) \quad (10)$$

By Fermat's theorem, the necessary conditions for maxima of the continuous objective function can occur either at its critical points or at points on its boundary. Therefore, the global maximizer of the above generally non-convex optimization problem is given by the following alternatives:

- boundary points of  $\mathcal{P}^2$ :  $P_1 = 0$  or  $P_1 = P$ .
- extreme points on the boundary of  $\mathcal{P}^2$ : i.e., the values  $P_1 \in [0, P]$  resulting from  $\frac{\partial \mathcal{J}(P_1)}{\partial P_1} = 0$ .

Specifically, we have the following result:

**Theorem 1:** For the two-beam RBF, the optimum sum-rate maximizing beam power allocation  $\mathbf{P}^* = (P_1^*, P_2^*)$  is given by:

$$\begin{cases} P_1^* = \arg \max_{P_1 \in \{0, P, P'\}} \mathcal{J}(P_1) \\ P_2^* = P - P_1^* \end{cases} \quad (11)$$

where  $P_1 \in [0, P]$  and

$$P' = \begin{cases} (-B \pm \sqrt{B^2 - 4A\Gamma})/2A & \text{if } A \neq 0 \\ -\Gamma/B & \text{if } A = 0 \end{cases} \quad (12a)$$

$$A = \gamma_{k_1 1} \gamma_{k_2 1} (\gamma_{k_2 1} - \gamma_{k_2 2}) (P \gamma_{k_1 2} + \sigma^2) + \gamma_{k_2 2} \gamma_{k_1 2} (\gamma_{k_1 1} - \gamma_{k_1 2}) (P \gamma_{k_2 1} + \sigma^2) \quad (12b)$$

$$B = \gamma_{k_1 1} (P \gamma_{k_1 2} + \sigma^2) (P \gamma_{k_2 1} \gamma_{k_2 2} + 2 \gamma_{k_2 1} \sigma^2 - \gamma_{k_2 2} \sigma^2) + \gamma_{k_2 2} (P \gamma_{k_1 2} + \sigma^2) (2 \gamma_{k_1 2} - \gamma_{k_1 1}) (P \gamma_{k_2 1} + \sigma^2) \quad (12c)$$

$$\Gamma = \gamma_{k_1 1} \sigma^2 (P \gamma_{k_1 2} + \sigma^2) (P \gamma_{k_2 2} + \sigma^2) - \gamma_{k_2 2} (P \gamma_{k_2 1} + \sigma^2) (P \gamma_{k_1 2} + \sigma^2)^2 \quad (12d)$$

*Proof:* See Appendix I.A. ■

Hence, the optimal power control is not ‘water-filling’, but either TDMA-mode (the BS transmits to only one user/beam with non-zero power) or SDMA-mode in which the transmit power values to multiple users are positive and allocated according to (12a).

1) *Beam power control in extreme interference cases:* To gain more intuition on the optimal power allocation scheme, we investigate two extreme cases in terms of interference. Define the interference factors  $\alpha_{k_m} = \frac{\sum_{j \neq m} \gamma_{k_m j}}{\gamma_{k_m m}}$ . In the 2-beam case, we have  $\alpha_{k_1} = \frac{\gamma_{k_1 2}}{\gamma_{k_1 1}}$  and  $\alpha_{k_2} = \frac{\gamma_{k_2 1}}{\gamma_{k_2 2}}$ . For non-interfering beams (i.e.,  $\alpha_{k_1} = \alpha_{k_2} = 0$ ), the optimal beam power allocation is given by the water-filling power allocation

$$P_1^* = \min \left\{ P, \left[ \frac{P}{2} + \frac{(\gamma_{k_1 1} - \gamma_{k_2 2}) \sigma^2}{2 \gamma_{k_1 1} \gamma_{k_2 2}} \right]^+ \right\} \text{ and } P_2^* = P - P_1^* \quad (13)$$

where  $[x]^+ = \max(0, x)$ . Note that SDMA with equal power allocation is optimal when both users experience the same channel conditions ( $\gamma_{k_1 1} = \gamma_{k_2 2}$ ).

In the case of fully-interfering beams (i.e.,  $\alpha_{k_1} = \alpha_{k_2} = 1$ ), TDMA mode is of course optimal as the solution to (7) under the assumption WLOG  $\gamma_{k_1 1} > \gamma_{k_2 2}$  is

$$P_1^* = P \quad \text{and} \quad P_2^* = 0 \quad (14)$$

2) *Optimality conditions for TDMA transmission mode:*

The beam power solution stated in Theorem 1 implies that the optimum transmission mode is either TDMA ( $P_1 = 0$  or  $P$ ) or SDMA with  $P_1 = P'$ . In this section, we are interested in identifying the region of TDMA optimality and providing the relevant conditions. We first derive conditions requiring knowledge of the interference factors  $\alpha_{k_i} \in (0, 1]$  only. These conditions can be used as practical design rules, especially in distributed resource allocation scenarios. Formally, we have that

**Lemma 1:** *If  $\alpha_{k_i} \geq 0.5$ , the optimum power allocation is  $P_1^* = P$  and  $P_2^* = 0$  (TDMA transmission mode).*

*Proof:* See Appendix I.B. ■

**Corollary 1:** *A sufficient condition for TDMA optimality is*

$$\alpha_{k_1} + \alpha_{k_2} \geq 1 \text{ or equivalently } \left( \frac{1}{\cos^2 \theta_1} \right)^2 + \left( \frac{1}{\cos^2 \theta_2} \right)^2 \geq 3 \quad (15)$$

where  $\theta_i = \angle(\bar{\mathbf{h}}_{k_i}, \mathbf{q}_i)$  is the angle (misalignment) between the direction of the normalized channel  $\bar{\mathbf{h}}_{k_i}$  and beam  $\mathbf{q}_i$ .

*Proof:* The first condition is a direct result of Lemma 1 by summing up the interference factors and the equivalent second relation is derived by using  $\alpha_{k_i} = \tan^2 \theta_i$ . ■

Additionally, if BGI knowledge is allowed (strategy 3), a (sharper) sufficient TDMA optimality condition is the following:

**Lemma 2:** *The optimum power allocation is TDMA mode ( $P_1^* = P$ ) if*

$$\frac{P \gamma_{k_1 1}}{\sigma^2} \geq \frac{1 - \alpha_{k_1} - \alpha_{k_2}}{\alpha_{k_1} \alpha_{k_2}} \quad (16)$$

*Proof:* See Appendix I.C. ■

B. *Beam Power Allocation for more than two beams*

For the general case of  $\mathcal{B} > 2$  beams, an analytical treatment of (7) does not unfortunately seem tractable, because of the lack of convexity. Therefore, we propose a suboptimal - yet efficient - iterative algorithm which aims to increase the system throughput by distributing the total power over the beams. Our algorithm tries to identify the extreme points of the sum rate by finding the power vector  $\mathbf{P}$  that maximizes (7). The extreme points of the objective function are found by iteratively solving the Karush-Kuhn-Tucker (KKT) conditions for the beam power control problem. Note that the power vector to which our algorithm converges is not necessarily optimal (global maximum) for general channels, unless the Hessian of the Lagrangian of the objective function is a negative definite matrix. Nevertheless, simulation results show that significant sum-rate improvement can be provided.

1) *Iterative Beam Power Control Algorithm:* We resort to a suboptimal - yet efficient - iterative algorithm, inspired by iterative water-filling (IWF) algorithms [19], [20], as a means to identify the extreme points on the boundary of  $\mathcal{P}^M$ . In the proposed Iterative Beam Power Control Algorithm, each user iteratively maximizes its own rate by performing single-user water-filling and treating the multiuser interference from all the other users (beams) as noise.

a) *Proposed Algorithm:* Let  $\mathbf{P}^{(0)}$  be the initial point and  $\mathcal{I}(\mathbf{P}^{(i)}) = \sigma^2 + \sum_{j \neq m} P_j^{(i)} \gamma_{k_j j}$  be the interference function at  $i$ -th iteration. The steps of the algorithm are summarized in Table I.

Iterative Beam Power Control Algorithm	
<b>Step 1 (Initialization)</b>	Set $\mathbf{P}^{(0)} = \mathbf{0}$
<b>Step 2</b>	For iteration $i = 1, 2, \dots$ , compute $\forall k_m \in \mathcal{S}$ : $\lambda_{k_m}^{(i)} = \frac{\gamma_{k_m m}}{\mathcal{I}(\mathbf{P}^{(i-1)})} = \frac{\gamma_{k_m m}}{\sigma^2 + \sum_{j \neq m} P_j^{(i-1)} \gamma_{k_j j}}$
<b>Step 3 (Water-filling)</b>	let $\pi^{(i)}$ be the solution of: $\pi^{(i)} = \arg \max_{\pi \geq 0, \sum_m \pi_m \leq P} \sum_{k_m \in \mathcal{S}} \log_2 \left( 1 + \pi_m \lambda_{k_m}^{(i)} \right)$
<b>Step 4 (Update)</b>	let $\mathbf{P}^{(i)} = \pi^{(i)}$

TABLE I  
ITERATIVE BEAM POWER CONTROL ALGORITHM

Some observations are in order: at each iteration  $i$ , once  $\lambda_{k_m}^{(i)} = \frac{\gamma_{k_m m}}{(\sigma^2 + \sum_{j \neq m} P_j^{(i-1)} \gamma_{k_j j})}$  is calculated for each user  $k_m$

using  $P_j^{(i-1)}$ ,  $j \neq m$ , it is then kept fixed and treated as noise. Given the total power constraint  $P$ , the ‘water-filling step’ is a convex optimization problem similar to multiuser water-filling with common water-filling level. Thus, all transmit powers in  $\mathbf{P}$  assigned to beams are calculated simultaneously in order to maintain a constant water-filling level. The algorithm computes iteratively the beam power allocation that leads to sum rate increase and converges to a limit value greater or equal to the sum rate of equal power allocation. Formally, the power assigned to beam  $m$  at iteration  $i$  yields  $P_m^{(i)} = [\mu - 1/\lambda_{k_m}^{(i)}]^+$ , with  $\sum_{k_m \in \mathcal{S}} [\mu - 1/\lambda_{k_m}^{(i)}]^+ = P$ , where  $\mu$  is the common water-filling level. The beam power control for strategy 3 assigns transmit powers over the beams when the sum rate given by the iterative solution is higher than that of the boundary points.

*b) Convergence Issues:* As stated before, no global maximum is guaranteed due to the lack of convexity of sum-rate maximization problem. Therefore, we do not expect that the convergence point of the iterative algorithm be generally a global optimal power solution. Interestingly, it can be shown that the convergence leads to a Nash equilibrium, when considering that each user participates in a non-cooperative game. The convergence to an equilibrium point can be guaranteed since  $\mathcal{I}(\mathbf{P})$  is a standard interference function [19], [21]. The proof of existence of Nash equilibrium follows from an easy adaptation of the proof in [22]. However, the uniqueness of these equilibrium points cannot be easily derived for the case of arbitrary channels.

In this section, we derive analytically the convergence point of the 2-beam case using the iterative algorithm and compare it with the optimal beam power solution given by Theorem 1.

At the steady state, say iteration  $s$ , we have that

$$\begin{cases} P_1^{(s)} = \mu - 1/\lambda_{k_1}^{(s)} \\ P_2^{(s)} = \mu - 1/\lambda_{k_2}^{(s)} \end{cases} \quad \text{with} \quad \begin{cases} \lambda_{k_1}^{(s)} = \frac{\gamma_{k_1 1}}{P_2^{(s-1)} \gamma_{k_1 2} + \sigma^2} \\ \lambda_{k_2}^{(s)} = \frac{\gamma_{k_2 2}}{P_1^{(s-1)} \gamma_{k_2 1} + \sigma^2} \end{cases} \quad (17)$$

and  $\mu = \frac{P}{2} + \frac{1}{2\lambda_{k_1}} + \frac{1}{2\lambda_{k_2}}$  from the sum power constraint. Upon convergence of the algorithm, we have that  $P_i^{(s)} = P_i^{(s-1)}$ ,  $i = 1, 2$ , which results to the system of equations  $\mathbf{A}\mathbf{P}^T = \mathbf{b}$  with

$$\mathbf{A} = \begin{bmatrix} 2 - \gamma_{k_2 1}/\gamma_{k_2 2} & \gamma_{k_1 2}/\gamma_{k_1 1} \\ \gamma_{k_2 1}/\gamma_{k_2 2} & 2 - \gamma_{k_1 2}/\gamma_{k_1 1} \end{bmatrix}$$

$$\mathbf{b} = \begin{bmatrix} P + \sigma^2 \left( \frac{1}{\gamma_{k_2 2}} - \frac{1}{\gamma_{k_1 1}} \right) \\ P + \sigma^2 \left( \frac{1}{\gamma_{k_1 1}} - \frac{1}{\gamma_{k_2 2}} \right) \end{bmatrix}$$

For  $\det \mathbf{A} \neq 0 \rightarrow \alpha_{k_1} \neq 1$  and  $\alpha_{k_2} \neq 1$ , we have that  $\mathbf{P}^T = \mathbf{A}^{-1}\mathbf{b}$ , giving the ‘water-filling’ solution  $(P_1, P - P_1)$  with

$$P_1 = \frac{P\gamma_{k_2 2}(\gamma_{k_1 1} - \gamma_{k_1 2}) + \sigma^2(\gamma_{k_1 1} - \gamma_{k_2 2})}{2\gamma_{k_1 1}\gamma_{k_2 2} - \gamma_{k_2 2}\gamma_{k_1 2} - \gamma_{k_2 1}\gamma_{k_1 1}} \quad (18)$$

It can be observed that (18) is different from (12a). Fortunately, it still provides a heuristic power allocation algorithm and as shown through simulations in Section VII, there is not a significant reduction in sum rate by allocating the power over beams using this algorithm.

We would like to point out that one may additionally resort to Geometric Programming [18], which represents the state of the art in continuous power control for non-convex problems. These techniques are shown to be convergent and it turns out that they often compute the globally optimal power allocation. Interestingly, it can be shown that the heuristic iterative algorithm proposed in Section V-B.1 finds an equivalent interpretation, as applying the so-called *successive convex approximation* [18], [23] to the beam power control problem results in the same iterative algorithm.

### C. Beam Power Control in Specific Regimes ( $\mathcal{B} \geq 2$ )

The apparent non-convexity of the  $\mathcal{B}$ -beam case can be alleviated in certain SINR regimes, as a hidden convexity of the beam power allocation problem appears. We shall consider the beam power allocation for  $\mathcal{B} = M$  beams in three cases: 1) the high SINR regime, 2) the low SINR regime, and 3) approximation by the arithmetic-geometric means inequality.

1) *High SINR regime:* For SINR values higher than 0dB, the approximation  $\log(1+x) \approx \log(x)$  can be applied, and the objective function  $\mathcal{G}(\mathbf{P}) = \mathcal{R}(\mathcal{S}, \mathbf{P})$  becomes

$$\mathcal{G}(\mathbf{P}) \approx \sum_{m=1}^M \log_2(\text{SINR}_{k_m m}) = \log_2 \left( \prod_{m=1}^M \text{SINR}_{k_m m} \right)$$

A similar result has previously observed in [18] in the case of code division multiple access (CDMA) power control and solved by Geometric Programming, as the approximate high-SINR sum rate is a concave function of  $\log P_m$ .

2) *Low SINR regime:* In the low SINR regime, the sum rate is approximated by applying Taylor first-order series expansion, i.e.  $\log(1+x) \approx x$ . Therefore,

$$\mathcal{G}(\mathbf{P}) \approx \log_2 e \sum_{m=1}^M \text{SINR}_{k_m m} = \log_2 e \sum_{m=1}^M \frac{P_m \gamma_{k_m m}}{\sum_{j \neq m} P_j \gamma_{k_m j} + \sigma^2} \quad (19)$$

The objective function (19) is convex in each variable  $P_m$  since

$$\frac{\partial^2}{\partial P_m^2} \left( \sum_{m=1}^M \frac{P_m \gamma_{k_m m}}{\sum_{j \neq m} P_j \gamma_{k_m j} + \sigma^2} \right)$$

$$= \sum_{m \neq i} \frac{2P_m \gamma_{k_m m} \gamma_{k_m i}^2}{\left( \sum_{j \neq m} P_j \gamma_{k_m j} + \sigma^2 \right)^3} \geq 0 \quad (20)$$

Hence, the optimal beam power control strategy is found by the KKT conditions and can be solved numerically using efficient interior-point methods [24].

3) *Arithmetic-geometric means approximation:* From the arithmetic-geometric means inequality [25], the sum rate can be upper bounded as

$$\mathcal{R}(\mathbf{P}) = \log_2 \left( \prod_{m=1}^M (1 + \text{SINR}_{k_m, m}(\mathbf{P})) \right)$$

$$\leq M \log_2 \left( 1 + \frac{1}{M} \sum_{m=1}^M \frac{P_m \gamma_{k_m m}}{\sigma^2 + \sum_{j \neq m} P_j \gamma_{k_m j}} \right) = \mathcal{G}_{AGM}(\mathbf{P}) \quad (21)$$

Since the logarithm is a monotonically increasing function and the argument of the log-function of  $\mathcal{G}_{AGM}(\mathbf{P})$  is convex wrt each  $P_m$ , a close-form global optimal solution can be derived.

The sharpness of the above sum-rate approximation is quantified by the difference  $\delta = \mathcal{G}_{AGM}(\mathbf{P}) - \mathcal{R}(\mathbf{P})$ . For  $h = \frac{\max_i(1+\text{SINR}_{k_i})}{\min_j(1+\text{SINR}_{k_j})} > 1$  the following inequality stands

$$0 \leq \delta \leq \frac{M}{\log_2} K'(h, 1) \quad (22)$$

where  $K'(h, 1) = \log\left(\frac{h^{\frac{1}{h-1}}}{e \log h \frac{1}{h-1}}\right)$  is the first derivative of the Kantorovich constant [26]. Therefore, inequality (21) is tight for equal SINR values, and the approximation is better when the spread of  $(1+\text{SINR})$  values is small ( $h \rightarrow 1$ ).

## VI. BEAM POWER CONTROL WITH SINR FEEDBACK

In this section, we present a low-complexity, low-feedback variant of the two-stage linear beamforming framework. For that, we adopt *strategy 4* in the second stage, assuming thus that the scheduler has access only to the same amount of feedback information as in [14], namely  $\beta'_k = \beta_k = \text{SINR}_{k_m m}$ . Nevertheless, we further exploit this scalar information in view of rendering the precoding matrix more robust with respect to cases where not all  $M$  users can be served satisfactorily simultaneously with the same amount of power. The major challenge here is that when only SINR feedback is available, the transmitter cannot estimate the precise effect on interference and user rate (SINR) if the transmit power had been allocated differently over the beams. In this case we resort to a power control strategy based on the maximization of the expected sum rate.

### A. On/Off Beam Power Control

We propose a simple power allocation scheme, coined as On/Off Beam Power Control, in which the transmitter takes a binary decision between:

- TDMA transmission toward one selected user (the one with maximum  $\beta_k$  from stage 1).
- SDMA where all random equipowered beams are active, as in [14].

This beam power control can be seen as discretization of the continuous power control problem solved in Section V, since the power vector only accepts a binary solution.

The scheduler, based only on SINR feedback, compares the instantaneous achievable SDMA sum rate with the expected TDMA rate, and selects the transmission mode that maximizes the system throughput. Let  $\mathcal{R}_{SDMA} = \sum_{m=1}^M \log(1+\text{SINR}_{k_m m})$  denote the achievable SDMA sum rate that can be explicitly calculated at the BS, and  $\mathcal{R}_{TDMA}$  denote the expected TDMA transmission rate.

The On/Off Beam power control scheme results in the following binary mode decision denoted as  $\mathcal{F}$ :

$$\mathcal{F} = \begin{cases} \text{TDMA} & \text{if } \Delta\mathcal{R} > 0 \\ \text{SDMA} & \text{if } \Delta\mathcal{R} \leq 0 \end{cases} \quad (23)$$

where  $\Delta\mathcal{R} = \mathcal{R}_{TDMA} - \mathcal{R}_{SDMA}$ .

The expected TDMA rate can be evaluated as  $\mathcal{R}_{TDMA} = \mathbb{E}\{\log_2(1 + \frac{P}{\sigma^2} \gamma_{k_1})\}$ , where  $F_{\gamma_{k_1}}(x) = (1 - e^{-x})^K$ , which is given by the following closed-form expression:

**Proposition 1:** For any values of  $P$ ,  $M$ , and  $K$ , the average rate of TDMA-based random beamforming is given by

$$\mathcal{R}_{TDMA} = \frac{1}{\log 2} \sum_{k=1}^K \binom{K}{k} (-1)^k e^{k\sigma^2/P} \text{Ei}(-k\sigma^2/P) \quad (24)$$

where  $\text{Ei}(x) = -\int_{-x}^{\infty} \frac{e^{-t}}{t} dt$  is the exponential integral.

*Proof:* See Appendix I. ■

## VII. NUMERICAL RESULTS

In this section, we evaluate the sum-rate performance of the proposed beam power control algorithms through Monte Carlo simulations, based on the system model described Section II and considering that  $\mathcal{B} = M$  beams are generated. The achieved sum rate is compared with conventional SDMA-based random beamforming [14] where equal power is allocated over the beams.

We simulate a time-varying Rayleigh fading channel where the fading  $\mathbf{h}_k(t)$  is i.i.d. among users and for different antennas. We consider Clark-Jake's Doppler model, with autocorrelation function  $\mathbb{E}\{\mathbf{h}(t)\mathbf{h}(t + \ell T_s)\} = J_0(2\pi f_D \ell T_s)$  where  $f_D$  denotes the one-sided Doppler bandwidth (in Hz) and  $J_0(\cdot)$  is the Bessel function of the first kind. We set the frame duration equal to  $T_s = 1$  ms and carrier frequency is 2 GHz.

We first assess the performance of beam power control with BGI second-stage feedback (strategy 3). As a general comment, it ought to be mentioned that channel variations do not change the capacity scaling of the proposed schemes with respect to the conventional ones. Time variation results in slightly reduced throughput as compared with uncorrelated channels, however as the two-stage approach can be accomplished in the preamble phase, the reported feedback does not become severely outdated. Note that the second stage of feedback collects fresh CSIT, therefore the precoder design does not suffer from extra outdated degradation (compared with a single-stage feedback).

In Figure 1 we present the sum rate achieved using optimal power allocation vs. the number of active users  $K$  for the 2-beam case, SNR = 20 dB and 60 Hz Doppler shift. Single-beam random beamforming refers to the scheme proposed in [15] where only one random beam is generated (TDMA) at each slot. The gains of optimally allocating power across beams are more pronounced for systems with low to moderate number of users (up to 30), whereas for  $K$  increasing, the benefits of beam power control vanishes as the optimal solution advocates the use of equipowered beams, as expected. Figure 2 shows a sum-rate comparison as a function of the average SNR for  $K = 10$  users and  $f_D = 60$  Hz, illustrating that beam power allocation prevents the system from becoming interference-limited. Power control allows us to switch off beams, thus keeping a linear capacity growth in the interference-limited regime and converging to TDMA at high SNR. In Figure 3 we compare the achieved sum rate difference between the optimal

power allocation and the power solution given by our iterative algorithm at SNR = 10 dB and  $f_D = 100$  Hz. Use of the iterative algorithm, despite suboptimal, results in negligible throughput loss at all ranges of  $K$ . The performance of the iterative power control is further evaluated in Figure 4 for a 4-beam downlink showing substantial sum-rate enhancements for practical number of users.

We then evaluate the results of the on/off beam power control (strategy 4), which uses the same amount of feedback as the conventional RBF [14]. Note that no additional feedback is requested during the second stage, thus no delay is introduced in the scheduling protocol. In Figure 5 we plot the sum rate vs. the number of users for  $M = 2$  transmit antennas and SNR = 10 dB. The scheme is switching from TDMA mode at low  $K$ , where all transmit power is given to the highest  $\beta_k = \text{SINR}_{k,m}$  user, to SDMA-based RBF with equal power allocation. We also observe that the sum-rate gap between the optimal power control (with second-stage feedback) and on/off power control (no additional feedback) for  $K < 20$  users is approximately 0.4 bps/Hz. In Figure 6 we consider a 4-beam RBF scheme and show the sum rate performance of on/off beam power control as a function of average SNR. Although the throughput curve of conventional RBF converges to a finite ceiling at high SNR, the TDMA-SDMA binary decision capability of the beam on/off scheme provides a simple means to circumvent the interference-limited behavior of RBF with no extra feedback. We note also that TDMA mode is generally preferable from a sum-rate point of view in sparse networks, and the range of  $K$  in which TDMA is beneficial increases for SNR increasing.

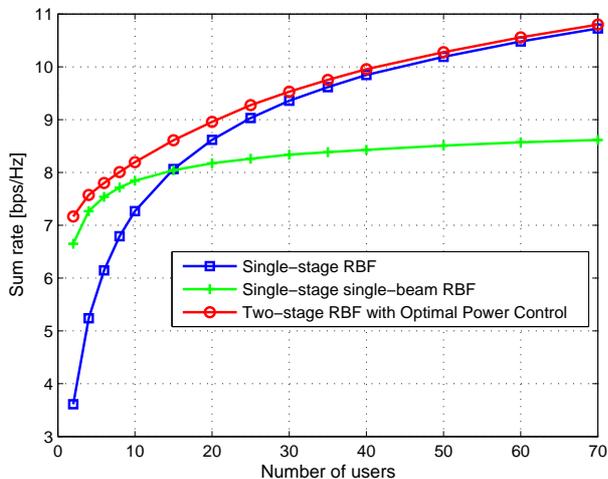


Fig. 1. Sum rate versus the number of users for Optimal Beam Power Control with  $M = 2$  transmit antennas, SNR = 20 dB and Doppler shift  $f_D = 60$  Hz. Significant throughput gains are achieved by optimally allocating the power across beams for systems with low to moderate number of users.

### VIII. CONCLUSION

The downlink of a multiuser MIMO system with limited feedback and more users than transmit antennas was considered. A two-stage scheduling and linear precoding framework,

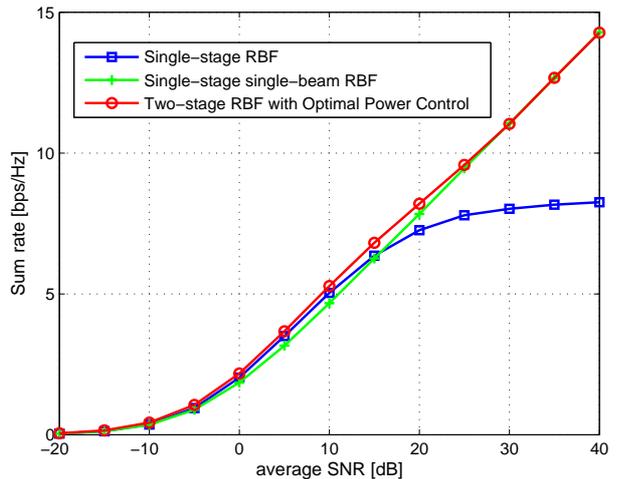


Fig. 2. Sum rate versus average SNR for Optimal Beam Power Control (strategy 3) with  $M = 2$  transmit antennas,  $K = 10$  users, and Doppler shift  $f_D = 60$  Hz. Beam power control circumvents the interference-limited behavior of RBF, achieving a linear sum-rate growth at high SNR.

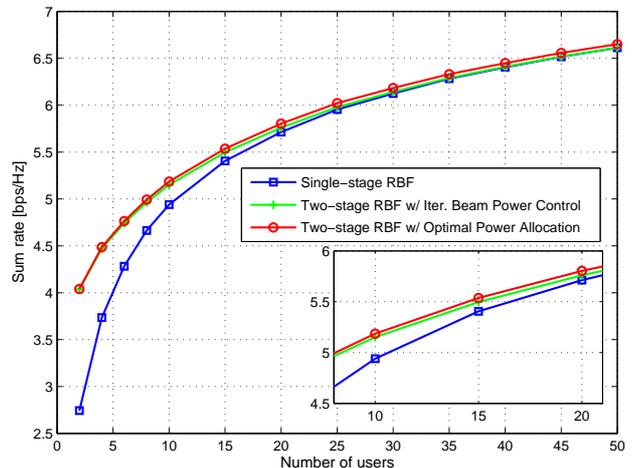


Fig. 3. Sum rate versus the number of users for Iterative Beam Power Allocation and Optimal Power Control with  $M = 2$  transmit antennas, SNR = 10 dB, and Doppler shift  $f_D = 60$  Hz. Use of suboptimal iterative algorithm results in negligible loss compared to the optimal solution.

which builds on previously proposed SDMA-based random beamforming, is introduced. This scheme divides the scheduling and the precoding design stages into two steps, where the final beamformer is designed based on refined CSIT fed back by the users selected in the first stage. Several beam power control strategies, with various levels of complexity and feedback load, are proposed in order to restore robustness of RBF in sparse networks. Their sum-rate performance is assessed, revealing substantial gains compared to RBF for systems with low to moderate number of users, at a moderate or zero cost of extra feedback.

Future research directions may include investigating CSIT quantization and the effect of feedback delay and estimation errors on the sum-rate performance. Fairness and quality of service are also issues of particular practical importance.

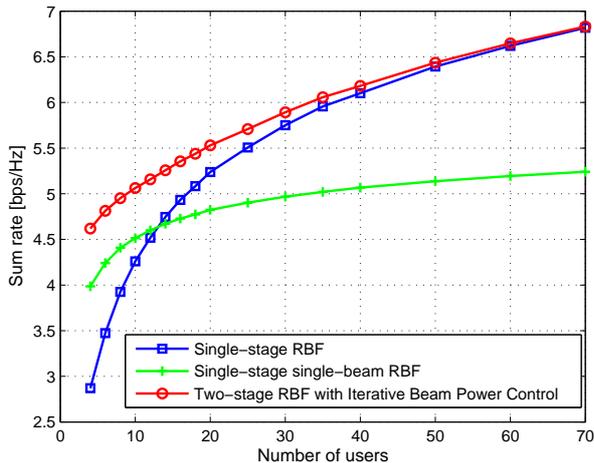


Fig. 4. Sum rate versus the number of users for Iterative Beam Power Allocation with  $M = 4$  transmit antennas, Doppler shift  $f_D = 100$  Hz and SNR = 10 dB.

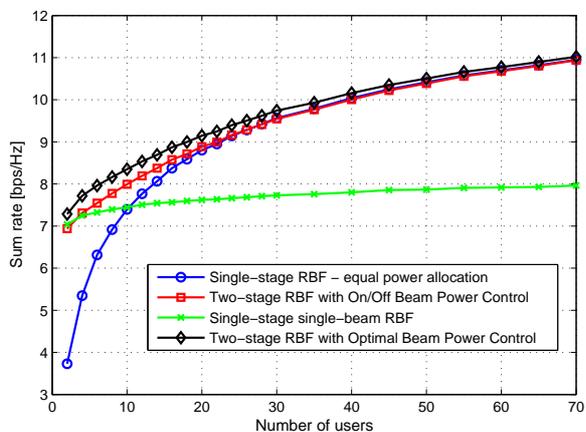


Fig. 5. Sum rate versus the number of users for On/Off Beam Power Control with  $M = 2$  transmit antennas and SNR = 20 dB. On/Off beam power control switches to TDMA mode at low  $K$ , and strategy 4 gives almost same performance as strategy 3 without the additional feedback.

## APPENDIX I

### A. Proof of Theorem 1

Since  $\mathcal{J}(P_1)$  is not always concave in  $P_1$ , the  $P_1$  that maximizes it is either the boundary points ( $P_1 = 0$  and  $P_1 = P$ ) or the solutions corresponding to  $\partial\mathcal{J}/\partial P_1 = 0$ . By differentiating the objective function with respect to  $P_1$ , we have

$$\frac{\partial\mathcal{J}}{\partial P_1} = AP_1^2 + BP_1 + \Gamma \quad (25)$$

where  $A, B, \Gamma$  are given by (12b), (12c) and (12d), respectively. Setting  $\frac{\partial\mathcal{J}}{\partial P_1} = 0$ , the possible values of  $P_1$  that maximize the throughput are the real-valued roots of the second-order polynomial  $AP_1^2 + BP_1 + \Gamma = 0$  (for  $A \neq 0$ ) that satisfy the constraint  $P_1 \in [0, P]$  or  $P_1 = -\Gamma/B$  for  $A = 0$ . Hence, the optimum  $P_1^*$  is the value among the boundary points ( $P_1 = 0$  and  $P_1 = P$ ) and the extreme points (roots of

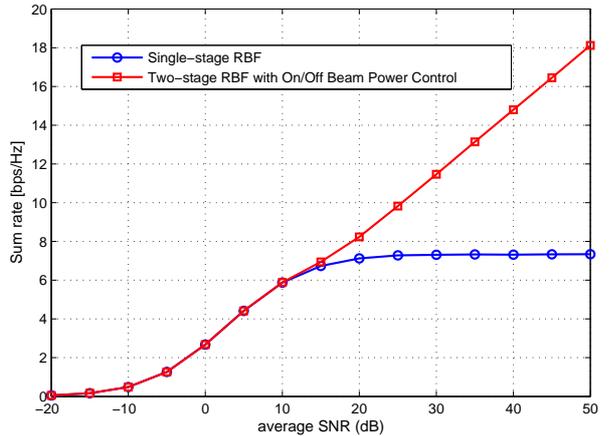


Fig. 6. Sum rate versus average SNR for On/Off Beam Power Control with  $M = 4$  transmit antennas and  $K = 25$  users. Interference-limited behavior (bounded sum rate) is avoided by selecting TDMA transmission mode for increasing SNR.

the polynomial) that maximizes  $\mathcal{J}(P_1)$ , which concludes the proof.

### B. Proof of Lemma 1

Let  $\mathcal{J}_i(P_i)$  ( $i = 1, 2$ ) represent the individual rate of user  $k_i$  given as

$$\mathcal{J}_i(P_i) = \log_2 \left( 1 + \frac{P_i \gamma_{k_i i}}{\sigma^2 + (P - P_i) \gamma_{k_i j}} \right) \quad (26)$$

$$= \log_2 \left( 1 + \frac{P_i}{\sigma^2 / \gamma_{k_i i} + \alpha_{k_i} (P - P_i)} \right), \quad j \neq i \quad (27)$$

The sum-rate maximizing beam power allocation problem can be rewritten as

$$\max_{\mathbf{P} \in \mathcal{P}^2} \mathcal{J}_1(P_1) + \mathcal{J}_2(P_2) \quad \text{subject to } P_1 + P_2 = P$$

We investigate now the behavior of the individual user rate objective function. By calculating the first and second derivative of  $\mathcal{J}_i(P_i)$  we have

$$\frac{\partial\mathcal{J}_i(P_i)}{\partial P_i} = \frac{\Delta + \alpha_{k_i} P_i}{\Delta(\Delta + P_i)} > 0 \quad (28)$$

$$\frac{\partial^2\mathcal{J}_i(P_i)}{\partial P_i^2} = \frac{d_1(\Delta + \alpha_{k_i} P_i)}{d_2} \quad (29)$$

with  $\Delta = \alpha_{k_i}(P - P_i) + \sigma^2/\gamma_{k_i i}$ ,  $d_1 = (2\alpha_{k_i} - 1)\Delta + \alpha_{k_i} P_i$ , and  $d_2 = \Delta^2(\Delta + P_i)^2$ . The sign of  $d_1$  determines the convexity or concavity of  $\mathcal{J}_i(P_i)$ . If  $d_1 > 0 \rightarrow P_i > \left(\frac{1}{\alpha_{k_i}} - 2\right)\Delta$ ,  $\mathcal{J}_i(P_i)$  is a convex function of  $P_i$ , and concave otherwise. Since  $\Delta > 0$ , for  $\alpha_{k_i} \geq 0.5$  the objective function  $\mathcal{J}_i(P_i)$  is convex  $\forall i$ , i.e.  $\frac{\partial\mathcal{J}_i(P_i)}{\partial P_i} > 0$ , hence the sum of two convex functions  $\mathcal{J}_1(P_1) + \mathcal{J}_2(P_2)$  is maximized for  $P_1^* = P$  and  $P_2^* = 0$ .

### C. Proof of Lemma 2

Let  $\mathcal{R}_{TDMA} = \log_2 \left( 1 + \frac{R\gamma_{k_1 1}}{\sigma^2} \right)$  denote the system throughput for TDMA mode. TDMA is optimal when  $\mathcal{R}_{TDMA} \geq \mathcal{R}(\mathbf{P}) \rightarrow \log_2 \left( \frac{\mathcal{A}(P_1)}{\mathcal{C}(P_1)} \right) \geq 0$ , where

$$\mathcal{A}(P_1) = \left( 1 + \frac{P\gamma_{k_1 1}}{\sigma^2} \right) (P\alpha_{k_1} \gamma_{k_2 2} + \sigma^2) ((P - P_1)\alpha_{k_1} \gamma_{k_1 1} + \sigma^2)$$

$$\begin{aligned} \mathcal{C}(P_1) &= (P\alpha_{k_1} \gamma_{k_1 1} + P_1\gamma_{k_1 1}(1 - \alpha_{k_1}) + \sigma^2) \\ &\quad \times (P\gamma_{k_2 2} + P_1\gamma_{k_2 2}(\alpha_{k_2} - 1) + \sigma^2) \end{aligned}$$

The region of TDMA optimality depends on the convexity of  $\Psi(P_1) = \mathcal{A}(P_1) - \mathcal{C}(P_1)$ . By differentiating twice we have that

$$\frac{\partial^2 \Psi(P_1)}{\partial P_1^2} = -2\gamma_{k_1 1} \gamma_{k_2 2} \left( \frac{P\gamma_{k_1 1}}{\sigma^2} \alpha_{k_1 1} \alpha_{k_2 2} + \alpha_{k_1 1} + \alpha_{k_2 2} - 1 \right) \quad (30)$$

For  $\frac{\partial^2 \Psi(P_1)}{\partial P_1^2} \leq 0$ ,  $\Psi(P_1)$  is concave wrt  $P_1$  ( $\Psi(P_1) \geq 0$ ), since  $\Psi(0) \geq 0$  and  $\Psi(P) = 0$ , which results in (16).

### D. Proof of Proposition 1

Since the cumulative distribution function of  $s = P|\mathbf{h}_k \mathbf{q}_1|^2 / \sigma^2$  is  $F_s(x) = 1 - e^{-x\sigma^2/P}$ , the average sum rate of TDMA-based random opportunistic beamforming is given by

$$\begin{aligned} \mathcal{R} &= \mathbb{E} \left\{ \log_2 \left( 1 + \max_{1 \leq k \leq K} \frac{P|\mathbf{h}_k \mathbf{q}_1|^2}{\sigma^2} \right) \right\} \\ &= \int_0^\infty \log_2(1+x) dF_s^K(x) dx \\ &\stackrel{(a)}{=} \frac{1}{\log 2} \int_0^\infty \frac{1}{1+x} (1 - F_s^K(x)) dx \\ &= \log_2 e \int_0^\infty \frac{1}{1+x} (1 - (1 - e^{-x\sigma^2/P})^K) dx \\ &\stackrel{(b)}{=} -\frac{1}{\log 2} \sum_{k=0}^{K-1} \binom{K}{k} (-1)^k \int_0^\infty \frac{e^{-xk\sigma^2/P}}{1+x} dx \\ &= \frac{1}{\log 2} \sum_{k=1}^K \binom{K}{k} (-1)^k e^{k\sigma^2/P} \text{Ei}(-k\sigma^2/P) \end{aligned}$$

where integration by parts is applied to obtain (a) and (b) follows from binomial expansion.

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