Distributed Power Allocation for Interfering Wireless Links Based on Channel Information Partitioning

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Abstract-Network-wide optimization of transmit power with the goal of maximizing the total throughput, promises significant system capacity gains in interference-limited data networks. Finding distributed solutions to this global optimization problem however, remains a challenging task. In this work, we first focus on the maximization of the weighted sum-rate capacity, as this allows the incorporation of QoS criteria in the objective function. For the case of two links, we are able to analytically characterize the optimal solution to the weighted sum-rate maximization problem. However, computing the optimal solution requires centralized knowledge of network information. We thus formulate a framework for distributed power optimization valid for N mutually interfering links, based on the concept of channel state partitioning. By assuming instantaneous knowledge of local information and statistical knowledge of non-local information, we derive a distributed power allocation algorithm, which we first analyze for the case of N = 2. Although a gain is observed over equal power allocation, the distributed algorithm shows a performance gap as compared to a centralized solution, as expected. We show however, that minimal information message passing (in this case one bit) between interfering links can help reduce this gap substantially. Finally, we also propose a method to incorporate user scheduling into the distributed power allocation algorithm.

Index Terms—Power control, Co-channel interference, Distributed, Resource allocation, Full spectrum reuse, Weighted sum rate, Scheduling, Multi-user Diversity

I. INTRODUCTION

Links operating on the same spectral resource are plagued by mutual interference which diminishes system capacity. Power control serves as a means to mitigate this effect and has been an extensively researched topic for the past 20 years. In traditional voice-centric wireless networks, power control was found to be an effective method to enhance the reliability of the system. A number of approaches have been proposed to address this problem, [1]–[5] to name a few. The key idea of earlier power control work is to either aim for a certain target received power, or to balance the transmit powers to achieve a minimum acceptable level for the signal-to-interference-plusnoise ratio (SINR) for each receiver. This is to guarantee a target outage probability for the communication link, which is the measure of Quality of Service (QoS) in connectionoriented voice networks.

In this work however, we investigate power allocation in the context of future data wireless networks enabled with link adaptation protocols. Based upon underlying channel conditions, such systems are able to adapt the transmit rate (or select transmit rates) through adaptive modulation and coding. Moreover, due to the elastic nature of data traffic (web browsing, email, etc.), guaranteeing a strict SINR requirement is not always required. Rather, maximizing the total amount of data transferred becomes a more relevant performance goal. However, having some form of Quality of Service (QoS) constraints on performance is none the less desirable for the operator, who may offer different levels of service to end users. In light of these arguments, we consider sum-rate and possibly weighted sum-rate of the system as our performance criterion, and formulate the power allocation problem to maximize the metric. The weighted sum-rate function proves useful for adaptive resource allocation policies, where, by virtue of the weights, a link can be more or less prioritized with respect to resources depending on QoS or fairness constraints. We characterize the optimal solution to this problem, which also encompasses the unweighted sum-rate maximization problem previously addressed in [6], [7]. However, finding this solution entails centralized processing of network-wide channel state information. Although this promises the maximum exploitable gain, it may be too costly from a signaling overhead point of view.

Consequently, sub-optimal distributed solutions to this problem are desirable if we hope to achieve some of the theoretical gains in practice. As one avenue, *game theoretic* results have been explored for this purpose (see for instance [8], among others). Typically, game theoretic algorithms represent the interfering links in the network as players of a non-cooperative game, where each tries to maximize its own utility function. Although the resulting power allocation strategies are very interesting and distributed by nature, such approaches do not always lead to globally (or "socially") optimum solutions. As a remedy, pricing mechanisms have been looked at, which aim at penalizing the interference created to other links, in order to make the game outcome more socially optimum [8]– [12]. However, the pricing function needs to be optimized as

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well, and it typically depends on the global system layout and environment [8], [10]. Some game theoretic approaches also require communication of information between links to compute the pricing function [12]. As an alternative to game theoretic approaches, Geometric Programming (GP) techniques can be applied in the high SINR regime, which render the sum-rate maximizing power control problem convex [13], [14]. Finally, distributed sum-rate maximizing power control and scheduling algorithms were proposed in [15], taking advantage of a simplified interference model. Such approaches rely, however, on statistical averaging properties of large random networks and thus are not applicable for all networks [16]. In the course of this study, we will formulate a generalized distributed optimization problem by taking advantage of statistical knowledge of "non-local" information in the network. This formulation is independent of any underlying channel model or system architecture and thus can be applied to both cellular or ad-hoc scenarios.

The most important contributions of this work are as follows:

- We initially formulate the power allocation problem for maximizing a weighted sum-rate criterion, which would enable the incorporation of certain types of QoS constraints on the links.
- For the particular case of two cells or links (say for a two-cell network or a larger network with clusters of two cells), we are able to characterize the optimal power allocation.
- We then propose a framework for distributed sum-rate maximizing power control in an arbitrary network with several interfering cells or links, based on the concept of channel state knowledge partitioning according to *local* and *non-local* information.
- By considering the two-cell case in the above framework, we derive simple conditions for link activation, based on signal-to-noise ratio (SNR) and SINR.
- By allowing 1-bit information message passing between interfering links, substantial improvement in the capacity performance can be obtained through a simple modification of the proposed algorithm. This almost closes the gap between distributed and centralized control.
- Finally, we incorporate user scheduling into the power allocation algorithm, so as to exploit an added multi-user diversity gain.

Numerical results show that the fully distributed and near distributed power allocation algorithms largely outperform a system without power control, and are able to extract a significant amount of the performance gain exhibited by centralized power control.

The rest of this paper is organized as follows: In Section II we describe the system model considered in this work. We then formulate and characterize the centralized optimal power allocation problem for weighted sum-rate maximization in Section III. We then propose an optimization framework for the distributed power allocation problem in Section IV. In Section V, a simple distributed algorithm is presented for link activation as well as a modified algorithm which exploits



Fig. 1. Snapshot of network model, with N = 4 interfering pairs of transmitters and receivers. The cellular model (a) and the single-hop peer-to-peer or ad-hoc model (b) give rise to equivalent mathematical models. Dashed circles refer to silent users while solid circles refer to access points or users selected by the scheduler.

minimal message passing. We will also discuss joint use of user scheduling with power allocation. Numerical results in Section VI demonstrate the performance of these algorithms, after which we conclude the paper.

II. SYSTEM MODEL

Consider a wireless network with a collection of nodes, which can be both transmitters and receivers. Initially we consider that by virtue of a scheduling protocol, N transmitreceive active pairs are already simultaneously selected from these nodes to potentially communicate at a given time instant, while others remain silent. Later on we will incorporate scheduling into the power allocation problem as well, i.e., joint power allocation and user scheduling. In this network, each transmitter sends a message which is intended for its receiver only. This setup can be seen as an instance of the interference channel, the analysis of which is a famously difficult problem in information theory [17]. We also assume that there is no interference cancellation capability at the receivers, nor can they jointly decode signals. In such circumstances, the receiver is interfered by all other active links due to full reuse of the spectral resource, and this interference is treated as noise. In practical terms, the situation depicted above can be that of a cellular network with reuse factor one (e.g. the downlink, with transmitters being access points (AP) or base stations), or it can also depict a snapshot of a single-hop ad-hoc network (Fig.1).

In this work, the proposed power allocation algorithms would apply to single hop wireless networks which can be either single-hop ad-hoc networks or full-reuse cellular networks. When, we look at user scheduling, this would be more suited to a cellular network where there is a user population allowing us to exploit scheduling gains. A practical example of where the results presented herein might be applicable are fixed broadband wireless access networks [18]

A. Signal Model

We consider N synchronized links which are active on any given *spectral resource slot* (where resource slots can be time or frequency slots in TDMA/FDMA, or codes in orthogonal CDMA). Due to full spectral resource reuse, the receiver sees interference from the transmitters of all other links. Denoting the random channel gain between the transmitter of any arbitrary link *i* and the receiver of link *n* by $G_{n,i} \in \mathbb{R}^+$, the received signal of Y_n can be written as

$$Y_n = \sqrt{G_{n,n}} X_n + \sum_{\substack{i=1\\i\neq n}}^N \sqrt{G_{n,i}} X_i + Z_n, \tag{1}$$

where X_n is the intended signal from the transmitter, $\sum_{i=1,i\neq n}^{N} \sqrt{G_{n,i}}X_i$ is the sum of interfering signals from other transmitters, and Z_n is the noise. For convenience, Z_n is modeled as additive white Gaussian noise (AWGN) with power $\mathbb{E}|Z_n|^2 = \eta_n$.

III. OPTIMAL POWER ALLOCATION

A. Problem Formulation

In this section, we formulate the optimal power allocation problem for maximizing a certain metric based on the sum of individual link capacities. For this purpose, we define the *transmit power vector* P, which contains transmit power values used by each transmitter to communicate with its respective receiver:

$$\boldsymbol{P} = [P_1, P_2, \dots, P_n, \dots, P_N].$$

where $[\mathbf{P}]_n = P_n$. As in all realistic networks we impose a power constraint on each transmitter such that $P_{\min} \leq P_n \leq P_{\max}$, and we assume from here on that $P_{\min} = 0$. Thus the constrained set of transmit power vectors is given by:

$$\Omega = \{ \boldsymbol{P} \mid 0 \le P_n \le P_{\max} \forall n = 1, \dots, N \}.$$

The signal to interference-plus-noise ratio (SINR) at the receiver of link n is then given by

$$\gamma_n(\mathbf{P}) = \frac{G_{n,n}P_n}{\prod_{\substack{n \\ i \neq n}}^N G_{n,i}P_i},$$
(2)

where $P_n = \mathbb{E}|X_n|^2$. Assuming an ideal link adaptation protocol and perfect CSI at the transmitter, the rate of link n can then be expressed in bits/sec/Hz using the Shannon capacity [17] as

$$\mathcal{R}_n(\boldsymbol{P}) = \log_2\left(1 + \gamma_n(\boldsymbol{P})\right),$$
 (3)

which is clearly dependent upon the complete transmit power vector.

1) Weighted Sum-Rate Capacity: In this case, the objective function we consider is the weighted sum-rate, defined as

$$\mathcal{C}(\boldsymbol{P}) \stackrel{\Delta}{=} \sum_{n=1}^{N} w_n \mathcal{R}_n(\boldsymbol{P}). \tag{4}$$

Here, $w_n \ge 0$ is the weight associated with the receiver of link n. For the particular case of a cellular network, if there are U_n users in each cell n, the weights are associated with each user $u_n \in [1, \ldots, U_n]$ which may be scheduled at any given instant. This choice of objective function is of particular interest in adaptive resource allocation policies. Specifically, a resource allocation unit can prioritize users by adjusting their respective weights, so as to achieve some sort of fairness or to fulfill QoS constraints. For example, traffic queue states can be observed for each user and the weights set accordingly so as to minimize the delay. Another scheme can be imagined where the weights are adjusted according to the throughput the users have already experienced so as to obtain some sort of rate fairness. Thus, this choice of objective function finds relevance in scenarios where QoS constraints may need to be met. We also point out here that sum-rate maximization is a special case of (4) when $w_n = 1 \forall n$. We will touch upon this special case later on in the text.

2) Optimal Power Allocation Problem: Taking (4) as the objective function we want to maximize, the optimal power allocation problem can be stated as

$$\boldsymbol{P}^* = \arg\max_{\boldsymbol{P}\in\Omega} \mathcal{C}(\boldsymbol{P}). \tag{5}$$

This problem is known to be non-convex [13], and an optimal solution would require an exhaustive search over the feasible set of transmit powers which entails high complexity as well as centralized processing.

By considering N = 2, i.e., just two links, we obtain some more insight into the problem at hand. Thus, in the next section, we investigate the optimal solution to the weighted sum-rate maximization power allocation problem for two interfering links.

B. Weighted Sum-Rate Optimal Power Allocation for N = 2

For two links, problem (5) can be written as

$$\boldsymbol{P}^* = \arg \max_{\boldsymbol{P} \in \Omega} \left(w_1 \mathcal{R}_1(\boldsymbol{P}) + w_2 \mathcal{R}_2(\boldsymbol{P}) \right), \tag{6}$$

We will now characterize the optimal solution to the power allocation problem for weighted sum-rate maximization. We first present the following lemma:

Lemma 1: The optimal solution to the weighted sum-rate maximizing power allocation problem (5), has at least one link operating at P_{max} .

Proof: This is straightforward from the proof for Lemma 1 in [6].

Letting

$$J(P_1, P_2) = w_1 \log_2(1 + \frac{G_{1,1}P_1}{\eta_1 + G_{1,2}P_2}) + w_2 \log_2(1 + \frac{G_{2,2}P_2}{\eta_2 + G_{2,1}P_1}),$$

through Lemma 1 we may let one of the links operate at maximum power by setting $P_2 = P_{\text{max}}$. Our task is then reduced to finding the optimal P_1 . The derivative of $J(P_1, P_{\text{max}})$ w.r.t P_1 can be expressed as

$$\frac{\partial J(P_1, P_{\max})}{\partial P_1} = \frac{aP_1^2 + bP_1 + c}{f(P_1)},$$

where

$$a = w_1 G_{1,1} G_{2,1}^2,$$

$$b = 2w_1 G_{1,1} \eta_2 G_{2,1} + G_{1,1} G_{2,1} G_{2,2} P_{\max}(w_1 - w_2)$$

$$c = w_1 G_{1,1} \eta_2^2 + w_1 G_{1,1} \eta_2 P_{\max} G_{2,2}$$

$$-w_2 P_{\max} G_{2,2} G_{2,1} \eta_1 - w_2 P_{\max}^2 G_{2,2} G_{2,1} G_{1,2},$$

$$f(P_1) = (\eta_2 + P_1 G_{2,1} + P_{\max} G_{2,2}) \cdot (\eta_2 + P_1 G_{2,1})$$

$$\cdot (\eta_1 + P_{\max} G_{1,2} + P_1 G_{1,1}).$$

We see that $f(P_1)$ is always positive, and in order to find P_1 such that $\frac{\partial J(P_1, P_{\max})}{\partial P_1} = 0$, we need to solve $aP_1^2 + bP_1 + c = 0$.

Note that when $w_1 = w_2$, i.e. the links are symmetric, a, b > 0 and this results in the scenario already treated in [6], [7]. In this case the optimal power allocation is *binary*, as expressed in Lemma 2.

Lemma 2: The optimal sum-rate capacity maximizing power allocation for 2 interfering links when $w_1 = w_2$, i.e.

$$P^* = \arg \max_{P \in \Omega} \sum_{n=1}^{2} \mathcal{R}_n(P),$$

lies in the binary set

$$\Omega^{B} = \{ \boldsymbol{P} \mid [\boldsymbol{P}]_{n} \in \{0, P_{\max}\} \}.$$
Proof: See [6], [7]¹.

When $w_1 > w_2$, the links are no longer symmetric. In this case a, b > 0, and P_1 is either 0 or P_{\max} if P_2 is set to P_{\max} . However, when $w_1 < w_2$, b may no longer be positive and thus the potential non-binary solution may also be possible as well:

$$P_1' = \frac{-b \pm \sqrt{b^2 - 4aa}}{2a}$$

For P_2 , a similar analysis can be carried out to see that when $w_1 > w_2$, we need to check P'_2 , obtained similar to P'_1 by simply inverting the indices of a, b, and c. Only positive real solutions which satisfy the power constraint need to be considered. This leads us to state the following theorem:

Theorem 1: The optimal power allocation for weighted sum-rate capacity maximization of 2 interfering links is given in (8)

As an example, consider the weights $w_1 = 0.1369$, $w_2 = 0.4544$, and the following channel gain matrix:

$$\mathbf{G} = \left(\begin{array}{cc} 0.9611 & 0.2004\\ 0.0940 & 0.5219 \end{array}\right),$$

where the *i*, *j*-th entry of the matrix *G* represents the channel gain $G_{i,j}$. We take the maximum power to be $P_{\max} = 1$, and assume from here on that the noise powers are the same for all links, $\eta_1 = \eta_2 = \eta = 0.1$. By employing the conditions in (8), allocating the power $(P_1^*, P_2^*) = (0.1203, 1)$ yields a weighted sum-rate of $C(P_1^*, P_2^*) = 1.2040$, which is slightly better than $C(P_1, P_2) = 1.1981$, obtained by the best binary



Fig. 2. Variation of transmit powers with changing weights for 2 interfering links. Channel gains are taken as $G_{1,1} = 0.9611$, $G_{1,2} = 0.2004$, $G_{2,2} = 0.5219$, $G_{2,1} = 0.0940$, and noise power is considered to be $\eta_1 = \eta_2 = 0.1$. Weight of link 1 is varied and $w_2 = 1 - w_1$. Binary power allocation is no longer optimal for the weighted sum rate maximization problem.

allocation, here $(P_1, P_2) = (0, 1)$. We also show the effect of varying the weights on the optimal power allocation in Fig. 2. Here we vary w_1 and take $w_2 = 1 - w_2$. We observe that for certain values of weights, intermediate power values (other than 0 or P_{max}) are indeed optimal for weighted sumrate maximization, which is in contrast to the equal weights (or no weights) case where binary power allocation is always optimal [6]. However, we also compare the weighted sum-rate obtained by searching over the optimal power allocation set (8), to searching over only the binary power allocation given by (7). Interestingly, Fig. 3 shows that although binary power allocation is not optimal, the difference between the two in terms of weighted sum-rate is quite small. Although we do not generalize this here, we will take advantage of this observation in the next section, where we propose a distributed algorithm for power allocation based on binary solutions. Moreover, it is also interesting to explore binary power allocation for the case of N > 2 and this has been studied in [7].

Note that in order to compute the optimal power allocation for weighted sum-rate maximization (8), centralized knowledge of the link state information and link weights is required. This is hard to realize in practice, as feeding back and processing all network information presents significant signaling and computational overhead. Thus, in the next section we propose a framework for a distributed solution of the optimal power allocation problem.

IV. DISTRIBUTED POWER ALLOCATION

Distributed optimization is an important problem as it enables the implementation of an otherwise unpractical centralized solution, especially for large systems. Finding good distributed optimization algorithms however proves to be a formidable task, as the objective function being optimized usually depends on all system parameters. Obtaining the optimal solution would thus require the gathering and processing

¹Note that this result was also reported independently in [19].

$$(P_1^*, P_2^*) = \begin{cases} \arg \max_{(P_1, P_2) \in \Omega^B} J(P_1, P_2) & w_1 = w_2 \\ \arg \max_{(P_1, P_2) \in \{\Omega^B \cup (P_{\max}, P_2')\}} J(P_1, P_2) & w_1 > w_2 \\ \arg \max_{(P_1, P_2) \in \{\Omega^B \cup (P_1', P_{\max})\}} J(P_1, P_2) & w_1 < w_2 \end{cases}$$

$$(8)$$



Fig. 3. Variation of weighted sum-rate with changing weights for 2 interfering links. By searching over the optimal power allocation set a very small gain is obtained as compared to just searching over binary power allocation.

of all system information, which is difficult in practice. In order to obtain a distributed solution, one can imagine however compromising on the amount of information available, so that a pragmatic, though sub-optimal solution is obtained.

For this purpose, we introduce the idea of channel state partitioning where the network channel information is divided into two classes: local information of which we can have instantaneous knowledge, and non-local information of which, we assume only statistical knowledge is available. Clearly, the notion of local and non-local information is receiver dependent. For the power allocation problem being considered, each link would make a decision based on local information, i.e. what the transmitter or receiver can measure locally, plus information fed back from the receiver to the transmitter. The resulting algorithms would be sub-optimal compared to the centralized solution, as some kind of assumption would have to be made about other links' behavior. Nonetheless, we argue that this is a practical form of distributed control in terms of both complexity and information exchange. In what follows, we formulate the distributed power allocation problem under statistical knowledge of non-local information. Note that this statistical knowledge can be acquired a priori, during a network calibration preamble.

A. Network Capacity Maximization Framework Under Statistical Knowledge

As stated, we assume that each transmitter has instantaneous local knowledge. Let us denote the set of complete network information by $\mathcal{G} = \{G_{i,j}\} \forall i, j$ and for ease of analysis,

from now on we assume that the noise is equal for all links i.e., $\eta_n = \eta \ \forall \ n$. The local information of which transmitter nhas instantaneous knowledge is given by $\mathcal{G}_n^{\text{local}}$. Thus, non-local information for transmitter n can be denoted as $\tilde{\mathcal{G}}_n = \mathcal{G} \setminus \mathcal{G}_n^{\text{local}}$, of which we assume only statistical knowledge. Based on this framework, link n then tries to maximize the *expected network capacity* defined as

$$\overline{\mathcal{C}}_{n}(\boldsymbol{P}) \stackrel{\Delta}{=} \mathbb{E}_{\tilde{\mathcal{G}}_{n}|\mathcal{G}_{n}^{\text{local}}} \left\{ \sum_{m=1}^{N} w_{m} \log_{2} \left(1 + \frac{G_{m,m}P_{m}}{\eta + \sum_{\substack{i=1\\i \neq m}}^{N} G_{m,i}P_{i}} \right) \right\}.$$
(9)

 $\mathbb{E}_{\tilde{\mathcal{G}}_n|\mathcal{G}_n^{\text{local}}}\{\cdot\}$ is the expectation operator averaging the capacity over all realizations of $\tilde{\mathcal{G}}_n$, conditioned on the knowledge of $\mathcal{G}_n^{\text{local}}$. The distributed power allocation problem under this framework can thus be written as

$$\boldsymbol{P}_{n}^{*} = \left[\arg\max_{\boldsymbol{P}\in\Omega} \overline{\mathcal{C}_{n}}(\boldsymbol{P})\right]_{n} \quad \forall \ n = 1, \dots, N.$$
(10)

B. Local v.s. Non-Local Channel Knowledge Partitioning: One Example

Clearly the choice of local and non-local information will significantly impact the distributed solution of the power allocation problem. The sets of local and non-local information can be partitioned in a number of ways, depending on the knowledge each link has. Optimal partitioning is actually an exciting open research problem. For the problem at hand, we let *local* information be $\mathcal{G}_n^{\text{local}} = \{G_{n,j}, w_n \forall j\}$. This means that a transmitter has knowledge of the direct channel, the interference from other cells to its intended receiver, and the weight of the user it is serving². This is a natural choice for local information, as these values can be measured at the receiver and fed back to the transmitter. Practically, channel information can be periodically fed back by the receiver to the transmitter through a pilot/dedicated channel. Thus the non-local information at the transmitter is given by $\tilde{\mathcal{G}}_n$ = $\{G_{i,i}, w_i \forall j, i \neq n\}$. Under this knowledge, the expected network capacity that transmitter n tries to maximize is given

²In another instance, assuming further restriction of the feedback channel, we may define $\mathcal{G}_n^{\text{local}} = \{G_{n,n}, w_n\}$, in which case the knowledge of interference appears only through its statistics.

by

$$\overline{\mathcal{C}_{n}}(\boldsymbol{P}) \stackrel{\Delta}{=} w_{n} \log_{2} \left(1 + \frac{G_{n,n}P_{n}}{\eta + \sum_{\substack{i=1\\i \neq n}}^{N} G_{n,i}P_{i}} \right) \\
+ \mathbb{E}_{\tilde{\mathcal{G}}_{n}|\mathcal{G}_{n}^{\text{local}}} \left\{ \sum_{\substack{m \neq n}}^{N} w_{m} \log_{2} \left(1 + \frac{G_{m,m}P_{m}}{\eta + \sum_{\substack{i=1\\i \neq m}}^{N} G_{m,i}P_{i}} \right) \right\}.$$
(11)

From the power allocation vector resulting from this maximization, link n uses P_n^* as the transmission power to its respective user. However, calculation of the expected capacity from all other links is not so trivial. In the next section, we thus focus again on the two-link case, which offers insight into the potential gain offered by this distributed approach. We propose a simple distributed algorithm to solve this problem, as well as a modified version of this algorithm incorporating 1-bit information exchange between neighboring links to enhance performance.

V. DISTRIBUTED POWER ALLOCATION FOR TWO LINKS

We now consider problem (10) specifically for two links. The case of two links, though not realistic, allows us to analytically explore the performance of the distributed approaches proposed later. The algorithms developed can then be used in a wider network with more links, where links are previously paired up in clusters of two links. Forming of the clusters should favor strongly interfering links, for which a distributed resource allocation technique will exhibit the largest benefits. Networks with sparsely deployed cells might be a case where clustering with dominant interferer will provide significant gains. For example, in a cellular network, adjacent cells are often the dominant interferers as the pathloss degradation between them is the least. A potential clustering method would be to determine the pairs of cells that interfere the most with each other based on average pathloss statistics.

Notice also that the proposed framework exploits statistical information about other links, *including* the weights of other links. Guaranteeing QoS usually requires the weights to be adapted at each scheduling instant, making the weights instantaneous parameters. If the weights correspond to a grade of service that a user has purchased, then we can assume the weights to be independent of the channel gains. Moreover, as the grade of service of all users is known to the network, we also assume knowledge of the *average weight* $\mathbb{E} \{w_n\} = \overline{w}$, a user may have. Focusing on link 1, we have knowledge of $G_{1,1}$, $G_{1,2}$ and w_1 (Fig. 4). We can write the expected network capacity as a function of the transmit powers as

$$\overline{\mathcal{C}}_{1}(P_{1}, P_{2}) = w_{1} \log_{2} \left(1 + \frac{G_{1,1}P_{1}}{\eta + G_{1,2}P_{2}} \right) + \overline{w} \mathbb{E} \left\{ \log_{2} \left(1 + \frac{G_{2,2}P_{2}}{\eta + G_{2,1}P_{1}} \right) \right\}, (12)$$



Fig. 4. A 2 cell/link scenario with mutual interference. Local information of link *n* is given by $\mathcal{G}_n^{\text{local}} = \{G_{n,i}, w_n \forall i\}$, i.e. the direct channel and interfering channel at the receiver.

where the expectation is taken over the distribution of other link channel gains, namely $G_{2,2}$ and $G_{2,1}$. The expected capacity for link 2 can be expressed similarly, by inverting the indices. Thus, each link will search over all possible power values to find the optimal expected capacity.

However, from (8) we know the centralized optimal power solution set for weighted sum-rate maximization. Motivated from this result, we adopt the reduced optimization search space given by (8) for the distributed problem as well. However, we point out that the centralized optimal power allocation (5) is not necessarily optimal for the distributed problem formulation (10) as the objective functions in the two cases are not the same. The distributed power allocation problem for weighted sum-rate maximization can thus be written as

$$\boldsymbol{P}_{i}^{*} = \left[\arg\max_{(P_{1},P_{2})\in\Omega'}\overline{\mathcal{C}_{i}}(P_{1},P_{2})\right]_{i} \quad \forall \ i=1,2$$
(13)

where $\Omega' = \Omega^B \cup (P_{\max}, P'_2) \cup (P'_1, P_{\max})$. Each link would thus need to independently search over five possible power allocation points to find the one that maximizes (13). However, evaluating the non-binary values for the powers still requires knowledge of instantaneous information of the other link, e.g. link 1 would require knowledge of $G_{2,2}, G_{2,1}$ and w_2 . Motivated by the result exhibited in Fig. 3, we adopt the binary power allocation given by (7). In this case, we can formally write the distributed optimization problem for equal link weights as

$$\boldsymbol{P}_{i}^{*} = \left[\arg\max_{(P_{1},P_{2})\in\Omega^{B}}\overline{\mathcal{C}_{i}}(P_{1},P_{2})\right]_{i} \quad \forall i = 1,2$$
(14)

The advantage gained from this simplification is that a completely distributed algorithm can be derived, as the powers can now only be either 0 or P_{max} , as shown below.

A. Fully Distributed Power Allocation

As already stated, by adopting binary power control a link will either transmit at P_{max} (from now on assumed to be 1

Algorithm 1 Distributed Power Allocation

1: Steps performed at link 1: 2: $\mathbf{if}_{(\gamma_{\underline{1}}([1,1]) \geq 2^{(\beta_1[\overline{\mathcal{R}}_2(0,1)-\overline{\mathcal{R}}_2(1,1)])} - 1)}$ or $(\gamma_1([1,0]) \geq 2^{(\beta_1[\overline{\mathcal{R}}_2(0,1)-\overline{\mathcal{R}}_2(1,1)])} - 1)$ $2^{\left(\beta_1 \frac{1}{\overline{\mathcal{R}}_2(0,1)}\right)} - 1$) then $P_1 = 1$ 3: 4: else $P_1 = 0$ 5: 6: **end if** 7: Steps performed at link 2: 8: **if** $(\gamma_{\underline{2}}([1,1]) \ge 2^{(\beta_2[\overline{\mathcal{R}}_1(1,0)-\overline{\mathcal{R}}_1(1,1)])} - 1)$ or $(\gamma_2([0,1]) \ge 2^{(\beta_2[\overline{\mathcal{R}}_1(1,0)-\overline{\mathcal{R}}_1(1,1)])} - 1)$ $2^{(\beta_2 \overline{R}_1(1,0))} - 1)$ then $P_2 = 1$ 9: 10: else 11: $P_2 = 0$ 12: end if



cell under consideration lies at a random point (x,y) drawn from a uniform distribution over the cartesian plane.

for simplicity) or remain inactive. Thus, solving problem (14) is equivalent to each link determining if it should be active or not, depending on knowledge of local information.

A cell i needs to consider the following cases to determine which power allocation maximizes the expected capacity defined in (12):

- 1) Expected capacity of both cells being active: $\overline{C}(1,1)$.
- 2) Expected capacity of only cell *i* being active: $\overline{C}(0,1)$ or $\overline{C}(1,0)$.

Focusing on link 1, the activity conditions can thus be summarized as follows:

$$P_1 = \begin{cases} 1 & \text{if } \overline{\mathcal{C}}(1,1) \ge \overline{\mathcal{C}}(0,1) \\ 1 & \text{if } \overline{\mathcal{C}}(1,0) \ge \overline{\mathcal{C}}(0,1) \\ 0 & \text{otherwise} \end{cases}$$

Note that there is no need to compare the expected capacity of both cells being active and only cell 1 being active, as cell 1 will be active in either case. By simple manipulation of the above conditions, link 1 will be active if either

$$SINR_{1} = \gamma_{1}([1,1]) \ge 2^{\left(\beta_{1}[\overline{\mathcal{R}}_{2}(0,1)-\overline{\mathcal{R}}_{2}(1,1)]\right)} - 1$$
 (15)

or

$$\operatorname{SNR}_{1} = \gamma_{1}([1,0]) \ge 2^{\left(\beta_{1}\overline{\mathcal{R}}_{2}(0,1)\right)} - 1,$$
 (16)

where $\overline{\mathcal{R}}_2(0,1)$ and $\overline{\mathcal{R}}_2(1,1)$ are the expected capacities of link 2 under the indicated power allocations and $\beta_1 = \frac{\overline{w}}{w_1}$. By symmetry, the conditions for link 2 can be expressed in a similar fashion by changing the respective indices. The steps performed at each link are given in Algorithm 1.

In what follows, based on a simplified distance pathloss channel model, we derive the expected capacities. The utility of such a model is that it allows us to examine scenarios in which large-scale attenuation dominates, as well as allowing us to investigate the expected capacities in the high and low interference regimes. However, in order to capture the complete propagation environment, other factors that contribute to average SINR need to be considered e.g. shadowing and fastfading. 1) Random Exponential Pathloss Channel Model: Assume that users are located according to a uniform spatial distribution over the cell area. Let the cell radius be R, and the distance between cells D (Fig. 5). An exponential pathloss model is assumed for the channel gains, with pathloss exponent ξ ; and thus $G_{n,i} = d_{n,i}^{-\xi}$, where $d_{n,i}$ is the distance between transmitter i and receiver n.

Fig. 5. 2 cells of radius R at a distance D from each other. A user in the

We first calculate the distribution of the distance r of the direct path, assuming the cell under consideration to be centered at the origin of the cartesian plane (Fig. 5). The joint distribution of x and y is given by

$$f(x,y) = \frac{1}{\pi R^2}$$
 for $0 \le x^2 + y^2 \le R^2$.

Since

$$r = \sqrt{x^2 + y^2}, \quad \theta = \tan^{-1}\frac{y}{x},$$

we can easily find the Jacobian

$$J(x,y) = \left| \begin{array}{cc} \frac{\delta r}{\delta x} & \frac{\delta r}{\delta y} \\ \frac{\delta \theta}{\delta x} & \frac{\delta \theta}{\delta y} \end{array} \right| = \frac{1}{r}.$$

Then we have

for

$$f(r,\theta) = f(x,y)|J(x,y)^{-1}| = \frac{r}{\pi R^2}$$

$$0 \le r \le R, \quad 0 \le \theta \le 2\pi.$$

With no interference the expected capacity is given (in bits/sec/Hz) by,

$$\overline{\mathcal{R}}(0,1) = \mathbb{E}\left\{\log_2\left(1+\frac{r^{-\xi}}{\eta}\right)\right\}$$

$$= \int_0^{2\pi} \int_0^R \log_2\left(1+\frac{r^{-\xi}}{\eta}\right) f(r,\theta) dr d\theta$$

$$= \int_0^{2\pi} \int_0^R \log_2\left(1+\frac{r^{-\xi}}{\eta}\right) \frac{r}{\pi R^2} dr d\theta$$

$$= \frac{1}{\ln(2)} \left[\frac{\xi}{2} - \frac{1}{2}\xi_2 F_1(-\frac{2}{\xi},1;1-\frac{2}{\xi};-\frac{R^{-\xi}}{\eta}) + \ln\left(\frac{R^{-\xi}+\eta}{\eta}\right)\right], \quad (17)$$



Fig. 6. Variation of expected capacities with distance between cells based on exponential pathloss model, with pathloss exponent 4. The expected capacity with interference will approach that without interference as the distance between cells is increased.

where $_2F_1$ denotes the hypergeometric function.

For the case of interference being present, the interfering channel distance is given by $v = \sqrt{r^2 + D^2 - 2rD\cos\theta}$ (Fig. 5). Thus, we have

$$\begin{aligned} \overline{\mathcal{R}}(1,1) &= & \mathbb{E}\left\{\log_2\left(1+\frac{r^{-\xi}}{\eta+v^{-\xi}}\right)\right\} \\ &= & \int_0^{2\pi}\int_0^R \log_2\left(1+\frac{r^{-\xi}}{\eta+v^{-\xi}}\right)f(r,\theta)\mathrm{d}r\mathrm{d}\theta. \end{aligned}$$

Although a closed form for this integral is too complicated to derive, it can be easily evaluated numerically to find the expected capacity when both cells are active. In Fig. 6, we plot the expected capacities $\overline{\mathcal{R}}(0,1)$ and $\overline{\mathcal{R}}(1,1)$ as a function of the distance D between cells (normalized w.r.t. 2R) for R = 500m and $\xi = 4$. Clearly, as the distance D increases the effect of interference diminishes and the two capacities approach each other, as expected.

Practically, $\overline{\mathcal{R}}(0,1)$ and $\overline{\mathcal{R}}(1,1)$ for any channel model can be calculated offline, by generation of a sufficient number of channel realizations, and plugged into conditions (15) and (16) to determine if the cell should be active. Thus, based on simple conditions and in a fully distributed way, each link decides based on local channel information whether it should transmit or not based on criteria (15) and (16). We call this algorithm *Fully Distributed Power Allocation* (FDPA).

B. Capacity Enhancement with 1-bit Message Passing

The FDPA algorithm presented in the previous section is completely distributed, i.e. it requires no real-time information exchange from other links. However, due to each link being ignorant of the other link, a sub-optimal decision is taken and in certain cases a very detrimental result would be each link shutting itself off, resulting in zero network capacity.

It is thus interesting to explore if somehow a minimum amount of information exchange could be used to enhance performance. We let this amount of information be one bit.

Algorithm 2 1-Bit Distributed Power Allocation

1:	Steps performed at Link 1:
2:	if $(\gamma_1([1,1]) \ge 2^{(\beta_1[\overline{\mathcal{R}}_2(0,1)-\overline{\mathcal{R}}_2(1,1)])} - 1)$ or $(\gamma_1([1,0]) \ge 2^{(\beta_1[\overline{\mathcal{R}}_2(0,1)-\overline{\mathcal{R}}_2(1,1)])} - 1)$
	$2^{\left(\widehat{\beta_1 \mathcal{R}_2}(0,1)\right)} - 1)$ then
3:	$P_1 = 1$
4:	$msg_bit = 1$
5:	else
6:	$P_1 = 0$
7:	$msg_bit = 0$
8:	end if
9:	Steps Performed at Link 2:
10:	if $msg_bit = 0$ then
11:	$P_2 = 1$
12:	else if $\gamma_2([1,1]) \ge 2^{(\beta_2[\mathcal{R}_1(1,0)-\mathcal{R}_1(1,1)])} - 1$ then
13:	$P_2 = 1$
14:	else
15:	$P_2 = 0$
16:	end if

More precisely, a link is allowed to send a 1-bit message to the other link. The most natural choice of information to send would be the result of its distributed (using FDPA criteria) optimization solution. We call this algorithm *1-Bit Distributed Power Allocation* (1-BDPA) and describe it as follows:

- Link 1 performs the optimization (14) based on criteria (15) and (16), and sends a 1-bit message to the other link to indicate whether it is active or not.
- 2) Link 2 then performs the optimization (14) to calculate P_2 , under the knowledge of P_1 .

If the message bit is a 0, then link 2 will obviously be active. If a 1 is sent, then link 2 needs only to consider if both cells being active gives better performance than the expected capacity of the other link. Clearly this algorithm will perform better than FDPA as with the 1-bit signal from link 1, a more *informed* decision can be made by link 2, thus avoiding shutting down both links simultaneously. Details are given in Algorithm 2.

C. Power Allocation and Scheduling

In cellular networks, there are normally a number of users in each cell requesting data from the AP. In this context, *user scheduling* can be exploited to obtain *multi-user diversity* gain [20]. The idea is to schedule a user which has comparatively better channel conditions than other users, so that higher throughput can be achieved.

In order to obtain a multi-user diversity gain, user scheduling can also be incorporated into the power allocation framework. This is easily done by observing that for a cell to be active and thus contribute capacity to the system, either of the conditions (15) and (16) should be satisfied. Thus, scheduling a user with the maximum SNR or SINR increases the probability of satisfying these conditions. If we suppose that there are U_n users in cell n and an equal number of users per cell, then the activation conditions for cell 1 can be written as

$$\max_{u_1 \in [1,2,...,U_1]} \operatorname{SINR}_1(u_1) \ge 2^{\left(\beta_{u_1} | \mathcal{R}_{(U_2)}(0,1) - \mathcal{R}_{(U_2)}(1,1) \right]\right)} - 1$$
(18)

$$\max_{u_1 \in [1,2,\dots,U_1]} \operatorname{SNR}_1(u_1) \ge 2^{\left(\beta_{u_1} \overline{\mathcal{R}}_{(U_2)}(0,1)\right)} - 1,$$
(19)

where $\overline{\mathcal{R}}_{(U_2)}(0, 1)$ and $\overline{\mathcal{R}}_{(U_2)}(1, 1)$ are the expected capacities based on employing the max-SNR and max-SINR scheduling policies. These will be different from the previously calculated expected capacities because scheduling in general changes the distributions of the channel gains. In this case, the U_n order statistics of the expected capacities have to be calculated. Similarly, $\beta_{u_n} = \frac{\overline{w}_{U_n}}{\overline{w}_{u_n}}$, where \overline{w}_{U_n} is the U_n order statistic of the average weights, and w_{u_n} is the weight associated with user u_n . Although these can be analytically calculated, they can also be easily obtained through sufficient Monte-Carlo simulations. Thus, the scheduling rule is to find the max-SNR and max-SINR users and see which one satisfies its respective condition. If both satisfy their respective conditions then the user which offers higher *expected* capacity is scheduled, i.e. either the max-SINR or the max-SNR user.

VI. NUMERICAL RESULTS

As stated previously, the formulation of the distributed power allocation is independent of the system architecture (cellular or ad-hoc). Thus for ease of simulation, we adopt a cellular network layout for evaluating the performance of the proposed power allocation algorithms. This will also allow us to investigate user scheduling jointly with power allocation. In this case, we consider the downlink, i.e., the transmitter is the AP, and the receiver is the user terminal (UT). We set both link weights equal to 1, as this will simplify presentation of the numerical results, thereby allowing us to focus more on the performance of the proposed techniques. Monte-Carlo simulations over random UT positions are carried out for a network with an operating frequency of 1.8 GHz and with cell radius R = 500 meters. A UT position is drawn randomly from a uniform distribution over the cell area. Gains for all inter-cell and intra-cell AP-UT links are based on the COST-231 [21] path loss model, including log-normal shadowing with standard deviation of 10 dB, as well as fast fading which is assumed i.i.d. with distribution $\mathcal{CN}(0,1)$. The peak power constraint is given by $P_{\text{max}} = 1$ Watts. In order to compute the expected capacity of the other cell, offline calculations based on an adequate number of channel realizations are done for when both cells are active or just the other cell is active.

A. 2-Cell Network

We first consider the performance of FDPA and 1-BDPA compared with the "no power control" (i.e. both cells always on at $P_{\rm max}$) and centralized "optimal allocation" (i.e. exhaustive search over all points) for a network with two cells. To gain insight into the effects of power allocation we vary the distance between the two cells. Denoting the distance between APs by D, we vary the ratio $\frac{D}{2R}$, 2R being the distance between neighboring APs in a reuse one cellular system. When $\frac{D}{2R} < 1$ then the cells overlap and this results in severe interference, akin to that in ad-hoc networks. When $\frac{D}{2R} > 1$ the cells are further apart and thus the effects of interference



Fig. 7. 2-cell Network: Comparison of average network capacity per cell for the fully distributed algorithm (FDPA) and 1-bit message passing approach (1-BDPA) with optimal and no power allocation. The two algorithms exhibit marked gain over no power allocation, with the 1-bit message passing approach providing a significant amount of capacity gain. All the approaches converge when the separation between links increases, as interference decreases and both cells transmitting at full power becomes optimal.

diminish. In Fig. 7 we plot the average network capacity per *cell* versus $\frac{D}{2R}$. It can be seen that power allocation provides the most benefit when $\frac{D}{2R}$ is small, i.e. when there is strong interference. Turning off one of the cells will then provide more overall capacity than when both cells are transmitting. The FDPA algorithm achieves 50% of the gain offered by optimal power allocation, whereas with 1-BDPA a substantial amount of the gain is exploited. As $\frac{D}{2R}$ increases, the gain from power allocation decreases and all the schemes converge to the same capacity. This is quite straightforward due to the fact that increasing the distance between the cells diminishes the effect of interference, and both cells become more or less "shielded" from interference. This can equivalently be seen from Fig. 6 where the expected capacity with interference increases as $\frac{D}{2R}$ increases. Thus, from a network capacity maximization point of view, both links should transmit at full power when $\frac{D}{2R}$ becomes large.

In Fig. 8 we depict the percentage of erroneous decisions made in the power allocation by each algorithm as compared to the optimal solution, where an erroneous decision is defined as a deviation from the centralized binary power allocation. FDPA makes a significant amount of errors in the high interference case. This is due to the fact that under severe interference both cells can become inactive as both cells may come to the conclusion that they will not contribute enough capacity to outweigh the interference caused. This is demonstrated in the curve labeled "FDPA: both cells off" which shows that FDPA turns both cells off 28% of the time in the high interference scenario, whereas, clearly at least one cell should be active. This type of error becomes more rare in the low interference causing too much interference and thus both cells being active



Fig. 8. Percentage Error of FDPA and 1-BDPA compared with the optimal binary power allocation. "FDPA: both cells off" shows that both cells are turned off 28% of the time in the high interference scenario, thus resulting in zero sum-rate. Allowing 1-bit signaling reduces the number of errors made and thus 1-BDPA outperforms FDPA.

becomes the optimal thing to do. We see that with 1-BDPA, in the high interference scenario the percentage of errors is relatively smaller. This is due to the fact that it can exploit the 1-bit information exchange to make a better decision, which in the severe interference case is to keep one, but not both, of the cells active. At the other extreme, when cells are far apart, the error percentage is small due to the fact that both cells are kept active in the presence of low interference.

B. 4-cell Network

Here we look at a simple clustering approach for a small 4-cell network and investigate the performance of the FDPA algorithm. For simplicity, we do not search for dominant interferers at each scheduling instant, but instead two adjacent cells are kept paired together to form a cluster throughout the entire simulation. The 4 hexagonal cells are arranged on a 2 dimensional grid, and 2 cells with sides touching are paired together. In this case, a cell can be paired with one of two possible cells, but as the network is symmetric, the choice of cell will not have an impact on the end results. The FDPA algorithm is then run over each cluster independently. The approach is compared to an exhaustive search over all possible binary power allocation and no power allocation, and results are presented in fig. 9. The distance between each of the cells is increased similar to the 2-cell case. Again, we see here that the FDPA approach provides gain in the high interference scenarios and as the distance between cells increases the gain from power control diminishes.

C. Scheduling

Finally, we compare the performance of power allocation and user scheduling in Fig. 10 for U = 1, 5 and 10. We see a gain in absolute capacity values when employing user



Fig. 9. 4-cell Network: Comparison of average network capacity per cell for the fully distributed algorithm (FDPA) with exhaustive search and no power allocation. Using the clustering approach, the FDPA algorithm still achieves marked gains over no power allocation.

scheduling. Notice that as the number of users increases, the gain from power allocation diminishes. This is due to the fact that the probability of finding users which have good direct gains, while still being sufficiently protected from interference, increases by the process of scheduling alone. Thus the curves for full reuse, 1-BDPA, and optimal power allocation will lie closer together. However, FDPA starts to suffer when user scheduling is employed. This can be due to the fact that it still results in both links being inactive, although through user scheduling full reuse becomes more and more likely. The rate of increase in expected capacity of user scheduling is overshadowed by the damaging effect of making wrong decisions.

VII. CONCLUSIONS

In this work, we have formulated the weighted sum-rate maximizing power allocation problem for mutually interfering links, which is a generalization of previous results on sumrate maximization. For the case of two links, we analytically characterized the optimal solution set to this problem. Obtaining this solution however requires centralized processing. We thus proposed a framework for distributed weighted sum-rate maximizing power control, exploiting statistical knowledge of non-local information. We again analyzed the particular case of two links, deriving simple conditions on SNR and SINR for link activation. Based on these conditions, computationally simple distributed algorithms were proposed which were shown to exploit a major part of the gain offered by the centralized optimal power allocation. Moreover, we also demonstrated how user scheduling can be incorporated into the power allocation algorithm. Through numerical results, the proposed power allocation algorithms exhibited significant sum-rate gains over no power allocation.



(a) With one user per cell



(b) With five users per cell



(c) With ten users per cell

Fig. 10. Effect of power allocation and user scheduling on average network capacity. Incorporating user scheduling makes full reuse more probable in terms of optimality for sum-rate maximization.

REFERENCES

- J. Zander, "Distributed cochannel interference control in cellular radio systems," *IEEE Trans. Veh. Technol.*, vol. 41, pp. 305–311, Aug. 1992.
- [2] J. F. Whitehead, "Signal-level-based dynamic power control for cochannel interference management," in *Proc. IEEE Vehicular Technology Conference*, NJ, USA, May 1993, pp. 499–502.
- [3] G. J. Foschini and Z. Miljanic, "A simple distributed autonomous power control algorithm and its convergence," *IEEE Trans. Veh. Technol.*, vol. 42, 1993.
- [4] R. D. Yates, "A framework for uplink power control in cellular radio systems," *IEEE J. Select. Areas Commun.*, vol. 13, pp. 1341–1347, 1995.
- [5] N. Bambos, "Toward power-sensitive network architectures in wireless communications: Concepts, issues and design aspects," *IEEE Personal Commun. Mag.*, vol. 5, pp. 50–59, June 1998.
- [6] A. Gjendemsjø, D. Gesbert, G. E. Øien, and S. G. Kiani, "Optimal power allocation and scheduling for two-cell capacity maximization," in *Proc. RAWNET (WiOpt)*, Boston, USA, 2006.
- [7] A. Gjendemsjø, D. Gesbert, G. E. Øien, and S. G. Kiani, "Binary power control for multi-cell capacity maximization," *to appear in IEEE Trans. Wireless Comm.*, accepted Aug. 2007.
- [8] D. Goodman and N. Mandayam, "Power control for wireless data," *IEEE Personal Commun. Mag.*, vol. 7, pp. 48–54, Apr. 2000.
- [9] C. Saraydar, N. B. Mandayam, and D. J. Goodman, "Efficient power control via pricing in wireless data networks," *IEEE Trans. Commun.*, vol. 50, pp. 291–303, Feb. 2002.
- [10] M. Xiao, N. B. Shroff, and E. K. P. Chong, "A utility-based powercontrol scheme in wireless cellular systems," *IEEE/ACM Trans. Networking*, vol. 11, pp. 210–221, Apr. 2003.
- [11] C. Zhou, M. L. Honig, and S. Jordan, "Utility-based power-control for a two-cell CDMA data network," *IEEE Trans. Wireless Commun.*, vol. 4, pp. 2764–2776, Nov. 2005.
- [12] J. Huang, R. Berry, and M. L. Honig, "Distributed interference compensation for wireless networks," *IEEE J. Select. Areas Commun.*, vol. 24, pp. 1074–1084, May 2006.
- [13] M. Chiang, "Geometric programming for communications systems," *Foundations and Trends in Communications and Information Theory*, vol. 2, no. 1/2, pp. 1–156, Aug. 2005.
- [14] C. W. Tan, D. P. Palomar, and M. Chiang, "Solving nonconvex power control problems in wireless networks: Low SIR regime and distributed algorithms," in *Proc. IEEE GLOBECOM*, St. Louis, MO, Nov.-Dec. 2005, pp. 3445–3450.
- [15] S. G. Kiani, G. E. Øien, and D. Gesbert, "Maximizing multi-cell capacity using distributed power allocation and scheduling," in *Proc. IEEE WCNC*, Hong Kong, China, March 2007, pp. 1690 – 1694.
- [16] J. Abouei, M. Ebrahimi, and A. K. Khandani, "A new decentralized power allocation strategy in single-hop wireless networks," in *Proc. CISS*, Princeton, NJ, March 2007.
- [17] T. M. Cover and J. A. Thomas, *Elements of Information Theory*. New York: John Wiley & Sons, 1991.
- [18] H. Bölcskei, A. J. Paulraj, K. V. S. Hari, R. U. Nabar, and W. W. Lu, "Fixed broadband wireless access: State of the art, challenges, and future directions," *IEEE Commun. Mag.*, vol. 39, pp. 100–108, Jan. 2001, invited paper.
- [19] M. Ebrahimi, M. A. Maddah-Ali, and A. K. Khandani, "Power allocation and asymptotic achievable sum-rates in single-hop wireless networks," in *Proc. CISS*, Princeton, NJ, March 2006.
- [20] R. Knopp and P. Humblet, "Information capacity and power control in single-cell multiuser communications," in *Proc. IEEE International Conference on Communications*, Seattle, WA, June 1995, pp. 331–335.
- [21] Urban Transmission Loss Models for Mobile Radio in the 900 and 1800 MHz Bands, EURO-COST Std. 231, 1991.