Distributed Communication Control Mechanisms for Ad hoc Networks

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***Abstract*—We considered a single hop ad-hoc network consisting of N source-destination pairs. Each transmitter is endowed with a finite buffer and accepts packets from a Poisson distributed arrival process. The channel is described by a Markov chain. We investigate distributed algorithms for joint admission control, rate and power allocation aiming at maximizing the individual or the global throughput defined as the average information rate successfully received. The decisions are based on the statistical knowledge of the channel and buffer states of the other communication pairs and on the exact knowledge of their own channel and buffer states. The problems are formulated as a cooperative and noncooperative games and reduced to the mathematical framework of the variational inequalities problems. The proposed algorithms provide sizable improvements with respect to straightforward extension of decentralized algorithms for multiple access channels to ad hoc networks.**

I. Introduction

Ad hoc networks consist of self-organizing nodes with dynamic topologies and distributed communications and networking capabilities. The delocalization of control mechanisms plays a fundamental role in supporting the self-forming and self-haling properties of ad hoc networks. Thus, on the one hand, it is apparent the need of distributed control algorithms. On the other hand, the benefits of cross-layer design [12] and joint optimization of distinct control functions are well known and have recently attracted many studies, mainly for single cell networks.

Centralized cross-layer approaches for resource allocation have been proposed both for the uplink (multiple access channel - MAC) and the downlink (broadcast channel -BC). Knowledge of both channel state information (CSI) and queue state information (QSI) enables throughput-optimal policies, i.e. policies which achieve the ergodic capacity region of a fading channel network [6], [7] (see e.g. maximum weighting matching scheduling [8]). Other properties, besides the throughput optimality, like the average queueing delay, have been also object of studies. Due to space constraints, an exhaustive overview of the literature exceeds the scope of this work. The interested reader can refer to [12], [9] and references therein.

Decentralized algorithms for control mechanisms as resource allocation (e.g. power, rates, subcarriers, etc.), scheduling, admission control have also received considerable attention and are typically based on game settings [10], [11]. A seminal work [1] combining both the decentralization and the cross-layer design instances is based on the theoretical framework provided in [3]. In [1], a MAC fading channel with channel states distributed according to a Markov chain is considered. Furthermore, each transmitter is endowed with a queue fed by a Poisson process. Decentralized selfish and cooperative games, eventually correlated, are proposed to optimize a utility function under the constraints on the maximum average queueing delay and maximum average power. Under the assumption of fixed transmission rate for all users and the assumption that reliable communications are always possible in this decentralized context, the utility function in [1] is the average maximum achievable rate. The proposed algorithms enable power allocation and admission control (accept or reject incoming packets in the queues). In a system with decentralized control mechanisms where each transmitter is not aware of the interferers' presence and effects and it is intrinsically subject to outage, the assumption of reliable communications is rather strong. Additionally, the constraint of a fix transmission rate in any channel condition does not allow for an optimal utilization of the channel and a more efficient use of the channel is expected by controlling and adapting the transmission rates to the CSI.

This work investigates *distributed* algorithms in single-hop ad hoc networks for joint power and rate allocation, scheduling and admission control. We use similar approach as in [1] for characterizing the network and the nodes with obvious modifications to model the peculiarities of ad hoc networks. Namely, we consider interfering channels instead of MAC. Furthermore, we define a different utility function that accounts for the intrinsic probability of having outage events in networks with slow fading and decentralized control mechanisms. The proposed utility function maximizes the system throughput defined as the average information rate successfully received. In order to further improve the network performance, we remove the assumption of fixed transmission rate and include in the controlling mechanisms also the rate allocation. Since practical applications are sensitive also to outage probability and the analyzed system does not foresee retransmission mechanisms, the controlling mechanism accounts also for a maximum outage probability along with a maximum average delay and a maximum average transmission power. This work proposes both decentralized policies where each transmitter aims at maximizing its own average information rate successfully received (selfish game) or the system throughput (team game) under the assumption of single user decoding at the receiver (point-to-point channel with interference noise) or multiuser decoding (compound channel). The performance of the various policies is assessed against the policy in [1] in terms of throughput, outage probability, and drop rate (fraction of arriving packets not accepted in the queue). Improvements between the 19% and the 68% for the throughput have been obtained. Interestingly, iterative optimization algorithms with different random initial points yield to the same equilibrium when a low complexity best response approach is applied and single user decoding is utilized at the receivers. This encourages to believe that the obtained equilibrium is also Pareto optimal. On the contrary, when multiuser decoding is applied at the receivers, multiple equilibria were obtained with considerable differences in terms of throughput. Multiplicity of the equilibria points, convergence of the best response approach eventually to a Nash equilibrium have been only partially addressed in this work and are still object of investigation.

The boldface capital/small letters are used for matrices/vectors respectively. A superscript for a matrix/row-vector shows the index of corresponding column-vector/element. A subscript for a matrix/row-vector shows the index of corresponding row-vector/element.

II. System Model

We consider a system consisting of N_{T} arbitrary source-destination pairs sharing the same medium. For example, we may assume that these N_{T} pairs are chosen from a larger number of nodes in an ad hoc network. The time is uniformly slotted. We assume on our model that (i) one node cannot transmit and receive at the same time and (ii) the transmitters are distinct while one node can be the destination of different information streams. Thus, there are N_{T} transmitters and in general N_{R} receivers, with $N_{\mathrm{R}} \leq N_{\mathrm{T}}$, in the system. The channel is block fading with duration of a block equal to a time slot. Furthermore, codewords are completely transmitted during a single time slot. The channel in time slot $t \in \mathbb{N}$ is described by and $N_{\mathrm{T}} \times N_{\mathrm{R}}$ matrix $\boldsymbol{Y}(t)$ whose (i,j) element $y_i^j(t)$ is the power attenuation of the channel between transmitter i and receiver j. Throughout this work we refer to them as the channel states (CS). The matrix of channel states is shown in Figure (1). The row i includes the states of the channels from the transmitting node i to all the destination nodes. This is the vector of known CS information at node i and it is denoted by $\boldsymbol{y}_i(t)$. The j-th column includes the states of the channels from all the transmitting nodes to the receiver j. This is the column vector denoted by $\boldsymbol{y}^j(t)$. It contains all the CS information necessary to determine the signal to interference and noise ratio (SINR) at the destination node j at time slot t. Furthermore, each power attenuation y_i^j is modelled as an ergodic Markov chain taking values in the discrete set E of cardinality L. For the sake of notation, we define a bijection between the set E and the set of the natural numbers $\{0,1,...,L-1\}$, $\varphi : E \to \{0,1,...,L-1\}$. Let its inverse be $\psi = \varphi^{-1}$. The Markov chain of y_i^j is defined by the transition matrix $\boldsymbol{T}(i,j)$ whose (k,ℓ) element $T_k^\ell(i,j)$ is the probability of transition from the CS $\psi(k)$ to the state $\psi(\ell)$. The conditional probability nature of $T_k^\ell(i,j)$ reflects

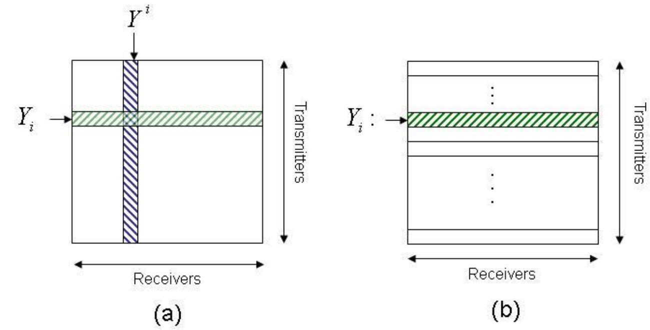

Figure 1. Definition of (a) RS and (b) NS

on the fact that $\sum_{\ell=1}^{L} T_k^\ell(i,j) = 1$. We assume throughout that $\boldsymbol{T}(i,j)$ is irreducible and aperiodic as in [1]. The steady CS probability distribution of the channel between transmitter i and destination j is given by the column vector $\boldsymbol{\pi}(i,j)$.

At each node, packets arrive from the upper layer according to an independent and identically distributed arrival process $\gamma_i(t), t \in \mathbb{N}$ with arrival rate λ_i. Here $P(\gamma_i(t))$ is the probability of receiving $\gamma_i(t)$ packets at time instant t. The packets have constant length.

Each transmitter is endowed with a buffer of finite length. We denote by B_i the maximum length of the buffer at node i and by $q_i(t)$ number of queuing packets at the beginning of slot t. In the following, we address the variable $q_i(t)$ also as the queue state (QS). In a given time slot we assume that all the arrivals from the upper layer occur after transmission of packets to the network.

In each time slot, on the basis of the available information at time t transmitter i decides (a) the transmission power level $p_i \in \mathcal{P}_i$, where $\mathcal{P}_i$ is a finite set of nonnegative reals including zero; (b) the number of packets to transmit $\mu_i \in \mathcal{M}_i$, with $\mathcal{M}_i = \{0,1,...,M_i\}$ and $M_i \leq B_i$; (c) to accept or reject new packets arriving from upper layers. We denote with $c_i = 1$ and $c_i = 0$ the decision of accepting and rejecting the packets, respectively. Therefore, the action of the node i at time slot t is described by the triplet $a_i(t) = (p_i(t), \mu_i(t), c_i(t))$.

The information available at node i at time t is given by the pair $x_i(t) = (\boldsymbol{y}_i(t), q_i(t))$, i.e. the CSs from transmitter i to all receivers and the number of the packets in the queue at the beginning of time slot t (QS). We refer to the pair $x_i(t)$ as the transmitter state (TS). Additionally, each transmitter knows the statistics of the other channels and the statistics of the arrival process in the buffer.

For further studies it is convenient to define two other state variables for transmitter i, namely the receiver state (RS), and the network state (NS). RS is given by the pair $x^i(t) = (\boldsymbol{y}^i(t), q_i(t))$. In order to define the NS, we divide the information of the matrix into two sets: (i) row vector $\boldsymbol{y}_i$ which is the TS of user i (ii) remaining row vectors in the matrix. We denote the latter set by $\boldsymbol{y}_{-i}$. Additionally, q_{-i} contains the queue states of users other than user i. Therefore, NS is $x_{-i} = (\boldsymbol{y}_{-i}, q_{-i})$. In a similar way, we can denote the set of actions of other users by a_{-i}.

A complete characterization of transmitter i requires the diagram of its TS. The TS is a combination of channel state $\boldsymbol{y}_i(t)$ and queue state $q_i(t)$. The CS transitions are independent of the buffer state. They are further independent of the action. As already mentioned each CS can independently be defined as a Markov chain. Then, the TS is also a Markov chain with transition probabilities $P_{\boldsymbol{y}_i(t)\boldsymbol{y}_i(t+1)} = \prod_{j=1}^{N_{\rm T}} T_{\varphi(y_i^j(t))}^{\varphi(y_i^j(t+1))}(i,j) \times P_{q_i(t)q_i(t+1)}$, i.e. the product of the transition probabilities from the CS $y_i^j(t)$ to the CS $y_i^j(t+1)$ or, equivalently,from $\varphi(y_i^j(t))$ to $\varphi(y_i^j(t))$ multiply by the probablity of transition from $q_i(t)$ to $q_i(t+1)$. Unlike the CS, the queue state depends on the transmitter action. In fact, the dynamics of the queue length are given by $q_i(t+1) = \min([q_i(t) + c_i(t)\gamma_i(t) - \mu_i(t)]^+, B_i)$ and can be described as Markov decision chains (MDC). Its transition probability is denoted by $P_{q_i(t)a_i(t)q_i(t+1)}$, the probability of transition from the queue state $q_i(t)$ to the queue state $q_i(t+1)$ when action $a_i(t)$ is adopted by the transmitter. Since the channel state is independent of the queue state, the transmitter state can also be described by a MDC with transition probability $P_{x_i(t)a_i(t)x_i(t+1)}$, i.e. the probability of transition from the state $x_i(t)$ to state $x_i(t+1)$ when the action $a_i(t)$ is adopted. Here, $P_{x_i(t)a_i(t)x_i(t+1)} = P_{y_i(t)y_i(t+1)}P_{q_i(t)a_i(t)q_i(t+1)}$.

The signal of the user of interest is impaired by the interfering signals and additive white Gaussian noise with variance σ^2. We denote by d_i the index of destination node for traffic of transmitter i. When the power level choices of the active transmitters are $\boldsymbol{p} = (p_1, p_2, ..., p_N)$, the RS vector for transmitter i is $x^i(t) = (\boldsymbol{y}^i(t), q_i(t))$, and the receiver performs single user decoding, the maximum instantaneous achievable rate for the i-th communication pair is given by

$$r_i^{\rm SU}(x^i(t), \boldsymbol{p}) = \log_2(1 + {\rm SINR}_i^{\rm SU}(x^i(t), \boldsymbol{p})) \qquad (1)$$

where ${\rm SINR}_i^{\rm SU}$ is the signal to interference and noise ratio of node i at its destination d_i given by

$${\rm SINR}_i^{\rm SU}(x^i(t), \boldsymbol{p}) = \begin{cases} \dfrac{y_i^{d_i}(t)p_i(t)}{\sigma^2 + \sum\limits_{\substack{j\neq i\\ q_j(t)>0}} y_j^{d_i}(t)p_j(t)}, & q_i(t) > 0 \\ 0, & \text{otherwise.} \end{cases}$$

If the receiver performs successive interference cancellation (SIC) decoding and, additionally, knows the transmission rate of the decodable interferes in the set $\mathcal{D}_i \equiv \{\mu_1, \ldots \mu_\ell\}$, the maximum instantaneous achievable rate for the i-th communication pair is given by

$$r_i^{\rm SIC}(x^i(t), \boldsymbol{p}) = \log_2(1 + {\rm SINR}_i^{\rm SIC}(x^i(t), \boldsymbol{p}), \qquad (2)$$

where

$${\rm SINR}_i^{\rm SIC}(x^i(t), \boldsymbol{p}) = \begin{cases} \dfrac{y_i^{d_i}(t)p_i(t)}{\sigma^2 + \sum\limits_{\substack{j\neq i\\ q_j(t)>0\\ j\notin\mathcal{D}_i}} y_j^{d_i}(t)p_j(t)}, & q_i(t) > 0 \\ 0, & \text{otherwise.} \end{cases}$$

In the following, we will write shortly $r_i(x^i(t), \boldsymbol{p})$ when it is irrelevant to specify the decoding approach.

III. Problem Statement

At each time slot, a node chooses its action without having a global view of the channel states and the other users' interference. There is no coordination among transmitters' actions and only local information is available at each node. Therefore, for any choice (p_i, μ_i), there is no guaranty that the μ_i packets can be received correctly when the TS is x_i. In such scenario, we aim at maximizing the throughput, i.e. the average number of packets successfully received by the destination. We will consider two different approaches: $(A - self)$ each user independently optimizes its strategy to maximize its own throughput (selfish game); $(A - coop)$ each user independently from others optimizes its strategy to maximize the joint throughput of the whole network (team game). Each approach can be investigated for two different kinds of receivers: (a) receivers performing single user decoding; (b) receiver performing SIC decoding. Approach $A{-}x$ with $x \in \{self, coop\}$ and decoding j, $j \in \{SU, SIC\}$, is addressed as $A{-}x{-}j$.

Let R be the rate required to transmit a packet in a time slot. The probability that $\mu_i(t)$ packets can be transmitted successfully in a time slot t by source i is

$$\Pr\{r_i(x^i(t), \boldsymbol{p}) \geq \mu_i(t)R\} \qquad (3)$$

and the average throughput for source i is

$$\limsup_{T\to+\infty} \frac{1}{T}\sum_{t=0}^{T-1} E(\Pr\{r_i(x^i(t), \boldsymbol{p}) \geq \mu_i(t)R | x_i(0) = \beta_i\}\mu_i(t)R \qquad (4)$$

where the expectation is conditioned to $x_i(0) = \beta_i$, the initial TS of user i.

For physical and QoS reasons the transmitters are subjected to constraints on the average transmitted powers, on the average queue length, and eventually on the maximum outage probability. More specifically, the average power of transmitter i is constrained to a maximum value $\overline{p}_i$ and the following upper bound is enforced

$$\limsup_{T\to+\infty} \frac{1}{T}\sum_{t=0}^{T-1} E\{p_i(x_i(t), a_i(t)) | x_i(0) = \beta_i\} \leq \overline{p}_i \qquad (5)$$

where $p_i(x_i(t), a_i(t))$ is the power, eventually zero, transmitted by the source i at time instant t when the action $a_i(t)$ is selected. The expectation is conditioned to the initial TS $x_i(0) = \beta_i$ of transmitter i. Similarly, in order to keep the average delay of the packets limited, the average queue length is constrained by the following bound:

$$\limsup_{T\to+\infty} \frac{1}{T}\sum_{t=0}^{T-1} E\{q_i(t) | x_i(0) = \beta_i\} \leq \overline{q}_i \qquad (6)$$

where $\overline{q}_i$ is maximum allowed average queue and the expectation is conditioned to $x_i(0) = \beta_i$. Finally, the outage probability in the steady state is bounded by $\overline{P}_i^{out}$

$$\lim_{T\to+\infty} \mathbf{Pr}\{r_i(x^i(t), \boldsymbol{p}) < \mu_i(t)R | x_i(0) = \beta_i\} \leq \overline{P}_i^{out}. \qquad (7)$$

Note that the constraint on the outage probability holds in the steady state while in the transient of the MDC we assume that the system can eventually tolerate higher outage probability. Throughout, a policy of transmitter i is a deterministic or probabilistic application from the space of TS $\mathcal{X}_i$ to the action space $\mathcal{A}_i$. Since the policies in stationary conditions of the Markov chain are dominating [1], in this paper we assume that the policy of a transmitter at time t is only a function of its current state and we omit the time indices. Then, a probabilistic (or mixed) policy of transmitter i is $u_i(a_i|x_i)$, i.e. the probability that mobile i chooses the action a_i when the state is x_i. The class of decentralized policies of mobile i is denoted by $\mathcal{U}_i$.

A. Problem Statement as an N-player Game

Let us formulate the previous problem as a stochastic N_{T}-player game. We denote by g_i the cardinality of the product set $\mathcal{K}_i = \mathcal{X}_i \times \mathcal{A}_i = \{(x_i, a_i) : x_i = (\mathbf{y}_i, q_i) \in \mathbf{x}_i, a_i = (p_i, \mu_i, c_i) \in \mathbf{a}_i(x_i)\}$ and by $< x_i, a_i >_{n_i}$ the n_i-th element of $\mathcal{K}_i$. The payoff matrix $\boldsymbol{C}^{(i)}$ of transmitter i is a $g_1 \times g_2 \times \dots g_n$ matrix having N_{T}-dimensions and its element $c^{(i)}_{n_1,n_2\dots n_{N_{\mathrm{T}}}}$ is the payoff of transmitter i when correspondingly to TS x_i it performs action a_i while the remaining users adopt the strategies $< x_k, a_k >_{n_k}$ with $k \neq i$.

In the selfish approach, when n_i is such that $< x_i, a_i >_{n_i} =< x_i, (\underline{p}_i, \underline{\mu}_i, \underline{c}_i) >$ and $\underline{p} = \{\underline{p}_1, ..., \underline{p}_{N_{\mathrm{T}}}\}$

$$c^{(i)}_{n_1,n_2\dots n_{N_{\mathrm{T}}}} = R\underline{\mu}_i 1_{(r_i(x^i,\underline{p}) \geq \underline{\mu}_i R)}, \tag{8}$$

i.e. the payoff is nonzero and equal to $R\underline{\mu}_i$ if transmitter i can transmit $\underline{\mu}_i$ packets with power $\underline{p}_i$ reliably. In the cooperative case

$$c^{(i)}_{n_1,n_2\dots n_{N_{\mathrm{T}}}} = R\sum_{j=1}^{N_{\mathrm{T}}} \underline{\mu}_j 1_{(r_j(x^j,\underline{p}) \geq \underline{\mu}_j R)}. \tag{9}$$

Let $z_i = z_i(x_i, a_i)$ be the joint probability that transmitter i performs action a_i while being in state x_i. It can be expressed by the row vector $\boldsymbol{z}_i = (z_i^1, z_i^2, ..., z_i^{g_i})$. Then, the payoff ρ_i equals the average throughput in (4) and it is given by the multilinear form

$$\rho_i = \sum_{n_1=1}^{g_1}\sum_{n_2=1}^{g_2} \dots \sum_{n_{N_{\mathrm{T}}}=1}^{g_{N_{\mathrm{T}}}} c^i_{n_1 n_2\dots n_{N_{\mathrm{T}}}} z_1^{n_1} z_2^{n_2} \dots z_{N_{\mathrm{T}}}^{n_{N_{\mathrm{T}}}} = \boldsymbol{z}_i \boldsymbol{f}^i \tag{10}$$

where

$$\boldsymbol{f}^i = \sum_{n_1=1}^{g_1} \dots \sum_{n_{i-1}=1}^{g_{i-1}} \sum_{n_{i+1}=1}^{g_{i+1}} \dots \sum_{n_{N_{\mathrm{T}}}=1}^{g_{N_{\mathrm{T}}}} c^{(i)}_{n_1\dots n_{i-1} k n_{i+1}\dots n_{N_{\mathrm{T}}}} \times z_1^{n_1} \dots z_{i-1}^{n_{i-1}} z_{i+1}^{n_{i+1}} \dots z_N^{n_N}.$$

The constrained optimization problem defined in (4)-(7) can be expressed in terms of joint probabilities $\boldsymbol{z}_i$ as

$$\max_{z_i(x_i,a_i)} \sum_{x_i \in \mathbf{x}_i} \sum_{a_i \in \mathbf{a}_i} z_i(x_i, a_i) Pr\{r_i(x^i, \boldsymbol{p}) \geq \mu_i R\} \mu_i R \tag{11a}$$

Subject to:

$$\sum_{x_i \in \boldsymbol{x}_i} \sum_{a_i \in \boldsymbol{a}_i} z_i(x_i, a_i)[\delta_{r_i}(x_i) - P_{x_i a_i r_i}] = 0 \quad \forall r_i \in \boldsymbol{x}_i \tag{11b}$$

$$\sum_{x_i \in \boldsymbol{x}_i} \sum_{a_i \in \boldsymbol{a}_i} p(x_i, a_i) z_i(x_i, a_i) \leq \overline{p}_i \tag{11c}$$

$$\sum_{x_i \in \boldsymbol{x}_i} \sum_{a_i \in \boldsymbol{a}_i} q_i z_i(x_i, a_i) \leq \overline{q}_i \tag{11d}$$

$$\sum_{x_i \in \boldsymbol{x}_i} \sum_{a_i \in \boldsymbol{a}_i} Pr\{r_i(x^i, \boldsymbol{p}) < \mu_i R\} z_i(x_i, a_i) \leq \overline{P}_i^{out} \tag{11e}$$

$$z_i(x_i, a_i) = z_i((\boldsymbol{y}_i, q_i), (\mu_i, p_i, c_i)|q_i \leq \mu_i) = 0 \tag{11f}$$

$$z_i(x_i, a_i) \geq 0;\ \forall (x_i, a_i) \in \mathcal{K}_i; \quad \sum_{(x_i,a_i) \in \mathcal{K}_i} z_i(x_i, a_i) = 1 \tag{11g}$$

where $P_{x_i a_i r_i}$ is the probability to move from state x_i to state r_i when action a_i is performed. Additionally, (11b) guarantees that the graph of the obtained MDP is closed; (11d)-(11e) correspond to the constraints (5)-(7), respectively; (11f) eliminates the invalid pairs in $\mathcal{K}_i$ such that the number of packets to be sent is not higher than the number of packets in the queue.

Note that if the joint probabilities $\boldsymbol{z}_k$, with $k \neq i$ had been known the payoff ρ_i would have reduced to a linear equation and the optimal $\boldsymbol{z}_i = \boldsymbol{z}_i^*$ would have been solution of a linear program.

The optimal policy $u_i^*(a_i|x_i)$ of transmitter i can be immediately derived from $\boldsymbol{z}_i^*$ in the steady state of the MDC system by the relation $u_i(a_i|x_i) = \frac{z^*(x_i,a_i)}{\sum_{a_i' \in \boldsymbol{a}_i} z^*(x_i,a_i')}$.

IV. Analysis of the Game

For the sake of simplicity, the Nash equilibrium problem is represented as follows:

$$\min_{\mathbf{z}_i} \quad -\boldsymbol{z}_i \boldsymbol{f}^{(i)} \tag{12a}$$

$$(b)\ \boldsymbol{A}_i \boldsymbol{z}_i^T + \boldsymbol{a}_i = 0 \qquad (c)\ \boldsymbol{B}_i \boldsymbol{z}_i^T + \boldsymbol{b}_i \leq 0 \qquad (d)\ \boldsymbol{z}_i^T \geq 0$$

where 12-(b) corresponds to the set of N_i^{eq} linear equality constraints and 12-(c) corresponds to the N_i^{ineq} linear inequality constraints.

A. Nash Equilibrium

The system of N-player constrained stochastic game (12) can be presented in the frame of a single non-linear complementarity problem (NLCP) or a variational inequality problem (VIP). In the following, we define our game in the frame of a nonlinear complementarity problem. Let us introduce the N_{T} Lagrangians

$$\boldsymbol{\mathcal{L}}_i(\boldsymbol{z}_i, \boldsymbol{u}_i, \boldsymbol{v}_i) = -\rho_i + \boldsymbol{u}_i(\mathbf{A}_i \boldsymbol{z}_i^T + \boldsymbol{a}_i) + \boldsymbol{v}_i(\boldsymbol{B}_i \boldsymbol{z}_i^T + \boldsymbol{b}_i)$$

where $\boldsymbol{u}_i$ and $\boldsymbol{v}_i$ are row vectors of Lagrangian multipliers and $i = 1, \ldots N_{\mathrm{T}}$. A Nash equilibrium necessarily satisfies the Karush-Kuhn-Tucker conditions

$$\boldsymbol{\theta}_{1i} = \nabla_{\boldsymbol{z}_i} \boldsymbol{\mathcal{L}}_i = -\boldsymbol{f}^i + \boldsymbol{u}_i \boldsymbol{A}_i + \boldsymbol{v}_i \boldsymbol{B}_i$$
$$\boldsymbol{\theta}_{2i} = \nabla_{\boldsymbol{u}_i} \boldsymbol{\mathcal{L}}_i = \boldsymbol{A}_i \boldsymbol{z}_i^T \qquad \boldsymbol{\theta}_{3i} = \nabla_{\boldsymbol{v}_i} \boldsymbol{\mathcal{L}}_i = -\boldsymbol{B}_i \boldsymbol{z}_i{}^T$$
$$\boldsymbol{z}_i \geq 0 \qquad \boldsymbol{u}_i \geq 0 \qquad \boldsymbol{v}_i \geq 0$$
$$\boldsymbol{z}_i \boldsymbol{\theta}_{1i} = 0 \qquad \boldsymbol{u}_i \boldsymbol{\theta}_{2i} = 0 \qquad \boldsymbol{v}_i \boldsymbol{\theta}_{3i} = 0.$$

Let $\mathbf{w}_i = (\boldsymbol{z}_i, \mathbf{u}_i, \mathbf{v}_i)$ $\boldsymbol{\Theta}_i(\boldsymbol{w}_i) = (\boldsymbol{\theta}_{1i}^T, \boldsymbol{\theta}_{2i}^T, \boldsymbol{\theta}_{3i}^T)^T$, and by concatenating different transmitters' vectors $\boldsymbol{w} = (\boldsymbol{w}_1, \boldsymbol{w}_2, ..., \boldsymbol{w}_{N_{\mathrm{T}}})$ and $\boldsymbol{\Theta}(\boldsymbol{w}) = (\boldsymbol{\theta}_1^T, \boldsymbol{\theta}_2^T, ..., \boldsymbol{\theta}_{N_{\mathrm{T}}}^T)^T$, we obtain the nonlinear complementarity problem

$$\boldsymbol{\Theta}(\boldsymbol{w}) \geq 0 \qquad \boldsymbol{w}^T \geq 0 \qquad \boldsymbol{w}\boldsymbol{\Theta}(\boldsymbol{w}) = 0 \tag{14}$$

in the $\sum_1^N [g_i + N_i^{eq} + N_i^{ineq}]$ unknowns $\boldsymbol{w}$.

The existence of Nash equilibria for a general class of constrained stochastic games, where players have independent state processes, is proven in [3]. Therefore, the solution set of the NLCP in (14) is nonempty for any finite number of users.

B. Symmetric Network

Let us consider the case as all the transmitters have the same statistics for the channels and the arrival processes and their sets of actions are the same. Additionally, the constraint parameters are identical. Equivalently, they have the same objectives and constraints. In such a case, an optimal policy is identical for all users. An NLCP in $g_i + N_i^{eq} + N_i^{ineq}$ unknown is obtained from (14) omitting index i and removing identical equations. In [5], an algorithm based on extragradient method for variational inequality problems is proposed. It converges whenever the solution set is not empty as our game [3]. The algorithm is iterative and based on quadratic programming. Its complexity depends on both the number of iterations and the number of projections at each iteration required for the convergence to the solution of the quadratic programming.

C. Best Response Algorithm

Because of the complexity of standard algorithms for NLCP, it is of great interest to investigate simpler approaches. As already mentioned, the game reduces to a LP when the strategies of $N_{\mathrm{T}} - 1$ players is known. Thus, a low complexity iterative algorithm is obtained by choosing a transmitter i and the policies u_{-i} of the remaining transmitters arbitrarily and solving the corresponding LP. In this way, a complete set of policies is obtained. In the iterative steps, a different transmitter is selected, its policy at the previous step ignored and determined by linear programming based on the policies obtained at the previous step.

1) symmetric case: In the symmetric case, the algorithm is initialized assigning an arbitrary identical policy $u(0)(a|x)$ to $N_{\mathrm{T}} - 1$ transmitters and determining the optimal policy for the remaining one. The new policy is assigned to $N_{\mathrm{T}} - 1$ nodes for the following step. At each iteration we solve an LP and obtain a new policy $u(t+1)(a|x)$ using the policy $u(t)(a|x)$ for evaluating the payoff. Note that, if the algorithm converges, its fixed points are Nash equilibria of the N_{T}-player game. The convergence of the best response algorithm to Nash equilibrium is guaranteed only in a symmetric case, while in the general case the algorithm could converge to a point which is not a Nash equilibrium.

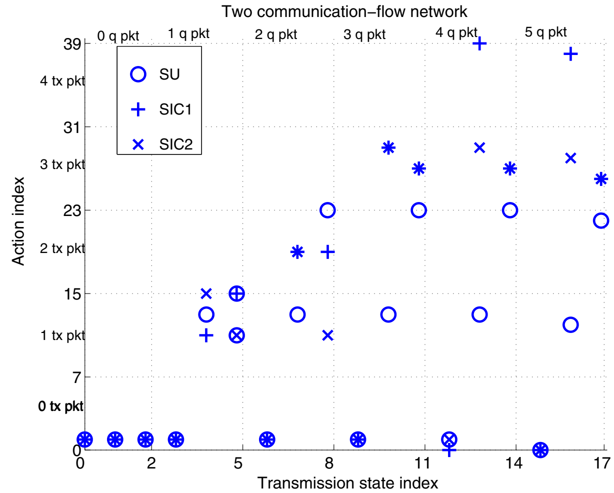

Figure 2. Policies in a symmetric two user network: $A - self - SU$ (circle mark) versus $A - self - SIC$ (cross and plus marks)

V. Numerical Results

In this section, we consider the two scenarios with parameters detailed in Table I. The CS varies according to a Markov chain with the following transition probabilities: $T_0^0(i,j) = \frac{1}{2}, T_0^1(i,j) = \frac{1}{2}, T_{L-1}^{L-1}(i,j) = \frac{1}{2}, T_{L-1}^{L-2}(i,j) = \frac{1}{2}; (2 \leq k \leq L-2) T_k^k(i,j) = \frac{1}{3}, T_k^{k-1}(i,j) = \frac{1}{3}, T_k^{k+1}(i,j) = \frac{1}{3}$. This means that at each time slot the channel preserves its state or changes by one unit.

The packet arrival process is described by a Poisson distribution with average rate $\lambda_i = 1$.

We perform a two-level admission control; one is done in our offline algorithm and set the variable c_i to 1/0 corresponding to the acceptance/rejection decision. However, as we only use one admission control flag c_i for all the possible number of packet arrivals, there exist situations where the remaining space of the queue is less than the number of packets arrived at the time. Therefore, a second (realtime) control is needed in order to drop the packets when the queue is full.

In the following, we compare the equilibrium policies and the performance of such strategies in the network. The performance measures are: (i) Throughput (TP), i.e. the number of packets per time slot correctly decoded by the receiver, (ii) Outage rate, i.e. the fraction of transmitted packets which can not be decoded correctly, (iii) Drop rate, i.e. the fraction of arriving packets from upper layer which are rejected due to the admission control.

Let us focus on scenario 1 of Table I. In Figure 2, we describe the equilibrium policies obtained by the proposed algorithm in case of selfish game when SIC and SU decoding is performed at the receiver. The action index is presented in abscissa while the state index is represented in ordinate. Since the state index needs to address the pair of CS and QS, the indexing approach is presented in Table II. Similarly, the

	N_{T}	N_{R}	B_i	L	M_i	$\lvert\mathcal{P}_i\rvert$	$\bar{p}_i$	$\bar{q}_i$
Scn1	2	2	5	3	5	4	1.5	3
Scn2	3	3	5	3	5	4	1.5	3

Table I
NETWORK PARAMETERS

state index	0 1 2 3 4 5 6 ... 17
queue state	0 0 0 1 1 1 2 ... 6
channel state	0 1 2 0 1 2 0 ... 2

Table II
LABELLING OF STATES

action index	0 1 2 3 4 5 6 7 8 9 ... 47
Num of packets	0 0 0 0 0 0 0 0 1 1 1 ... 6
power level	0 0 1 1 2 2 3 3 0 0 1... 3
accept/reject	0 1 0 1 0 1 0 1 0 1 0... 1

Table III
LABELLING OF POLICIES

	TP	Outage Rate	Drop rate
$A-self-SU$	0.49	0.42	0.15
$A-self-SIC1$	0.64	0.24	0.16
$A-self-SIC2$	0.69	0.19	0.15
$A-coop-SU$	0.5	0.4	0.17
Policy in [1]	0.41	0.35	0.37

Table IV
COMPARISON OF $A-self-SU$, $A-self-SIC$, AND $A-coop-SU$ IN TERMS OF PERFORMANCE

table III describe the mapping between action indices and the triplets (μ_i, p_i, c_i).

As apparent from Figure 2, the best response algorithm converges to a single solution in $A-self-SU$ model while for $A-self-SIC$ model two distinct solutions are obtained.

An interesting question on the general behavior of both cases is whether a decoupling between the decision on the rate μ_i and the decision on the power p_i is possible. For example, the case where (I) decision on μ_i is not affected by the CSs and is an increasing function of the QS, and (II) the power level is independent from the queue level and only a function of CS. However, the optimal policy in Figure 2 does not make any decoupling between rate and power allocations.

The optimal policy for $A-self-SU$ does not transmit packet when the channel is in the worst situation and the decision on the power level is irrelevant. For the two other CSs, namely medium and good, the decision on μ_i is affected by both CS and QS and is a non-decreasing function of both.

The optimal policy for $A-self-SIC$ has the same trends in the worse CS. In medium and good states of channel, the optimal policies of SIC transmit equal or more number of packets, comparing to SU. However, this yields a significantly lower probability of outage as evident from Table IV.

Unlike $A-self-SIC$, the optimal policy of $A-coop-SU$ transmits less packets but with lower probability of outage. However, the decrease in the probability of outage is not as significant as in $A-self-SIC$.

Table IV compares the performance of the policy obtained by the approach in [1] and our proposed policies.

$A-self-SU$ shows sizable improvements of throughput and drop rate compared to the policy in [1]. The $A-self-SIC$ policies outperform considerably the $A-self-SU$ ones. The improvement in terms of throughput of $A-coop-SU$ over the $A-self-SU$ is not relevant.

In order to analyze the performance of the proposed algorithm in a network with more than 2 users, the optimal policy of $A-self-SU$ for Scenario 2 was determined (Figure 3). In this case, the unique optimal policy corresponding to a unique Nash equilibrium becomes less and less sensitive to the queue length. In other words, μ_i has a slower increase, compared to the two communication flow network, in response to the QS increase. This fact suggests a limited performance for distributed resource allocation when the network scales.

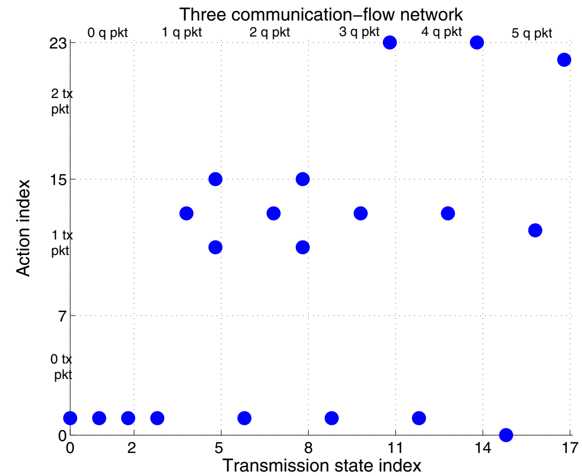

Figure 3. Optimal policies for scenario 2

VI. ACKNOWLEDGEMENTS

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