Structured Spatio-temporal Sample Covariance Matrix Enhancement with Application to Blind Channel Estimation in Cyclic Prefix systems

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***Abstract*—Multichannel aspect allows the introduction of blind channel estimation techniques. Most existing such techniques for frequency-selective channels are quite complex. In this paper, we consider the blind channel estimation problem for Single Input Multi Output (SIMO) cyclic prefix (CP) systems. We have shown before [1] that blind channel estimation becomes computationally much more attractive and more straight forward to analyze in terms of performance in CP systems. Inspired by the iterative sample covariance matrix (SCM) structure enhacement techniques of Cadzow and others [2], we propose here an algorithm to structure the sample block circulant covariance matrix by enforcing two essential properties: rank and FIR structure. These two properties are exhibited by the true covariance matrix in the case of FIR SIMO channels with spatially white noise and CP transmission. The proposed enhancement procedure leads to an interesting enhanced SCM, even for the single CP symbol case. A simulation study for some classical channel estimators that depend on the SCM (with and without structuring) is presented, indicating that structuring allows for considerable performance gain in terms of the channel normalized mean square error (NMSE) over a wide SNR range.**

I. Introduction

A wealth of blind channel estimation techniques have been introduced for spatio-temporal channels over the past decade, based on the singularity of the received signal power spectral density matrix [3]. This singularity can be exploited to separate the white noise contribution. The main problem characteristic in fact that allows channel identifiability is the minimum phase characteristic of the Single-Input Multiple-Output (SIMO) or MIMO matrix channel transfer function of the spatio-temporal channel. Spatio-temporal channels arise in mobile communications when multiple antennas or polarizations or beams are used at the receiver. Physical multi-channels can also arise in xDSL systems when the receiver has access to a complete cable bundle. Other problem formulations that lead to multi-channel models are the use of oversampling at the receiver or the decoupling of inphase and in-quadrature components when real symbols get modulated or the reception of multiple signal copies in ARQ protocols. A variety of blind symbol/channel estimation strategies can be developed depending on the amount of a priori information that gets formulated on the unknown symbols. In general, the less structure that gets exploited about the symbol alphabet, the less problems tend to be encountered with local minima. Of course, more estimation accuracy is obtained by exploiting more information. A reasonable strategy is hence to exploit a progressive range of algorithms exploiting increasing a priori information levels. The algorithm at the next level can be initialized with the estimate obtained at the previous level of a priori information. The memory introduced by a convolutive channel leads to the requirement of having to treat all available data in a contiguous observation interval in one shot if no suboptimality is allowed. This leads to problem formulations with large convolution matrices, large covariance matrices and high complexity. Attempts have been made by our own group to introduce asymptotic approximations, by approximating large Toeplitz convolution matrices by circulant matrices, to allow transformation to the frequency domain, or by others by introducing approximate DFT operations. Cyclic prefixes have been introduced in a number of existing systems such as OFDM systems for ADSL and wireless LANs. Recently, Orthogonal Frequency Division Multiple Access (OFDMA) has been adopted as a multiple access scheme for the Frequency Domain Duplexing Down-Link (FDD-DL)in LTE (Long Term Evolution) systems. The introduction of a cyclic prefix renders the transformation to the frequency domain clean and exact even for a finite data length. The resulting algorithmic simplifications will be detailed for a number of classical blind channel estimation methods. Furthermore, the same framework can be used to analyze the performance of the algorithms and the algorithmic simplifications also translate into much simplified performance expressions, which allow a direct and insightful analytical performance comparison between a number of algorithms.

This paper is organized as follows. Section II starts with the description of the basic baseband SIMO-cyclic prefix model. In section III we develope a unified framework for cyclic prefix system channel estimators. Section IV defines some classical blind channel estimators within the framework introduced in section III. The algorithm for structuring the covariance matrix is developed in section V. In section VI, we provide the experimental results and finally a conclusion is drawn in Section VII.

II. SIMO Cyclic Prefix Block TX Systems

Consider a SIMO system with M outputs:

$$\begin{aligned} \underbrace{\mathbf{u}[m]}_{M\times 1} &= \sum_{j=0}^{L_h-1} \underbrace{\mathbf{h}[j]}_{M\times 1}\ \underbrace{a[m-j]}_{1\times 1} + \underbrace{\mathbf{w}[m]}_{M\times 1} \\ &= \underbrace{H(q)}_{M\times 1}\ \underbrace{a[m]}_{1\times 1} + \underbrace{\mathbf{w}[m]}_{M\times 1} \end{aligned} \quad (1)$$

where $H(q) = \sum_{j=0}^{L} \mathbf{h}[j]\, q^{-j}$ is the SIMO system transfer function corresponding to the z transform of the impulse response $\mathbf{h}[.]$. Equation (1) mixes time domain and z transform domain notations to obtain a compact representation. In $H(q)$, z is replaced by q to emphasize its function as an elementary time advance operator over one sample period. Its inverse corresponds to a delay over one sample period: $q^{-1}\mathbf{a}[n] = \mathbf{a}[n-1]$.

Consider a (OFDM or single-carrier) CP block transmission system with N samples per block. The introduction of a cyclic prefix of K samples means that the last K samples of the current block (corresponding to N samples) are repeated before the actual block. If we assume w.l.o.g. that the current block starts at time 0, then samples $\mathbf{a}[N-K]\cdots\mathbf{a}[N-1]$ are repeated at time instants $-K, \ldots, -1$. This means that the output at sample periods $0, \ldots, N-1$ can be written in matrix form as

$$\begin{bmatrix} \mathbf{u}[0] \\ \vdots \\ \mathbf{u}[N-1] \end{bmatrix} = \mathrm{U}[0] = \mathrm{H}\ \mathrm{A}[0] + \mathrm{W}[0] \quad (2)$$

where the matrix H is not only (block) Toeplitz but even (block) circulant: each row is obtained by a cyclic shift to the right of the previous row. Consider now applying an N-point FFT to both sides of (2) at block m:

$$F_{N,M}\mathrm{U}[m] = F_{N,M} H\, F_N^{-1} \quad F_N \mathrm{A}[m] + F_{N,M}\mathrm{W}[m] \quad (3)$$

or with new notations:

$$\mathrm{Y}[m] \quad = \quad \mathcal{H} \qquad \mathrm{X}[m] \quad + \quad \mathrm{V}[m] \quad (4)$$

where $F_{N,M} = F_N \otimes I_M$ (Kronecker product: $A \otimes B = [a_{ij}B]$), F_N is the N-point $N \times N$ DFT matrix, $\mathcal{H} = \mathrm{diag}\{\mathbf{h}_0, \ldots, \mathbf{h}_{N-1}\}$ is a block diagonal matrix with diagonal blocks $\mathbf{h}_n = \sum_{l=0}^{L_h-1} \mathbf{h}[l]\ e^{-j2\pi \frac{1}{N} nl}$, the $M \times 1$ channel transfer function at tone n (frequency $= n/N$ times the sample frequency). In OFDM, the transmitted symbols are in $\mathrm{X}[m]$ and hence are in the frequency domain. The corresponding time domain samples are in $\mathrm{A}[m]$. The OFDM symbol period index is m. In Single-Carrier (SC) CP systems, the transmitted symbols are in $\mathrm{A}[m]$ and hence are in the time domain. The corresponding frequency domain data are in $\mathrm{X}[m]$. The components of W are considered white noise, hence the components of V are white also. At tone (subcarrier) $n \in \{0, \ldots, N-1\}$ we get the following input-output relation

$$\underbrace{\mathbf{y}_n[m]}_{M\times 1} = \underbrace{\mathbf{h}_n}_{M\times 1}\ \underbrace{x_n[m]}_{1\times 1} + \underbrace{\mathbf{v}_n[m]}_{M\times 1} \quad (5)$$

where the symbol $x_n[m]$ belongs to some finite alphabet (constellation) in the case of OFDM.

III. Frequency domain Framework for CIR Estimation

The basic idea relies on the fact that to get the cost function or information for the temporal channel response it suffices to sum up the cost functions or information over the tones after transforming back to the time domain. To be a bit more explicit, let $\mathbf{h}$ be the vectorized channel impulse response then there exists transformation matrices G_k (containing DFT portions)such that

$$\mathbf{h}_k \;=\; G_k\, \mathbf{h}\ . \quad (6)$$

To be more accurate, G_k is of size $M \times ML_h$ such that it contains the first ML_h elements of the kth block row of $F_{N,M}$. Now, if at tone k we have a cost function of the form

$$\mathbf{h}_k^H\ Q_k\ \mathbf{h}_k \quad (7)$$

then this induces a cost function for the overall channel impulse response of the form

$$\mathbf{h}^H \left[\sum_{k=0}^{N-1} G_k^H\ Q_k\ G_k\right] \mathbf{h} \quad (8)$$

and similarly for Fisher information matrices. So in what follows, we shall concentrate on the cost function for a given tone.

IV. Blind SIMO Channel Estimation

A. Subchannel Response Matching (SRM)/Cross Relation method (CR)

The subchannel response Matching (SRM) estimator which was (re)invented four times in [5],[6],[7],[8], is based on a linear parametrization of the noise subspace in terms of the channel coefficients [9] so that $P^{\perp}_{\mathbf{h}_k} = P_{\mathbf{h}_k^{\perp H}}$ where $\mathbf{h}_k^{\perp}$ is given by

$$\mathbf{h}^{\perp}_{k\,_{bal,min}} = \begin{bmatrix} -\mathbf{h}_{k,2} & \mathbf{h}_{k,1} & 0 & \cdots & 0 \\ 0 & -\mathbf{h}_{k,3} & \mathbf{h}_{k,2} & \cdots & \vdots \\ \vdots & & \ddots & \ddots & 0 \\ \mathbf{h}_{k,M} & 0 & \cdots & 0 & -\mathbf{h}_{k,1} \end{bmatrix}. \quad (9)$$

In (9) we choose the minimum number of rows in $\mathbf{h}_k^{\perp}$ which has a size $(M - \delta_{M,2}) \times M$. This noise parametrization is balanced in the sense that every subchannel appears the same number of times, in this case twice. A balanced $\mathbf{h}_k^{\perp}$ leads to $tr\{\mathbf{h}_k^{\perp}\mathbf{h}_k^{\perp H}\} = \alpha||\mathbf{h}_k||^2$, where $\alpha = 2-\delta_{M,2}$. In the noiseless case, $\mathbf{y}_k = \mathbf{h}_k x_k$ and we have $\mathbf{h}_k^{\perp}\mathbf{y}_k = \mathcal{Y}_k\mathbf{h}_k = 0$. Based on this relation the channel at tone k can be uniquely determined up to a scale factor [7],[8], as the unique right singular vector of $\mathcal{Y}_k$ corresponding to the singular value zero. When noise is present, $\mathcal{Y}_k\mathbf{h}_k \neq 0$ and the SRM criterion is solved in the least-squares sense $||\mathbf{h}_k^{\perp}\mathbf{y}_k||_2^2 = tr\{\mathbf{h}_k^{\perp}\mathbf{y}_k\mathbf{y}_k^H\mathbf{h}_k^{\perp H}\}$. By the law of large numbers, asymptotically this criterion can be replaced by its expected value: $tr\{\mathbf{h}_k^{\perp} S_{\mathbf{y_k y_k}} \mathbf{h}_k^{\perp H}\}$. Practically, $S_{\mathbf{y_k y_k}}$ is not available so it is replaced by the sampled spectrum per each tone $\hat{S}_{\mathbf{y_k y_k}}$ which is computed directly from the fourier transformed version of the received data as we will show in the next section. Moreover, the SRM criterion can be written in the form shown in (8) where $Q_k = \sum_{i=1}^{M} D_i \hat{S}^*_{\mathbf{y_k y_k}} D_i^H$ where $\mathcal{D}_{i+1} = \mathcal{C}\mathcal{D}_i\mathcal{C}$.

$$\mathcal{D}_\mathbf{1} = \begin{bmatrix} 0 & 1 & 0 & \cdots \\ -1 & 0 & \cdots & \\ 0 & \vdots & \ddots & \\ \vdots & & & \end{bmatrix}. \tag{10}$$

$$\mathcal{C} = \begin{bmatrix} 0 & \cdots & 0 & 1 \\ 1 & 0 & \cdots & 0 \\ 0 & \ddots & & \vdots \\ \vdots & 0 & 1 & 0 \end{bmatrix}. \tag{11}$$

Then, we attempt to minimize the sum of the SRM criteria (cost functions) over all tones jointly to obtain an estimate of the channel impulse response $\mathbf{h}$. The SRM cost function is shown below:

$$\min_h \mathbf{h}^H \left[\sum_{k=0}^{N-1} G_k^H \left\{ \sum_{i=1}^{M} D_i \hat{S}^*_{\mathbf{y_k y_k}} D_i^H \right\} G_k \right] \mathbf{h} \tag{12}$$

We denote the matrix between the braces in (12) by Q_{SRM}, it has the size of $ML_h \times ML_h$. The estimated channel impulse response $\hat{\mathbf{h}}_{SRM}$ that is obtained by solving (12) is the eigen vector that corresponds to the minimum eigen value of Q_{SRM}. This solution has a scalar ambiguity that can be solved by forcing a least square constraint as follows: $\min_\alpha ||\mathbf{h}^0 - \alpha\hat{\mathbf{h}}_{SRM}||^2$. This yields the following solution:

$$\hat{\hat{\mathbf{h}}}_{SRM} = \frac{\hat{\mathbf{h}}_{SRM}^H \mathbf{h}^0}{||\hat{\mathbf{h}}_{SRM}||^2} \hat{\mathbf{h}}_{SRM} \tag{13}$$

B. Noise Subspace Fitting (NSS)

The sampled spectrum per each tone $\hat{S}_{\mathbf{y_k y_k}}$ can be decomposed into signal and noise subspace contributions:

$$\begin{aligned} \hat{S}_{\mathbf{y_k y_k}} &= \hat{S}_{\mathbf{y}_k,\mathcal{S}} + \hat{S}_{\mathbf{y}_k,\mathcal{N}} \\ &= \hat{E}_{s,k}\hat{\Lambda}_{s,k}\hat{E}_{s,k}^H + \hat{E}_{n,k}\hat{\Lambda}_{n,k}\hat{E}_{n,k}^H \end{aligned} \tag{14}$$

The basic idea of the NSF is to fit the estimated noise subspace that we obtain from the sampled spectrum to the true noise subspace which is spanned by the columns of $\mathbf{h}_k^{\perp H}$.

$$\min_{h_k,T} ||\mathbf{h}_k^{\perp H} - \hat{E}_{n,k}T||_F \tag{15}$$

where $||X||_F = tr\{X^H X\}$. This criterion differs from the original subspace fitting strategy proposed in [10], which would propose $\min_{h_k,T} ||\hat{E}_{n,k} - \mathbf{h}_k^{\perp H} T||_F$ as criterion. We propose (15) because it leads to a simpler optimization problem. Both approaches can be made to be equivalent by the introduction of column space weighting. The cost function in (15) is separable. In particular, it is quadratic in T. Minimization w.r.t. T leads to $T = \hat{E}_{n,k}^H \mathbf{h}_k^{\perp H}$ and $\mathbf{h}_k^{\perp H} - \hat{E}_{n,k}\hat{E}_{n,k}^H \mathbf{h}_k^{\perp} = P^{\perp}_{\hat{E}_{n,k}} \mathbf{h}_k^{\perp H}$ where $P^{\perp}_{\hat{E}_{n,k}} = I - P_{\hat{E}_{n,k}} = P_{\hat{E}_{s,k}}$ and $P_{\hat{E}_{n,k}}, P_{\hat{E}_{s,k}}$ denote respectively the projection matrix on the noise subspace ($\hat{E}_{n,k}$) and the signal subspace ($\hat{E}_{s,k}$).

Hence,

$$\begin{aligned} \min_{h_k,T} ||\mathbf{h}_k^{\perp H} - \hat{E}_{n,k}T||_F &= \min_{h_k} ||P_{\hat{E}_{s,k}} \mathbf{h}_k^{\perp H}||_F \\ &= \min_{h_k} tr\{\mathbf{h}_k^{\perp} \hat{E}_{s,k}\hat{E}_{s,k}^H \mathbf{h}_k^{\perp H}\} \end{aligned} \tag{16}$$

Similar to the case of SRM, the NSF criterion can be written in the form shown in (8) where $Q_k = \sum_{i=1}^{M} D_i \hat{E}_{s,k}\hat{E}_{s,k}^H D_i^H$ and D_i is the same as for the SRM criterion. Again, we attempt to minimize the NSF jointly over all the tones subject to the least square constraint to avoid introducing N constraints and to exploit the correlation exists between the different tones. Therefore, the NSF cost functions takes the following form:

$$\min_h \mathbf{h}^H \left[\sum_{k=0}^{N-1} G_k^H \left\{ \sum_{i=1}^{M} D_i \hat{E}_{s,k}\hat{E}_{s,k}^H D_i^H \right\} G_k \right] \mathbf{h} \tag{17}$$

Following the same discussion as in case of SRM we get the following solution:

$$\hat{\hat{\mathbf{h}}}_{NSF} = \frac{\hat{\mathbf{h}}_{NSF}^H \mathbf{h}^0}{||\hat{\mathbf{h}}_{NSF}||^2} \hat{\mathbf{h}}_{NSF} \tag{18}$$

The substanial computational power saving offered by our farmework namely, working per tones instead of working in the time domain, is elucidated by remarking that we perform Eigen Value Decomposition (EVD) of N matrices $\hat{S}_{\mathbf{y_k y_k}}$ each of size $(M \times M)$ while working in the time domain requires the EVD of a huge matrix $\hat{S}_{UU}$of size $(MN \times MN)$. Knowing that the number of operations required to perform the EVD of a matrix is proportional to the cubic of its size and the number of tones N for some systems (eg. LTE downlink) may reach to 2048, then the great advantage of our framework in terms of computational power saving becomes evident.

V. Block Toeplitz Covariance Matrix Enhancement

Here we go back to sample covariance refinements suggested by Cadzow in the eighties [2] and which we tried to exploit to enhance the dereverberation of the acoustic channel [4]. The idea is to iteratively reinforce several structural properties, the reinforcement of which consists of a projection onto a convex set. The iterations then converge to the joint reinforcement of all properties. Theoretically, the matrix valued vector signal spectrum is of the form

$$S_{\mathbf{yy}}(z) = \mathbf{h}(z)\, S_{xx}(z)\, \mathbf{h}^\dagger(z) + S_{\mathbf{vv}}(z) \tag{19}$$

where $.^\dagger$ denotes paraconjugate which is defined as $h^\dagger(z) = h(1/z^*)^H$ and $S_{\mathbf{vv}}(z) = \sigma_v^2 I$ is the white noise spectrum. Saking for the simplicity of notations, we omit the index k in $\mathbf{S_{yy}}$. The signal part of the spectrum, $\mathbf{h}(z)\, S_{xx}(z)\, \mathbf{h}^\dagger(z)$ is singular, not because of spectral poverty as in the SISO case, but because of limited rank in the matrix dimension. In the SISO case, a stationary signal covariance matrix can only be singular if the signal consists of a number of (complex) sinusoids, with their number being smaller than the covariance matrix dimension. Singularity in the MIMO case has nothing to do with spectral poverty but with matrix singularity of the matrix spectrum at every frequency.

Inspired by [2], (19) suggests the following procedure. First we start with the sample spectrum at each tone:

$$\begin{array}{l}\widehat{S}_{\mathbf{yy}}(z_n) = \frac{1}{P}\sum_{m=1}^{P} \mathbf{y}_n[m]\mathbf{y}_n^H[m] \ , \\ n = 0, \dots, N-1\end{array} \tag{20}$$

where P is the number of OFDM symbols over which we compute the sample spectrum and $z_n = e^{j2\pi n/2N}$, with the following properties: $\widehat{S}_{\mathbf{y}}^{\dagger}(z_n) = \widehat{S}_{\mathbf{y}}^H(z_n)$ (Hermitian transpose).

Now, at each frequency bin n, $S_{\mathbf{y}}(z_n)$ is of the form

$$\begin{aligned} S_{\mathbf{y}}(z_n) &= S_{\mathbf{y},\mathcal{S}}(z_n) + S_{\mathbf{y},\mathcal{N}}(z_n) \\ &= \mathbf{h}(z_n)\, S_x(z_n)\, \mathbf{h}^{\dagger}(z_n) + \sigma_v^2\, I_M \\ &= V_{max,n}\, (\lambda_{max,n} - \sigma_v^2)\, V_{max,n}^H + \sigma_v^2\, I_M \end{aligned} \tag{21}$$

where $S_{\mathbf{y},\mathcal{S}}(z_n)$, $S_{\mathbf{y},\mathcal{N}}(z_n)$ are the signal and noise components of $S_{\mathbf{y}}(z_n)$, and $\lambda_{max,n}$ and $V_{max,n}$ are its maximum eigenvalue and corresponding eigenvector. Now, the $\widehat{S}_{\mathbf{y}}(z_n)$ can be forced to the closest (in Frobenius norm) matrix of the form in (21) by computing its spatial eigen decomposition. Let $\widehat{\lambda}_{1,n} \geq \widehat{\lambda}_{2,n} \geq \cdots \geq \widehat{\lambda}_{M,n}$ be its eigenvalues, hence $\widehat{\lambda}_{max,n} = \widehat{\lambda}_{1,n}$, $\widehat{V}_{max,n} = \widehat{V}_{1,n}$. Then we get $\widehat{S}_{\mathbf{y}}(z_n) = \widehat{S}_{\mathbf{y},\mathcal{S}}(z_n) + \widehat{S}_{\mathbf{y},\mathcal{N}}(z_n) = \widehat{V}_{max,n}\, (\widehat{\lambda}_{max,n} - \widehat{\sigma}_v^2)\, \widehat{V}_{max,n}^H + \widehat{\sigma}_v^2\, I_M$ with $\widehat{\sigma}_v^2 = \frac{1}{N(M-1)} \sum_{n=0}^{N-1} \sum_{i=2}^{M} \widehat{\lambda}_{i,n}$ due to the spatiotemporal white noise assumption. Note that in fact at every frequency bin only $\lambda_{max,n}$ and $V_{max,n}$ need to be computed since $\sum_{i=2}^{M} \widehat{\lambda}_{i,n} = \mathrm{tr}\{\widehat{S}_{\mathbf{y}}(z_n)\} - \widehat{\lambda}_{max,n}$. Since the noise spectrum $\widehat{S}_{\mathbf{y},\mathcal{N}}(z_n) = \widehat{\sigma}_v^2\, I_M$ is fairly simple, there is no further structure to be imposed. The signal spectrum $\widehat{S}_{\mathbf{y},\mathcal{S}}(z_n) = \widehat{V}_{max,n}\, (\widehat{\lambda}_{max,n} - \widehat{\sigma}_v^2)\, \widehat{V}_{max,n}^H$ on the other hand is supposed to be spectrum of a FIR correlation sequence. This FIR character can be imposed by windowing in the time domain. The resulting source whitened signal spectrum $\widehat{S}_{\mathbf{y},\mathcal{S}}(z_n)$ then undergoes IFFT to obtain the corresponding matrix correlation sequence. The frequency-wise rank structure enforcement will have destroyed the FIR character of the correlation sequence, which can then simply be enforced in the time domain by proper windowing (without forgetting the symmetry structure of the first block column of the block circulant matrix). The operations of eigen structure enforcement in frequency domain and FIR structure enforcement in the time domain can then be iterated untill convergence. Typically a few iterations suffice. We are now ready to state the following iterative process:

1) Compute the matrix spectrum $\widehat{S}_{\mathbf{y}}(z_n)$ at each frequency bin as illustrated in (20).
2) Compute the eigendecomposition of the spectrum $\widehat{S}_{\mathbf{y}}(z_n)$ at each frequency bin $n = 0, 1, \dots, N-1$. Determine the noise variance $\widehat{\sigma}_v^2 = \frac{1}{N(M-1)} \sum_{n=0}^{N-1} \sum_{i=2}^{M} \widehat{\lambda}_{i,n}$ and the signal part of the spectrum $\widehat{S}_{\mathbf{y},\mathcal{S}}(z_n) = \widehat{V}_{max,n}\, (\widehat{\lambda}_{max,n} - \widehat{\sigma}_v^2)\, \widehat{V}_{max,n}^H$.
3) Compute the acoustic channel correlations

$$\begin{bmatrix} \widehat{r}_{\mathbf{y}}(0) \\ \widehat{r}_{\mathbf{y}}(1) \\ \vdots \\ \widehat{r}_{\mathbf{y}}^H(1) \end{bmatrix} = \frac{1}{N} (F_N^* \otimes I_M) \begin{bmatrix} \widehat{S}_{\mathbf{y}}(z_0) \\ \widehat{S}_{\mathbf{y}}(z_1) \\ \vdots \\ \widehat{S}_{\mathbf{y}}(z_{N-1}) \end{bmatrix} \tag{22}$$

Put the correlations outside the range $n \in \{0, 1, \dots, L_h-1\}$ to zero to obtain the Hermitian of the following block row $[\widehat{r}_{\mathbf{y}}(0)\ \widehat{r}_{\mathbf{y}}^H(1) \cdots \widehat{r}_{\mathbf{y}}^H(L_h-1)\ 0\ \cdots\ 0\ \widehat{r}_{\mathbf{y}}(L_h-1) \cdots \widehat{r}_{\mathbf{y}}(1)]$.

4) Compute the spectrum of the thus windowed correlation sequence

$$\begin{bmatrix} \widehat{S}_{\mathbf{y}}(z_0) \\ \widehat{S}_{\mathbf{y}}(z_1) \\ \vdots \\ \widehat{S}_{\mathbf{y}}(z_{N-1}) \end{bmatrix} = (F_N \otimes I_M) \begin{bmatrix} \widehat{r}_{\mathbf{y}}(0) \\ \widehat{r}_{\mathbf{y}}(1) \\ \vdots \\ \widehat{r}_{\mathbf{y}}^H(1) \end{bmatrix} \tag{23}$$

Go back to step 2 untill convergence. Note that the IFFTs and FFTs in (22) and (23) can be carried out efficiently in Matlab by reshaping the $N \times 1$ vectors of $M \times M$ blocks into $N \times M^2$ matrices.

After convergence, we make use of the refined spectrum we get at step (4) to get an enhanced channel impulse response estimation within the framework described in the previous section.

VI. Experimental Results

We run our simulations within the framework of an SIMO-OFDM system where each OFDM symbol is composed of 128 tones. The performance of the different deterministic channel estimators (structured and non structured) are evaluated by means of the NMSE vs. SNR. The NMSE is defined as $\frac{||\mathbf{h}^0 - \hat{\mathbf{h}}||^2}{||\mathbf{h}^0||^2}$ and the SNR is defined as $\frac{\sum_{k=0}^{N-1} ||\mathbf{h}_k||^2 \sigma_{x_k}^2}{\sigma_v^2 MN}$. The symbols are drawn from QPSK constellation and the NMSE is averaged over 10000 Monte-Carlo runs of the noise, symbols and the channel. We consider Rayleigh fading channel realizations where each one is composed of five i.i.d. channel coefficients. It is worthy to note that for SNR less than 20 dB the algorithm always converges typically after three or four iterations while at higher SNR the convergence is guranteed at no more than ten iterations. However, we consider a convergence is achieved when the following condition is fulfilled: $\frac{\widehat{\sigma}_{v,i}^2 - \widehat{\sigma}_{v,i-1}^2}{\widehat{\sigma}_{v,i}^2} \leq 0.1$ where i denotes the number of the current iteration at which the convergence is checked. Figure 1 shows the performance of both SRM and NSF with and without structuring where three antennas have been utilized at the receiver and the sampled spectrum has been computed from just one OFDM symbol. We remark that SRM yields better performance than NSF. This is due to the fact that when we work with one OFDM symbol then SRM is a weighted version of NSF with the weight being the largest eigen value of the sampled spectrum at each tone. However, when structuring is used both estimators show at least 3 dB gain even at very high SNR. It is also obvious that after structuring the performance of both estimators is congruent whatever is the SNR. To elaborate more the advantage of our structuring algorithm we plot in Figure 2 the BER versus SNR where we have used the estimated channels by various algorithms to equalize the received signal using MMSE equalizer and a hard decision decoding to extract the received bits. This result shows that our algorithm outperforms the non-structured ones by more than 2 dB at BER = 10^{-2}.

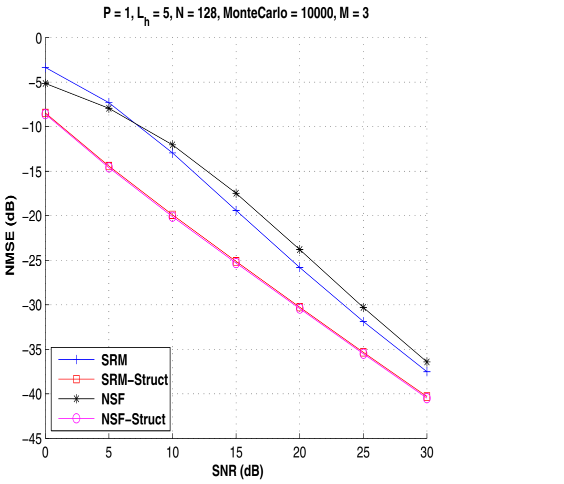

Fig. 1. The NMSE versus SNR for structured and non-structured estimators.

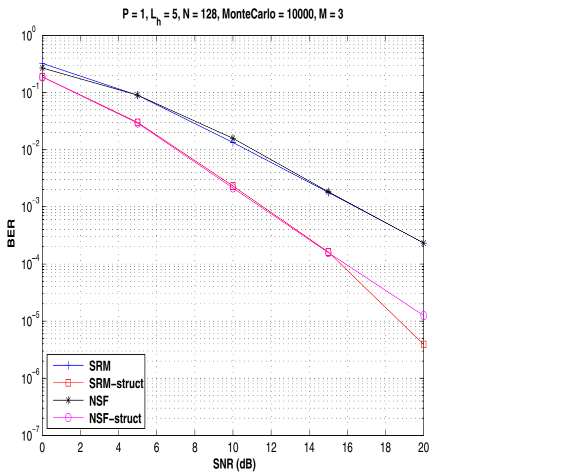

Fig. 2. The BER versus SNR for structured and non-structured estimators.

VII. Conclusions

To sum up, we have shown in this article the capabiltiy to exploit the classical blind deterministic channel estimators with a great computational power saving within the cyclic prefix systems. This is accomplished by minimizing the sum of the cost functions at different tones instead of minimizing the ordinary cost function in the time domain. Moreover, we propose a spatio-temporal based algorithm to enhance the sample covariance matrix upon which a class of well-known estimators rely. The enhancement is achieved by enforcing both the rank and the FIR structure properties. The simulations show that the proposed algorithm has the potential to provide a 5 dB gain (in terms of NMSE) at low to moderate SNR while it still has the capabiltiy to provide a noticeable gain at high SNR.

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