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Spectrum pooling based on centralized and distributed resource allocation strategies for cognitive radio networks

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THESE

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Spécialité: Communication et Electronique

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Spectrum Pooling basé sur des stratégies d'allocation de ressources centralisées et distribuées pour les réseaux radio cognitifs

Thèse soutenue le 8 Décembre 2008, devant le jury composé de :

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Abstract

The last ten years has seen an explosion in uses of wireless technologies. This, in turn, has driven a demand for more spectrum to support these uses. A recent spectrum license auction by the Federal Communications Commission (FCC), which regulates all civilian uses of wireless technologies in the United States, found that the existing spectrum utilization can be improved through opportunistic access to the licensed bands without interfering with the existing users. Notably, it is suggested that Dynamic Spectrum Access Networks as well as cognitive radio networks, will provide high bandwidth to mobile users via heterogeneous wireless architectures and dynamic spectrum access techniques. Cognitive radio networks, however, impose several research challenges due to the broad range of available spectrum as well as diverse Quality-of-Service (QoS) requirements of applications. These heterogeneities must be captured and handled dynamically as mobile terminals roam between wireless architectures and along the available spectrum pool.

In this dissertation, we study spectrum pooling strategies based on centralized and distributed resource allocation techniques. Throughout this work, we consider different system models in which cognitive users compete for a chance to transmit simultaneously or orthogonally with the primary system. On the basis of these models, we define the specific resource allocation problem addressed in this work in view of maximizing network capacity and at the same time, insuring a QoS for the primary system. In particular, we analyze the resource allocation problem and offer insights into user selection strategies and spectrum sensing in a cognitive radio network environment.

We initially investigate the problem of orthogonal communication scenarios between the primary system and cognitive users. For the first time, our study attempts to quantify the asymptotic (with respect to the band) achievable gain of using orthogonal spectrum pooling communications in terms of spectral efficiency. We then derive the total spectral efficiency as

well as the maximum number of possible pairwise communications of such a spectrum pooling system.
Having looked at orthogonal communication scenario, we then extend the cognitive protocol to allow the cognitive users to transmit *simultaneously* with the primary. We proceed to propose algorithms for simultaneous communication schemes to maximize the sum network capacity. These approaches allow cognitive radios to support and guarantee QoS when sharing spectrum while limiting the interference to the incumbent user. In the first approach, we employ a cognitive radio protocol where a virtual noise-threshold is used as a proxy for the primary user to allow cognitive users to profit from the primary user resources in an opportunistic manner, and at the same time, to maintain a guarantee of service to the primary user when cognitive communication is considered. The key idea is that the primary user has a certain quality of service to fulfill. This gives the secondary user a transmitting opportunity since the primary user will not, in any case, use all its rate as long as it has its quality of service satisfied.
The previous approach relies on a virtual noise threshold assumption. The approach is reminiscent of the interference temperature concept. However, as a practical matter, the FCC abandoned the interference temperature approach due to the fact that it is not a workable concept and would result in increased interference in the frequency bands where they were to be used. Accordingly, to determine the spectrum band allocation that meets the QoS requirements of different users, we propose, in the second approach, a different way to efficiently protect primary systems from secondary system interference, based on outage probability. We particularly propose a joint distributed algorithm for power allocation and user selection that tends to decrease control overhead in large cognitive radio networks.
In the end, we look at the problem of sensing in spectrum pooling scenarios. The proposed approach is based on an information-theoretic sub-space analysis for the detection of vacant sub-bands in a large spectrum context. We also investigate empirical techniques and compare results to UMTS real-world measurements as well as to other simulated signals in order to analyze the robustness of the proposed approach in presence of increased levels of noise.

Contents

List of Figures

Acronyms

Here are the main acronyms used in this document. The meaning of an acronym is usually indicated once, when it first occurs in the text. The English acronyms are also used for the French summary.

FCC	Federal Communications Commission
UHF	Ultra High Frequency
UWB	Ultra Wide Band
AWGN	Additive White Gaussian Noise
CDMA	Code Division Multiple Access
3GPP	Third Generation Partnership Project
CSIR	Channel State Information at the Receiver
CSIT	Channel State Information at Transmitter
FDMA	Frequency Division Multiple Access
KKT	Kraush-Kuhn-Tucker optimality conditions
LOS	Line-of-Sight
NLOS	Non Line-of-Sight
PDF	Probability Density Function
CDF	Cumulative Distributive Function
bps	bits per second
ML	Maximum Likelihood
i.i.d.	independent and identically distributed
MAC	Multiple Access Control
OFDMA	Orthogonal Frequency Division Multiple Access
OFDM	Orthogonal Frequency Division Multiplexing
WRAN	Wireless Regional Area Network
UMTS	Universal Mobile Telecommunications System
MIMO	Multiple-Input Multiple-Output
SDMA	Space Division Multiple Access
SINR	Signal-to-Interference-plus-Noise Ratio

SIR	Signal-to-Interference Ratio
SIC	Successive Interference Cancelation
SNR	Signal-to-Noise Ratio
TDMA	Time Division Multiple Access
TDD	Time Division Duplex
FDD	Frequency Division Duplex
CRN	Cognitive Radio Network
PU	Primary User
SU	Secondary User
QoS	Quality of Service
BS	Base Station
CH	Cluster Head
RHS	Right Hand Side
LHS	Left Hand Side

Notations

We regroup here the principle notations and symbols used on the different chapters of this document. As far as possible, we have tried to conserve the same notations from one chapter to another. However, some notations have different definitions depending on when they occur in the text.

General Notations

$\mathbb{R}$	Set of real numbers
$\mathbb{E}$	Expectation operator
$\mathcal{CN}$	Complex Normal Distribution
σ^2	Thermal noise variance

Chapter 2 : System Model and Resource Allocation

N	Number of primary users
M	Number of secondary users' pairs
$\tilde{M}$	Number of secondary users' pairs allowed to transmit
Ψ	Set of indices of all presently active secondary users
$h^i_{pu,n}$	Channel gain from the i^{th} primary user indexed by pu to the desired user n
R	Cell radius
R_p	Primary system protection area radius
$h_{j,n}$	Channel gain from secondary users j to the desired user n
p_{BS}	Received power for the primary user from the base station
p_j	Transmit power used for secondary user j
$P_{\max}$	Maximum power constraint
G^2_{pu}	Primary user pathloss gain profile estimate
P_{out}	Outage probability
R^i_{pu}	Transmit data rate for the i^{th} primary user

$I(\mathbf{x};\mathbf{y})$	Mutual information of the channel between the transmitted vector $\mathbf{x}$ and the received vector $\mathbf{y}$
C_{pu}^i	Instantaneous capacity of the i^{th} primary user
C_j	Instantaneous capacity of the j^{th} secondary user
$\mathcal{C}$	Sum network capacity

Chapter 3: Spectral Efficiency of Orthogonal Spectrum Pooling Systems

N	Number of sub-bands
L	Number of secondary users
$\mathcal{T}_l$	Transmitter l
$\mathfrak{R}_l$	Receiver l
C_l	Instantaneous capacity per sub-band for user l
P_l^i	Transmit power of user l on sub-band i
$\overline{P}$	Average power constraint
γ_0	Lagrange's multiplier
h_l^i	The block fading process of user l on the sub-band i
S_l^i	Symbol transmitted by user l on the sub-band i
P_l^i	Power allocation of user l on the sub-band i
n_l^i	The additive Gaussian noise at the ith sub-band
Ψ_l	Set of the number of sub-bands sensed occupied by user l
Ω_l	Set of the remaining idle sub-bands sensed by user l
Φ_l	Spectral efficiency per band of user l
Δ_l	Band factor gain of user l

Chapter 4: Cognitive Radio using Virtual Noise

N	Number of sub-bands
h_k^i	The block fading process of user k on the sub-band i
s_k^i	Symbol transmitted by user k on the sub-band i
p_k^i	Transmit power of user k on sub-band i
n^i	Additive Gaussian noise on the ith sub-band
σ_v^2	Virtual noise threshold
$\overline{P}$	Average power constraint
γ_0, λ	Lagrange's multipliers
R_p	Primary user transmit rate at the receiver
R_s	Secondary user transmit rate at the receiver
$C_{k,N}$	Instantaneous capacity per sub-band for user k
C_{sum}	Sum capacity

$F(x)$	The cumulative density function of x

Chapter 5: Joint Distributed Resource Allocation

N	Number of primary users
M	Number of secondary users' pairs
$\tilde{M}$	Number of secondary users' pairs allowed to transmit
Ψ	Set of indices of all presently active secondary users
$h^i_{pu,n}$	Channel gain from the i^{th} primary user indexed by pu to the desired user n
R	Cell radius
R_p	Primary system protection area radius
$h_{j,n}$	Channel gain from secondary users j to the desired user n
p_{BS}	Received power for the primary user from the base station
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$I(\mathbf{x};\mathbf{y})$	Mutual information of the channel between the transmitted vector $\mathbf{x}$ and the received vector $\mathbf{y}$
C^i_{pu}	Instantaneous capacity of the i^{th} primary user
C_j	Instantaneous capacity of the j^{th} secondary user
$\mathcal{C}$	Sum network capacity

Chapter 1

Introduction

1.1 Overview

New technologies in the area of adaptive wireless networks have the potential to utilize the large amount of unused spectrum in an intelligent way while not interfering with other incumbent devices in frequency bands already licensed for specific uses. These technologies fall in the category of adaptive, spectrum agile and cognitive radios techniques, which are enabled by the rapid and significant advancements in radio technologies. Observing that in some locations or at some times of day, 70% of the allocated spectrum may be sitting idle, the FCC has recently recommended [1] that significantly greater spectral efficiency could be realized by deploying wireless devices that can coexist with the licensed (primary) users, generating minimal interference while taking advantage of the available resources. The current approach for spectrum sharing is regulated so that wireless systems are assigned fixed spectrum allocations, operating frequencies and bandwidths, with constraints on power emission that limits their range. Therefore, most communication systems are designed in order to achieve the best possible spectrum efficiency within the assigned bandwidth using sophisticated modulation, coding, multiple antennas and other techniques. The most advanced systems are approaching Shannon's channel capacity limit [2], so further increase in capacity would require additional system bandwidth. The growing interest in cognitive radio network (CRN) research from signal processing and communication communities has spurred an increasing number of papers in the recent years. There are a large number of proposals for all communication layers, but the system infrastructure has not been clearly defined. In addition, most of these research results rely on theoretical analysis or computer simulations. In order to influence and convince regulators, these theoretical results should be demonstrated in realistic scenarios.

Thus, with the tremendous growth of wireless applications, many spectrum segments have been allocated following the spectrum property rights model. These licensees (primary users) have the exclusive rights to exploit this authorized spectrum for commercial or public use. However, recent measurements have shown that some of these licensed spectrum resources were not fully exploited depending on the locations and time [1, 3] (see Figure 1.1). As a result, the FCC has recently suggested [1] that significantly greater spectral efficiency could be realized by considering *cognitive radio* [4]. Such a scheme would define at least two classes of spectrum users. The first would be primary user (PU) who already possess a license to use a particular frequency. The second would be secondary user (SU) or cognitive user consisting of "unlicensed" user[1]. Primary users would always have full access to the spectrum when they need it while secondary users could use the spectrum when it would not harmfully interfere with the primary user. Clearly, the introduction of intelligent radios poses many new technical challenges in protocol design, power allocation, interference metrics, environment awareness, sensing, new distributed algorithms and quality of service (QoS) guarantees [5]. In particular, one of the greatest challenges is to build a radio capable of intelligently finding and handling the available frequency band without any compromise on the QoS. Overcoming these issues becomes very challenging due to the scarcity of radio resource (i.e., spectrum), the inherent transmission impairments of wireless links (multipath, fading, noise) and user mobility. Thus, there are many challenges across all layers of a cognitive radio system design, from its application to its implementation [6, 7].

[1] The term "unlicensed" is a misnomer that has created serious confusion in the regulatory treatment of these devices. The term will, however, continue to appear throughout this article because it has been so widely adopted in this use that attempting to substitute a more appropriate term proves both cumbersome and confusing.

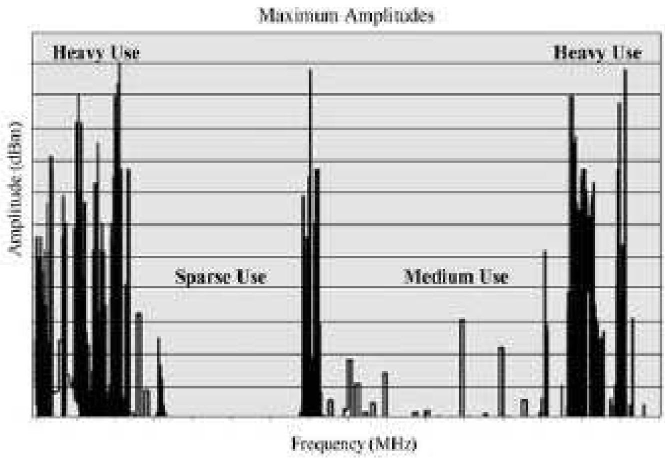

Figure 1.1: Spectrum utilization.

1.2 Challenges

The rising demand in wireless communication for free available spectrum goes along with increasing restrictions to spectrum utilization, i.e., Quality-of-Service (QoS) requirements, as for instance in consumer electronics or other multimedia applications. Efforts such as the DARPA neXt Generation (XG) communications program [8, 9] also known as Dynamic Spectrum Access Networks (DSANs), the National Science Foundation Programmable Wireless Networking (NSF-ProWiN) program [10], the agile spectrum policy initiatives conducted in the US, Canada and the European Union, and the standardization work taking place under the IEEE auspices [6, 11] indicate the level of activity in the field which has the potential to unleash tremendous spectrum capacity for a plethora of new applications. These approaches allow cognitive radios to support and guarantee QoS when sharing spectrum without requiring direct information exchange in observing past spectrum utilization. A recent spectrum license auction by the Federal Communications Commission, which regulates all civilian uses of wireless technologies in the United States, generated almost 14 billion dollars [12]. To combat this overcrowding, the FCCs most recent report on the wireless industry found that all uses of licensed wireless services, from mobile telephone use to fixed wireless data services, continued to grow at an astounding rate. At the same time, a multi-billion dollar industry has grown in the use of "unlicensed" spectrum. In fact, the FCC has maintained precisely such a hierarchy, assuring licensees superior rights to users of unlicensed spectrum. But recent recommendations by the FCCs Spectrum Task Force, as well as proposals supported by technology companies and advocates of the commons school of spectrum reform, have prompted the FCC to consider new alternatives [5]. Indeed, because users of unlicensed spectrum enjoy economies of scale and do not pay for expensive spectrum licenses, unlicensed spectrum can offer a less expensive and more readily deployable form of wireless service than licensed spectrum albeit at a trade off for quality of service and protection from interference. With this rise in intensive use, the FCC has also faced pressure to open more spectrum for unlicensed use. At the heard of these challenges, lies the ability to exploit the resource as efficiently as possible. This is exactly the issue tackled in this work where the cognitive radio behavior is studied and evaluated with a particular attention to primary system QoS and control overhead reduction.

1.3 Scenario and Assumptions

Throughout this dissertation, we focus our study on non-real time data transmission services since they are delay insensitive. Such an opportunistic data transmission is facilitated by the time-varying user interference profile owing to mobility, fading, and power control [13]. This work was motivated by the fact that future personal communication systems are supposed to support a variety of services, including real-time and non-real time services (web browsing, e-mail, fax, file transfer, etc.). However, although we focus on non-real time services, we emphasize that our analysis also holds for accommodating real time data traffic.
Another technique that has been increasingly popular is Time Division Duplex (TDD) in which the same carrier is used for both links in different time slots. One property of such systems is that, since the same frequency is used, the channel characteristics are nearly the same in both links, provided the channel does not change too rapidly.
Cooperative communication has been known recently as a way to overcome the limitation of wireless systems. In some recent works, the cognitive radios are allowed to cooperate for sensing the spectrum, so that the hidden terminal issues are addressed [14, 15]. In most of the approaches that can be found in the literature, the need may exist for centralized knowledge of all channel for all nodes in the network. To circumvent this problem, the design of so-called *distributed* resource allocation techniques is crucial. Distributed optimization refers to the ability for each user to manage its

local resources (e.g. rate and power control, user scheduling) based only on locally observable channel conditions such as the channel gain between the access point and a chosen user, and possibly locally measured noise and interference. In this thesis, we study the spectrum sharing methods where multiple systems coexist and interfere with each other. Chapter 2 is devoted to the discussion about the different scenarios in cognitive radio systems. We will discuss cooperation in centralized networks and decentralized networks separately. The centralized network is a network whose size is fixed by the coverage area of the access point or base station. The decentralized network has a size that can be scaled up more flexibly by allowing intermediate nodes in the transmission path as a relay. Also, the de-centralized manner of cognitive radio broadens the scope of its applications. Cooperative decentralized systems are usually modeled as cooperative ad-hoc networks [16]. With cooperation, systems can support more users because of the improved spectrum-management strategy. An extensive analysis on cooperation in both networks is discussed in this dissertation. We also propose new schemes to deal with the coexistence issue of the cognitive network while limiting the interference to the incumbent user.

1.4 Contributions

In this dissertation, we attempt to define schemes for accessing to the radio spectrum and posing several constraints in the management and in the sharing strategies for such a precious resource. Within this setting, we consider different system models in which cognitive users compete for a chance to transmit simultaneously or orthogonally with the primary system. On the basis of these models, we define the specific resource allocation problem and offer insights into how to design user selection strategies in a cognitive radio network environment. We initially investigate the problem of orthogonal communication scenarios between the primary system and cognitive users. Thus, within this setting, a device transmits over a certain time or frequency band only when no other user does. Next, we extend the cognitive protocol to allow cognitive users to transmit simultaneously with the primary user in the same frequency band as long as the level of interference with the primary user remains within an acceptable range. We first introduce the notion of the *virtual noise threshold* which represents a proxy for the primary user to allow cognitive user to profit from the primary user resources in an opportunistic manner, and at the same time, to maintain a guarantee of service to the primary user when cognitive communication is considered. Then, we

investigate the problem of joint power allocation and user selection in a CRN consisting of multiple secondary transmitters and receivers communicating simultaneously in the presence of the primary system. We emphasize on the capability of the proposed approach to allow cognitive devices to support and guarantee QoS for the primary user, while sharing spectrum. In resolving the question set forth above, we should, as a general rule, favor primary system QoS rather than attempting to enhance unlicensed spectrum access. This is not, as we will argue in this dissertation, at the expense of exclusively licensed services, since we must still ensure that these unlicensed services do not interfere with existing licensed services.
In Chapter 2, we begin by presenting the system model and assumptions used throughout most of this dissertation. We consider an ad-hoc channel architecture in which the primary and the cognitive user wish to communicate, subject to mutual interference. We introduce the scope of resource allocation, focusing on power allocation. We define the figure of merit used throughout this work as the sum rate. We then formulate the optimization problem for sum-rate maximization, for which we will investigate solutions and algorithms in later chapters.

In Chapter 3, we consider a generic spectrum pooling scenario where users communicate in an orthogonal manner enabling public access to the new spectral ranges without sacrificing the transmission quality of the actual license owners. For the first time, our analysis attempts to quantify the achievable gain of using spectrum pooling with respect to classical radio devices. The goal here is to obtain a characterization of the achievable spectral efficiency as well as the maximum number of possible pairwise communications within such a scenario.

The work in this chapter has been submitted for publication in:

M. Haddad, M. Debbah and A. Hayar ,“**Spectral Efficiency for Cognitive Radio Systems**”, *IEEE Global Telecommunications Conference (GLOBECOM 2007)*, Washington, D.C, USA, 2007.

M. Haddad, A. Hayar and M. Debbah,“**Spectral Efficiency of Spectrum Pooling Systems**”, *IET Special Issue on Cognitive Spectrum Access*, Vol. 2, No. 6, pp. 733-741, July 2008.

In Chapter 4, we present two cognitive radio protocols which are believed to be potential promising candidates for future cognitive radio network deployment and offer insights into how to design such scenario in a cognitive radio network environments. The first part of the chapter is a description of the cognitive radio protocol based on virtual noise threshold. We also propose an algorithm to gather the primary user channel state information (CSI) in both cooperative and non cooperative scenario. In the second part, we characterize the fundamental performance of the proposed optimal power allocation policy in terms of the achievable rate. The key idea within this chapter is that, using a virtual noise-threshold as a proxy for the primary user a cognitive radio can vary its transmit power in order to maximize the sum capacity, while maintaining a guarantee of service to the primary user.

The work in this chapter has been submitted for publication in:

M. Haddad, M. Debbah and A. Hayar ,"**Distributed Power Allocation for Cognitive Radio**", *International Symposium on Signal Processing and its Applications (ISSPA 2007)*, Sharjah, United Arab Emirates, 2007.

M. Haddad, M. Debbah and A. Hayar ,"**Optimal Power Allocation For Cognitive Radio Based on a Virtual Noise Threshold**", *The First International Workshop on Cognitive Wireless Networks (CWNets 2007)*, Vancouver, Canada, 2007.

Results in Chapter 4 showed that cognitive protocols can be extended to allow the SU to transmit *simultaneously* with the PU in the same frequency band. This is exactly the setup in Chapter 5, where the cognitive radio behavior is generalized to allow secondary users to transmit simultaneously with the primary system *as long as the level of interference to primary users remains within an acceptable range* by means of outage probability. Specifically, it is proposed in this chapter to combine cognitive radio with multi-user diversity technology to achieve strategic spectrum sharing and self-organizing communications. Our analysis treats both uplink and downlink scenarios. We also address the problem of user selection strategy in the context of CRN where both centralized and distributed solutions are presented.

The work in this chapter has been submitted for publication in:

M. Haddad, A. Hayar, G. E. Øien and S. G. Kiani,"**Uplink Distributed**

Binary Power Allocation for Cognitive Radio Networks", *Crown-Com 2008*, Singapore, May 2008.

M. Haddad, A. Hayar and G. E. Øien,"**Downlink Distributed Binary Power Allocation for Cognitive Radio Networks**", *PIMRC 2008*, Cannes, France, September 2008.

B. Zayen, M. Haddad, A. Hayar and G. E. Øien "**Binary Power Allocation for Cognitive Radio Networks with Centralized and Distributed User Selection Strategies**", *Phycom Journal, ELSEVIER* vol. 1, Issue 3, 2008.

Chapter 2

Scenarios and System Models

In this chapter, we begin by presenting the system model and assumptions used throughout most of this dissertation. We consider both cooperative and non cooperative architecture in which the primary and the cognitive user wish to communicate, subject to mutual interference. However, we draw the reader's attention that some of the results presented in later chapters also carry forward to the distributed case. Due to users fully sharing the same spectral resource, co-channel interference is experienced from concurrent transmissions. The advantage of such models is that it is independent of the underlying radio interface and can be used to evaluate the system performance for a number of radio access mechanisms, e.g. TDMA, OFDMA, orthogonal-CDMA, etc. We then formulate the power allocation problem for sum-rate maximization in a cognitive radio network, for which we will investigate solutions and algorithms in later chapters.

2.1 Performance metric

The end goal of this dissertation is to propose practical and distributed schemes for CRN resource allocation with the view of (1) improving the spectral efficiency, and (2) maintain a QoS guarantee for the primary system which will thus be our main figure of merit. In order to characterize the achievable performance limit of such systems, the system performance criterion is directly dependent on the application requirements. Therefore, in order to differentiate real-time service from non real-time service, three capacities measures can be found in the literature. A comprehensive review of these concepts can be found in [17].
Usually, when the coherence time T_c is smaller than the codeword length (fast fading), the relevant performance metric is the *ergodic capacity*, namely [18]:

$$C_{erg} = \mathbf{E}\left\{\log_2\left(1 + \mathrm{SNR}\,|h|^2\right)\right\} \qquad (b/s/Hz) \tag{2.1}$$

The ergodic capacity was developed for non-real time applications. It determines the maximum achievable rate over all fading states without a delay constraint. The corresponding optimum power allocation is the well-known water-filling allocation [19]. However, for real-time applications in a slow fading environment, substantial delay can occur when averaging over the fading states and the decoding error probability can not be made arbitrarily small. To address this issue, the notion of *information outage probability* defined as the probability that the instantaneous mutual information of the channel is below the transmitted code rate was introduced in [20]. Accordingly, the outage probability is:

$$P_{out}(R) = P\left\{I(\mathbf{x};\mathbf{y}) \leq R\right\} \tag{2.2}$$

where $I(\mathbf{x};\mathbf{y})$ is the mutual information of the channel between the transmitted vector $\mathbf{x}$ and the received vector $\mathbf{y}$ and R is the target data rate in $(bits/s/Hz)$. Additional definitions related to outage probability are those of:

- *Zero outage capacity* [21]: also called *delay-limited capacity.* It represents the maximum data-rate R for which the minimum outage probability is zero.

- *Outage capacity* [22]: is the maximum target rate that can be achieved over the channel with an outage probability less than ϵ.

Note that we should in fact use [bit/s/Hz], since the capacity is normalized with respect to the bandwidth; however, due to the bandwidth normalization, we can use the term "capacity" or "rate" with unit [bit/s] throughout the dissertation without causing any confusion. Accordingly, capacity expressions are given in [bits/s/Hz], where normalization to the respective bandwidth W is performed if required in the rest of the dissertation. The sum here is done over the stationary instantaneous distribution of the fading channel on each user l.

In what follows, we present the two cognitive radio protocols we have considered in the first part of this dissertation. Specifically, we present two schemes employing varying degrees of cooperation : (1) cooperative, where the cognitive user to get the primary user's channel state information (CSI) communicate with the primary base station (BS) and (2) non-cooperative, where the cognitive system is assumed to communicate with its own BS independently from the primary system.

2.2 The cooperative scenario

Cooperative communication has been known recently as a way to overcome the limitation of wireless systems. In some recent works, the cognitive radios are allowed to cooperate for sensing the spectrum, so that the hidden terminal issues are addressed [14].

2.2.1 The single operator scenario

Let us first consider a single operator scenario where the primary and the cognitive users attempt to communicate with a common base station, subject to mutual interference (see Fig. 2.1). A particularly noteworthy target in this context, where the primary user is supposed to be oblivious to the presence of potential secondary users, is how the cognitive user would gather the instantaneous CSI of the primary user without any cooperation with the primary system.
To do so and because there are many channel responses that are proportional to the number of cognitive users in a multi users system, the feedback overhead may be too large and thus reverse-link channel capacity may be greatly wasted. To reduce the feedback information in such cases, we propose a new communication scenario wherein no feedback is assumed.

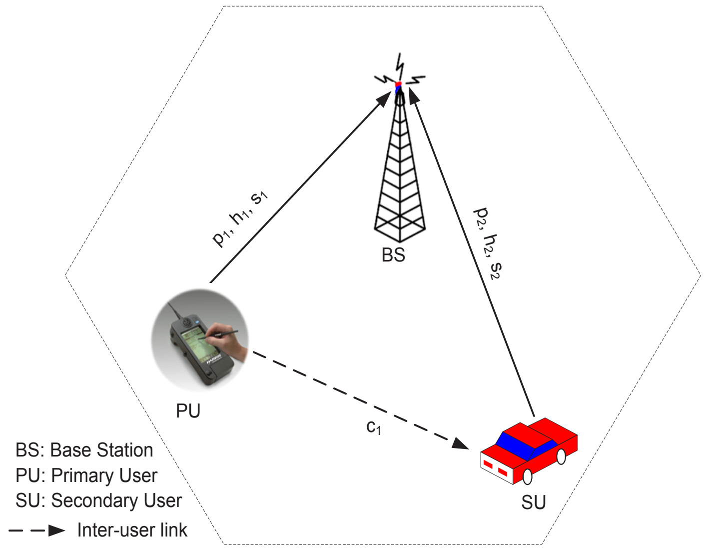

Figure 2.1: The single cognitive radio channel.

2.2.2 The heterogeneous network scenario

As mentioned before, there exist temporally unused spectrum holes in the licensed spectrum band. Hence, CRN networks can be deployed to exploit these spectrum holes through cognitive communication techniques. As a general framework, we propose in Figure 2.2 a heterogeneous architecture where the cognitive network coexists with the primary network at the same location and on the same spectrum band. Although the main purpose of the CRN is to determine the best available spectrum, CRN functions in the licensed band are mainly aimed at the detection of the presence of primary users. The channel capacity of the spectrum holes depends on the interference at the nearby primary users. Thus, the interference avoidance with primary users is the most important issue in this architecture. Furthermore, if primary users appear in the spectrum band occupied by cognitive users, users should vacate the current spectrum band and move to the new available spectrum immediately, called *spectrum handoff*. In particular in our work, we consider a system where the primary and the cognitive users are supposed to communicate to different receivers, subject to mutual interference. Such an accurate modeling of the cognitive radio channel is a key to understand the actual benefits brought by cognitive radio in the later chapters. Though the system model presented herein is related to a sin-

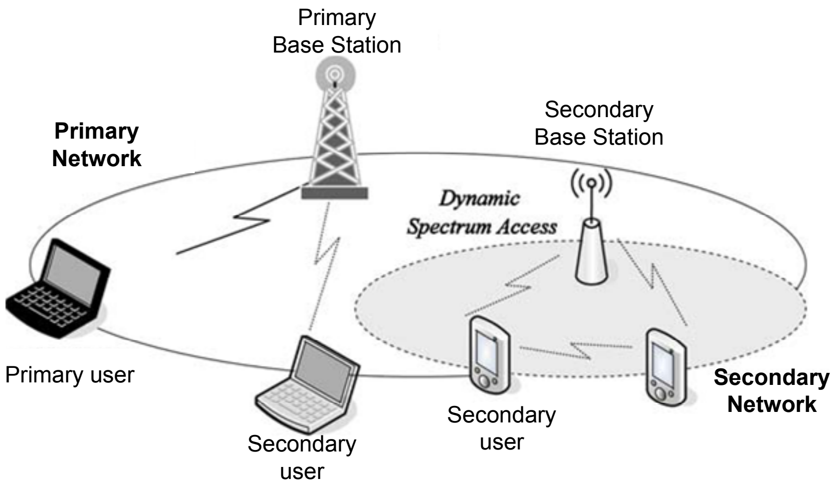

Figure 2.2: Dynamic spectrum access in cognitive radio network.

gle operator model, notations and analysis will carry to the heterogeneous model case as well.

2.2.3 System Model

Consider the baseband discrete-frequency model at a receiver l with N sub-bands

$$y_l^i = h_l^i \sqrt{p_l^i(h_l^i)} s_l^i + n_l^i, \qquad \text{for} \quad i = 1, ..., N \tag{2.3}$$

where:

- the sub-band index i lies from 1 to N,
- the user index l is taken equal to 1 for the primary user and lies from 2 to L for secondary users,
- h_l^i: is the block fading process of user l on the sub-band i,

- s_l^i: is the symbol transmitted by user l on the sub-band i,
- $p_l^i(h_l^i)$: is the power allocation of user l on the sub-band i,
- n_l^i: is the additive Gaussian noise at the ith sub-band.

We further assume that the channel h_l stays constant over each block fading length (i.e. coherent communication). The assumption of coherent reception is reasonable if the fading is slow in the sense that the receiver is able to track the channel variations. We statistically model the channel gains h_l to be i.i.d distributed over the L Rayleigh fading coefficients and $\mathbb{E}\left\{|h_l|^2\right\} = 1$ for $l = 1, ..., L$. The additive Gaussian noise n_l at the receiver is i.i.d circularly symmetric and $n_l \sim \mathcal{CN}(0, N_0))$ for $l = 1, ..., L$.

2.2.4 Problem formulation

We propose to study the instantaneous capacity per sub-band in bits/s/Hz, also called *spectral efficiency*, namely [23]:

$$C_l = \frac{1}{N}\sum_{i=1}^{N}\log_2\left(1 + \frac{p_l^i \mid h_l^i \mid^2}{N_0}\right); \qquad l = 1, ..., L \tag{2.4}$$

The goal here is to maximize the sum capacity over the system expressed as:

$$C_{sum,N} = \sum_{l=1}^{L} C_{l,N} \tag{2.5}$$

where L represents the number of users transmitting.

Let us now assume the baseband discrete-time model within a coherence time period T when each user l for $l = 2, ..., L$ has N sub-bands or sub-channels:

$$y_l^i(k) = c_{l-1,l}^i(k)\sqrt{p_{l-1}^i(h_{l-1}^i)}s_{l-1}^i(k) + n_{l-1}^i(k), \tag{2.6}$$

where $c_{l-1,l}^i(k)$ is the block fading process from user $l-1$ to user l on the ith sub-band, at time k. We further assume that $0 \leq k \ll \beta T$ and $\beta < 1$, i.e. the coherence time is sufficiently large so that the channel stays constant for samples and jumps to a new independent value (block-fading model).
In chapter 3, the cognitive protocol behavior is assumed to allow only one user to simultaneously transmit over the same sub-band by virtue of orthogonal power allocation. The received signal of user l can therefore be written as:

$$y_l^i(k) = \begin{cases} \underbrace{c_{l-1,l}^i(k)\sqrt{p_{l-1}^i}s_{l-1}^i(k)}_{\text{signal}} + \underbrace{n_{l-1}^i(k)}_{\text{noise}}, & \text{if} \quad p_{l-1}^i \neq 0 \\ n_{l-1}^i(k), & \text{otherwise} \end{cases} \tag{2.7}$$

2.3 The non-cooperative scenario

In a realistic network, centralized system coordination is hard to implement, especially in fast fading environments and in particular if there is no fixed infrastructure for SUs, i.e., no back-haul network over which overhead can be transmitted between users. In fact, centralized channel state information for a dense network involves immense signaling overhead and will not allow the extraction of diversity gains in fast-fading channel components. To alleviate this problem, we present a CRN with secondary users attempt to communicate independently from the primary system. In particular, we expose the core problem of resource allocation in the multiuser context addressed in chapter 5.

2.3.1 Joint Distributed Binary User Resource Allocation

Consider a CRN that consists of a primary user, a base station, and M pairs of secondary users randomly distributed over the system (see Fig. 2.3). The channel gains are i.i.d random variable. Our study treats both downlink and uplink communication systems. In the coverage area of the primary system, there is an *interference boundary* within which no SUs can communicate in an ad-hoc manner. Thus, as can be seen in Figure 2.3, for the impairment experienced by the primary system to be as small as possible, a SU must be able to detect very reliably whether it is far enough away from a primary base station, i.e., in the area of possible cognitive radio operation. A peak transmit power constraint $p_{\max}$ is imposed at each SU and to simplify exposition, we shall assume that it is identical for all transmitters.

2.3.2 System Model

We consider a wireless CRN with a collection of users randomly distributed over the system. Users can be both transmitters and receivers. By virtue of a scheduling protocol, N PUs and M pairs of secondary users are simultaneously selected from these users to communicate at a given time instant, while others remain silent. The channel gains are assumed i.i.d random variable.

Each PU user is allocated a unique resource slot from the N sub-carriers so that he transmits in an orthogonal manner with respect to other PUs within his coverage area, i.e. no interference between different PUs like in the Orthogonal Frequency-Division Multiple Access-based (OFDMA-based). In order to facilitate the problem formulation of the joint resource allocation problem, we state the following notations:

- the index of PUs (or equivalently the number of sub-bands) i lies between 1 and N,
- the index of SUs j lies between 1 and M,
- $h_{pu,n}$ denotes the channel gain from the PU indexed by pu to a desired SU n,
- $h_{pu,pu}$ denotes the channel gain between the base station (BS) and the PU,
- $h_{j,n}$ denotes the channel gain from SU j to a desired SU n,
- the data destined from the primary system is transmitted with power p_{pu}.
- the data destined from SU j is transmitted with power p_j.

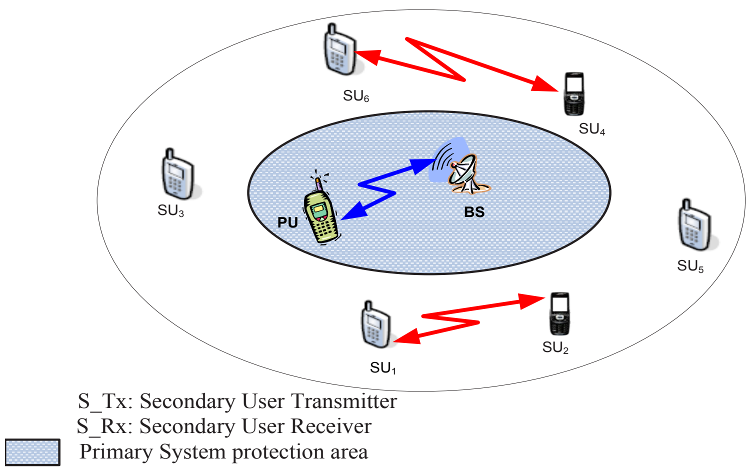

Figure 2.3: The Cognitive Radio Network.

In the coverage area of the primary system, there is an *interference boundary* within which no SUs can communicate in an ad-hoc manner. Thus, as can be seen in Figure 2.3, for the impairment experienced by the primary system to be as small as possible, a SU must be able to detect very reliably whether it is far enough away from a primary base station, i.e., in the area of possible cognitive radio operation. We further assume that only one PU is allowed to transmit during a resource slot. Nevertheless, with the same goal of sum secondary capacity maximization, we emphasize that the proposed study is still valid for the multi-PU case. The algorithm is simply run independently over all the PUs in parallel since we suppose that PUs are assumed to operate in an orthogonal manner with respect to other PUs (OFDMA-based system). For OFDMA, the proposed algorithm is simply run independently over the N sub-carriers in parallel. With the same goal of cognitive capacity maximization, the proposed algorithm can be easily extended to multi-carrier systems.

2.3.3 Problem formulation

Secondary users offer the opportunity to improve the system throughput over the system by detecting the PU activity and adapting their transmissions accordingly while avoiding the interference to the PU by satisfying the QoS constraint on outage. The motivation behind the proposed technique is that, via opportunistically adapting their transmit power with the guide of the proposed strategy, SUs can maximize the achievable sum rate under the constraint of maintaining the outage probability of the PU not degraded. The expression of the i^{th} PU (on the i^{th} sub-band) instantaneous capacity is

$$C_{pu}^{i} = \log_2\left(1 + \frac{p_{pu} \mid h_{pu,pu}^{i} \mid^2}{\sum_{j=1}^{M} p_j \mid h_{j,pu} \mid^2 + \sigma^2}\right); \qquad \text{for } i = 1, ..., N \tag{2.8}$$

where σ^2 is the ambient noise variance. Although this is not a restriction of the proposed analysis and for the sake of simplicity, we assume that only one PU is allowed to transmit within the interference boundary. The multi-PU case will be discussed afterwards in detail. Subsequently, unless stated otherwise, we will find it convenient to drop the primary user index i. On the other hand, by making SUs access the primary system spectrum, the j^{th} SU experiences interference from the PU and all neighboring co-channel SU links that transmit on the same band. Accordingly, the j^{th} SU instantaneous capacity is given by:

$$C_j = \log_2 (1 + \mathrm{SINR}_j)\,; \quad \text{for} \quad j = 1, ..., M \tag{2.9}$$

where

$$\mathrm{SINR}_j = \frac{p_j \mid h_{j,j} \mid^2}{\sum_{\substack{k=1 \\ k \neq j}}^{M} p_j \mid h_{k,j} \mid^2 + p_{pu} \mid h_{bs,j} \mid^2 + \sigma^2} \tag{2.10}$$

SUs need to recognize their communication environment and adapt the parameters of their communication scheme in order to maximize the per-user cognitive capacity, expressed as

$$C_{sum} = \frac{1}{\tilde{M}} \sum_{j=1}^{\tilde{M}} C_j\,, \tag{2.11}$$

while minimizing the interference to the primary users, in a *distributed* fashion. The sum here is made over the $\tilde{M}$ SUs allowed to transmit. Moreover, we assume that the coherence time is sufficiently large so that the channel stays constant over each scheduling period length. We also assume that SUs know the channel state information (CSI) of their own links, but have no information on the channel conditions of other SUs. No interference cancelation capability is considered. Power control is used for SUs both in an effort to preserve power and to limit interference and fading effects.

2.3.4 Non-cooperative Optimization Problem

Our goal within this work is thus to determine, under the assumption that the PU is oblivious to the presence of the cognitive users, what would be the cognitive system capacity (which can also be viewed as the total increase in system capacity due to the SUs' activity) and, at the same time, the maximum number of cognitive communication links allowed in such a system. We present a distributed algorithm for power allocation in the sense that it requires a SU to decide *distributively* to either transmit data or stay silent over the channel coherence time depending on a specified threshold. The optimization problem can therefore be expressed as follows:

$$\text{Find } \{p_1*, ..., p_M*\} = \arg \max_{p_1, \cdots, p_M} C_{sum} \tag{2.12}$$

subject to:

$$\begin{cases} p_j \in \{0, p_{max}\}, \quad \text{for} \quad j = 1, ..., M \\ \\ p^i_{out} = Prob\left\{C^i_{pu} \leq R^i_{pu} \mid R^i_{pu}, q\right\} \leq q, \;\; \text{for} \quad i = 1, ..., N \end{cases}$$

where R^i_{pu} is the i^{th} PU transmitted data rate. By writing the outage probability as:

$$p^i_{out} = Prob\left\{C^i_{pu} \leq R^i_{pu}\right\} \leq q, \qquad \forall\, i = 1, ..., N \tag{2.13}$$

the following holds: $C^i_{pu} \leq R^i_{pu} \Rightarrow C^i_{pu} \leq \max_i R^i_{pu}, \forall\, i$.

Within this setting, if $Prob\left\{C^i_{pu} \leq \max_i R^i_{pu}\right\} \leq q, \forall\, i$, we have also guaranteed that $p^i_{out} \leq q, \forall\, i$. Following this trend, the outage probability condition in (2.13) can be resumed to the worst case primary system constraint. As a consequence of this assumption, we will find it convenient to drop the sub-band index i in chapter 5 and consider only the worst case. Alternatively, the information about the outage failure can be carried out by a band manager that mediates between the primary and secondary users [24], or can be directly fed back from the PU to the secondarys transmitter through collaboration and exchange of the CSI between the primary and secondary users as proposed in [25].
In chapter 5, we will focus on only one sub-band. As a consequence of this assumption, we will find it convenient to drop the sub-band index i in the rest of our derivation in chapter 5. Nevertheless, with the same goal of sum secondary capacity maximization, the proposed algorithm can be easily extended to multi-band systems. The central theme of this dissertation thus arises: How to deal with the coexistence issue of the cognitive network while limiting the interference to the incumbent user, within reasonable complexity and signaling constraints? and how could secondary users exploit the primary user spectrum without interfering with During the course of this thesis, we will present practical scheme and constructive results which demonstrate the value of CRN resource allocation and provide insight into solving this problem. Moreover, we will also focus on *distributed* algorithms requiring only local information, which would be the first step to making some of these strategies realizable in practice.

In the first instance, we try to gain an insight into the behavior of expected SUs' interference in large wireless networks. Specifically, it is proposed to combine cognitive radio with multi-user diversity technology to achieve strategic spectrum sharing and self-organizing communications. The motivation behind such a strategy is to derive a simple algorithm where a secondary user can decide to either transmit data or stay silent over the channel coherence time depending on a specified threshold, without affecting the primary users' QoS.

Chapter 3

Spectral Efficiency of Orthogonal Spectrum Pooling Systems

In this chapter, we investigate the idea of using spectrum pooling *to reuse locally unused spectrum to increase the total system capacity. We consider a multiband/wideband system in which the primary and cognitive users wish to communicate to different receivers, subject to mutual interference. We assume that each user knows only his channel and the unused spectrum through adequate sensing. The basic idea under the proposed scheme is based on orthogonal transmissions in a spectrum pooling scenario. The idea is quite simple: a cognitive radio will listen to the channel and, if sensed idle, will transmit during the voids. It turns out that, although its simplicity, the proposed scheme showed very interesting features with respect to the spectral efficiency and the maximum number of possible pairwise cognitive communications. We impose the constraint that users successively transmit over available bands through selfish water filling. Notice that, although a water filling power allocation strategy is adopted in this analysis, we emphasize that this is not a restriction of the proposed protocol. For the first time, our study has quantified the asymptotic (with respect to the band) achievable gain of using spectrum pooling in terms of spectral efficiency compared to classical radio systems. We then derive the total spectral efficiency as well as the* *maximum number of possible pairwise communications of such a spectrum pooling system.*

3.1 Introduction

The basic idea within the chapter is based on *spectrum pooling*. The notion of spectrum pooling was first mentioned in [26]. It basically represents the idea of merging spectral ranges from different spectrum owners (military, trunked radio, etc.) into a common pool. It also reflects the need for a completely new way of spectrum allocation as proposed in [27]. The goal of spectrum pooling is to enhance spectral efficiency by overlaying a new mobile radio system on an existing one without requiring *any changes* to the actual licensed system. Motivated by the desire for an effective and practical scheme, our study treats the problem of spectrum pooling from sensing to achievable performance. We consider an asynchronous TDD communication scenario in which hierarchical strategy is adopted. Typically, one primary user and multiple cognitive users detecting unoccupied spectrum bands and adapting the transmission to those bands while avoiding the interference to the primary user. However, contrary to the work addressed in next chap-

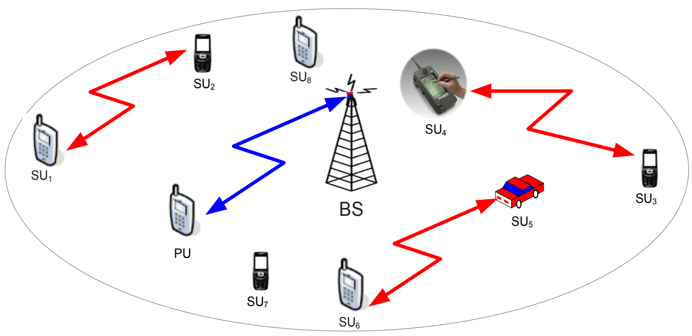

Figure 3.1: The cognitive radio network for spectrum pooling.

ter, in this contribution, we impose as a first step that only one user can simultaneously transmit over the same sub-band using successive water filling (see Figure 3.1). Especially OFDM based WLANs like IEEE802.11a and HIPERLAN/2 are suitable for an overlay system like spectrum pooling as they allow a very flexible frequency management on a carrier-by-carrier basis. We examine the total spectral efficiency of the spectrum pooling system and show that the overall system spectral efficiency can be considerably enhanced by considering cognitive communications with respect to the traditional system (without cognition). In particular, it is of major interest, in this context, to quantify the spectral efficiency gain in order to show the interest behind using spectrum pooling terminals with respect to classical systems (without cognition). In fact, although spectrum polling have spurred great interest and excitement, many of the fundamental theoretical questions on the limits of such technologies remain unanswered. The merits of our approach lie in the simplicity of the proposed scheme and, at the same time, its efficiency. Results showed very interesting performance in terms of the number of cognitive users allowed to transmit as well as the system spectral efficiency gain we get. Such an accurate and simple system modeling presents a key to understand the actual benefits brought by spectrum pooling technology.

The rest of the chapter is organized as follows: In Section 3.3, we describe the spectrum pooling protocol. In Section 3.4, we address the problem of sensing. Section 3.5 details the spectral efficiency analysis adopted throughout this chapter when the number of sub-bands is limited. In Section 3.6, we investigate the asymptotic performance of such a system in terms of spectral efficiency. Performance evaluation is provided in Section 3.7 and Section 3.7 concludes the chapter.

3.2 The channel model

The baseband discrete-frequency model at the receiver $\mathfrak{R}_l$ (see Figure 3.2) is:

$$y^i_{\mathfrak{R}_l} = h^i_l \sqrt{p^i_l(h^i_l)} s^i_l + n^i_l, \qquad \text{for} \quad i = 1, ..., N \qquad \text{and} \qquad l = 1, ..., L \quad (3.1)$$

where:

- the sub-band index i lies from 1 to N,
- the user index l is taken equal to 1 for the primary user and lies from 2 to L for secondary users,
- h^i_l: is the block fading process of user l on the sub-band i,

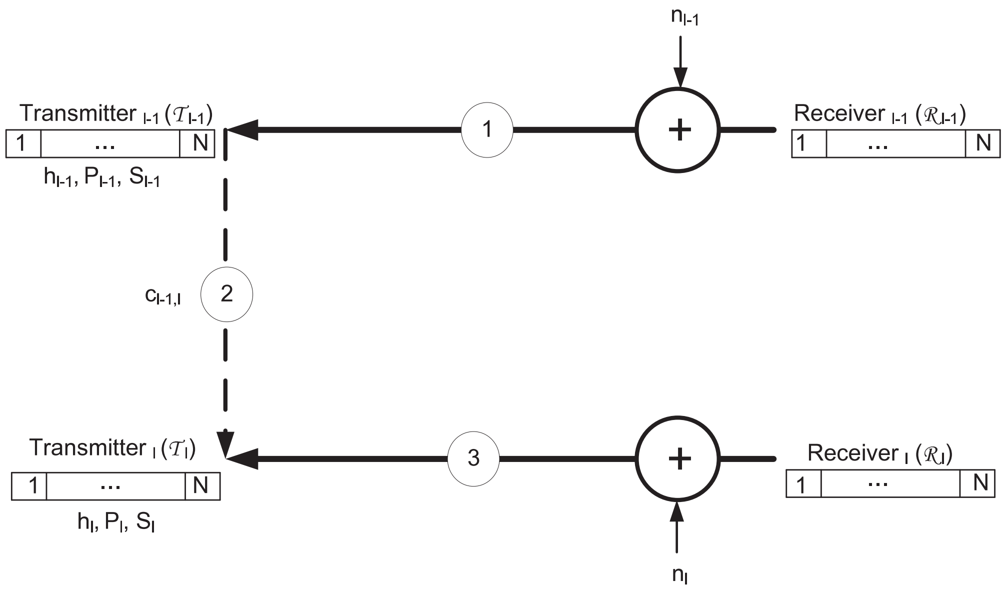

Figure 3.2: The cognitive radio channel in a wideband/multiband context with N sub-bands.

- s_l^i: is the BPSK signal transmitted by user l on the sub-band i ,
- $p_l^i(h_l^i)$: is the power control[1] of user l on the sub-band i,
- n_l^i: is the white additive Gaussian noise at the ith sub-band.

As already considered in the last chapter, we further assume that the channel h_l stays constant over each block fading length (i.e. coherent communication). The assumption of coherent reception is reasonable if the fading is slow in the sense that the receiver is able to track the channel variations. We statistically model the channel gains h_l to be i.i.d distributed over the L Rayleigh fading coefficients and $\mathbb{E}\left\{|h_l|^2\right\} = 1$ for $l = 1, ..., L$. The additive Gaussian noise n_l at the receiver is i.i.d circularly symmetric and $n_l \sim \mathcal{CN}(0, N_0))$ for $l = 1, ..., L$.

[1]Throughout the rest of the chapter, we will find it convenient to denote by p_l^i the power allocation policy of user l on sub-band i, rather than $p_l^i(h_l^i)$.

3.3 The Spectrum pooling Protocol

We consider an asynchronous TDD communication scenario in which one primary system and $L-1$ cognitive users wish to communicate to different receivers, subject to mutual interference. The basic idea under the proposed protocol is quite simple: the cognitive users listen to the wireless channel and determine, either in time or frequency, which part of the spectrum is unused. Then, they *successively* adapt their signal to fill detected voids in the spectrum domain. Each transmitter $\mathfrak{T}_l$ for $l = 1, ..., L$ estimates the pilot sequence of the receiver $\mathfrak{R}_l$ in order to determine the channel gain h_l (see links (1) and (3) in Fig. 3.2). Notice here that since we are in a TDD mode, when we estimate the channel in one way, we can also know it the other way. Thus, each user l is assumed to know only his own channel gain h_l and the statistical properties of the other links (probability distribution). We further assume that the channel does not change from the instant of estimation to the instant of transmission.
A particularly noteworthy target in this context, when we employ a "*listen-before-talk*" strategy, is to reliably detect the sub-bands that are currently accessed by a specified user in order to be spared from the coming users transmission. This knowledge can be obtained from two manners: In a centralized mode where the proposed system would require information from a third party (i.e. central database maintained by regulator or another authorized entity) to schedule users coming. Alternatively, an extra signalling channel is dedicated to perform the collision detection so that cognitive users will not transmit at the same moment. Specifically, the primary user comes first in the system and estimates his channel gain. The second user comes in the system randomly, for instance in a Poisson process manner, and estimates his channel link. Such an assumption could be further justified by the fact that in an asynchronous context, the probability that two users decide to transmit at the same moment is negligible as the number of users is limited. Thus, within this setting, the primary user is assumed not to be aware of the cognitive users. It communicates with his receiver in an ad-hoc manner while a set of spectrum pooling transmitters that are able to reliably sense the spectral environment over a wide bandwidth, decide to communicate with theirs respective receivers only if the communication does not interfere with the primary user. Accordingly, under our opportunistic approach, a device transmits over a certain sub-band only when no other user does. Such an assumption is motivated by the fact that when $\mathfrak{R}_l$ sends his pilot sequence to $\mathfrak{T}_l$, it will not interfere with $\mathfrak{T}_{l-1}$ for $l = 2, ..., L$. The sensing operation will be discussed in the next section. Throughout

the rest of the chapter, we will adopt this framework to analyze the achievable performance of such a system in terms of spectral efficiency as well as the maximum number of possible pairwise communication within this scenario. Such an accurate and simple system modeling presents a key to understand the actual benefits brought by spectrum pooling technology. In fact, although cognitive radios have spurred great interest and excitement in industry, many of the fundamental theoretical questions on the limits of such technologies remain unanswered.
Moreover, in order to characterize the achievable performance limit of such systems, three capacity measures can be found in the literature. A comprehensive review of these concepts can be found in [17]. The relevant performance metric of the proposed protocol is the instantaneous capacity per sub-band in bits/s/Hz, also called *spectral efficiency*, namely [23]:

$$C_l = \frac{1}{N} \sum_{i=1}^{N} \log_2 \left(1 + \frac{p_l^i \mid h_l^i \mid^2}{N_0} \right); \qquad l = 1, ..., L \tag{3.2}$$

The sum here is done over the stationary instantaneous distribution of the fading channel on each user l. The instantaneous capacity determines the maximum achievable rate over all fading states without a delay constraint. In this work, we allocate transmit powers for each user (over a total power budget constraint) in order to maximize its transmission rate. In fact, when channel state information is made available at the transmitters, users know their own channel gains and thus they will adapt their transmission strategy relative to this knowledge. The corresponding optimum power allocation is the well-known *water filling* allocation [2] expressed by[2]:

$$p_l^i = \left(\frac{1}{\gamma_0} - \frac{N_0}{\left| h_l^i \right|^2} \right)^+ \tag{3.3}$$

where γ_0 is the Lagrange's multiplier satisfying the average power constraint per sub-band:

$$\frac{1}{N} \sum_{i=1}^{N} p_l^i = \overline{P} \tag{3.4}$$

Without loss of generality, throughout the rest of the chapter, we take $\overline{P} = 1$. The water-filling strategy affords a significant performance gain over the constant-power strategy at low SNR. The intuition is that when there is

[2] $(x)^+ = \max(0, x)$.

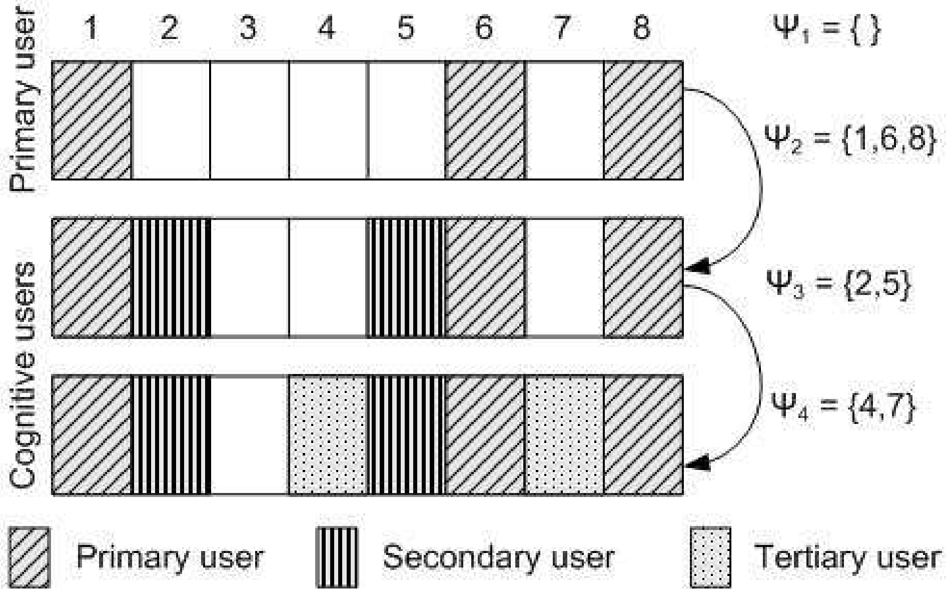

Figure 3.3: One primary user and two cognitive users in a spectrum pooling system with 8 sub-bands.

little transmit power, it is much more effective to expend it on the strongest sub-channel of the system rather than spread the power evenly across all modes. On the contrary, in high SNR regime the transmitter tends to spread its power among all its available sub-bands. Thus, in the first case the opportunistic link has plenty of free dimensions, while in the second one, it is effectively limited. This power allocation behavior has been also reported in [28] and [29]. Notice that, although a water filling power allocation strategy is adopted in this analysis, we emphasize that this is not a restriction of the proposed protocol. In fact, as mentioned before, one important task when implementing spectrum pooling is that cognitive users operate on the idle sub-bands of the licensed system delivering a binary channel assignment as shown in Fig. 3.3. Hence, our study is valid for any binary power control without resorting to the restriction assumption of successive water filling. For clarity sake, let us take the following example with $N = 8$ sub-bands. As shown in Figure 3.3, the primary user is always prioritized above cognitive users by enjoying the entire band while cognitive users adapt their signal to fill detected voids with respect to their order of priority. As a first step, the primary user maximizes its rate according to its channel process. As mentioned before in expression (3.3), only user with a channel gain h^i above a certain threshold equal to $\gamma_0.N_0$ transmits on the sub-band i (Ψ_2). User 2, comes in the system randomly, senses the spectrum and decides to transmit only on sub-bands sensed idle. Thus, following its fading gains, user 2 adapts its signal to fill these voids in the spectrum domain in a complementary fashion (Ψ_3). Similarly, user 3 will sense the remaining sub-bands from user 1 and user 2 and decides to transmit during the remaining voids (Ψ_4).

3.4 Sensing issue

So far, we have focused on pairwise communications between transmitters and receivers (see links 1 and 3 in Fig. 3.2). Let us now investigate the *inter-transmitter* communications (link 2 in Fig. 3.2) in order to analyze the problem of sensing. To this effect, let us assume the baseband discrete-time model within a coherence time period T when each user l for $l = 2, ..., L$ has N sub-bands as described in Figure 3.2:

$$y_l^i(k) = c_{l-1,l}^i(k)\sqrt{p_{l-1}^i(h_{l-1}^i)}s_{l-1}^i(k) + n_{l-1}^i(k), \tag{3.5}$$

where $c_{l-1,l}^i(k)$ is the block fading process from user $l-1$ to user l on the ith sub-band, at time k. We further assume that $0 \leq k \ll \beta T$ and $\beta < 1$, i.e. the coherence time is sufficiently large so that the channel stays constant for samples and jumps to a new independent value (block-fading model).
The proposed sensing techniques hinge on the assumption that all devices operate under a unique standard so that they know the pilot sequence used by the other users. As stated above, in this work, the spectrum pooling behavior is assumed to allow only one user to simultaneously transmit over the same sub-band. The received signal at user l can therefore be written as (see link 2 in Fig. 3.2):

$$y_l^i(k) = \begin{cases} \underbrace{c_{l-1,l}^i(k)\sqrt{p_{l-1}^i}s_{l-1}^i(k)}_{\text{signal}} + \underbrace{n_{l-1}^i(k)}_{\text{noise}}, & \text{if} \quad p_{l-1}^i \neq 0 \\ n_{l-1}^i(k), & \text{otherwise} \end{cases} \tag{3.6}$$

By assuming that βT is an integer equal to M and by making βT sufficiently large, the mean received power over the detection duration at receiver $\mathfrak{R}_l$ is:

$$\lim_{M\to\infty} \frac{1}{M}\sum_{k=1}^{M} \left|y_l^i(k)\right|^2 = \begin{cases} \left|c_{l-1,l}^i\right|^2 p_{l-1}^i + N_0, & \text{if} \quad p_{l-1}^i \neq 0 \\ N_0, & \text{otherwise} \end{cases} \tag{3.7}$$

Accordingly, in order to determine which part of the spectrum is unused, cognitive user has just to detect the received power and compare it to the noise power N_0. However, in addition to the fact that it supposes that $M \to \infty$ (i.e. infinite time coherence period), the proposed method would be not efficient at low SNR-regime (see Figure 3.4). In fact, the quality of such a technique is strongly degraded with the reduction in the precision of the noise threshold [30][31]. The principal difficulty of this detection is to obtain a good estimation of the noise variance. In the setting of spectrum pooling mechanism, we would need a channel sensing method that continuously senses the channel. Thus, the channel sensing should be performed with a very high probability of correct detection (to assure very low probability of interference with the primary system). Weiss *et al.* proposed in [32] a distributed spectrum pooling protocol where all the nodes participate in channel sensing so that all cognitive users perform detection. Moreover, formulas for the calculation of the detection and false alarm probability in a spectrum pooling system have been derived in [33] for the general case of an arbitrary primary systems covariance matrix.

3.5 Spectral efficiency analysis

Let us first define the set of the number of sub-bands sensed occupied by user l by:

$$\Psi_l = \left\{ i \in \{1, ..., N\};\; p_{l-1}^i \neq 0 \right\} \tag{3.8}$$

where Ψ_l obeys to the following properties:

$$\begin{cases} \Psi_1 = \emptyset, \\ \bigcup_{l=1}^{L+1} \Psi_l \subseteq \{1, ..., N\}, \\ \bigcap_{l=1}^{L+1} \Psi_l = \emptyset \end{cases} \tag{3.9}$$

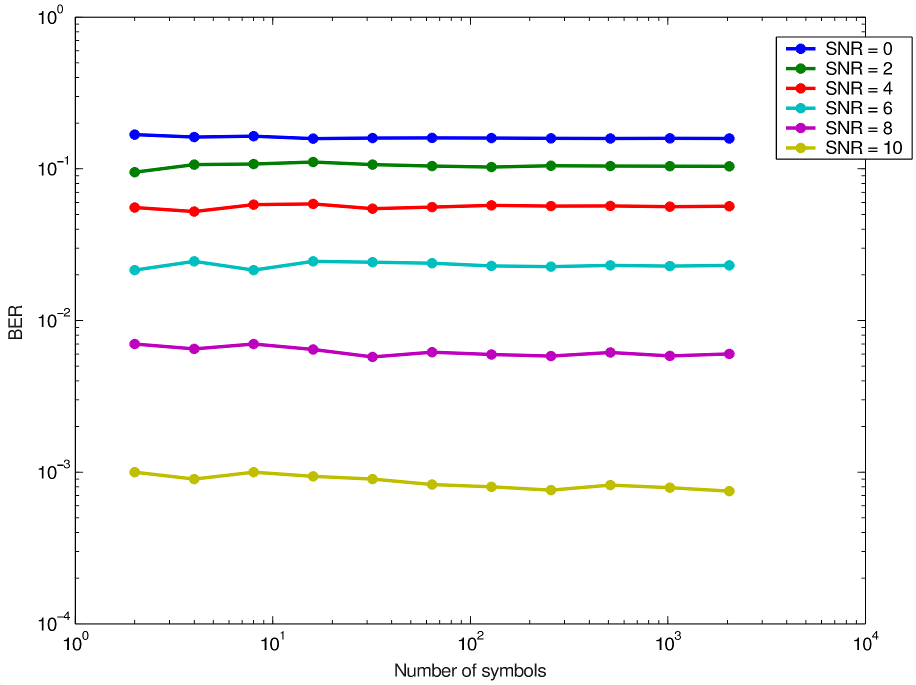

Figure 3.4: BER v.s number of symbols (M) in dB for BPSK in AWGN using power detection where SNR are in dB.

The spectral efficiency per sub-band of user l, given a number of sub-bands N, is:

$$C_{l,N} = \frac{1}{card(\Omega_l)} \sum_{i\in\Omega_l} \log_2\left(1+\frac{p_l^i \mid h_l^i \mid^2}{N_0}\right) \qquad bits/s/Hz \tag{3.10}$$

where Ω_l represents the set of the remaining idle sub-bands sensed by user l, namely:

$$\Omega_l = \left\{ i \in \{1,...,N\} \bigcap \overline{\bigcup_{k=1...l} \Psi_k} \right\} \tag{3.11}$$

For a given number of sub-bands N, the optimal power allocation which maximizes the transmission rate of user l is the solution to the following optimization problem:

$$\max_{p_l^1,...,p_l^{card(\Omega_l)}} C_{l,N}, \qquad \text{for} \quad l = 1,...,L$$

subject to the average power constraint per sub-band:

$$\begin{cases} \dfrac{1}{card(\Omega_l)} \displaystyle\sum_{i\in\Omega_l} p_l^i = 1, \\ \\ p_l^i \geq 0, \end{cases} \tag{3.12}$$

The resulting optimal power control policy is given by (3.3). Notice that the maximum number of users L allowed by such a system must satisfy the condition that $card(\Omega_L) \neq 0$.
Let us now derive the spectral efficiency of such a system. The spectral efficiency per band of user l is given by:

$$\Phi_{l,N} = \frac{1}{N} \cdot \sum_{i\in\Omega_l} \log_2\left(1+\frac{p_l^i \mid h_l^i \mid^2}{N_0}\right) \tag{3.13}$$

By multiplying and dividing (3.13) by $card(\Omega_l)$, we obtain[3]:

$$\Phi_{l,N} = \frac{card(\Omega_l)}{N}.C_{l,N}, \qquad \text{for} \quad l = 1,...,L \tag{3.14}$$

[3]Notice that since the primary user enjoys the entire bandwidth, we have: $card(\Omega_1) = N$.

As expected, when $l = 1$, the spectral efficiency without cognition is equal to the primary user spectral efficiency $C_{1,N}$. We define $\Delta_{l,N}$ as the band factor gain of user l for N sub-bands, namely:

$$\Delta_{l,N} \triangleq \frac{card(\Omega_l)}{N}, \qquad \text{for} \quad l = 1, ..., L \tag{3.15}$$

In other words, the band factor gain represents the fraction of the band unoccupied at user l. The spectral efficiency per band of user l can therefore be expressed by:

$$\Phi_{l,N} = \Delta_{l,N} \cdot C_{l,N}, \qquad \text{for} \quad l = 1, ..., L \tag{3.16}$$

and the sum spectral efficiency of a system with N sub-bands per user is given by:

$$\Phi_{sum,N} = \sum_{l=1}^{L} \Phi_{l,N} \tag{3.17}$$

3.6 Asymptotic Performance

Let us now study the achievable performance when devices operate in a wide-band context (i.e. $N \to \infty$). The spectral efficiency of user l for a large number of sub-bands in (3.10) becomes:

$$C_{l,\infty} = \int_0^\infty \log_2\left(1 + \frac{p_l(t) \cdot t}{N_0}\right) \cdot f(t)dt, \quad \text{for} \quad l = 1, ..., L \tag{3.18}$$

where p_l is subject to the average constraint:

$$\int_0^\infty p_l(t) \cdot f(t)dt = 1 \tag{3.19}$$

Although this is not a restriction of our approach, from now on we assume that the channel gains are i.i.d Rayleigh distributed. However, all theoretical results as well as the methodology adopted in this chapter can be translated immediately into results for any other probability distribution function of the channel model. In this way, the term $f(t)$ in (7.16) will be replaced by the appropriate probability distribution function. The spectral efficiency of user l for i.i.d Rayleigh fading is given by:

$$C_{l,\infty} = \int_0^\infty \log_2\left(1 + \frac{p_l(t) \cdot t}{N_0}\right) \cdot e^{-t}dt, \quad \text{for} \quad l = 1, ..., L \tag{3.20}$$

where p_l is subject to the average constraint:

$$\int_0^{\infty} p_l(t) \cdot e^{-t} dt = 1 \tag{3.21}$$

and γ_0 is the Lagrange's multiplier satisfying[4]:

$$\frac{1}{\gamma_0} \int_{\gamma_0 \cdot N_0}^{+\infty} e^{-t} dt - N_0 \cdot E_i\left(\gamma_0 \cdot N_0\right) = 1 \tag{3.22}$$

Numerical root finding is needed to determine different values of γ_0. Our numerical results, in section , show that γ_0 increases as N_0 decreases, and γ_0 always lies in the interval [0,1]. On the other hand, an asymptotic expansion of (7.17) in [34] shows that at very high SNR-regime, $\gamma_0 \rightarrow 1$.
Moreover, the spectral efficiency of user l can be computed for $l = 1, ..., L$ as follows:

$$\begin{aligned} C_{l,\infty} &= \int_0^{\infty} \log_2\left(1 + \frac{p_l(t) \cdot t}{N_0}\right) \cdot e^{-t} dt \qquad (3.23) \\ &= \int_{\gamma_0 N_0}^{\infty} \log_2\left(1 + \frac{\left(\frac{1}{\gamma_0} - \frac{N_0}{t}\right) \cdot t}{N_0}\right) \cdot e^{-t} dt \\ &= \int_{\gamma_0 N_0}^{\infty} \log_2\left(\frac{t}{\gamma_0 \cdot N_0}\right) \cdot e^{-t} dt \\ &= \frac{1}{\ln(2)} \cdot E_i\left(\gamma_0 \cdot N_0\right) \end{aligned}$$

In order to characterize the achievable performance of such system in terms of spectral efficiency, we define the spectral efficiency within the frequency bandwidth W, as [35]:

$$C_{l,\infty}(W) = \frac{1}{W} \int_{\frac{-W}{2}}^{\frac{W}{2}} \log_2\left(1 + \frac{p_l(f).\left|H_l(f)\right|^2}{N_0}\right) df \tag{3.24}$$

By identifying expression (3.20) with (3.24), we obtain a characterization of the frequency variation f as function of the channel gains t, namely:

$$f = -W \cdot e^{-t} + \frac{W}{2}, \tag{3.25}$$

[4] $E_i(x)$ is the exponential integral function defined as: $E_i(x) = \int_x^{+\infty} \frac{e^{-t}}{t} dt$.

Similar to our approach in the previous section, we define the band factor gain Δ_∞ as the fraction of the band sensed idle from user l to user $l+1$ over the total bandwidth W for an infinite number of sub-bands:

$$\Delta_\infty \triangleq \frac{\Delta f}{W} \tag{3.26}$$

where Δf represents the frequency interval where the fading gain in (3.25) is below a certain threshold equal to $\gamma_0 \cdot N_0$. By deriving the appropriate vacant band Δf when $t \in [0, \gamma_0 \cdot N_0]$ in (3.25), we obtain:

$$\Delta_\infty = 1 - exp\left(-\gamma_0 \cdot N_0\right) \tag{3.27}$$

Accordingly, the asymptotic spectral efficiency of user l is given by:

$$\Phi_{l,\infty} = \Delta_\infty \cdot C_{l,\infty}, \qquad \text{for} \quad l = 1, ..., L \tag{3.28}$$

Similar to the case where the number of sub-bands is fixed, when $l = 1$, the spectral efficiency without cognition is equal to the primary user spectral efficiency $C_{1,\infty}$. In particular, it is of major interest to quantify the spectral efficiency gain Δ_∞ in order to show the interest behind using spectrum pooling terminals with respect to classical systems (without cognition). To do so, following the same procedure and going from user 2 to L, we obtain the expression of the asymptotic spectral efficiency as function of $C_{1,\infty}$:

$$\Phi_{l,\infty} = \Delta_\infty^{l-1}.C_{1,\infty}, \qquad \text{for} \quad l = 1, ..., L \tag{3.29}$$

The overall asymptotic sum spectral efficiency for a system with L users is therefore:

$$\begin{aligned} \Phi_{sum,\infty} &= \sum_{l=1}^{L} \Phi_{l,\infty} \\ &= \sum_{k=0}^{L-1} \Delta_\infty^k C_{1,\infty} \\ &= \underbrace{\frac{1-\Delta_\infty^L}{1-\Delta_\infty}}_{\geq 1} \cdot C_{1,\infty} \end{aligned} \tag{3.30}$$

Thus, the sum spectral efficiency obtained by considering cognitive communications is greater than or equal to the spectral efficiency without cognition

$C_{1,\infty}$. Such a result, rather intuitive, justifies the increasing interest behind using cognitive radio terminals in future wireless communication systems since the sum spectral efficiency of such systems performs always better than classical communication systems (without cognition).
On the other hand, by substituting $C_{1,\infty}$ by its expression in (3.23), we obtain the final expression of the achievable sum spectral efficiency in such a system:

$$\Phi_{sum,\infty} = \frac{1}{\ln(2)} \cdot \frac{1-\Delta_\infty^L}{1-\Delta_\infty} \cdot E_i\left(\gamma_0 \cdot N_0\right) \tag{3.31}$$

This result is very interesting as, by only knowing the statistics of the channel gains (through γ_0) and the SNR (through N_0), one can derive the achievable spectral efficiency as well as the potential gain resulting from using spectrum pooling.

3.7 Performance evaluation

In order to validate our approach in the previous Section, we compare the theoretical expression of the sum spectral efficiency in (3.31) to expression in (3.17). We model L i.i.d Rayleigh channels (one for each user) and assume perfect sensing of the idle-sub-bands. Our numerical result in Figure 3.5, tends to validate the asymptotic analysis we adopt throughout the chapter. It clearly shows that the sum spectral efficiency in (3.17) matches expression (3.31) even for a moderate number of sub-bands N (from $N = 16$). Moreover, since the maximum number of users is not theoretically limited, we will consider only L that satisfies the condition that $card(\Omega_L) \neq 0$, otherwise, the L-th spectral efficiency would be negligible. Figure 3.6 characterizes the maximum number of users L as function of the received signal energy per information bit E_b/N_0 for different number of sub-bands N. As expected, we remark that the maximum number of users allowed to transmit increases with the number of sub-bands especially at low E_b/N_0 region. Furthermore, the maximum number of cognitive users ranges from 1 to 8. As an example, the proposed scheme, although its simplicity allows up to 4 cognitive users to benefit from the licensed spectrum at 8 dB for $N = 2048$ sub-bands.

In [36], we analyzed the different configurations of the sum spectral efficiency for a system with 5 users as function of the SNR. We showed that at low SNR region, the spectral efficiency is significantly increased with respect to the traditional system without cognition while, at high SNR regime, the maximum sum spectral efficiency reaches $C_{1,\infty}$. In this chapter however, we

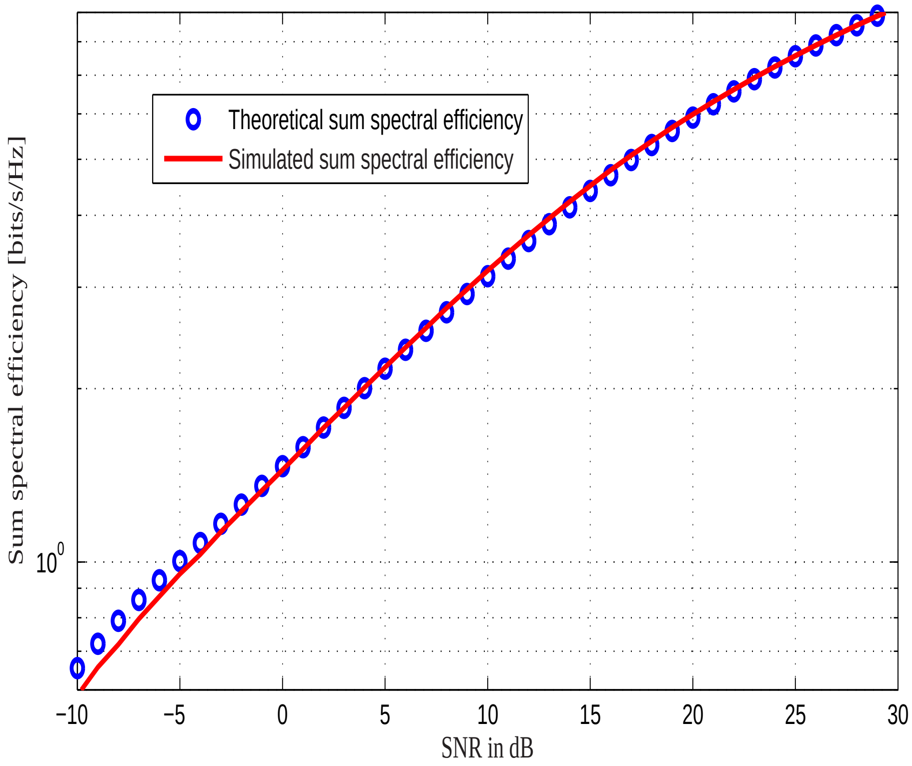

Figure 3.5: Comparison between theoretical expression of the sum spectral efficiency in (3.31) and simulated one in (3.17) for $L = 5$ and N=16.

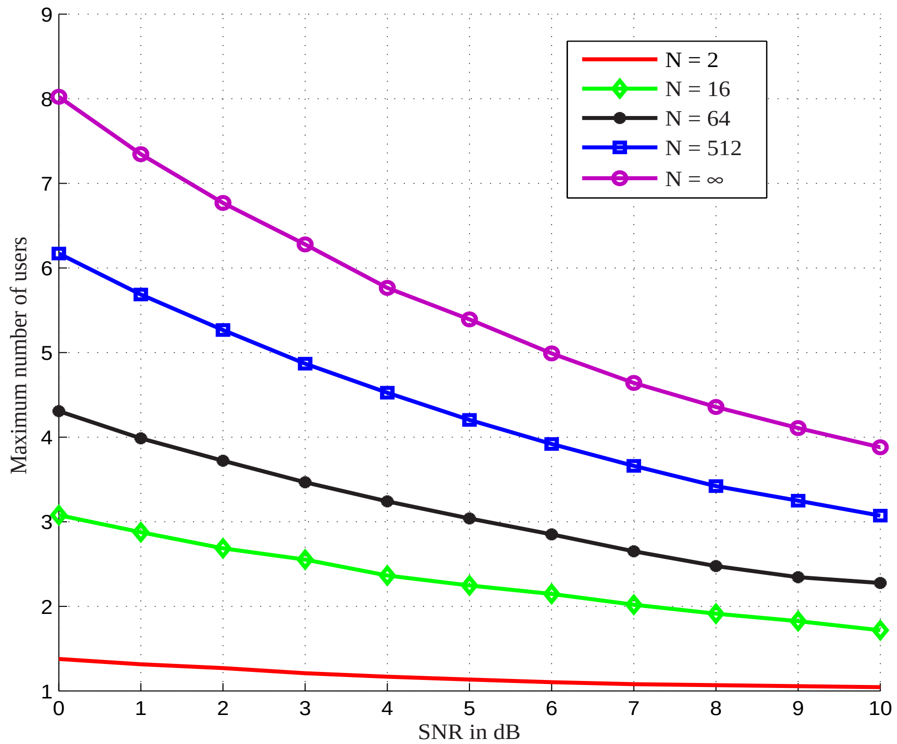

Figure 3.6: The maximum number of users for different number of sub-bands (N).

will focus on the sum spectral efficiency gains as function of E_b/N_0. In fact, the E_b/N_0 versus spectral efficiency characteristic is of primary importance in the study of the behavior of the required power in the wideband limit (where the spectral efficiency is small). The key idea behind doing so is to find the best tradeoff between transmitted energy per information bit and spectral efficiency [35]. It is also useful for the sake of comparing results obtained for different configurations to represent the fundamental limits in terms of received energy per information bit rather than the Signal-to-noise ratio. By replacing the SNR in (3.23) by its equivalent expression in terms of E_b/N_0, the spectral efficiency of the primary user becomes:

$$C_{1,\infty} = \frac{1}{\ln(2)} \cdot E_i \left(\frac{\gamma_0}{\frac{E_b}{N_0} \cdot C_{1,\infty}} \right) \tag{3.32}$$

In such a case, the explicit solution of the spectral efficiency versus E_b/N_0 is not feasible. In Figure 3.7, we plot the sum spectral efficiency gains (with respect to the configuration where only the primary user enjoys the entire band) as function of E_b/N_0 where solutions are given by the implicit equation in (3.32). The goal here is rather to quantify the spectrum pooling spectral efficiency gain from user to user. Simulation results were obtained through dichotomic algorithms in Figure 3.7. We found out that the maximum spectral efficiency gain can not exceed the range of 60% for a configuration with one primary user and 4 cognitive users. Notice that, as E_b/N_0 increases, all the configurations tend towards the configuration where only the primary user enjoys the entire band. This can be justified by the fact that, at high E_b/N_0 regime, the water-level $\frac{1}{\gamma_0}$ is becoming greater than the quantity $\frac{N_0}{|h|^2}$ and more power is poured within each sub-band (see equation(3.3)).

To proceed further with the analysis, we resort to performance comparison of the proposed scheme with respect to a traditional system where no cognition is used. As far as sum spectral efficiency comparison is concerned, this can be conducted by considering the two following configurations:

- *the non-cognitive radio configuration (NCR)*: where the primary user enjoys the entire bandwidth following an average power constraint per sub-band given by:

$$\frac{1}{N} \sum_{i=1}^{N} p_l^i = L \cdot \overline{P} \tag{3.33}$$

  where L is the maximum number of users at each SNR (as shown in

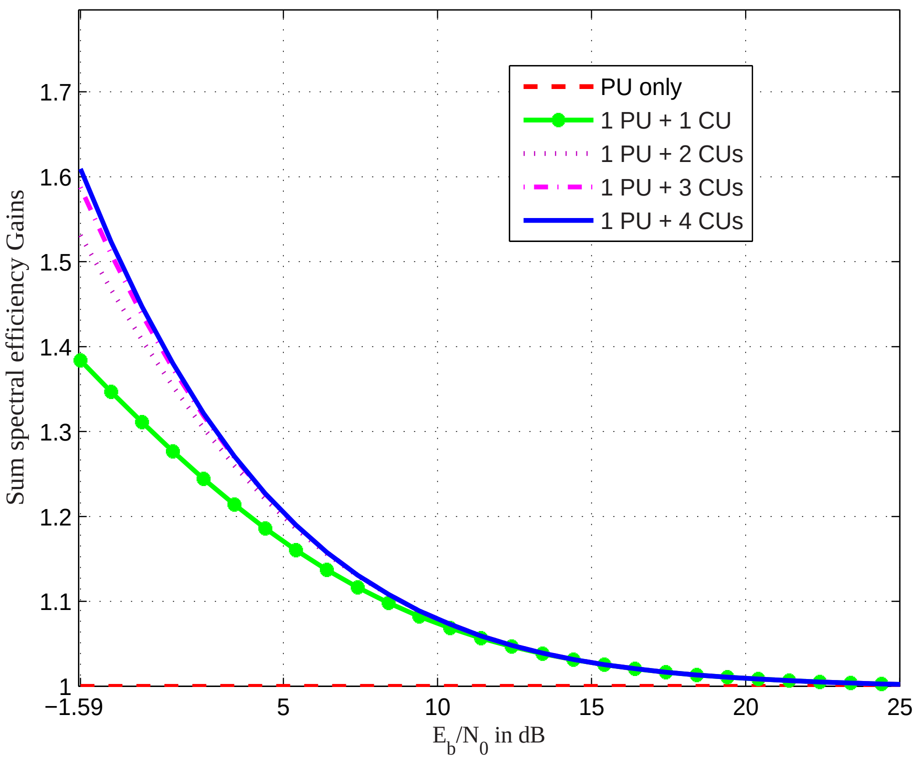

Figure 3.7: Sum spectral efficiency gains of the system with one primary user and 4 cognitive users (CU).

Fig. 3.6). The primary user can accordingly distribute $(N \cdot L \cdot \overline{P})$ over the N sub-bands in order to maximize its capacity,

- *the cognitive configuration*: where $(L-1)$ cognitive users coexist with the primary user while sharing the N sub-bands available. Each user has to maximize its capacity with respect to the average power constraint per band of $(card(\Omega_l) \cdot \overline{P})$ as in (4.2).

Figure 3.8 validates the expectation from the analysis in (3.30). It clearly shows that the spectrum pooling strategy performs always better than traditional communication system using the same spectral resources due to the multi-user diversity gain. In particular, the spectrum pooling system achieves 1 bit per second per hertz more than the NCR system. Let us now focus on the band factor gains expressions. So far, we have quantified the spectral efficiency gains of different configurations with five users. Let us now investigate how the simulated spectral efficiency gain (with a finite N) converges to the theoretical one (when N is assumed to be infinite). Let us first write the spectral efficiency of each user l as follows:

$$\Phi_{l,\infty} = \alpha_{l,\infty} \cdot C_{1,\infty}, \qquad \text{for} \quad l = 1, ..., L \tag{3.34}$$

where:

$$\alpha_{l,\infty} = \Delta_{\infty}^{l-1}, \qquad \text{for} \quad l = 1, ..., L \tag{3.35}$$

Note here that $\alpha_{l,\infty}$ represents the band factor gain from the primary user to user l. In Figure 3.9, numerical simulation is carried out by considering a system with four cognitive users. We compared simulated values of $\alpha_{l,N}$ based on equation (3.14) to theoretical values in (3.35) for each user l and for SNR = 10 dB. We remark that as N increases, the simulated band factor gain tends to $\alpha_{l,\infty}$. Moreover, simulation results show that $\alpha_{2,N}$ converges more rapidly to the associated theoretical gain factor value than for user 3 or user 4.

3.8 Conclusion

In this chapter, we have considered orthogonal communications in a generic spectrum pooling scenario enabling public access to the new spectral ranges without sacrificing the transmission quality of the actual license owners. For the first time, our analysis has quantified the achievable gain of using spectrum pooling with respect to classical radio devices. We found out that though its simplicity, the proposed scheme is effective to provide a

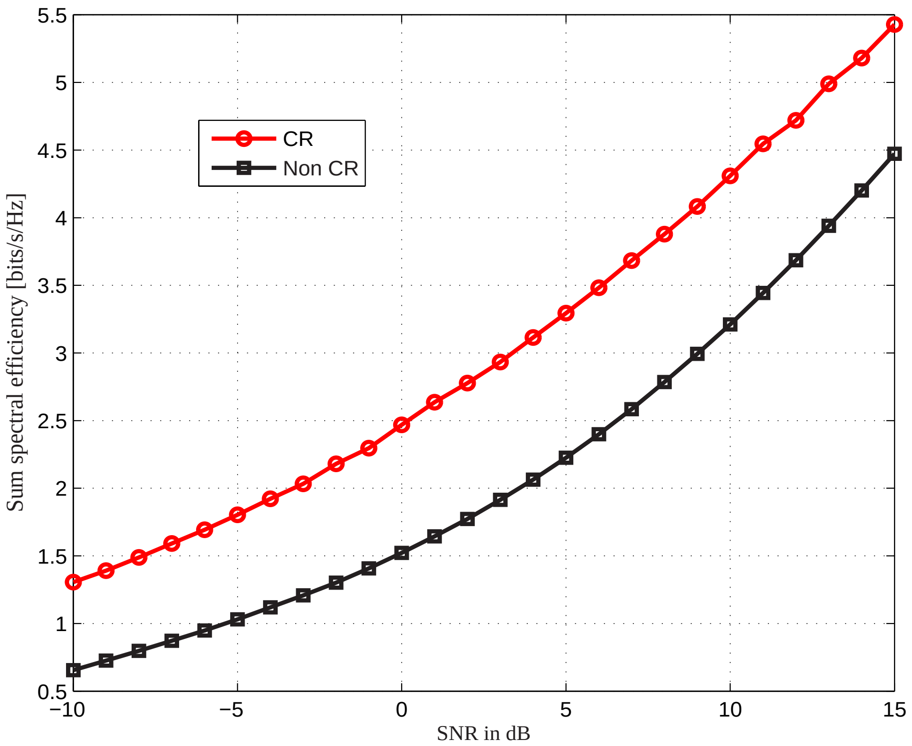

Figure 3.8: Sum spectral efficiency of a system using cognitive radio (CR) and a traditional system (Non CR) for $N = 512$.

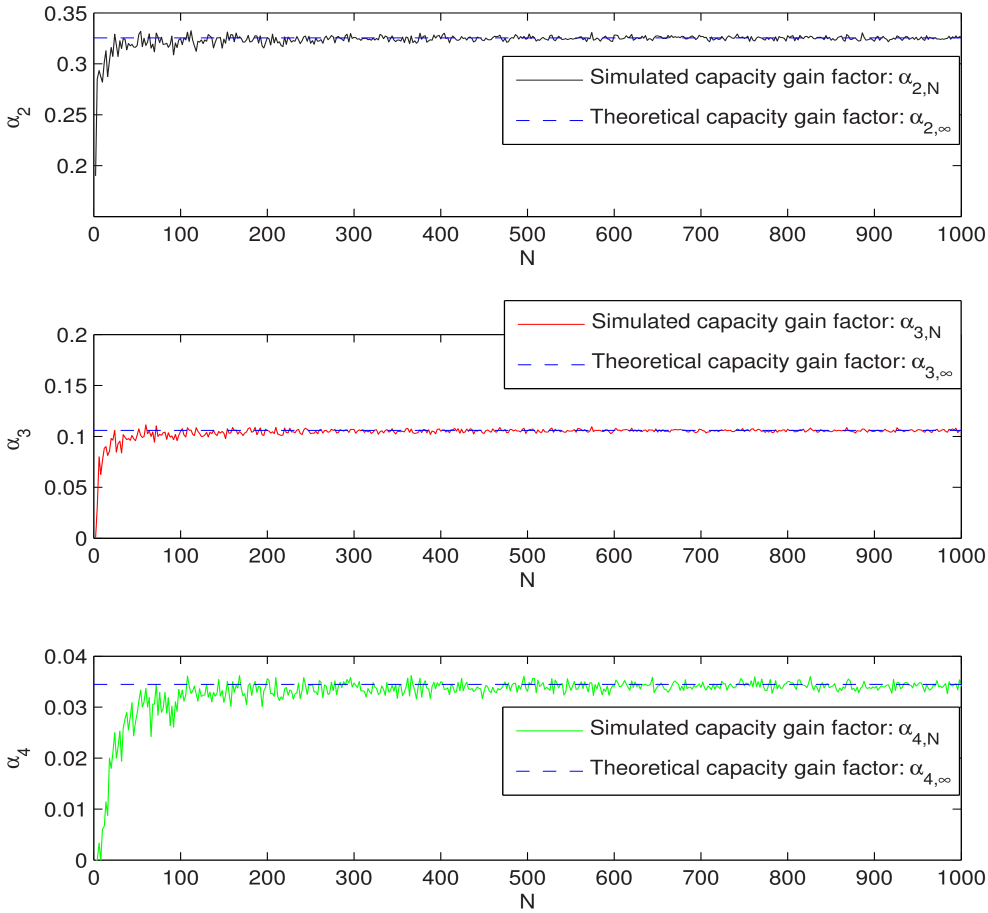

Figure 3.9: Convergence of band factor gains at SNR = 0 dB.

higher spectral efficiency gain than the classical scheme does. We further obtained a characterization of the achievable spectral efficiency as well as the maximum number of possible pairwise communications within such a scenario. Simulation results validate our theoretical claims and offer insights into how much one can gain from spectrum pooling in terms of spectral efficiency. As a future work, it is of major interest to generalize the problem to limited feedback in order to characterize the sum spectral efficiency gain of such cognitive protocols with respect to the proposed scenario. It would be further interesting to measure the throughput of the proposed protocol given a realistic primary system model (e.g., ethernet traffic) compared to an OFDM/TDD overlay cognitive radio system.

Chapter 4

Cognitive Radio using Virtual Noise

As we have seen, spectrum pooling promises significant system spectral efficiency gains in cognitive radio networks. However, the approach presented in the previous chapter relies on orthogonal communications between the primary system and cognitive users. In this chapter, we consider a multi-band/wideband system with two users, one primary (licensed) user and one secondary (cognitive) user attempting to communicate simultaneously in the same frequency band subject to mutual interference. At the core lies the idea that the primary user has a certain quality of service to fulfill. This gives the secondary user a transmitting opportunity since the primary user will not, in any case, use all its rate as long as it has its quality of service satisfied. We determine, under the assumption that the primary user is oblivious to the presence of the cognitive user, the optimal power allocation policy for such a system. The proposed strategy is proved to be the optimal one that achieves the maximum rate for both users under the constraint that the secondary user guarantees a quality of service for the primary user. We explicitly derive the capacity of the primary as well as the secondary user. Asymptotic analysis shows that (i) the sum system capacity of such a cognitive scenario using a virtual noise threshold as a proxy for the primary user performs always better than classical communication system where the primary user selfishly maximizes its capacity, and (ii) the sum system capacity is maximized for *a particular value of the virtual noise threshold, while maintaining a quality of service for the primary user.*

4.1 Introduction

Interference in wireless networks is known to hinder reliable communication and ultimately limit the achievable network capacity. Thus, in such environments, the capacity is a direct function of the total interference level seen at any receiver. Modeling interference for such specific scenarios has thus become a critical task and is receiving increasing attention in the literature. In light of the arguments presented above, here we consider a TDD-uplink communication scenario in which the primary and the secondary user wish to communicate, subject to mutual interference in a heterogeneous network where devices operate in a wideband/multiband context. One property of such systems is that, since the same frequency is used, the channel characteristics are nearly the same in both links, provided the channel does not change too rapidly. Under this scheme, we allow the secondary user to transmit simultaneously with the primary user as long as the primary user has not its quality of service affected. In particular, we impose that the secondary user can transmit simultaneously with the primary user as long as the level of interference with the primary user remains lower than a specified virtual noise threshold.

The notion of *virtual noise threshold* is chosen to be a proxy for the primary user to allow cognitive user to profit from the remaining vacant sub-bands and, at at the same time, it maintains a guarantee of service to the primary user when cognitive communication is considered. This strategy hinges on the assumption that, in any case, the primary user will not necessarily need all that rate. In fact, as long as all its target rate is achieved, he does not care about what it leaves unused.
The virtual noise level is in fact initially imposed by the primary system depending on the primary user requested rate. Based on this, the following questions may naturally arise. First, for a given virtual noise level σ_v^2, is the sum system capacity maximized for a particular value of σ_v^2, and how can we design a simple and efficient algorithm to approximate the theoretical optimal virtual noise level, while avoiding a feedback overhead to achieve such an optimization? In particular, is the constraint based on asymptotic assumption still holds for a finite number of sub-bands? Second and particularly noteworthy question in the context of cognitive radio, when we seek

to optimize the sum system capacity, what is the impact of this optimization on the primary user QoS? Will there be an inherent incompatibility between system capacity maximization and primary user QoS guarantee? In other words, does the primary user requested rate considerably degrades and, how much does the primary loose in terms of outage probability? Finally, do different coding strategies adversely affect the achievable throughput of the proposed cognitive radio protocol?

In this chapter, we present two cognitive radio protocols which are believed to be potential promising candidates for future cognitive radio network deployment. We provide analytical and simulated insights towards addressing these important questions. We propose an asymptotic constraint to achieve the maximum possible sum capacity and validate the proposed result with a finite number of sub-bands. In both cases, we address the achievable performance with and without SIC strategy. We determine, under the assumption that the primary user is oblivious to the presence of the cognitive user, the acceptable interference level within a given quality of service. We also give an interesting way to acquire the primary user's side information. The proposed strategy is proved to be the optimal in the sense that it achieves the maximum rate for each of the two users under the constraint that the secondary user maintains a guarantee of service to the primary user when cognitive communication is considered.

In the second part of the chapter, we explore the achievable rate of the primary and the secondary user as well. We derive and compare expressions for the capacity when single user decoding strategy is used and when the two users are jointly decoded using a successive interference cancelation (SIC) scheme. In both configurations, the sum capacity of the proposed cognitive protocol performs always better than classical communication system where the primary user selfishly maximizes its capacity. Simulations show that SIC does not provide a lot of capacity gain with respect to the case where only single user decoding is used. Our analysis treats the case where devices operate on a finite number of sub-bands and when the number of sub-bands grows large (wideband context). In the latter case, we show that the sum system capacity is maximized for a particular value of the virtual noise threshold.

The last part of the chapter is dedicated to the design of the virtual noise level. Our study offers some insights into how to characterize the virtual noise in real cognitive radio network by taking into account the primary system QoS guarantee via outage constraint consideration. Simulations results validate our theoretical claims and show that the empirical constraint converges to the optimal condition predicted from our asymptotic theory even at moderate number of sub-bands.

The remainder of the chapter is organized as follows: In section 4.2, we introduce the notion of the virtual noise threshold. We also describes the cognitive radio scenario and give a simple way for the secondary user to acquire the primary user's side information. Section 4.3 details the optimal power allocation policy for each user by considering two different coding schemes. We also determine what would be the average rate of the primary and the secondary user depending on the characteristic of the coding scheme used. In Section 4.4, we study the asymptotic behavior of the capacity as the number of sub-bands grows large. Optimum virtual noise threshold design as well as QoS issues are addressed in Section 4.5. Simulation results are provided in Section 4.6. Section 4.7 concludes the chapter.

4.2 The Cognitive Radio Protocol

Spectrum utilization can be improved by making a secondary user to access a spectrum hole unoccupied by the primary user at the right location and the right time. In current cognitive radio protocol proposals, the device listens to the wireless channel and determines, either in time or frequency, which part of the spectrum is unused. It then adapts its signal to fill this void in the spectrum or time domain. Thus, a device transmits over a certain frequency band or time only when no other user does, like in [14]. In the same context, authors showed in [36] how we can improve the overall system spectral efficiency on classical approaches by considering orthogonal cognitive communications if there is ordering in the transmission. The contribution of some recent studies [37] and [25] has extended the cognitive protocol to allow the cognitive users to transmit *simultaneously* with the primary users in the same frequency band. Similarly, in [38], a linear precoder based on Vandermonde matrices allows an OFDM radio to co-exist with similar pre-existing terminals without generating any additional interference. The idea is to exploit the redundancy of the OFDM cyclic prefix and frequency selectivity of the channel. This is exactly the question tackled in this work where the cognitive radio behavior is generalized to allow the secondary user to transmit simultaneously with the primary user as long as the level of interference with the primary user remains within an acceptable range.

4.2.1 Scenario Description

In traditional systems, when the primary user considers only the ambient noise level σ^2, it will exploit all the resources by water-filling [2] on this noise level and therefore, leaves no resources for the secondary user to transmit. In our case, the primary base station implicity over-estimates the noise level which is considered as thermal noise plus interference. In fact, whether the primary user considers only the ambient noise level σ^2, he will send at the maximum achievable rate and there will be no resources left for the cognitive user. The notion of virtual noise threshold is therefore chosen to be a proxy for the primary user to allow cognitive user to profit from the remaining vacant sub-bands like in [39]. Subsequently, we refer to the above defined over-estimate noise level as the *virtual noise* level. A key idea behind doing so is that, in any case, the primary user will not necessarily need all available rate. In fact, the primary user has a target rate to be achieved (as it has a certain QoS), and as long as that this target rate is satisfied, it does not care about what it leaves more. Accordingly, assuming that the primary user over-estimates the actual thermal noise, two questions are noteworthy in this context: (i) how can the secondary user benefit from that? (ii) in which over-estimation case does the secondary user maximize its rate?

Moreover, contrary to the recent work addressed in [37] and [25], in this contribution, it is proposed that each user is assumed to know only its own channel gain. In [37], the cognitive user is assumed to obtain an *a-priori* knowledge of the information that will be transmitted by the primary user. In [25], authors allow the primary and the secondary systems to *cooperate* and jointly design their encoder-decoder pairs. However, in practice, primary system should be unaware about the existence of the cognitive radio (unlicensed) system and operates according to the demands of the population of primary terminals. This implies that it is the role of cognitive radios to recognize their communication environment and adapt the parameters of their communication scheme to maximize the QoS for the secondary users, while minimizing the interference to the primary users in a *distributive* fashion. Accordingly, the virtual noise threshold has a double role:

- *it allows cognitive user to profit from the primary user resources in an* *opportunistic manner,*

- *it limits the interference to the incumbent (primary) user when cognitive communication is considered.*

We found out that a cognitive user can vary its transmit power in order to maximize its own capacity, while maintaining a guarantee of service to the primary user.
Consider a given virtual noise-threshold σ_v^2, the maximum achievable rate that the primary user can obtain over the N sub-bands is given by:

$$C_{1,N} = \frac{1}{N} \sum_{i=1}^{N} \log_2 \left(1 + \frac{p_1^i \mid h_1^i \mid^2}{\sigma_v^2} \right) \qquad (bits/s/Hz) \tag{4.1}$$

The optimal power allocation which maximizes the transmission rate of the primary user is solution of the optimization problem of:

$$\{p_1^{1*}, ..., p_1^{N*}\} = \arg \max_{p_1^1, ..., p_1^N} C_{1,N},$$

subject to:

$$\begin{cases} \dfrac{1}{N} \displaystyle\sum_{i=1}^{N} p_1^i = \overline{P}, \\ \\ p_1^i \geq 0, \qquad \text{for } \ i = 1, ..., N \end{cases} \tag{4.2}$$

where $\overline{P}$ is the average power constraint. In [2] authors looked at the problem of maximizing instantaneous capacity subject to an average power constraint, and showed that the optimum power allocation follows from Shannon's principle of water-filling, namely [1]:

$$p_1^{i*} = \left(\frac{1}{\gamma_0} - \frac{\sigma_v^2}{\left| h_1^i \right|^2} \right)^+, \qquad i = 1, ..., N \tag{4.3}$$

where γ_0 is the Lagrange's multiplier satisfying the average power in (4.2).

[1] $(x)^+ = \max(0, x)$.

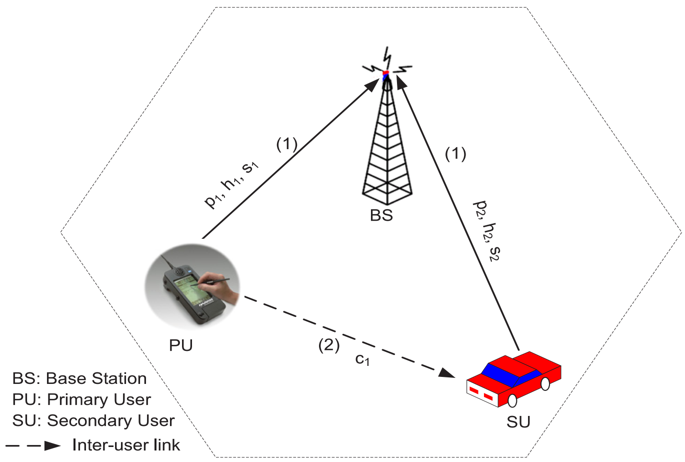

Figure 4.1: The Multiple Access Channel (MAC) cognitive uplink channel in a wideband/multiband context.

Within this chapter, we present two cognitive radio protocols which are believed to be potential promising candidates for future cognitive radio network deployment and offer insights into how to design such scenario in a cognitive radio network environments.

4.2.2 The Multiple Access Channel (MAC)

Let us consider a scenario where the primary and the cognitive users attempt to communicate with a common base station, subject to mutual interference (see Fig. 4.1). A particularly noteworthy target in this context, where the primary user is supposed to be oblivious to the presence of potential secondary users, is how the cognitive user would gather the instantaneous CSI of the primary user without any cooperation with the primary system.
To do so and because there are many channel responses that are proportional to the number of cognitive users in a multi users system, the feedback overhead may be too large and thus reverse-link channel capacity may be greatly wasted. To reduce the feedback information in such cases, we propose a new communication scenario wherein the power allocation policy of the PU, rather than his full channel state information, is fed back to the cognitive user. By feeding back the transmit power assignment, the feedback burden can be greatly reduced. As far as primary channel estimation is concerned, this can be conducted in three steps:

i) Each user estimates the pilot sequence transmitted by the BS in order to determine its channel gain via link (1) in Fig. 4.1. Note here that since we are in a TDD mode, when the users estimate the channel in one way, they can also know it on the reverse link,

ii) In the second frame, the primary user broadcasts a pilot sequence so that the BS estimates the channel h_1 (see link (1) in Fig. 4.1),

iii) In a third step, when the primary user sends its information with power p_1, this information can be overheard by the secondary user and it can estimate the power knowing the inter-user channel c_1^i in link (2). In other words, assuming that the two users operate on a unique standard, the secondary user first estimates the inter-user channel via link (2) and then can learn about the primary user power p_1^i at each sub-band i.

iv) Finally, the secondary user can gather the primary users' channel gains by reverse engineering. In fact, by only knowing the virtual noise threshold (σ_v^2) and the power allocation policy of the primary user (i.e. water-filling), the cognitive user can obtain the primary users' channel gains following equation (4.3). Given a set of power $\left\{p_1^1(h_1^1), p_1^2(h_1^2), ..., p_1^N(h_1^N)\right\}$ and the average power constraint in (4.2), we obtain a system of $(N+1)$ equations in $(N+1)$ unknowns, i.e., $\left\{\gamma_0, h_1 1, ..., h_1^N\right\}$. The steps of the proposed algorithm are described in Box 1. For brevity, only the key steps are detailed.

4.2.3 The interference channel

In a realistic network, such a system coordination may not be practical because it requires large signaling overhead for the feedback and exchange of full channel state information among multiple nodes. To this end, we consider a TDD-uplink communication scenario (see Fig. 4.2) where devices attempt to communicate with their own base station, like in the cognitive *X-channel* widely used in the literature [37] [25]. Accordingly, the primary and the primary and the cognitive users are supposed to communicate to different receivers, subject to mutual interference. Although such a scheme is akin to the general ad-hoc scheme, it can also be viewed as a hierarchical cell structure configuration of a microcell hotspot being operated within a macrocell using the same frequency band like in [13]. Although this is not a restriction of the proposed analysis and for the sake of simplicity,

Algorithm 1 : Primary user channel estimation

initialize $\left\{p_1^1(h_1^1), p_1^2(h_1^2), ..., p_1^N(h_1^N)\right\}$

1: BS broadcasts a preamble which contains σ_v^2 to all users in the system,

2: PU and SU estimate their own channels,

3: PU broadcasts a preamble to the BS overheard by SU,

4: SU estimates the inter-user channel,

5: SU $\leftarrow \left\{p_1^1(h_1^1), p_1^2(h_1^2), ..., p_1^N(h_1^N)\right\}$

6: Given σ_v^2 and the power allocation policy (water-filling), resolve the following system:

$$\begin{cases} \dfrac{1}{N}\displaystyle\sum_{i=1}^{N} p_1^i = \overline{P} \\ \\ p_1^{i*} = \left(\dfrac{1}{\gamma_0} - \dfrac{\sigma_v^2}{\left|h_1^i\right|^2}\right)^+, \qquad \text{for } i = 1, ..., N \end{cases}$$

7: SU $\leftarrow \left\{h_1^1, h_1^2, ..., h_1^N\right\}$

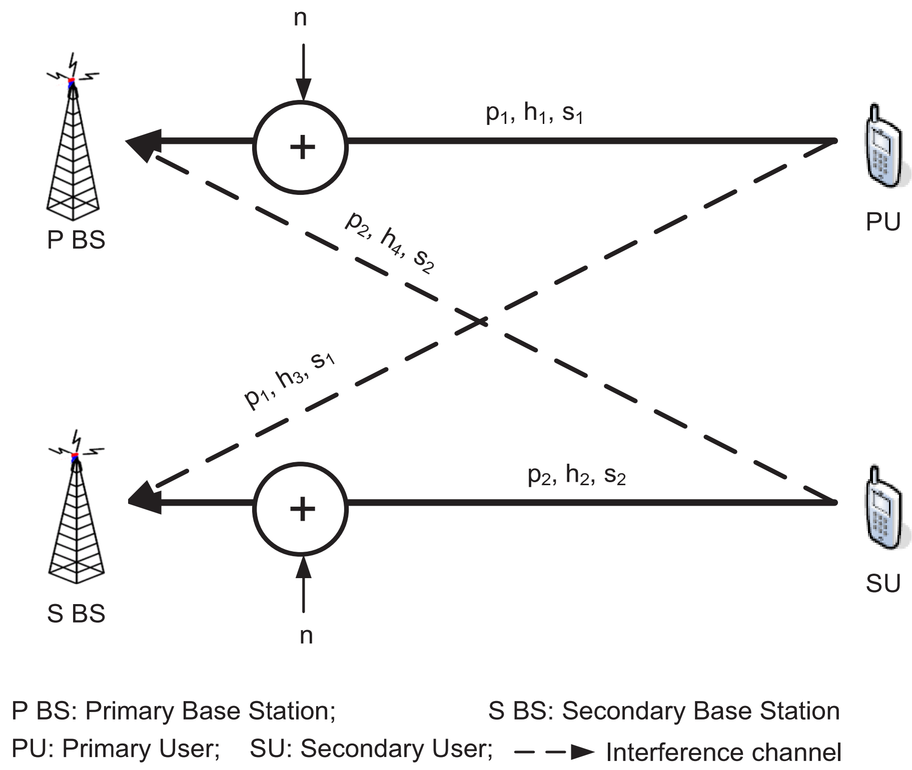

Figure 4.2: The interference cognitive radio uplink channel.

from now on we will treat the MAC framework. We draw the reader's attention however to the fact that the following study still stands for the interference channel model. Typically, the primary interferer contribution h_1, respectively the secondary interferer contribution h_2, is replaced in the interference term by the appropriate channel gain from the primary user to the secondary BS h_3, respectively from the secondary user to the primary BS h_4 (see Fig. 4.2). This knowledge can be gathered by users through communication exchange with the appropriate base stations.
We derive the optimum power allocation policies of each user in terms of maximizing their own capacity and determine what would be the average capacity rate of the primary and the secondary user in this setting.

4.3 The achievable rate for the Cognitive user

The secondary user offers the opportunity to improve the sum capacity over the system by reliably detecting primary user activity and adapting its transmission, while avoiding the interference to the primary user by satisfying constraint in (4.5). In fact, the spectrum utilization can be improved by making a secondary user to access to the primary user spectrum at the right location and the sub-band in question. In this section, we will first derive the optimal power allocation policy for both primary and secondary user depending on the characteristic of the coding scheme and then analyze the corresponding performance in terms of achievable capacity. On the other hand, one basic assumption throughout this chapter is that a cognitive user can vary its transmit power in order to maximize the capacity, while maintaining a guarantee of service to the primary user. Thus, when channel state information is made available at the primary user, it will adapt its transmission strategy relative to this knowledge by transmitting at the target rate less than the real data rate with an error-free transmission in order to maintain a guarantee of service. This implies the following inequality:

$$\log_2\left(1+\frac{p_1^i\left|h_1^i\right|^2}{\sigma_v^2}\right) \leq \log_2\left(1+\frac{p_1^i\left|h_1^i\right|^2}{p_2^i \mid h_2^i \mid^2 + \sigma^2}\right) \tag{4.4}$$

Reliable communication can therefore be achieved when the virtual noise threshold is higher than the cognitive interferer contributes, yielding:

$$\sigma_v^2 \geq p_2^i \mid h_2^i \mid^2 + \sigma^2, \qquad i = 1, ..., N \tag{4.5}$$

4.3.1 Two–User Multiple Access Channel

With the same goal of capacity maximization, let us focus on the achievable rate of a system in a non cognitive scenario (when the primary user selfishly water-fills on the ambient noise level σ^2) and compare it to the achievable rate of the proposed virtual noise-based system (when the primary user experiences the virtual noise σ_v^2). Consider the achievable rates in an AWGN multiple access channel with two users. Let R_p and R_s denote the primary and the secondary transmit rates at the receiver, respectively. Then both the primary and the secondary signal can be decoded if their respective data

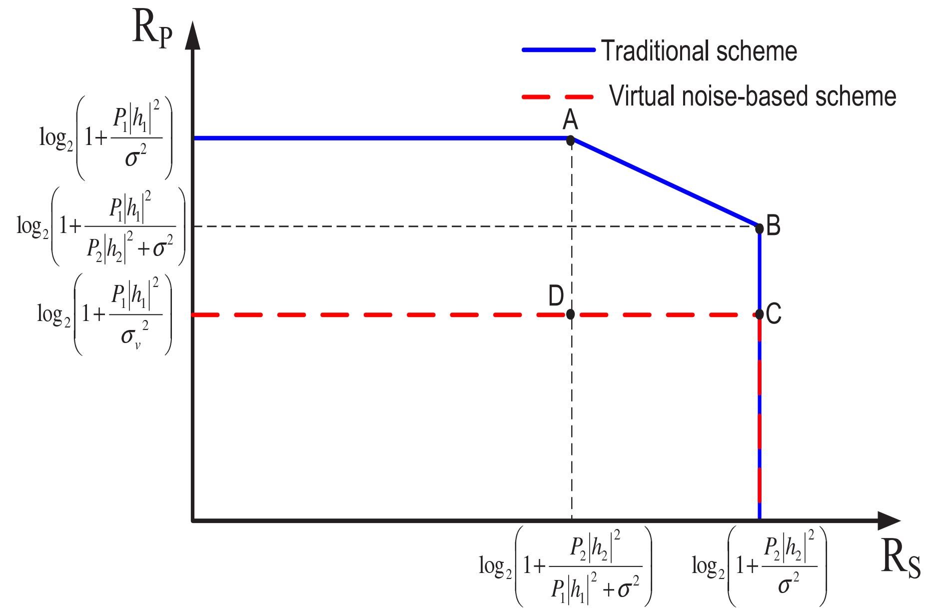

Figure 4.3: The region of achievable rate pairs $\mathcal{R} = (R_s, R_p)$ in a two–user multiple access channel.

rates R_p and R_s are chosen within the convex region defined by:

$$R_p \leq \frac{1}{N}\sum_{i=1}^{N}\log_2\left(1+\frac{p_1^i \mid h_1^i \mid^2}{\sigma_v^2}\right) \tag{4.6}$$

$$R_s \leq \frac{1}{N}\sum_{i=1}^{N}\log_2\left(1+\frac{p_2^i \mid h_2^i \mid^2}{\sigma^2}\right) \tag{4.7}$$

$$R_p + R_s \leq \frac{1}{N}\sum_{i=1}^{N}\log_2\left(1+\frac{p_1^i \mid h_1^i \mid^2}{\sigma_v^2}+\frac{p_2^i \mid h_2^i \mid^2}{\sigma^2}\right) \tag{4.8}$$

This convex region is illustrated on Fig. 4.3.

The strategies to achieve the rate pairs at the border segment AB involve Successive Interference Cancelation (SIC) at the receiver. This is done in two stages. Consider the rate pairs on point B. In the first stage, the receiver decodes the data of the primary user, treating the signal from the secondary user as interference. The maximum rate secondary user can achieve is precisely given by (4.7). Once the receiver decodes the data of the primary user, it can reconstruct primary users signal and subtract it from the aggregate received signal. An opposite strategy is used for the rates on the other corner point A. All the other rate points on the segment AB represent the optimal operating points of the channel. They can be obtained by time-sharing between the multiple access strategies in point A and point B as suggested in [2]. Nevertheless, in a traditional scheme (when the primary user experiences σ^2) no resources will be left for potential cognitive users. Considering the proposed virtual noise-based scheme, the primary user over-estimates the actual thermal noise. Following the above trend, the capacity region of the primary user is reduced as Fig. 4.3 shows. In what follows, we will study the impact of such an over-estimation on the primary user achievable rate with respect to traditional systems.

4.3.2 Achievable rate without SIC

Consider the rate pairs on point D in Fig. 4.3. The expression of the secondary user instantaneous capacity is given by:

$$C_{2,N} = \frac{1}{N}\sum_{i=1}^{N}\log_2\left(1 + \frac{p_2^i \mid h_2^i \mid^2}{p_1^i \mid h_1^i \mid^2 + \sigma^2}\right) \tag{4.9}$$

The optimal secondary user power allocation which maximizes the capacity of the system is solution of the optimization problem:

$$\{p_2^{1*}, ..., p_2^{N*}\} = \arg \max_{p_2^1,...,p_2^N} C_{2,N} \tag{4.10}$$

subject to[2]:

$$\begin{cases} \dfrac{1}{N}\displaystyle\sum_{i=1}^{N} p_2^i = 1 & \text{(4.11a)} \\ p_2^i \geq 0; \qquad \text{for} \quad i = 1, ..., N & \text{(4.11b)} \\ \sigma_v^2 \geq p_2^i \mid h_2^i \mid^2 + \sigma^2; \qquad \text{for} \quad i = 1, ..., N & \text{(4.11c)} \end{cases}$$

[2]Without loss of generality, we take $\overline{P} = 1$ in the average power constraint.

Theorem 4.1 *Consider the system model described in Section 4.2. The optimal secondary user power allocation solution of the optimization problem (4.10) under the constraints in (4.11) is:*

$$p_2^{i*} = \begin{cases} \dfrac{\sigma_v^2 - \sigma^2}{\left|h_2^i\right|^2}; & \text{if } \lambda \leq \dfrac{\left|h_2^i\right|^2}{p_1^i \mid h_1^i \mid^2 + \sigma_v^2} \\ \left(\dfrac{1}{\lambda} - \dfrac{p_1^i \mid h_1^i \mid^2 + \sigma^2}{\left|h_2^i\right|^2}\right)^+; & \text{otherwise} \end{cases} \tag{4.12}$$

where λ is the Lagrange multiplier solution of the following equation:

$$\frac{1}{N}\sum_{i=1}^{N}\left[\frac{\sigma_v^2 - \sigma^2}{\mid h_2^i \mid^2}\right] - \frac{1}{N}\sum_{i=1}^{N}\left[\frac{\sigma_v^2 - \sigma^2}{\mid h_2^i \mid^2} - \left(\frac{1}{\lambda} - \frac{p_1^i \mid h_1^i \mid^2 + \sigma^2}{\mid h_2^i \mid^2}\right)^+\right]^+ = 1 \tag{4.13}$$

The proof is given in Appendix 7.1. The first part of the theorem follows from the convex optimization problem by using Lagrange multipliers and Karush-Kuhn-Tucker conditions in [40]. A sketch of this proof can be found in Appendix 7.1. Accordingly, the optimal power allocation for this problem is shown to be a mixture of channel inversion and water-filling allocation. Note here that the proposed strategy prevents to obtain infinite power in extreme fading environments, i.e. for bad fading states h_2^i, the power allocation policy is the water-filling.

The second part of the theorem asses the viability of such a power control policy. The proof of this result follows directly from the three regions defined in the proof of *Theorem 4.2*. Formally, we can show that the secondary user power in (4.12) can be written as:

$$p_2^{i*} = \frac{\sigma_v^2 - \sigma^2}{\mid h_2^i \mid^2} - \left[\frac{\sigma_v^2 - \sigma^2}{\mid h_2^i \mid^2} - \left(\frac{1}{\lambda^*} - \frac{p_1^i \mid h_1^i \mid^2 + \sigma^2}{\left|h_2^i\right|^2}\right)^+\right]^+ \tag{4.14}$$

By taking the sum over the N sub-bands and by considering the average power constraint in (5.15b), we get the following constraint on λ which is more tractable in terms of calculations.

Discussion: Assume σ_v^2 and h_1^i fixed. For good h_2^i the optimal power-control law is the channel inversion policy. On the contrary, for bad values of h_2^i the optimal power-control law is *water-filling* on the inverse of the channel gain. This stands in contrast to the traditional case of channel inversion policy where more power is allocated when the channel is bad than when the channel is good. Notice that in the event of deep fades, i.e. h_2^i tends to be zero, the proposed policy prevents from obtaining infinite power since the secondary user power allocation is zero.

For the sake of completeness and for future use, we obtain a closed-form expression of the secondary user capacity, while considering the optimal power control in *Theorem 4.1.*

Theorem 4.2 *Consider the optimal secondary user power allocation given in* Theorem 4.1*. The secondary user capacity is then given by:*

$$C_{2,N} = \frac{1}{N}\sum_{i=1}^{N} \log_2\left(\max\left\{\min\left[T_1, T_2(\lambda)\right], 1\right\}\right) \tag{4.15}$$

where:

$$T_1 = 1 + \frac{\sigma_v^2 - \sigma^2}{p_1^i \mid h_1^i \mid^2 + \sigma^2} \tag{4.16}$$

and

$$T_2(\lambda) = \frac{\mid h_2^i \mid^2}{\lambda\left(p_1^i \mid h_1^i \mid^2 + \sigma^2\right)} \tag{4.17}$$

The proof is given in Appendix 7.2. The result follows from a direct derivation of the expression of the optimal secondary user power allocation given in *Theorem 4.1.* Accordingly, we only have to compute the quantity $\max\left\{\min\left[T_1, T_2(\lambda)\right], 1\right\}$ with respect to the lagrange multiplier λ to obtain the secondary user capacity.

4.3.3 Achievable rate with SIC

Let us now focus on the secondary user capacity when the secondary user is assumed to have perfect knowledge of the primary user's side information. The BS can then decode the primary user's signal at the appropriate rate and its contribution to the interference is then perfectly subtracted using SIC. The expression of the secondary user instantaneous capacity on point C in Fig. 4.3 is given by:

$$C_{2,N} = \frac{1}{N}\sum_{i=1}^{N}\log_2\left(1+\frac{p_2^i \mid h_2^i \mid^2}{\sigma^2}\right) \tag{4.18}$$

Similar to the case without SIC, the optimal secondary user power allocation solution of the problem (4.10) under the constraints in (4.11) is:

$$p_2^{i*} = \begin{cases} \dfrac{\sigma_v^2-\sigma^2}{\left|h_2^i\right|^2}; & \text{if} \quad \lambda' \leq \dfrac{\left|h_2^i\right|^2}{\sigma_v^2} \\ \left(\dfrac{1}{\lambda'}-\dfrac{\sigma^2}{\left|h_2^i\right|^2}\right)^+; & \text{otherwise} \end{cases} \tag{4.19}$$

where λ' is the Lagrange's multiplier satisfying the average power constraint in (4.11).

Moreover, the secondary user capacity is the same than for the case without SIC, where:

$$C_{2,N} = \frac{1}{N}\sum_{i=1}^{N}\log_2\left(\max\left\{\min\left[T_1', T_2'(\lambda')\right], 1\right\}\right) \tag{4.20}$$

where:

$$T_1' = \frac{\sigma_v^2}{\sigma^2} \tag{4.21}$$

and

$$T_2'(\lambda') = \frac{\mid h_2^i \mid^2}{\lambda'\sigma^2} \tag{4.22}$$

and λ' is solution of the following equation:

$$\frac{1}{N}\sum_{i=1}^{N}\left[\frac{\sigma_v^2-\sigma^2}{\mid h_2^i \mid^2}\right] - \frac{1}{N}\sum_{i=1}^{N}\left[\frac{\sigma_v^2-\sigma^2}{\mid h_2^i \mid^2} - \left(\frac{1}{\lambda'}-\frac{\sigma^2}{\mid h_2^i \mid^2}\right)^+\right]^+ = 1$$

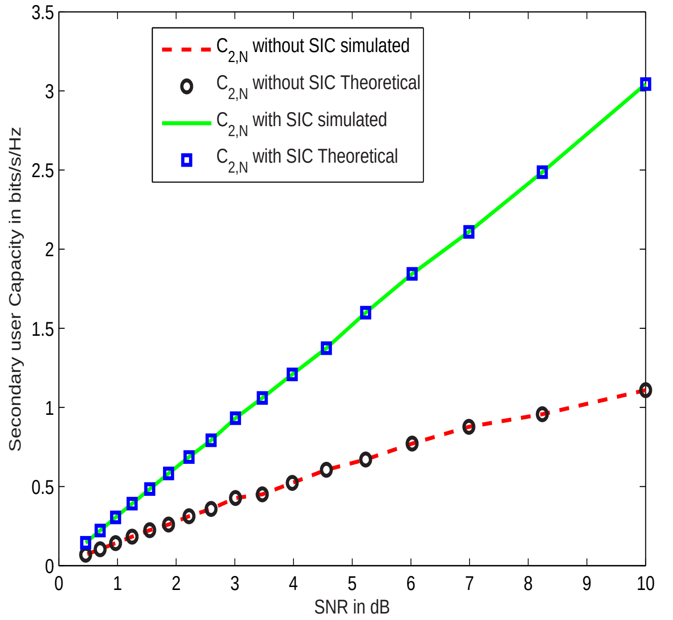

Figure 4.4: Simulated secondary capacity in (4.9) respectively (4.18) vs Theoretical secondary capacity in (4.15) respectively (4.20) for $\sigma_v^2 = 1$ and $N = 10$.

The following expressions follows from the same reasoning as in the previous subsection.
We compare our exact analytical expressions with Monte Carlo simulations; the latter are carried out by generating 10 i.i.d Rayleigh distributed channels and evaluating (4.20) and (4.15) respectively. As expected, the comparison shows an excellent agreement between analysis and simulation. Indeed, Figure 4.4 clearly shows that the theoretical secondary user capacity curve in (4.9), respectively (4.18) and theoretical ones in (4.15), respectively (4.20) perfectly match.

4.4 Asymptotic Performance

So far, we have derived optimum power allocations for each user in order to maximize their own capacity given a virtual noise threshold σ_v^2 given a finite number of sub-bands N. To proceed further with the analysis, we resort to asymptotic analysis when devices operate in a wide-band context i.e. under the assumption that $N \to \infty$. We investigate the performance of such system in terms of achievable rates assuming independent fading. The goal here is to prove the utility of using such cognitive radio scheme with respect to traditional communication systems. Without loss of generality and for the sake of simplicity, our study will address the case where the BS uses SIC strategy. Nevertheless, simulation results in Section 4.6 shows that *Theorem 4.3* still holds for the SIC case.

By making N sufficiently large, each user can compute the virtual noise level *independently* based only the channel model statistic. In fact, within this setting, the average power constraint in (4.5) becomes:

$$\int_0^\infty p_2(t).f(t)dt = 1$$

where $f(t)$ is the probability density function (p.d.f) of the channel model. By substituting p_2 by its expression in (4.12), the virtual noise threshold σ_v^2 is solution of the following equation

$$\frac{1}{\lambda'}\int_{\lambda'\sigma^2}^{\lambda'\sigma_v^2} f(t)\ dt - \sigma^2 \int_{\lambda'\sigma^2}^{\lambda'\sigma_v^2} \frac{f(t)}{t}\ dt + \left(\sigma_v^2 - \sigma^2\right)\int_{\lambda'\sigma_v^2}^{\infty} \frac{f(t)}{t}\ dt = 1$$

Obviously, such an approach can be immediately translated into results for any other probability distribution function of the channel model by replacing by the appropriate probability distribution function.

Theorem 4.3 *The sum capacity of cognitive systems using a virtual noise threshold as a proxy for the primary user performs always better than classical communication system (where the primary user selfishly maximizes its capacity).*

The proof is given in Appendix 7.3. It follows from an approximation of the expression of the sum capacity by taking N sufficiently large. Such a result shows the feasibility of allowing secondary users using locally unused spectrum for their transmissions with dynamic transmit powers and proves the fundamental constraint on the cognitive radio's noise-threshold.

4.5 Virtual noise threshold Design

As mentioned before, the virtual noise level is initially imposed by the primary system depending on the primary user requested rate. Based on this,

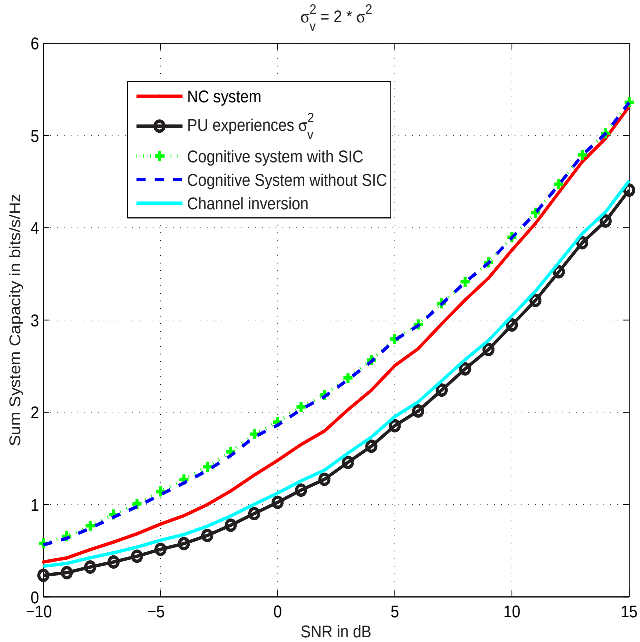

Figure 4.5: Sum system capacity for different configurations with $\sigma_v^2 = 2 * \sigma^2$ and $N = 10$. The sum capacity of the proposed cognitive scheme using virtual noise optimal power performs always better than classical system (Non Cognitive (NC) system).

the following questions may naturally arise. When we seek to optimize the sum system capacity, what is the impact of this optimization on the primary user QoS? Will there be an inherent incompatibility between system capacity maximization and primary user QoS guarantee? In other words, does the primary user requested rate considerably degrades and, how much does the primary loose in terms of outage probability? Finally, do different coding strategies adversely affect the achievable throughput of the proposed cognitive radio protocol?
The motivation of doing so in an environment where two senders share common resources is to obtain a characterization of the optimum virtual noise threshold so that it could be designed in the standard and optimized accordingly. Particularly noteworthy in this setting is that cognitive radios are considered as lower priority users of spectrum allocated to the primary user [6]. It turns out necessary to study the primary user achievable rate

when considering the optimal virtual noise in order to guard against primary user's QoS degradation.
In this part, we provide analytical and simulated insights towards addressing these important questions. We propose an asymptotic constraint to achieve the maximum possible sum capacity and validate the proposed result with a finite number of sub-bands. In both cases, we address the achievable performance with and without SIC strategy.

4.5.1 Optimum virtual noise threshold

Let us first investigate the variation of the sum capacity as function of σ_v^2 when the number of sub-bands grows sufficiently large, i.e. under the constraint that $N \rightarrow \infty$.

Theorem 4.4 *The sum capacity of a cognitive system using virtual noise threshold admits a unique optimal virtual noise level. The optimum virtual noise threshold that maximizes the sum system capacity is satisfied if and only if the primary user Lagrange multiplier γ_0 is equal to the secondary user Lagrange multiplier λ', namely*

$$C_{sum,\infty} \textit{ is maximized} \Leftrightarrow \gamma_0 = \lambda'. \tag{4.23}$$

Proof 4.1 *The sum system capacity is given by:*

$$C_{sum,\infty} = \underbrace{\int_{\gamma_0\sigma_v^2}^{\infty} \log_2\left(\frac{t}{\gamma_0.\sigma_v^2}\right).f(t)dt}_{(4.24.a)} + \underbrace{\int_{\lambda'\sigma^2}^{\lambda'\sigma_v^2} \log_2\left(\frac{t}{\lambda'\sigma^2}\right).f(t)dt}_{(4.24.b)} + \underbrace{\int_{\lambda'\sigma_v^2}^{\infty} \log_2\left(\frac{\sigma_v^2}{\sigma^2}\right).f(t)dt}_{(4.24.c)} \tag{4.24}$$

By differentiating each of the three expressions in (4.24) with respect to

σ_v^2 *we obtain*[3]*:*

$$\begin{aligned}\frac{\partial(4.24.a)}{\partial\sigma_v^2} &= \frac{\partial}{\partial\sigma_v^2}\int_{\gamma_0\sigma_v^2}^{\infty}\log_2\left(\frac{t}{\gamma_0.\sigma_v^2}\right).f(t)dt \\ &= \frac{F(\gamma_0.\sigma_v^2)-1}{\ln(2).\sigma_v^2}.\end{aligned}$$

$$\begin{aligned}\frac{\partial(4.24.b)}{\partial\sigma_v^2} &= \frac{\partial}{\partial\sigma_v^2}\int_{\lambda'\sigma^2}^{\lambda'\sigma_v^2}\log_2\left(\frac{t}{\lambda'.\sigma^2}\right).f(t)dt \\ &= \lambda' f\left(\lambda'.\sigma_v^2\right).\log_2\left(\tfrac{\sigma_v^2}{\sigma^2}\right)\end{aligned}$$

$$\begin{aligned}\frac{\partial(4.24.c)}{\partial\sigma_v^2} &= \frac{\partial}{\partial\sigma_v^2}\int_{\lambda'\sigma_v^2}^{\infty}\log_2\left(\frac{\sigma_v^2}{\sigma^2}\right).f(t)dt \\ &= \frac{1-F(\lambda'.\sigma_v^2)}{\ln(2).\sigma_v^2} - \lambda' f\left(\lambda'.\sigma_v^2\right).\log_2\left(\frac{\sigma_v^2}{\sigma^2}\right)\end{aligned}$$

Setting the derivative equal to zero, yields:

$$\frac{\partial C_{sum,\infty}(\sigma_v^2)}{\partial\sigma_v^2} = \frac{1}{\ln(2).\sigma_v^2}\left[F(\gamma_0.\sigma_v^2)-F(\lambda'.\sigma_v^2)\right] = 0 \tag{4.25}$$

The optimum value is then obtained when :

$$F(\gamma_0.\sigma_v^2)-F(\lambda'.\sigma_v^2) = 0 \tag{4.26}$$

The c.d.f F is a strictly increasing and continuous (bijective) function. The sum capacity $C_{sum,\infty}$ is then maximized if and only if

$$\gamma_0 = \lambda'.$$

[3] $F(x)$ is the cumulative density function (c.d.f) of x that can be defined in terms of the probability density function f as follows: $F(x) = \int_{-\infty}^{x} f(t)\,dt$.

The existence and the uniqueness of such a solution follows from the bijectivity of the c.d.f. On the other hand, simulation results in Section 4.6 show that the extremum value in (4.25) corresponds indeed to a maximum (see Fig. 4.8). ■

Theorem 4.4 points out that, by making N sufficiently large, the optimum virtual noise level is only a function of the SNR (through σ^2) and the channel statistics (through the Lagrange multipliers). Accordingly, the virtual noise level is a parameter that can be designed in the standard based on the statistics of the channel and can be optimized based on the channel statistics.

On the other hand, although only SIC case is treated in *Theorem 4.4*, we emphasize that our analysis is still valid for the case without SIC as Figure 4.7 clearly shows. The latter observation is particularly verified in the high SNR region where the Lagrange multipliers are approximatively the same in the two configurations (see Fig. 4.8). Such an accurate modeling of the virtual noise is a key to understand the actual benefits brought by cognitive radio technology.

4.5.2 QoS issues

The rising demand in Cognitive Radio Networks poses the problem to support Quality of Service requirements for the primary system. In fact, CRN as defined in the Wireless Regional Area Network (WRAN) standard [6] are inherently opportunistic. On the other hand, opportunism cannot be undertaken at the expense of QoS. Overcoming these issues becomes more and more challenging due to the fact that QoS, in its basic sense, is a metric of efficiency. There are a large number of proposals for all communication layers that goes along with increasing restrictions to spectrum utilization [24][41], but the QoS issue still has not been clearly defined. In addition, it is unclear how the secondary system opportunism, for instance real time, is compatible with the support of QoS for both CR systems and primary systems.

This issue requires special attention in two possible scenario:

- A secondary device might cooperate with the primary user. Through explicit signaling, the secondary device would learn when it can operate and when to interrupt service. Traditionally, it is the regulator who grants permission for secondary access and defines the signaling protocol. If a primary licensee has sufficient flexibility, it may choose to grant secondary access instead, presumably for a fee [24]. This would be a form of secondary market,

- Alternatively, a secondary device would attempt to coexist with the primary user, such that the presence of secondary devices goes unnoticed. Secondary device would then access spectrum opportunistically, when they determine that doing so would not adversely affect primary user QoS according to the virtual noise constraint in (4.5). This approach allows cognitive radios to support and guarantee QoS for the primary user, while sharing spectrum without requiring direct information exchange.

In what follows, we will adopt the latter framework and determine what would be the average rate loss for the primary user in terms of outage probability when considering the optimal virtual noise threshold.

4.6 Simulations and Results

Monte-Carlo simulations were carried out using N i.i.d channels with a Rayleigh distribution to measure the performance of the proposed protocol. We first compare the achievable rate with different possible configurations for $\sigma_v^2 = 2 * \sigma^2$. As expected, our scheme outperforms traditional scenarios where the primary user considers only the ambient noise level σ^2. He will then exploit all the resources by water-filling on this noise level and therefore, leaves no resources for the secondary user to transmit. The impact of this result is two-fold:

- It tends to validate the asymptotic assumptions under which *Theorem 3* was derived, including the non-SIC strategy,

- It confirms the basic idea we claim throughout this contribution by introducing the notion of virtual noise level as a proxy for the primary user.

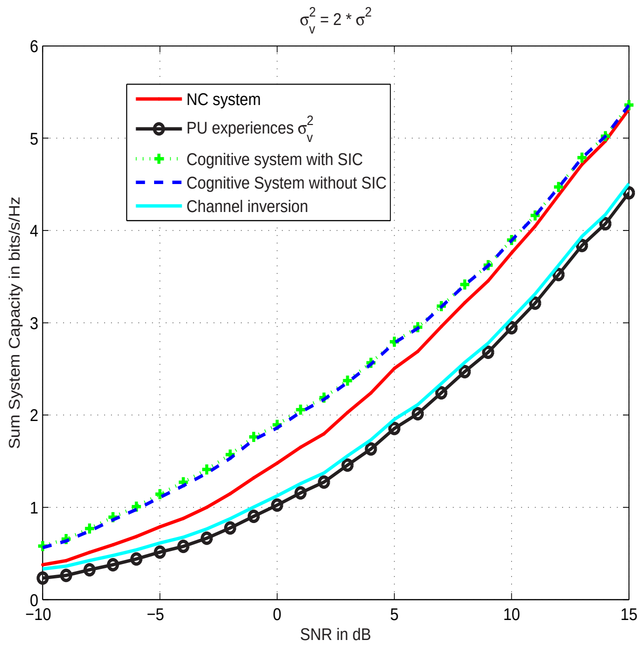

Figure 4.6: Sum system capacity for different configurations with $\sigma_v^2 = 2 * \sigma^2$ and $N = 10$ as function of the SNR in dB. The sum capacity of the proposed cognitive scheme using virtual noise optimal power performs always better than classical system (Non Cognitive (NC) system).

In order to study the impact of such an over-estimation on the primary user achievable rate, we also compare the capacity of the primary user when it considers σ_v^2 to its capacity in a Non Cognitive (NC) system (when the primary user selfishly water-fills on the ambient noise level σ^2). It turns out that the primary user capacity in the cognitive scenario degrades with respect to the NC scenario according to the increase of the SNR. Figure 4.6 also shows that SIC strategy do not give us a lot of sum capacity gain with respect to the case without SIC.

Let us now focus on the virtual noise design. It was shown in *Theorem 4.4* that the sum system capacity admits a unique maximum achieved when the primary user Lagrange multiplier is equal to the secondary user Lagrange multiplier. In Figure 4.7, we plot the sum system capacity for

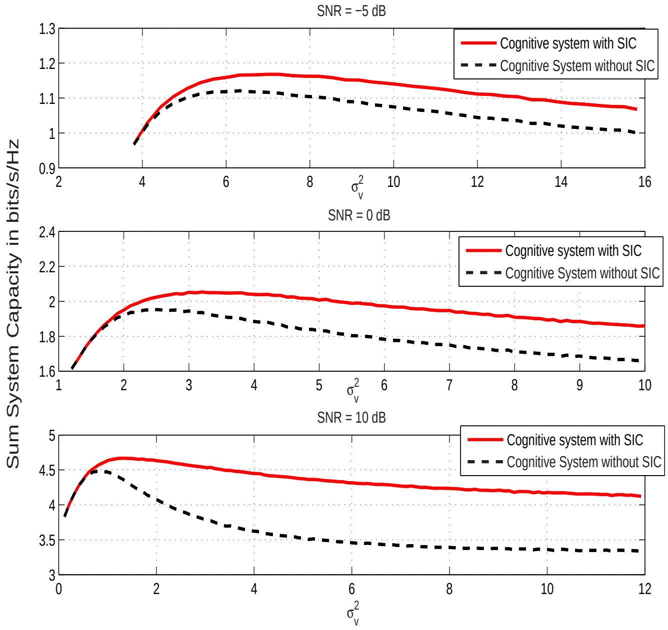

Figure 4.7: Sum system capacity for SIC and without SIC cases for $N = 100$ as function of the virtual noise.

SIC and without SIC cases for different values of the SNR. It is clear that the sum capacity admits a unique maximum for a particular value of the virtual noise level $\sigma_v^{2\,opt}$. Alternatively, notice that the $\sigma_v^{2\,opt}$ decreases when the SNR becomes higher. In order to further validate *Theorem 4.4*, we plot the difference between the primary user Lagrange multiplier and the secondary user Lagrange multiplier as function of the number of sub-bands at the optimum virtual noise of interest. Simulation results in Figure 4.8, where Lagrange multipliers were obtained through dichotomical root finder, validate the expectation from the analysis. First, it shows that *Theorem 4.4* (initially obtained for the SIC case) still holds for the non-SIC case especially at high SNR regime. Second, it illustrates that the empirical constraint converges to the optimal condition predicted from our asymptotic theory even at moderate number of sub-bands. This result tends to validate the assumptions under which the formula in *Theorem 4.4* was derived, including the non-SIC strategy.

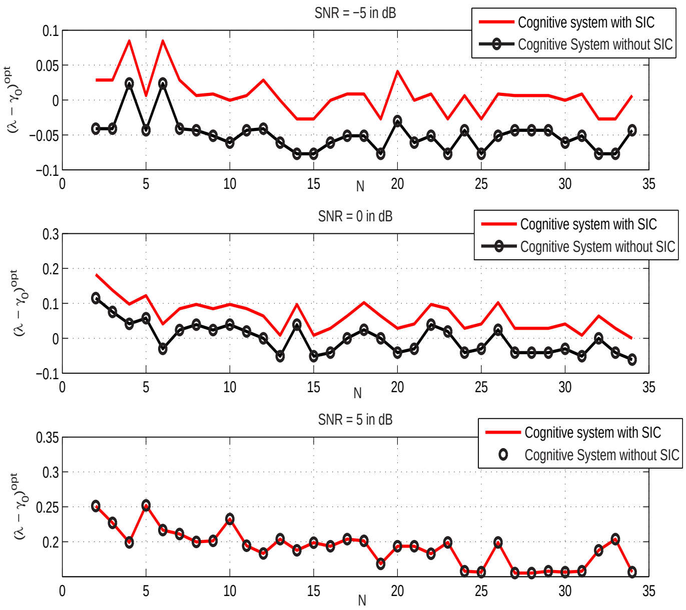

Figure 4.8: Difference between the primary user Lagrange multiplier and the secondary user's one for SIC and without SIC cases as function of the number of sub-bands N. Our results show that the asymptotic constraint in *Theorem 4.4* provides a good estimate of the empirical constraint, including the non-SIC strategy.

This naturally leads to the primary user QoS issues considered in Section 4.5.2. In particular, we focus on the achievable rate of the primary user in a non cognitive scenario (when it experiences the ambient noise σ^2) and compare it to the achievable rate of the proposed cognitive scenario (when it water-fills on $\sigma_v^{2\,opt}$). As can be seen in Fig. 4.9, the primary user capacity in a NC scenario is slightly better than its capacity using the optimal virtual noise level. Thus, the proposed cognitive system achieves almost 0.5 bit per second per hertz more than the NC system at 0-dB of signal-to-noise ratio and one bit per second hertz more than the NC system at 10-dB of SNR. In an opposite way, we see that the simulation results exhibit approximately 3-dB of capacity gain beyond traditional NC system even without SIC strategy. Moreover, it turns out that SIC strategy does not exhibit a significant capacity gain relative to the non-SIC scheme as shown in Figure 4.9. To

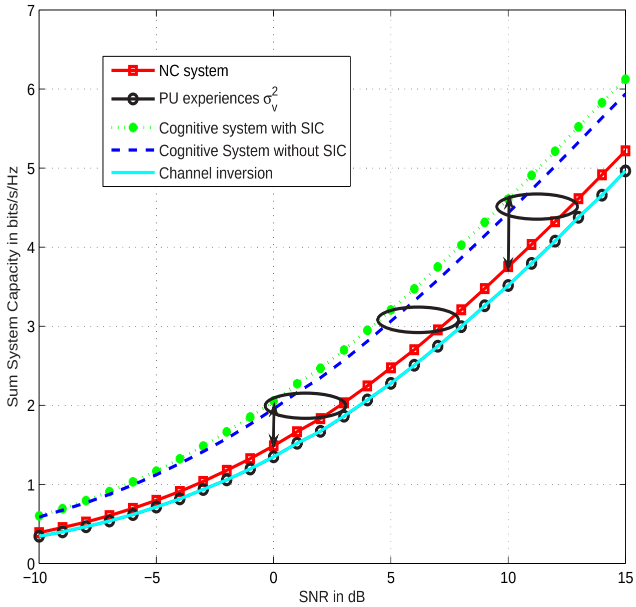

Figure 4.9: Sum system capacity that captures the optimal virtual noise σ_v^2 for $N = 10$. The proposed optimal technique offers approximately 3 dB of SNR gain and approximately 1 bps/Hz of capacity gain at 10 dB of SNR beyond traditional NC system even without SIC strategy.

proceed further with the primary user QoS analysis, we resort to outage capacity analysis in [20]. Suppose that the primary user has a fixed requested rate depending on its QoS. Then, as shown in Figure 4.10, a rate $R = 1$ bit/sec/Hz can be satisfied with an outage probability of 27% for the cognitive scenario and with an outage probability of 31% for the NC scenario. In the general case, we notice a difference of 4% between the two configurations in terms of outage probability which indicates that the proposed strategy completely guarantees a quality of service to the primary user. Such results show the feasibility of allowing secondary users using locally unused spectrum for their transmissions with dynamic transmit powers and prove the fundamental constraint on the cognitive radio's noise-threshold.

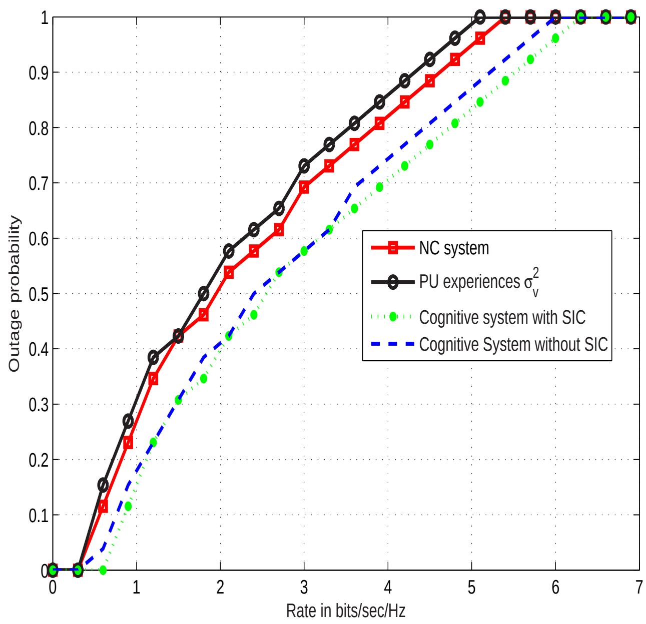

Figure 4.10: Outage probabilities for optimal σ_v^2 and $N = 10$. Notice a difference of 4% between the NC and the proposed cognitive systems in terms of outage probability.

4.7 Conclusion

Our contribution within this chapter is two-fold. The first part of this chapter is a description of the cognitive radio protocol based on virtual noise threshold. We also proposed an algorithm to gather the primary user channel state information (CSI). In the second part, we characterize the fundamental performance of the proposed optimal power allocation policy in terms of the achievable rate.

Our main result is that using a virtual noise-threshold as a proxy for the primary user, we showed that a cognitive radio can vary its transmit power in order to maximize the sum capacity, while maintaining a guarantee of service to the primary user. In this setting, we showed that the sum system capacity of such a cognitive scenario using a virtual noise threshold as a proxy for the primary user performs always better than classical communication systems (where the primary user selfishly maximizes its capacity). Moreover, we showed that the sum system capacity is maximized for a particular value of the virtual noise threshold. Simulation results validate our theoretical claims and offer insights into how to design the virtual noise in real CRN environments.

As a future work, it is of major interest to generalize the problem to multi-user systems in order to characterize the sum capacity gain of such cognitive networks. The related work can also be extended to the two-way channel context.

Chapter 5

Joint Distributed Resource Allocation

In current cognitive radio protocol proposals, secondary user devices listen to the wireless channel and determines, either in time or frequency, which part of the spectrum is unused. It then adapts its signal to fill this void in the spectrum domain. Thus, a SU device transmits over a certain time or frequency band only when no other user does, like in [14]. In the same context, it was shown in chapter 3 how we can improve the overall system spectral efficiency compared to classical approaches by considering a spectrum pooling *scenario. Results in chapter 4 showed however that cognitive protocols can be extended to allow the SU to transmit* simultaneously *with the PU in the same frequency band. This is exactly the setup in this work, where the cognitive radio behavior is generalized to allow secondary users to transmit simultaneously with the primary system* as long as the level of interference to primary users remains within an acceptable range *by means of outage probability. Specifically, it is proposed in this chapter to combine cognitive radio with multi-user diversity technology to achieve strategic spectrum sharing and self-organizing communications. Our analysis treats both uplink and downlink scenarios. We first present a distributed power allocation algorithm that attempts to maximize the throughput of the CRN. The algorithm is simple to implement, since a secondary user can decide to either transmit data or stay silent over the channel coherence time depend-*

ing on a specified threshold, without affecting the primary users' QoS. We then address the problem of user selection strategy in the context of CRN. Both centralized and distributed solutions are presented. Simulation results carried out based on a realistic network setting show promising results.

5.1 Introduction

Motivated by the desire for efficient spectral utilization, we present a novel algorithm based on binary power allocation for sum rate maximization in cognitive radio networks. At the core lies the idea of combining multi-user diversity gains with spectral sharing techniques and consequently maximizing the secondary user sum rate while maintaining a guaranteed quality of service to the primary system. In most of the approaches that can be

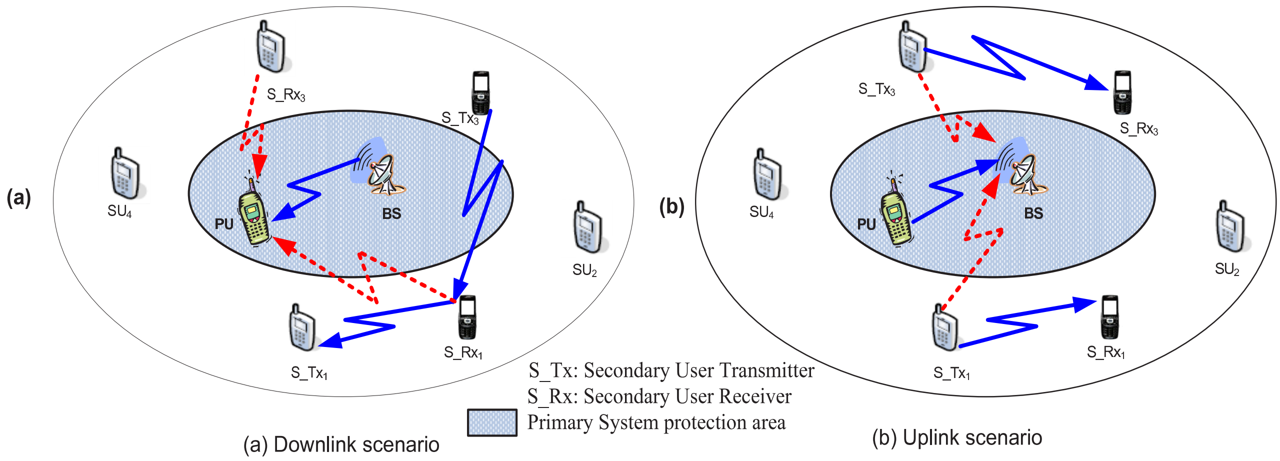

Figure 5.1: The Cognitive Radio Network with one primary user (PU) and $M = 4$ secondary users attempting to communicate with their respective pairs in an ad-hoc manner during an primary system transmission, subject to mutual interference.

found in the literature, the need may exist for centralized knowledge of all channel and interference state conditions for all nodes in the network. To circumvent this problem, the design of so-called *distributed* resource allocation techniques is crucial. Distributed optimization refers to the ability for each user to manage its local resources (e.g. rate and power control, user scheduling) based only on locally observable channel conditions such as the channel gain between the access point and a chosen user, and possibly locally measured noise and interference. A key example of multi-user resource allocation is that of power control, which serves as means for both battery savings at the mobile, and interference management. In this work,

we will focus on *binary* power control since it has the advantage of leading towards simpler or even distributed power control algorithms [42]. In [43], it was also shown that the *optimal* power control, with respect to the sum rate, is always binary for a two-cell network as well as in the low signal-to-interference ratio (SINR) regime for an $N-$cell (link) network. In the general case when the number of cells (links) > 2, it was also demonstrated by extensive computer simulations that a restriction to binary power levels yields only a negligible capacity loss [44].
A particularly noteworthy question in the context of cognitive radio, when we seek to optimize the sum system capacity, is to guarantee a QoS to PUs. There are a large number of proposals for all communication layers treating the increase of restrictions to spectrum utilization [24], but the QoS issue still has not been clearly defined. In addition, it is unclear how secondary system opportunism, is compatible with the support of QoS for both cognitive radio systems and primary systems. The FCC proposed the concept of "*interference temperature*" as a way to have unlicensed transmitters share licensed bands without causing harmful interference. Rather than merely regulate transmitter power at fixed levels, as in the past, the scheme would have governed transmitter power on a variable basis calculated to limit the energy at victim receivers, where interference actually occurs. As a practical matter, however, the FCC abandoned the interference temperature concept recently [45] due to the fact that it is not a workable concept and would result in increased interference in the frequency bands where they were to be used. While offering attractive promises, cognitive radios face various challenges, starting from defining the fundamental performance limits of this radio technology, in order to achieve the capability of using the spectrum in an opportunistic manner. Specifically, cognitive radio is required to determine the spectrum band allocation that meets the QoS requirements of different users. This decision can be made by assessing the channel capacity, known as the most important factor for spectrum characterization. In this contribution, we will propose a different way to efficiently protect primary systems from SU interference, based on outage probability. The notion of *information outage probability* defined as the probability that the instantaneous mutual information of the channel is below the transmitted code rate was introduced in [20]. Accordingly, the outage probability can be written as:

$$P_{out}(R) = P\left\{I(\mathbf{x};\mathbf{y}) \leq R\right\} \tag{5.1}$$

Where $I(\mathbf{x};\mathbf{y})$ is the mutual information of the channel between the transmitted vector $\mathbf{x}$ and the received vector $\mathbf{y}$ and R is the target data rate in

($bits/s/Hz$). Reliable communication can therefore be achieved when the mutual information of the channel is strong enough to support the target rate R. Thus, the cognitive transmitter can adapt its transmit power p within the range of $[0; P_{max}]$ to fulfill the two basic goals listed as follows:

- *Self-goal*: Trying to transmit as much information for himself as possible,
- *Moral-goal*: Maintaining the primary users outage probability unaffected.

The motivation behind doing so is that, in any case, the PU will not necessarily need all that rate. In fact, the primary user will experience the SUs interference, and as long as all his target rate (depending on his QoS) to be achieved, he does not care about what he leaves more. In what follows, we adopt this setting and consider a CRN in which primary and secondary users attempt to communicate, subject to mutual interference. We propose a distributed cognitive radio coordination that maximizes the CRN sum rate while minimizing the interference to the primary user. Our goal is to realize PU-SU spectrum sharing by optimally allocating SU transmit powers in order to maximize the total SU throughput under interference and noise impairments, and short term (minimum and peak) power constraints, while preserving the QoS of the primary system. In such approaches, users individually make a decision on their transmit power so as to optimize their contribution to the system throughput. At the core of the distributed concept lies the idea of making the interference more predictable by making the network larger or denser, and consequently the resource allocation problem of a given user is made more dependent on the local channel conditions of that user, thus facilitating distributed optimization. At first sight, joint resource allocation does not lend itself easily to distributed optimization because of the strong coupling between the locally allocated resources and the interference created elsewhere in the CRN. Hence the maximization of a SU capacity taken individually will not in general result in the best overall network capacity, although we suggest later cases for which the outcomes for the centralized and distributed capacity optimization will differ little. Following the above trend, we will explore a distributed joint resource allocation framework and then analyze what would be the loss when considering a distributed strategy in terms of the number of active users and the average rate with respect to a centralized strategy where the system rely on some form of centralized control to obtain gains at various layers of the communication stack. Our study treats both downlink and uplink communications.

In both cases, we will derive a distributed power allocation algorithm and address the QoS issues for the primary system from an outage point of view.

The next Section describes the cognitive radio network. In Section 5.3, the proposed distributed power control algorithm is investigated in both the high and low SINR regimes, respectively. Section 5.4 includes the primary users' QoS issues. Simulation results are provided in Section 5.5, and Section 5.6 concludes the chapter.

5.2 The Cognitive Radio Context

5.2.1 The System Model

We consider a wireless CRN with a collection of users randomly distributed over the geographical area considered. Users can be both transmitters and receivers. By virtue of a scheduling protocol, one PU and M pairs of secondary users are simultaneously selected from these users to communicate at a given time instant, while others remain silent. The channel gains are assumed i.i.d. random variables. Throughout this chapter, we will use the following notation:

- the index of SUs j lies between 1 and M,
- $h_{pu,n}$ denotes the channel gain from the PU indexed by pu to a desired SU n,
- $h_{pu,pu}$ denotes the channel gain between the base station (BS) and the PU,
- $h_{j,n}$ denotes the channel gain from SU j to a desired SU n,
- the data destined from the primary system is transmitted with power p_{pu}.
- the data destined from SU j is transmitted with power p_j.

In the coverage area of the primary system, there is an *interference boundary* within which no SUs can communicate in an ad-hoc manner. Thus, as can be seen in Figure 5.1, for the impairment experienced by the primary system to be as small as possible, a SU must be able to detect very reliably whether it is far enough away from a primary base station, i.e., in the area of possible cognitive radio operation. The expression of the PU instantaneous capacity is

$$C_{pu} = \log_2 \left(1 + \frac{p_{pu} \mid h_{pu,pu} \mid^2}{\sum_{j=1}^{M} p_j \mid h_{j,pu} \mid^2 + \sigma^2} \right) \tag{5.2}$$

where σ^2 is the ambient noise variance. On the other hand, by making SUs access the primary system spectrum, the j^{th} SU experiences interference from the PU and all neighboring co-channel SU links that transmit on the same band. Accordingly, the j^{th} SU instantaneous capacity is given by:

$$C_j = \log_2 (1 + \mathrm{SINR}_j)\,; \quad \text{for} \quad j = 1, ..., M \tag{5.3}$$

where

$$\mathrm{SINR}_j = \frac{p_j \mid h_{j,j} \mid^2}{\sum_{\substack{k=1 \\ k \neq j}}^{M} p_k \mid h_{k,j} \mid^2 + p_{pu} \mid h_{pu,j} \mid^2 + \sigma^2} \tag{5.4}$$

SUs need to recognize their communication environment and adapt the parameters of their communication scheme in order to maximize the cognitive capacity, expressed as

$$C_{sum} = \frac{1}{\tilde{M}} \sum_{j=1}^{\tilde{M}} C_j\,, \tag{5.5}$$

while minimizing the interference to the primary users, in a *distributed* fashion. The sum here is made over the $\tilde{M}$ SUs allowed to transmit. Moreover, we assume that the coherence time is sufficiently large so that the channel stays constant over each scheduling period length. We also assume that SUs know the channel state information of their own links, but have no information on the channel conditions of other SUs. No interference cancelation capability is considered. Power control is used for SUs both in an effort to preserve power and to limit interference and fading effects.

5.2.2 The Cognitive Radio protocol

Under this scheme, we allow SUs to transmit simultaneously with the PU as long as the interference from the SUs to the PU that transmits on the same band remains within an acceptable range. Specifically, we impose that SUs may transmit simultaneously with the PU as long as the PU in question does not have his QoS affected in terms of outage probability. We consider that PUs operate at a desired rate (depending on their respective QoS demands). Based on PU channel statistics, we determine the outage failure, in other words the probability that the PU of interest is actually under that rate. From a practical point of view, the outage probability as well as the requested rate can be broadcasted before the start of the communication by the primary system base station, and is used as a preamble for the PU to get informed which data rate is requested. This preamble can also be overheard by SUs who can then learn about these outage values.

One basic assumption throughout this chapter is that a SU can vary its transmit power, under short term (minimum and peak) power constraints, in order to maximize the cognitive capacity, while maintaining a QoS guarantee to the primary user. The idea of the binary on/off power control is simple, as well as yielding quasi-optimal results in a number of cases [44]. As such, it constitutes a promising tool for making spectrum sharing a reality. Besides complexity reduction, an important additional benefit of binary power control is to allow distributed optimization.

5.3 Binary power control algorithm

Secondary users offer the opportunity to improve the system throughput by detecting the PU activity and adapting their transmissions accordingly while avoiding the interference to the PU by satisfying the QoS constraint on outage. The motivation behind the proposed technique is that, by opportunistically adapting their transmit power with the guide of the proposed strategy, SUs can maximize the achievable sum rate under the constraint of maintaining the outage probability of the PU not degraded. Our goal within this work is thus to determine, under the assumption that the PU is oblivious to the presence of the cognitive users, what would be the cognitive system capacity (which can also be viewed as the total increase in system capacity (or spectral efficiency) due to the SUs' activity) and, at the same time, the maximum number of cognitive communication links allowed in such a system. We present a distributed algorithm for power allocation in the sense that it requires a SU to decide *distributively* to either transmit data or stay silent over the channel coherence time depending on a specified SNR threshold. The optimization problem can therefore be expressed as follows:

$$\text{Find } \{p_1*, ..., p_M*\} = \arg \max_{p_1,\cdots,p_M} C_{sum} \tag{5.6}$$

$$\sum_{j\in\Psi}\log_2\left(1+\frac{p_j \mid h_{j,j}\mid^2}{\sigma^2+p_{pu}\mid h_{pu,j}\mid^2+\sum\limits_{\substack{k\in\Psi\\k\neq j}}p_k\mid h_{k,j}\mid^2}\right)<\sum_{\substack{j\in\Psi\\j\neq m}}\log_2\left(1+\frac{p_j \mid h_{j,j}\mid^2}{\sigma^2+p_{pu}\mid h_{pu,j}\mid^2+\sum\limits_{\substack{k\in\Psi\\k\neq j\neq m}}p_k\mid h_{k,j}\mid^2}\right) \tag{5.8a}$$

$$\log_2\left(1+\mathrm{SINR}_m\right)+\sum_{\substack{j\in\Psi\\j\neq m}}\log_2\left(1+\frac{p_j \mid h_{j,j}\mid^2}{\sigma^2+p_{pu}\mid h_{pu,j}\mid^2+\sum\limits_{\substack{k\in\Psi\\k\neq j}}p_k\mid h_{k,j}\mid^2}\right)<\sum_{\substack{j\in\Psi\\j\neq m}}\log_2\left(1+\frac{p_j \mid h_{j,j}\mid^2}{\sigma^2+p_{pu}\mid h_{pu,j}\mid^2+\sum\limits_{\substack{k\in\Psi\\k\neq j\neq m}}p_k\mid h_{k,j}\mid^2}\right) \tag{5.8b}$$

$$\Rightarrow\left(1+\mathrm{SINR}_m\right)\prod_{\substack{j\in\Psi\\j\neq m}}\left(1+\frac{p_j \mid h_{j,j}\mid^2}{\sigma^2+p_{pu}\mid h_{pu,j}\mid^2+\sum\limits_{\substack{k\in\Psi\\k\neq j}}p_k\mid h_{k,j}\mid^2}\right)<\prod_{\substack{j\in\Psi\\j\neq m}}\left(1+\frac{p_j \mid h_{j,j}\mid^2}{\sigma^2+p_{pu}\mid h_{pu,j}\mid^2+\sum\limits_{\substack{k\in\Psi\\k\neq j\neq m}}p_k\mid h_{k,j}\mid^2}\right) \tag{5.8c}$$

---

subject to:

$$\begin{cases} p_j \in \{0, P_{max}\}, \quad \text{for} \quad j = 1, ..., M \\ \\ P_{out} = Prob\,\{C_{pu} \leq R_{pu} \mid R_{pu}, q\} \leq q \end{cases} \tag{5.7}$$

where R_{pu} is the PU transmitted data rate. The key idea within the proposed iterative algorithm is, as in [42], to subsequently limit p_j to $\{0, P_{max}\}$, i.e., to switch "off" transmission in SUs' links which do not contribute enough capacity to outweigh the interference degradation caused by them to the rest of the network. We propose an adaptation of the distributed algorithm which allows a subset of controlled size $\tilde{M}$ of the total number of SUs M to transmit simultaneously on the same sub-band. It turns out necessary to limit the number of SUs interfering with the primary user so as to guarantee the QoS for the primary system. A SU should be deactivated if this action results in an increase in the cognitive capacity of SUs or if its transmission violates the PU outage constraint. Let Ψ be the set of indices of all presently active SUs.Denoting the SU which is to be potentially turned off by m, the network capacity with and without SU turned off is given by the LHS and the RHS of (5.8a) respectively, and after simple manipulations (5.8c).

5.3.1 At high SINR regime

The CRN described in the previous subsection can be modeled by interference channels, due to the fact that SUs employ the same spectral resource in each link, giving rise to an interference-limited system. At high SINR regime, in all "on" SU, and assuming an interference-limited system, we can simplify the condition (5.8c) as

$$\mathrm{SINR}_m = \frac{p_m \mid h_{m,m} \mid^2}{p_{pu} \mid h_{pu,m} \mid^2 + \sum\limits_{\substack{k\in\Psi \\ k\neq m}} p_k \mid h_{k,m} \mid^2} < \frac{\prod\limits_{\substack{j\in\Psi \\ j\neq m}} \left(p_{pu} \mid h_{pu,j} \mid^2 + \sum\limits_{\substack{k\in\Psi \\ k\neq j}} p_k \mid h_{k,j} \mid^2 \right)}{\prod\limits_{\substack{j\in\Psi \\ j\neq m}} \left(p_{pu} \mid h_{pu,j} \mid^2 + \sum\limits_{\substack{k\in\Psi \\ k\neq j\neq m}} p_k \mid h_{k,j} \mid^2 \right)} \tag{5.9}$$

$$\Downarrow$$

$$\frac{p_m \mid h_{m,m} \mid^2}{p_{pu} \mid h_{pu,m} \mid^2 + \sum\limits_{\substack{k\in\Psi \\ k\neq m}} p_k \mid h_{k,m} \mid^2} < \frac{\prod\limits_{\substack{j\in\Psi \\ j\neq m}} \sum\limits_{\substack{k\in\Psi\cup\{pu\} \\ k\neq j}} p_k \mid h_{k,j} \mid^2}{\prod\limits_{\substack{j\in\Psi \\ j\neq m}} \sum\limits_{\substack{k\in\Psi\cup\{pu\} \\ k\neq j\neq m}} p_k \mid h_{k,j} \mid^2} \tag{5.10}$$

Suppose that devices operate in a dense network, i.e. a large number of SUs is distributed over a restricted geometrical area. Dense networks lend themselves to simplified modeling of the total interference experienced by any user, thanks to the large number of interference sources being averaged at the receiver [46]. Based on the observation that interference to any user in a large dense network is only weakly dependent on the user's position, we can approximate the interference term by an average interference gain, denoted by G^2 which is independent of the user location multiplied, by the total transmit power of active interferers:

$$\sum_{j=1}^{M} p_j \mid h_{n,j} \mid^2 \simeq G^2 \sum_{j=1}^{M} p_j = G^2 M P_{max}\,, \quad \text{for all} \quad n \tag{5.11}$$

The constant G^2 depends only on the average amplitude of the channel gain and the pathloss. Though only an approximation, this model is supported by simulations. Accordingly, condition (5.10) becomes

$$\frac{p_m \mid h_{m,m} \mid^2}{\sum\limits_{\substack{k\in\Psi\cup\{pu\} \\ k\neq m}} p_k \mid h_{k,m} \mid^2} < \frac{\prod\limits_{\substack{j\in\Psi \\ j\neq m}} G^2 \sum\limits_{\substack{k\in\Psi\cup\{pu\} \\ k\neq j}} p_k}{\prod\limits_{\substack{j\in\Psi \\ j\neq m}} G^2 \sum\limits_{\substack{k\in\Psi\cup\{pu\} \\ k\neq j\neq m}} p_k} \tag{5.12}$$

Let us denote by $\tilde{M} = card\{\Psi\}$ and suppose[1] that $K = \frac{p_{pu}}{P_{max}}$. As all "on" SUs transmit with P_{max}, the m^{th} SU will be active only if

$$\frac{| h_{m,m} |^2}{\sum\limits_{\substack{k \in \Psi \cup \{pu\} \\ k \neq m}} | h_{k,m} |^2} > \left(\frac{\tilde{M} + K - 1}{\tilde{M} + K - 2} \right)^{\tilde{M}-1} \tag{5.13}$$

As the number of SUs increases, we get (as in [42])

$$\lim_{\tilde{M} \to \infty} \left(\frac{\tilde{M} + K - 1}{\tilde{M} + K - 2} \right)^{\tilde{M}-1} = \left(\frac{\tilde{M} + K - 1}{\tilde{M} + K - 2} \right)^{\tilde{M}+K-2} \cdot \left(\frac{\tilde{M} + K - 1}{\tilde{M} + K - 2} \right)^{1-K}$$

$$= e = 2.718281...$$

Thus, for a large network size, a SU will be active if its experimental signal-to-interference ratio is more than e, namely

$$\mathrm{SIR}_m = \frac{p_m \mid h_{m,m} \mid^2}{\mid h_{i,m} \mid^2 + \sum\limits_{\substack{k \in \Psi \\ k \neq m}} p_k \mid h_{k,m} \mid^2} > e \tag{5.14}$$

5.3.2 At low SINR regime

The restriction to binary power levels yields in general only a negligible capacity loss. In addition, as stated before, it was shown in [44] that in the low-SINR regime, i.e., where the approximation $\ln(1 + x) \simeq x$ holds with good accuracy, binary power control is in fact always optimal. In the low SINR regime and starting from (5.8a), we get (5.15a). After simple manipulations and following (5.15c), the m^{th} SU will now be active if

$$\mathrm{SINR}_m < \frac{\sum\limits_{\substack{j \in \Psi \\ j \neq m}} p_j \mid h_{j,j} \mid^2}{P_{max} G^2 (\tilde{M} + K - 2) + \sigma^2} \tag{5.16}$$

$$\simeq \frac{P_{max} G^2 (\tilde{M} - 1)}{P_{max} G^2 (\tilde{M} + K - 2) + \sigma^2}$$

[1] Notice that for the case of uplink $K = 1$ since the PU is transmitting with $p_{pu} = P_{max}$. However, in the downlink scenario, $K > 1$ since the power transmitted by the BS is generally greater than P_{max}.

$$\frac{p_m \mid h_{m,m} \mid^2}{\sigma^2 + p_{pu} \mid h_{pu,m} \mid^2 + \sum\limits_{\substack{k\in\Psi\\k\neq m}} p_k \mid h_{k,m} \mid^2} < \sum_{\substack{j\in\Psi\\j\neq m}} \frac{p_j \mid h_{j,j} \mid^2}{\sigma^2 + p_{pu} \mid h_{pu,j} \mid^2 + \sum\limits_{\substack{k\in\Psi\\k\neq j\neq m}} p_k \mid h_{k,j} \mid^2} - \sum_{\substack{j\in\Psi\\j\neq m}} \frac{p_j \mid h_{j,j} \mid^2}{\sigma^2 + p_{pu} \mid h_{pu,j} \mid^2 + \sum\limits_{\substack{k\in\Psi\\k\neq j}} p_k \mid h_{k,j} \mid^2} \tag{5.15a}$$

$$\frac{p_m \mid h_{m,m} \mid^2}{\sigma^2 + p_{pu} \mid h_{pu,m} \mid^2 + \sum\limits_{\substack{k\in\Psi\\k\neq m}} p_k \mid h_{k,m} \mid^2} < \sum_{\substack{j\in\Psi\\j\neq m}} \left(\frac{p_j \mid h_{j,j} \mid^2}{\sigma^2 + \sum\limits_{\substack{k\in\Psi\cup\{pu\}\\k\neq j\neq m}} p_k \mid h_{k,j} \mid^2} - \frac{p_j \mid h_{j,j} \mid^2}{\sigma^2 + \sum\limits_{\substack{k\in\Psi\cup\{pu\}\\k\neq j}} p_k \mid h_{k,j} \mid^2} \right) \tag{5.15b}$$

$$\frac{p_m \mid h_{m,m} \mid^2}{\sigma^2 + p_{pu} \mid h_{pu,m} \mid^2 + \sum\limits_{\substack{k\in\Psi\\k\neq m}} p_k \mid h_{k,m} \mid^2} < \sum_{\substack{j\in\Psi\\j\neq m}} \frac{p_j \mid h_{j,j} \mid^2}{\sigma^2 + \sum\limits_{\substack{k\in\Psi\cup\{pu\}\\k\neq j\neq m}} p_k \mid h_{k,j} \mid^2} \tag{5.15c}$$

where we use the same dense average network assumptions as in (5.11). Suppose, as in the high SINR regime, that we are in an interference-limited context. This would suggest that $\sigma^2 \ll P_{max}G(\tilde{M} + K - 2)$ in the RHS of (5.16). As the number of SUs increases, we get

$$\lim_{\tilde{M}\to\infty} \left(\frac{\tilde{M} - 1}{\tilde{M} + K - 2} \right) = 1 \tag{5.17}$$

Thus, as previously done, a SU will be active if its experimental SIR is more than 1:

$$\text{SIR}_m = \frac{p_m \mid h_{m,m} \mid^2}{\mid h_{i,m} \mid^2 + \sum\limits_{k\in k,m} p_k \mid h_{k,m} \mid^2} > 1 \tag{5.18}$$

We thus confirm, as intuition would expect, that SUs under better SINR conditions would transmit only above a higher threshold than in the low-SINR regime.

5.4 Primary system QoS issues

In the current study, we adopt a QoS guarantee to the PU by means of an outage constraint. This knowledge can be obtained from two manners: In a centralized mode where the proposed system would require information from a third party (i.e. central database maintained by regulator or another authorized entity) to schedule SUs coming. In a realistic network, centralized system coordination is hard to implement, especially in fast fading environments and in particular if there is no fixed infrastructure for SUs, i.e., no back-haul network over which overhead can be transmitted between users. In fact, centralized channel state information for a dense network involves immense signaling overhead and will not allow the extraction of diversity gains in fast-fading channel components. Then, it may be desirable to design a scheme that can mitigate the interference with low signaling overhead. To alleviate this problem, we propose a distributed method where SUs can get rid of PU knowledge. In a distributed framework, the information about the outage failure can be carried out by a band manager that mediates between the primary and secondary users [24], or can be directly fed back from the PU to the secondary transmitters through collaboration and exchange of the CSI between the primary and secondary users as proposed in [25]. To proceed further with the analysis and for the sake of emphasis, we introduce the PU average channel gain estimate G_{pu} based on the following decomposition:

$$h_{pu,pu} \triangleq G_{pu} * h'_{pu,pu}$$

where $h'_{pu,pu}$ is the random component of channel gain and represents the *normalized* channel impulse response tap. This gives us the following PU outage probability expression:

$$\begin{aligned} P_{out} &= Prob\left\{ \log_2\left(1 + \frac{p_{pu} G_{pu}^2 \mid h'_{pu,pu} \mid^2}{\sum_{j=1}^{\tilde{M}} p_j \mid h_{j,pu} \mid^2 + \sigma^2}\right) \leq R_{pu} \right\} \leq q \qquad (5.19) \\ &\simeq Prob\left\{ \frac{p_{pu} G_{pu}^2 \mid h'_{pu,pu} \mid^2}{G_{su}^2 \sum_{j=1}^{\tilde{M}} p_j + \sigma^2} \leq 2^{R_{pu}} - 1 \right\} \leq q \\ &\simeq Prob\left\{ \mid h'_{pu,pu} \mid^2 \leq \left(2^{R_{pu}} - 1\right)\left(\frac{\tilde{M} G_{su}^2 P_{max} + \sigma^2}{G_{pu}^2 p_{pu}}\right) \right\} \leq q \end{aligned}$$

From now on we assume for simplicity of analysis that the channel gains are i.i.d rayleigh distributed. However, the results can be immediately translated into results for any other channel model by replacing by the appropriate probability distribution function. Continuing from (5.19), we have:

$$P_{out} \simeq \int_0^{\left(2^{R_{pu}}-1\right)\left(\frac{\tilde{M}G_{su}^2 P_{max}+\sigma^2}{G_{pu}^2 p_{pu}}\right)} \exp(-t)dt \leq q \qquad (5.20)$$

Finally, we get the following outage constraint:

$$P_{out} \simeq 1-\exp\left[-\left(2^{R_{pu}}-1\right)\left(\frac{\tilde{M}G_{su}^2 P_{max}+\sigma^2}{G_{pu}^2 p_{pu}}\right)\right] \leq q \qquad (5.21)$$

and the maximum number $\tilde{M}$ of active "on" SUs that transmit with P_{max} is given by

$$0 \leq \tilde{M} \leq \frac{-\log(1-q)}{(2^{R_{pu}}-1)} . \frac{G_{pu}^2 p_{pu}}{G_{su}^2 P_{max}} - \frac{\sigma^2}{G_{su}^2 P_{max}} \qquad (5.22)$$

By writing $\mathrm{SNR} = \frac{G_{su}^2 P_{max}}{\sigma^2}$, equation (5.22) can be expressed as:

$$0 \leq \tilde{M} \leq \frac{-\log(1-q)}{(2^{R_{pu}}-1)} . \frac{G_{pu}^2 p_{pu}}{G_{su}^2 P_{max}} - \frac{1}{\mathrm{SNR}} = \tilde{M}_{theorie} \qquad (5.23)$$

The LHS in (5.23) prevents from obtaining a negative number of users when the SNR decreases significantly. The formula in (5.23) points out that the number of SUs allowed to transmit increases as their SNR increases. The algorithm can be implemented using a centralized controller who observes global network and makes decisions, or through a distributed algorithm where each SU performs a distributed voting process. Recent results in [47] show that the heuristic based algorithms perform similarly to the global optimum (in a cellular-based context), and the centralized and distributed algorithms perform almost similarly.

The pseudo-code for the proposed approach is given in Algorithm 1 where $\tilde{M}_{theorie}$ is the number of SUs allowed to transmit ruled by (5.23). An iterative approach is adopted to obtain an algorithm for power allocation. The algorithm is first initialized with a full power allocation vector. Each SU simultaneously measures his SIR and depending on wether the SU is on high or low SINR, respectively, he remains active or inactive during the next iteration based on (5.18), respectively (5.14). Similarly, at every iteration, inequality (5.18) and (5.14) are evaluated for the SU in question based on the power allocation resulting from the previous iteration, and the power allocation vector is updated. Within each iteration, each PU verifies the outage probability constraint based on the resulting power allocation. The algorithm is run until the secondary sum capacity stabilizes or for a given number of iterations.

Algorithm 2 Cognitive Radio Power Allocation(SINR, rate, target outage probability)

$p_j^{(1)} = P_{max} \ \ \forall j$ and $\tilde{M}^{(1)} = M$
for $it = 1 : IT_{max}$ **do**
  while $\tilde{M}^{(it)} < \tilde{M}_{theorie}$ **do**
    for $j = 1 : M$ **do**
      $\triangleright$ **at high SINR regime**
      if $\mathrm{SINR}_j^{(it)} > e$ **then**
        $p_j^{(it+1)} \leftarrow P_{max}$
      else
        $p_j^{(it+1)} \leftarrow 0$
      end if
      $\triangleright$ **at low SINR regime**
      if $\mathrm{SINR}_j^{(it)} > 1$ **then**
        $p_j^{(it+1)} \leftarrow P_{max}$
      else
        $p_j^{(it+1)} \leftarrow 0$
      end if
    end for
    if $P_{out}^{(it+1)} \geq q$ **then**
      $\tilde{M}^{(it+1)} \leftarrow \tilde{M}^{(it)} - 1$
    end if
  end while
end for

Fairness Issues

As we focus on capacity maximization schemes, it is expected that fairness issues will arise with regard to some SUs that might experience long periods of silence due to prolonged detrimental fading conditions or a poor user spatial distribution. However, we draw the reader's attention to the fact that the solutions akin to the cellular scheduling scenario, giving various levels of fairness-capacity trade-off, can be used also in this context, e.g. use of proportional-fair type measures [48]. Hence, we may alternatively use a capacity measure for each SU that is normalized by the throughput of total SUs in the network. We can adopt a fairness strategy based on next rule where SUs with large transmit time are dropped after a given time. Moreover, when multiple orthogonal units are employed, a SU that is inactive for one code, frequency, or time slot may be active on another. Investigations of the fairness-capacity trade-off are however, are out of the scope of this work.

5.5 Numerical Results

To go further with the analysis, we resort to realistic network simulations. Specifically, we consider a cognitive radio network as described in Figure 5.1 with one PU and M secondary users attempting to communicate during a transmission, subject to mutual interference. A hexagonal cellular system functioning at 1.8 GHz with a primary cell of radius $R = 1000$ meters and a primary protection area of radius $R_p = 600$ meters is considered. Secondary transmitters may communicate with their respective receivers of distances $d < R_p$ from the BS. Channel gains are based on the COST-231 path loss model [49] including log-normal shadowing with standard deviation of 10 dB, plus fast-fading assumed to be i.i.d. circularly symmetric with distribution $\mathcal{CN}(0,1)$. The peak power constraint is given by $P_{max} = 1$ Watt while the power ratio K is taken equal to 10 for the downlink and equal to 1 for the uplink. This is justified in the light of the fact that the power control transmitted by the BS is generally taken almost ten times the primary user transmitted power in multiple possible standards. Figure 5.2 and 5.3 capture the number of active SUs in the downlink and the uplink respectively for different rates and outage probability. As expected, it is shown that increasing the target data rate, less SUs are allowed to transmit. As an example, in the downlink, 9 SUs are allowed to transmit at a rate equal to 0.1 bits/s/Hz and a target outage probability $q = 1\%$ while only 7 SUs are active at a rate equal to 0.5 bits/s/Hz and for the same outage

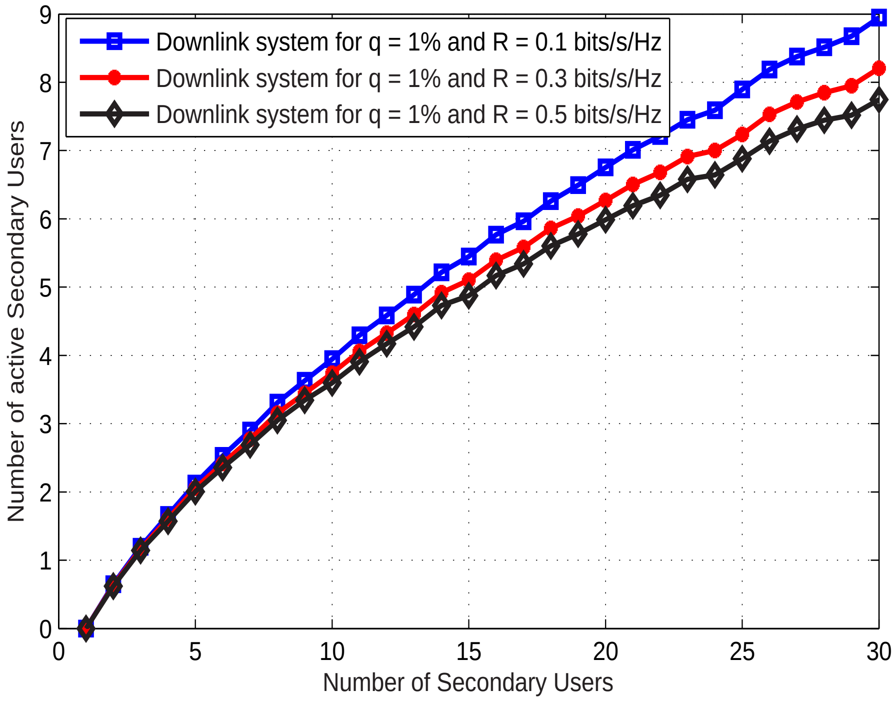

Figure 5.2: Number of active secondary users vs. number of SUs for different rates and outage probability in the downlink.

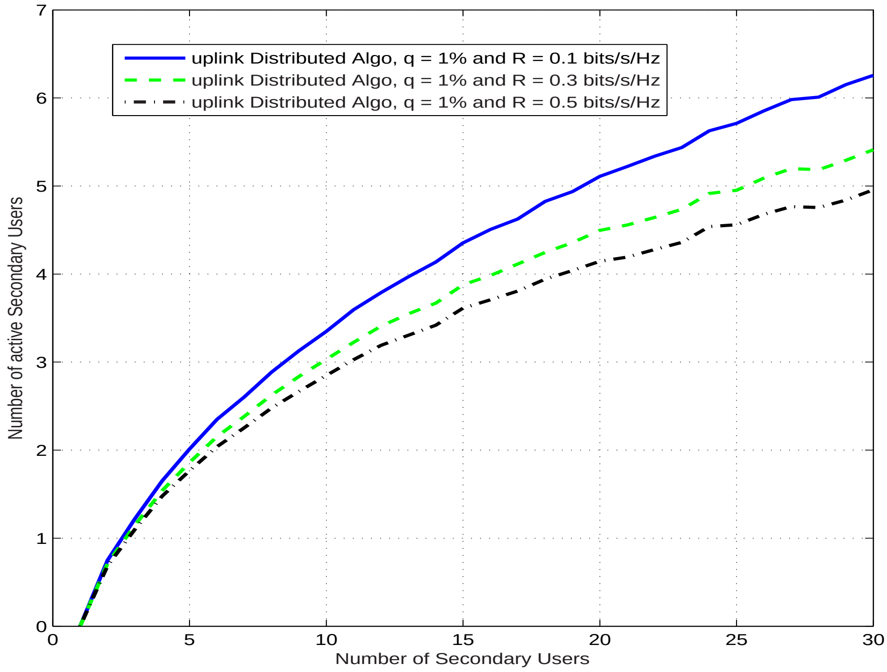

Figure 5.3: Number of active secondary users vs. number of SUs for different rates and outage probability in the uplink.

probability. On the other hand, in the uplink, at a rate $R = 0.1$ bits/s/Hz and an outage probability $q = 1\%$, we get 3 and 5 active SUs for 10 and 20 potential SUs, respectively. Although not shown here, we also remark that, asymptotically, i.e., as the number of SUs goes large, the number of active SUs keeps constant due to the influence of interference impairments on the PU's QoS. This tends to confirm the intuition from formula (5.23) where the number of active SUs is always upper-bounded by $\tilde{M}_{theorie}$.
Figure 5.4 and 5.5 depict the sum secondary user capacity per user in the downlink and the uplink respectively. As expected, it is found that the capacity of the uplink system outperforms that of downlink system. On the other side, increasing the number of SUs yields significantly increase in capacity because the increase in degree of freedom more than compensates for the decrease in SINR due to interference. However, reaching a certain number of SUs, the sum SU capacity per user decreases as the number of SUs increases. Notice here that, as the primary cell radius R and the primary protection area radius R_p decrease, the sum secondary user capacity per user becomes more sensitive to the interference impairments leading to a significant decrease in the sum secondary rate. The current curve claims that in CRN, when one attempts to maximize the number of active SUs, the cognitive capacity degrades asymptotically. Typically, there is a fundamental trade-off between cognitive capacity maximization and number of active SUs maximization.

5.6 Conclusion

In this chapter, we explored the idea of combining multi-user diversity gains with spectral sharing techniques to maximize the secondary user sum rate while maintaining a QoS to a primary user. Both uplink and downlink scenarios are treated. Our contribution within this chapter is two-fold. In the first part of the chapter, we derived a distributed algorithm for power allocation under a cognitive capacity maximization criterion and minimum and peak power constraints. We found out that a secondary user can self-adapt its spectrum assignment to approximate a new optimal assignment in order to maximize the system spectral efficiency. We also investigated the QoS issues from an outage point of view. Both theoretical and simulation results based on a realistic network setting are shown to exhibit interesting features in terms of CRN deployment while maintaining QoS for the primary system by means of outage probability. In particular, we showed that in such CRN, one should make a trade-off between cognitive capacity maximization and number of active SUs maximization.

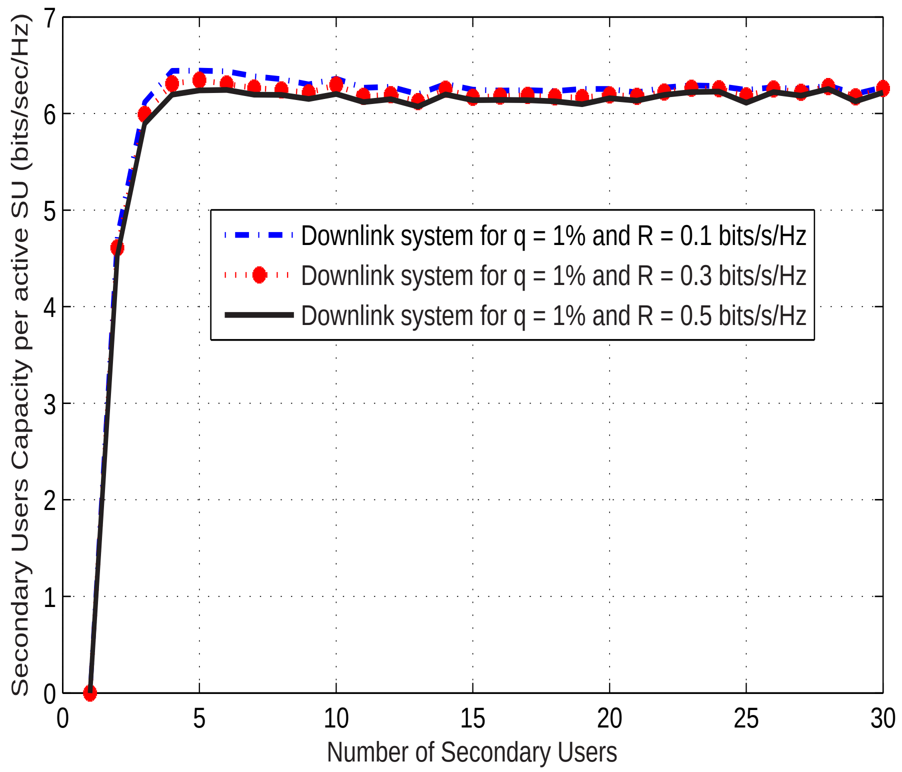

Figure 5.4: Sum secondary user capacity per user vs. number of SUs for different rates and outage probability in the downlink.

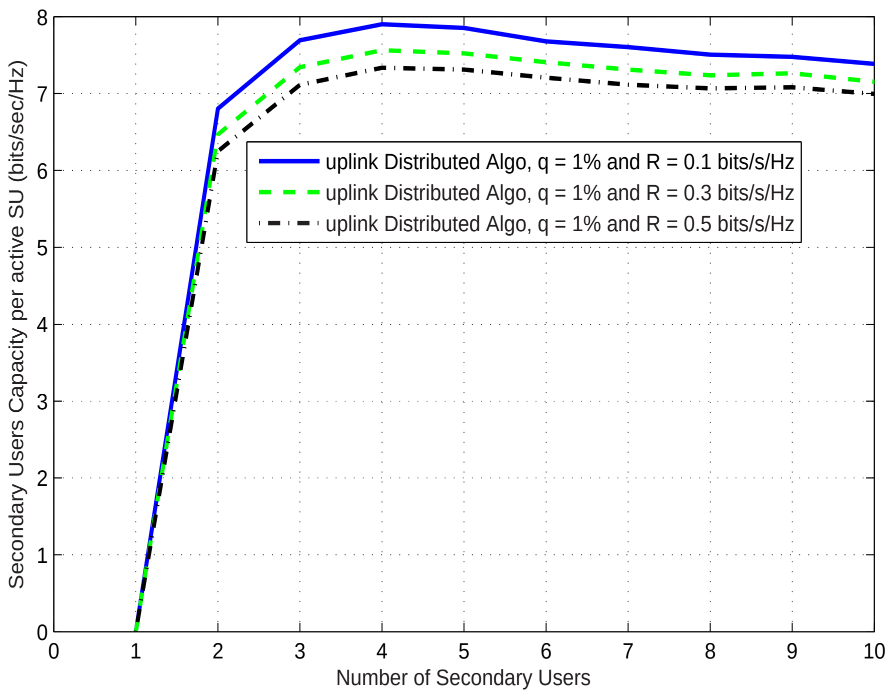

Figure 5.5: Sum secondary user capacity per user vs. number of SUs for different rates and outage probability in the uplink.

Chapter 6

Conclusions and Future Work Directions

In this thesis, we have studied resource allocation techniques in spectrum pooling cognitive radio networks. We have also attempted to define schemes for accessing to the radio spectrum and posing several constraints in the management and in the sharing strategies for such a precious resource. Within this setting, We have considered different system models in which cognitive users compete for a chance to transmit simultaneously or orthogonally with the primary system. On the basis of these models, We have also defined the specific resource allocation problem and offer insights into how to design such scenario in a cognitive radio network environments.

We have initially investigated the problem of orthogonal communication scenarios between the primary system and cognitive users whereby a device transmits over a certain time or frequency band only when no other user does. Typically, we have considered a generic spectrum pooling scenario where users communicate in an orthogonal manner enabling public access to the new spectral ranges without sacrificing the transmission quality of the actual license owners. For the first time, our analysis quantified the achievable gain of using spectrum pooling with respect to classical radio devices in terms of the spectral efficiency as well as the maximum number of possible pairwise communications within such a scenario.

Next, we have extended the cognitive protocol to allow cognitive users to transmit simultaneously with the primary user in the same frequency band as long as the level of interference with the primary user remains within an acceptable range. We have first introduced the notion of the *virtual noise threshold* which represents a proxy for the primary user to allow cognitive user to profit from the primary user resources in an opportunistic manner, and at the same time, to maintain a guarantee of service to the primary user when cognitive communication is considered. The proposed strategy was proved to be the optimal one that achieves the maximum rate for both users under the constraint that the secondary user guarantees a quality of service for the primary user. We have explicitly derived the capacity of the primary as well as the secondary user. Asymptotic analysis shows that (i) the sum system capacity of such a cognitive scenario using a virtual noise threshold as a proxy for the primary user performs always better than classical communication system where the primary user selfishly maximizes its capacity, and (ii) the sum system capacity is maximized for a particular value of the virtual noise threshold, while maintaining a quality of service for the primary user.
Then, we have investigated the problem of joint power allocation and user selection in a CRN consisting of multiple secondary transmitters and receivers communicating simultaneously in the presence of the primary system. Both uplink and downlink scenarios are treated. We have derived a distributed algorithm for power allocation under a cognitive capacity maximization criterion and minimum and peak power constraints. We found out that a secondary user can self-adapt its spectrum assignment to approximate a new optimal assignment in order to maximize the system spectral efficiency. We have also investigated the QoS issues from an outage point of view. As a result, we have showed that in such CRN, one should make a trade-off between cognitive capacity maximization and number of active SUs maximization. Next, we have explored the user selection strategies where centralized and distributed strategies were presented. Both theoretical and simulation results based on a realistic network setting provide substantial throughput gains, thereby illustrating interesting features in terms of CRN deployment while maintaining QoS for the primary system by means of outage probability.

Future Work Directions

Although we have tried to solve some of the problems linked to cognitive radio radio network resource allocation, a number of issues arise as a consequence. The most notable of these, is the notion of cooperation in mesh cognitive radio architecture. Cooperative communication has been known recently as a way to overcome the limitation of wireless systems. In some recent works, the cognitive radios are allowed to cooperate for sensing the spectrum, so that the hidden terminal issues are addressed [14] [15]. Extending these algorithms to mesh networks or developing new algorithms based on this framework would be the next step.
Specifically, for future future work, it would be of great concern to study the spectrum sharing methods where multiple systems coexist and interfere with each other in such systems. Cooperation can be employed for both centralized and decentralized networks separately with different degree of information exchange. Obviously, one of the greatest challenges is to build a radio capable of intelligently finding and handling the available frequency band without any compromise on the primary system quality of service.
Moreover, an important issue in the design of mesh networks is to propose a general theory upon information theory where the constraints of delay/protocol overhead are taken into account in the notion of capacity which also remain to be investigated. Specifically, we still need to provide cross-layer designs for more general classes of communications schemes (typically for slow to highly mobile networks with a smooth transition between the two). Accordingly, it is of major interest to look at the problem of spectral efficiency maximization in a cross-layer point of view. The problem would then be to find the appropriate parameters and the corresponding metrics which would best fit the system model considered (number of secondary users, spectral efficiency, bit-error, primary system quality of service, overhead...). Following the above trend, it is interesting to explore distributed joint resource allocation framework to reduce the feedback information between users and then analyze performances of such schemes with respect to a centralized strategy where the system rely on some form of centralized control to obtain gains at various layers of the communication stack.
Extensions of the problem for multi-primary users with limited CSI are also problems of timely relevance that require further research. Moreover, fairness issues between cognitive users, which have not been taken into account in our work, need to be incorporated in order to provide substantial throughput while satisfying certain QoS constraints between cognitive users.
In order to conclude, we might say that the theoretical limits of cognitive ra-

dio systems are relatively understood nowadays. However, the gap between the current practical schemes and the theoretical limits is still significant, making the design of cognitive radio networks an open and exiting issue. Notably, proposals such as ultra-wide band (UWB) and interference temperature have called into question the validity of the FCCs hierarchy and required reexamination of the source of authority for the FCCs unlicensed spectrum access rules.

Chapter 7

Appendix

APPENDIX 7.1
Optimal Power allocation

Appendix 7.1 *The optimization problem being convex, the optimal solution is computed by applying the Lagrangian. By considering the inequality constraint in (5.15c) as an equality we obtain:*

$$\begin{aligned}\mathbb{L}\left(p_2^i, \lambda, \mu_i\right) &= \frac{1}{N}\sum_{i=1}^{N}\log_2\left(1+\frac{p_2^i \mid h_2^i \mid^2}{p_1^i \mid h_1^i \mid^2 + \sigma^2}\right) - \lambda\left(\frac{1}{N}\sum_{i=1}^{N} p_2^i - 1\right) - \\ &\sum_{i=1}^{N}\mu_i\left(p_2^i \mid h_2^i \mid^2 + \sigma^2 - \sigma_v^2\right);\end{aligned} \tag{7.1}$$

By differentiating the Lagrangian with respect to p_2^{i*} and setting the derivative equal to zero, we get:

$$\frac{\partial \mathbb{L}\left(p_2^{i*}\right)}{\partial p_2^{i*}} = \frac{1}{N}\frac{1}{\ln(2)}\frac{\mid h_2^i \mid^2}{p_1^i \mid h_1^i \mid^2 + p_2^{i*} \mid h_2^i \mid^2 + \sigma^2} - \frac{\lambda}{N} - \mu_i \mid h_2^i \mid^2 = 0 \tag{7.2}$$

Solving for p_2^{i*} in equation (7.2) leads to the solution[1]*:*

$$p_2^{i*} = \frac{1}{\lambda + N\mu_i \mid h_2^i \mid^2} - \frac{p_1^i \mid h_1^i \mid^2 + \sigma 2}{\mid h_2^i \mid^2}; \qquad \forall \ i \tag{7.3}$$

[1]For the sake of simplicity, we will omit the term in $\frac{1}{\ln(2)}$ for the rest of our calculations.

The constraints in (4.11) lead to the following Karush-Kuhn-Tucker (KKT) conditions as in [40]:

$$\begin{cases} \mu_i^* \left(p_2^{i*} \mid h_2^i \mid^2 + \sigma^2 - \sigma_v^2\right) = 0 & \forall \; i \\ \\ \mu_i^* \geq 0 \end{cases} \tag{7.4}$$

From equation (7.3), we obtain:

$$\lambda^* = \frac{\mid h_2^i \mid^2}{p_2^{i*} \mid h_2^i \mid^2 + p_1^{i*} \mid h_1^i \mid^2 + \sigma^2} - N\mu_i^* \mid h_2^i \mid^2 \tag{7.5}$$

Following equation (7.5), if $\lambda^* \leq \dfrac{\mid h_2^i \mid^2}{p_1^{i*} \mid h_1^i \mid^2 + \sigma_v^2}$ *we obtain* $\mu_i^* > 0$ *leading by considering KKT condition in (7.4) to the following solution:*

$$p_2^{i*} = \frac{\sigma_v^2 - \sigma^2}{\left|h_2^i\right|^2} \qquad \forall \; i \tag{7.6}$$

Let us now consider the case where $\lambda^* > \dfrac{\mid h_2^i \mid^2}{p_1^{i*} \mid h_1^i \mid^2 + \sigma_v^2}$.

This means that we have $\mu_i^* = 0$ *leading to the following solution:*

$$p_2^{i*} = \left(\frac{1}{\lambda^*} - \frac{p_1^i \mid h_1^i \mid^2 + \sigma^2}{\left|h_2^i\right|^2}\right)^+ ; \qquad \text{if} \qquad \lambda^* > \frac{\left|h_2^i\right|^2}{p_1^i \mid h_1^i \mid^2 + \sigma_v^2} \qquad \forall \; i \tag{7.7}$$

This concludes the first part of the proof.

The Second part of the theorem comes from a direct derivation by writing the secondary user power allocation as:

$$p_2^{i*} = \frac{\sigma_v^2 - \sigma^2}{\mid h_2^i \mid^2} - \left[\frac{\sigma_v^2 - \sigma 2}{\mid h_2^i \mid^2} - \left(\frac{1}{\lambda^*} - \frac{p_1^i \mid h_1^i \mid^2 + \sigma^2}{\left|h_2^i\right|^2}\right)^+\right]^+ \tag{7.8}$$

By taking the sum over the N sub-bands and by considering the average power constraint in (5.15a), we get the desired result. ■

APPENDIX 7.2
PROOF OF THEOREM 4.2

Appendix 7.2 *By replacing by the optimal power constraint p_2^*, we get the piecewise continuous functions on the three regions with respect to λ:*

$$1+\frac{p_2^{i*}\left|h_2^i\right|^2}{p_1^i \mid h_1^i \mid^2 + \sigma^2} = \begin{cases} 1+\dfrac{\sigma_v^2-\sigma^2}{p_1^i \mid h_1^i \mid^2 + \sigma^2}; & \text{if } \lambda \leq \dfrac{\left|h_2^i\right|^2}{p_1^i \mid h_1^i \mid^2 + \sigma_v^2} \\ \dfrac{\left|h_2^i\right|^2}{\lambda\left(p_1^i \mid h_1^i \mid^2 + \sigma^2\right)}; & \text{if } \dfrac{\left|h_2^i\right|^2}{p_1^i \mid h_1^i \mid^2 + \sigma_v^2} \leq \lambda \leq \dfrac{\left|h_2^i\right|^2}{p_1^i \mid h_1^i \mid^2 + \sigma^2} \\ 1; & \text{otherwise} \end{cases} \tag{7.9}$$

For the sake of simplicity, let us define the following variables:

$$\begin{cases} \alpha = \dfrac{\sigma_v^2-\sigma^2}{p_1^i \mid h_1^i \mid^2 + \sigma^2}, \\ \beta = \dfrac{\left|h_2^i\right|^2}{p_1^i \mid h_1^i \mid^2 + \sigma_v^2}, \\ \gamma = \dfrac{\left|h_2^i\right|^2}{p_1^i \mid h_1^i \mid^2 + \sigma^2}. \end{cases}$$

Equation (7.9) becomes then:

$$1+\frac{p_2^{i*}\left|h_2^i\right|^2}{p_1^i \mid h_1^i \mid^2 + \sigma^2} = \begin{cases} T_1 = 1+\alpha; & \text{if } \lambda \leq \beta \\ T_2 = \dfrac{\gamma}{\lambda}; & \text{if } \beta \leq \lambda \leq \gamma \\ 1; & \text{otherwise} \end{cases} \tag{7.10}$$

Let us now analyze the variation of the above function with respect to the three regions as function of λ. By computing the quantity ($T_1 - T_2$), we find that:

$$T_1 - T_2 = \frac{\gamma}{\beta} - \frac{\gamma}{\lambda} \tag{7.11}$$

- *In the first region, we have*

$$\left.\begin{array}{r} \text{if} \quad \lambda \leq \beta \Rightarrow \quad T_1 \leq T_2 \\ \\ \textit{and it is clear that} \;\; T_1 \geq 1 \end{array}\right\} \quad \Rightarrow \max\left\{\min\left[T_1, T_2(\lambda)\right], 1\right\} = T_1 \tag{7.12}$$

- *In the second region, we have*

$$\left.\begin{array}{r} \text{if} \quad \lambda \geq \beta \quad \Rightarrow \quad T_2 \leq T_1 \\ \\ \text{if} \quad \lambda \leq \gamma \quad \Rightarrow \quad T_2 \geq 1 \end{array}\right\} \quad \Rightarrow \lambda\left\{\min\left[T_1, T_2(\lambda)\right], 1\right\} = T_2 \tag{7.13}$$

- *In the third region, we have*

$$\left.\begin{array}{r} \text{if} \quad \lambda \geq \beta \quad \Rightarrow \quad T_2 \leq T_1 \\ \\ \text{if} \quad \lambda \geq \gamma \Rightarrow \quad T_2 \leq 1 \end{array}\right\} \quad \Rightarrow \max\left\{\min\left[T_1, T_2(\lambda)\right], 1\right\} = 1 \tag{7.14}$$

By comparing (7.10) with the results corresponding to each region defined above, we establish the desired result. ■

APPENDIX 7.3
PROOF OF THEOREM 4.3

Appendix 7.3 *Given a virtual noise level, we will study asymptotical performances of such a system in terms of the sum capacity when N is assumed to be infinite. Let us first compute the expression of the sum capacity by making $N \rightarrow \infty$ when cognitive communications are possible. The expression in (4.9) becomes*

$$C_{sum,\infty} = C_{1,\infty} + C_{2,\infty} \tag{7.15}$$

Let us first study achievable performances of each user. By making $N \to \infty$, the primary user capacity in (4.1) becomes

$$C_{1,\infty} = \int_0^{\infty} \log_2 \left(1 + \frac{p_1(t).t}{\sigma_v^2}\right).f(t)dt \tag{7.16}$$

where γ_0 is the Lagrange's multiplier satisfying the average power constraint, namely:

$$\frac{1}{\gamma_0}\int_{\gamma_0.\sigma_v^2}^{+\infty} f(t)dt - \sigma_v^2 \int_{\gamma_0.\sigma_v^2}^{+\infty} \frac{f(t)}{t}dt = 1 \tag{7.17}$$

Capacity of the primary user in (4.1) becomes:

$$\begin{aligned}
C_{1,\infty} &= \int_0^{\infty} \log_2 \left(1 + \frac{p_1(t).t}{\sigma_v^2}\right).f(t)dt \\
&= \int_{\gamma_0\sigma_v^2}^{\infty} \log_2 \left(1 + \frac{\left(\frac{1}{\gamma_0} - \frac{\sigma_v^2}{t}\right).t}{\sigma_v^2}\right).f(t)dt \\
&= \int_{\gamma_0\sigma_v^2}^{\infty} \log_2 \left(\frac{t}{\gamma_0.\sigma_v^2}\right).f(t)dt
\end{aligned}$$

Similarly, we compute the capacity of the secondary user

$$
\begin{aligned}
C_{2,\infty} &= \int_0^{\infty} \log_2\left(1+\frac{p_2(t).t}{\sigma^2}\right).f(t)dt \\
&= \int_{\lambda'\sigma^2}^{\lambda'\sigma_v^2} \log_2\left(1+\frac{\left(\frac{1}{\lambda'}-\frac{\sigma^2}{t}\right).t}{\sigma^2}\right).f(t)dt + \\
& \int_{\lambda'\sigma_v^2}^{\infty} \log_2\left(1+\frac{\left(\frac{\sigma_v^2-\sigma^2}{t}\right).t}{\sigma^2}\right).f(t)dt \\
&= \int_{\lambda'\sigma^2}^{\lambda'\sigma_v^2} \log_2\left(\frac{t}{\lambda'\sigma^2}\right).f(t)dt + \\
& \int_{\lambda'\sigma_v^2}^{\infty} \underbrace{\log_2\left(\frac{\sigma_v^2}{\sigma^2}\right)}_{\geq 0}.f(t)dt
\end{aligned}
$$

Let us now compute the sum capacity of a system where the primary user decides to maximize its rate selfishly. In other words, it will water-fill over the ambient noise level σ^2 and no resources will be left for potential cognitive users.

$$
\begin{aligned}
C'_{sum,\infty} &= \int_0^{\infty} \log_2\left(1+\frac{p_1(t).t}{\sigma^2}\right).f(t)dt \qquad (7.18)\\
&= \int_{\gamma_0'\sigma^2}^{\infty} \log_2\left(\frac{t}{\gamma_0'.\sigma^2}\right).f(t)dt.
\end{aligned}
$$

where γ_0' is the Lagrange's multiplier satisfying the average power constraint on σ^2. Now, in order to compute the difference between the sum capacity in the two configurations, let us compare the following difference

$$
\begin{aligned}
C_{sum,\infty} - C'_{sum,\infty} &\geq C_{1,\infty} - C'_{sum,\infty} \\
&= \int_{\gamma_0\sigma_v^2}^{\gamma_0'\sigma_v^2} \log_2\left(\frac{t}{\gamma_0.\sigma_v^2}\right).f(t)dt + \Theta
\end{aligned}
$$

where:

$$\Theta = \int_{\gamma_0 \sigma_v^2}^{\infty} \log_2\left(\frac{t}{\gamma_0 . \sigma_v^2}\right) . f(t) dt - \int_{\gamma'_0 \sigma^2}^{\infty} \log_2\left(\frac{t}{\gamma'_0 . \sigma^2}\right) . f(t) dt$$

Therefore, we have just to show that Θ *is positive. Numerical root finding is needed to determine different values of* γ_0 *and* γ'_0*. Our numerical results show that, as long as condition (4.18) is satisfied, we have* $\gamma_0 . \sigma_v^2 \leq \gamma'_0 . \sigma^2$ *with probability one. We then obtain*

$$\begin{aligned} \Theta \ &= \int_{\gamma_0 \sigma_v^2}^{\gamma'_0 \sigma^2} \log_2\left(\frac{t}{\gamma_0 . \sigma_v^2}\right) . f(t) dt + \\ &\int_{\gamma'_0 \sigma^2}^{\infty} \underbrace{\left[\log_2\left(\frac{t}{\gamma_0 . \sigma_v^2}\right) - \log_2\left(\frac{t}{\gamma'_0 . \sigma^2}\right)\right]}_{\log_2\left(\frac{\gamma'_0 . \sigma^2}{\gamma_0 . \sigma_v^2}\right)} . f(t) dt \end{aligned}$$

Thus, under our assumptions, Θ *is always positive and the sum capacity of cognitive system that considers virtual noise threshold performs always better than for traditional systems.* ■

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