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Simple Power–Controlled Modulation Schemes for Fixed–Rate Transmission over Fading Channels

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Abstract

This paper examines power–controlled modulation schemes for *fixed–rate* transmission over fading radio channels. We are primarily interested in transmitter diversity schemes for multiple–block (slot) transmission on narrowband *time–division duplex* systems such as DECT and PHS. We consider transmitter power control over one and two independently–faded blocks using variable (coded) modulation schemes. The main conclusion is that simple and *de facto* standard coded–modulation schemes can yield performance comparable to a non–fading AWGN channel at a fixed data rate.

1 Introduction

The purpose of this paper is to show the benefits of exploiting channel state feedback for *fixed–rate* data transmission over fading radio channels. We are mostly interested in illustrating this with some practical examples. The work of Goldsmith [1] has shown that channel state feedback can be used by the transmitter to reliably transmit at a high spectral efficiency under long–term power constraints on an ergodic fading channel. This can be achieved in practice by a combined variable–rate, variable–power coded–modulation scheme [2]. Equivalently, one may opt to spread the coded bits across a time interval (for example with the use of a bit–interleaver) over which the channel has varied greatly. Caire *et al.* have shown in [3] that traditional error control codes can be used effectively with large symbol alphabets to achieve low bit error rates (BER).

Roughly speaking these two techniques are equivalent, in the sense that they both exploit the ergodicity of the channel process. Both allow for information rates approaching channel capacity, depending on the desired BER. The amount of time needed for the average rate to be attained in the variable–rate scheme will be on the order of the interleaving delay required to achieve a high diversity order in the fixed–rate scheme.

In many systems (e.g. voice telephony), fixed rate transmission with small decoding delay is required. Here, we are not at liberty to exploit the ergodicity of the channel. Furthermore, in wireless loop applications [4], the channels are often static which renders ergodic channel assumptions completely meaningless. We may, however, perform partial averaging by coding across variations in the frequency-domain using either a slow-frequency hopping mechanism, spread-spectrum or multitone signalling. In such instances, Ozarow *et al.* have demonstrated that the fundamental performance is properly defined by an *information outage probability*, $P_{\mathrm{out}}(R)$ which is a function of the fixed code rate R, and that the channel capacity is zero, when no channel state feedback is available at the transmitter. For static fading systems where coded data is interleaved over a small number of independent blocks, Knopp and Humblet have shown in [5] that practical coding schemes can achieve block error rates approaching $P_{\mathrm{out}}(R)$, which can be seen as a practical lower–bound on the achievable probability of block error.

In a recent study by Caire *et al.*[6] it was shown that channel state feedback can be used effectively to reduce or drive $P_{\mathrm{out}}(R)$ to zero. The latter is typically the case when the fading process has more than one degree of freedom which allows for a non–zero channel capacity. It is shown that for transmission across two static faded blocks with independent fading levels, a loss in signal–to–noise ratio (SNR) on the order of 5dB in Rayleigh fading can be expected with respect to a non–fading AWGN channel. Here we build upon this work and give a few examples of simple power–controlled modulation schemes for transmission across one and two independently–faded blocks which essentially remove the effect of signal fading, under the assumption of a long–term power constraint.

2 System model

Consider the transmission of F complex sequences $\{c_{f,i}\},\quad f = 1,\cdots,F$ across F independent static fading channels, with channel strengths $\{\alpha_f\},\quad f = 1,\cdots,F$. The α_f are assumed independent and identically distributed with probability density and distribution functions denoted by $f_\alpha(u)$ and $F_\alpha(u)$ respectively. Each $c_{f,i}$ is assumed to be drawn independently from some finite variable-size constellation with $M_f(\boldsymbol{\alpha})$ signal points, where $\boldsymbol{\alpha} = \begin{pmatrix}\alpha_1 & \cdots & \alpha_F\end{pmatrix}$. We assume narrowband channels so that we may use a discrete–time channel model. This would correspond to multiple frequency-slot transmission in a system like DECT[7] or PHS[8]. The transmitted signal energy per symbol is denoted $\mathcal{E}_s$. We assume a two–way system where the transmitter has perfect knowledge of the $\{\alpha_f\}$, which can be achieved either by a feedback path via the opposite link or by power measurement of the opposite link in a *time–division duplex*(TDD) system. This is already employed to a certain extent in DECT and PHS. Using this information, the transmitter adjusts its power in each block by the power controller $\mathcal{P}_f(\boldsymbol{\alpha}),\ f = 1, \cdots, F$. The signal at the receiver is given by

$$r_{f,i} = \sqrt{\alpha_f \mathcal{P}_f(\boldsymbol{\alpha})\mathcal{E}_s} e^{j\phi_f} c_{f,i} + z_{f,i} \tag{1}$$

where ϕ_f is the uniformly–distributed random phase induced by channel f, and $z_{f,i}$ is a zero–mean complex circular–symmetric Gaussian random variable with variance N_0. We approximate the BER in each block by

$$\mathrm{BER}_f(\boldsymbol{\alpha}) = N_f(\boldsymbol{\alpha})\mathbf{Q}\left(\sqrt{.5\mathcal{P}_f(\boldsymbol{\alpha})\alpha_f d_f^2(\boldsymbol{\alpha}) R\gamma_b}\right), \tag{2}$$

where $N_f(\boldsymbol{\alpha})$ is a factor taking into account the number of nearest neighbours for the code (constellation) used in block f and average number of bit errors for the most likely error patterns. The $d_f^2(\boldsymbol{\alpha})$ is the minimum square Euclidean distance for the code (constellation) used in block f. R is the information rate per block in bits/symbol and $\gamma_b = \mathcal{E}_s/RN_0$ is the signal–to–noise ratio per information bit. Note the dependence of the code parameters on the channel strengths.

3 Transmission over one Block ($F = 1$)

We begin with transmission over one block, and choose a power controller of the form

$$\mathcal{P}(\alpha) = \begin{cases} K/\alpha & \alpha > \alpha_{\mathrm{T}} \\ 0 & \alpha \leq \alpha_{\mathrm{T}} \end{cases} \tag{3}$$

where K is a constant chosen such that the average transmit power is unity and is given by $K = \left(\int_{\alpha_{\mathrm{T}}}^{\infty} f_\alpha(u)du/u\right)^{-1}$. In unit-mean Rayleigh fading (i.e. $f_\alpha(u) = e^{-u}$) this yields $K = 1/\mathrm{E}_1(\alpha_{\mathrm{T}})$, where $\mathrm{E}_1(x) = \int_x^{\infty} e^{-u}du/u$ is the first–order exponential integral [9] . We see that for non–zero K we require $\alpha_{\mathrm{T}} > 0$. Here we have introduced an outage event which we control with the cutoff level α_{T}, which is in keeping with the results of [6]. When transmitting, the received power is kept constant, so we effectively have a non–fading channel and any traditional coding techniques can be applied. This type of power control was refered to as *truncated channel inversion* by Goldsmith [1], and is optimal in the sense of minimizing the information outage probability [6]. We may now write the BER as

$$\mathrm{BER} = .5F_\alpha(\alpha_{\mathrm{T}}) + (1 - F_\alpha(\alpha_{\mathrm{T}}))N\mathrm{Q}\left(\sqrt{.5KRd^2\gamma_b}\right) \tag{4}$$

under the assumption that when $\alpha \leq \alpha_{\mathrm{T}}$ the receiver chooses bits at random. This quantity can be minimized numerically by varying α_{T}. We show the minimum BER for 2 bits/symbol using uncoded QPSK transmission ($N = 1,\ d^2 = 2$) and Ungerböck 8-PSK TCM schemes [10] in Figure 1. With these simple schemes we lose 6-8 dB (at BER=10^{-3}) with respect to uncoded QPSK on a non-fading channel.

4 Transmission over $F = 2$ Blocks

The power controllers and codes (constellations) are now chosen such that

1. The instantaneous information rate per block is $R = .5(R_1 + R_2)$ bits/symbol
2. $\mathrm{E}\left[\mathcal{P}_1(\boldsymbol{\alpha}) + \mathcal{P}_2(\boldsymbol{\alpha})\right] = 2$
3. $\mathrm{BER}_f(\boldsymbol{\alpha}) = \mathrm{BER},\ \forall f\ \text{ s.t. }\ \mathcal{P}_f(\boldsymbol{\alpha}) > 0$

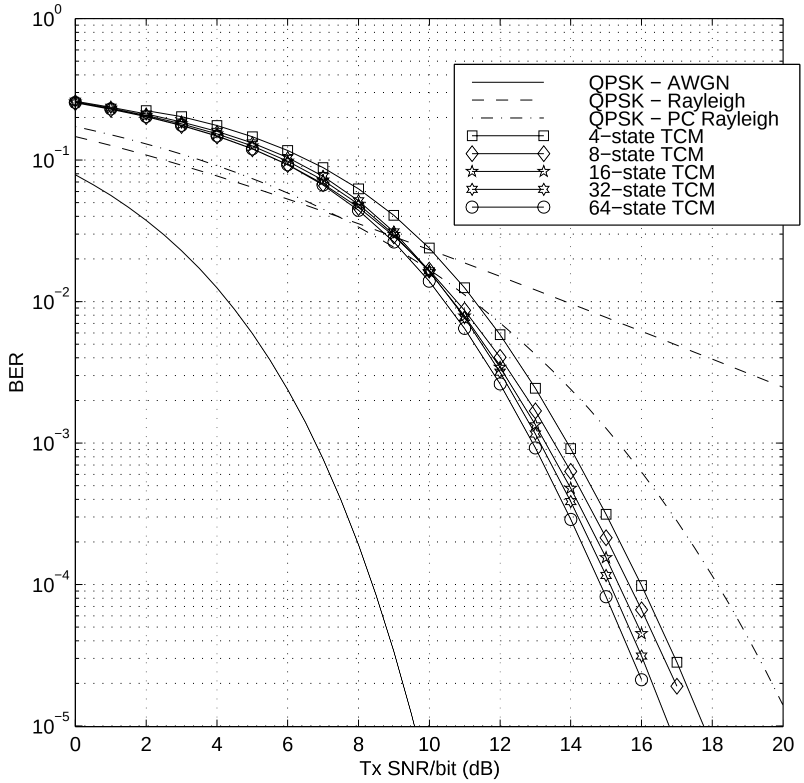

Figure 1: Transmission schemes for $F = 1$ block at 2 bits/symbol

The first constraint assures that we always transmit at a fixed information rate. The second is a long–term average power constraint. By making the BER constant in each block to which non–zero power is allocated, we are again able to employ coding schemes designed for non–fading AWGN channels, rather than specially designed codes such as [5]. Note that we no longer have an outage region, which is now possible due to the diversity introduced by multiple blocks. A pragmatic power control scheme inspired by the optimal schemes in [6] is shown for $F = 2$ blocks in Figure 2. We divide the (α_1, α_2) plane into 3 regions bordered by the horizontal and vertical axes and the lines $\alpha_1 = K_{\mathrm{T}}\alpha_2$ and $\alpha_1 = \alpha_2/K_{\mathrm{T}}$, where $0 < K_{\mathrm{T}} \leq 1$ is a constant to be determined. For the 2 regions where one block strength (say $\alpha_{\max}$) is $1/K_{\mathrm{T}}$ times stronger than the other we transmit with power $K_1/\alpha_{\max}$ in that block with a code of rate $2R$ bits/symbol. In the block with the weaker channel strength, we do not transmit at all. When the channel strengths of the two blocks are close to equal, we transmit with powers K_2/α_f in both with a code of rate R bits/symbol. Note that the degerate case with $K_{\mathrm{T}} = 1$ corresponds to choosing the strongest block (i.e. selection diversity at the transmission end.)

From the third constraint above, K_1 and K_2 are related by the equation

$$N_1 \mathbf{Q}\left(\sqrt{.5K_1 d_1^2 R\gamma_b}\right) = N_2 \mathbf{Q}\left(\sqrt{.5K_2 d_2^2 R\gamma_b}\right)$$

. If we are interested only in asymptotic equality (i.e. for high SNR) this reduces to $K_2 = \frac{d_1^2}{d_2^2}K_1$. The long–term power constraint imposes that K_1 and K_2 satisfy

$$1 = \int_0^\infty \left[K_2 \int_{K_{\mathrm{T}}\alpha_2}^{\alpha_2/K_{\mathrm{T}}} f(\alpha_1)\frac{d\alpha_1}{\alpha_1} + K_1 \int_{\alpha_2/K_{\mathrm{T}}}^{\infty} f(\alpha_1)\frac{d\alpha_1}{\alpha_1}\right] f(\alpha_2)d\alpha_2 \qquad (5)$$

Noting that $\int_0^\infty \mathrm{E}_1(\beta x)e^{-x}dx = \log(1 + 1/\beta)$ we have in unit–mean Rayleigh fading that $K_1 \log(1 + K_{\mathrm{T}}) - K_2 \log(K_{\mathrm{T}}) = 1$. The optimal value for K_{T} which maximizes K_1,K_2 (and minimizes BER) can

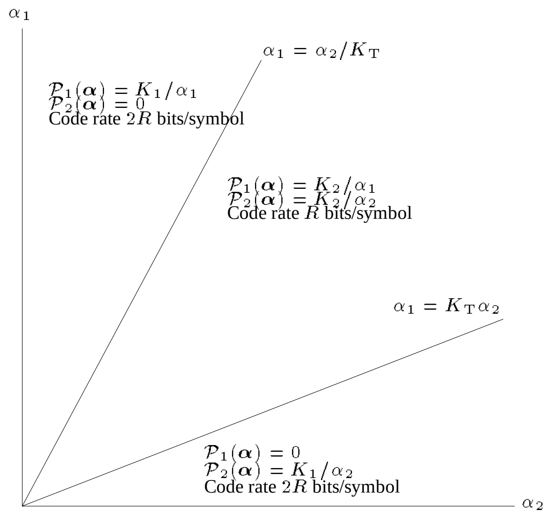

Figure 2: Two–block Power Control Scheme

be found numerically. For asymptotic equality of the BER in the two transmission modes we can neglect the multiplicative terms in front of the Q-function to obtain the maximum K_1,K_2 at $K_{\rm T} = d_1^2/(d_2^2 - d_1^2)$, if $d_2^2 \geq 2d_1^2$. Otherwise, $K_{\rm T} = 1$, which yields $K_1 = 1/\log 2$.

A few examples of schemes with a transmission rate of 2 bits/symbol and minimum BER are considered in Figure 3. We see that both variable modulation (using two modulation schemes) and selection diversity (using only one modulation scheme) effectively remove the effects of signal fading with only 2 independently faded blocks (at BER=10^{-3} we fall 2-5 dB from QPSK on a non–fading channel.) This is completely in keeping with the results of [6], and, as expected, the more general variable modulation scheme significantly outperforms selection diversity. The TCM schemes are standard Ungerböck codes [10] with 32QAM and 8PSK with the number of states chosen such that the trellis complexity is the same in both transmission modes. The performance curves for the TCM examples were found by computer simulation.

This two–block transmission scheme has an advantage in interference–limited (e.g. cellular) systems, since in a slotted system, some slots will remain empty a significant portion of the time, which will reduce interference levels. Alternately, in a multiuser setting, the unused slots can be occupied by other users.

5 Conclusions and Future Work

In this letter we have presented a few examples of simple power–controlled *fixed–rate* modulation schemes for slowly–fading radio channels. This type of signaling could prove to be beneficial in a multi–slot (narrowband/frequency-flat) transmission mode of a TDD system such as DECT or PHS. We have shown that with very simple *off–the–shelf* error control codes and ideal channel state feedback, we can expect error–rate performances comparable to those of non–fading AWGN channels. This is made possible by using transmitter diversity in conjunction with power control. Near–perfect channel state information should be reasonably simple to achieve in a slowly fading TDD scenario.

We only considered Rayleigh distributed fading statistics, since this represents a worst case setting, and therefore expect even more promising results for indoor or rural communications which exhibit a strong deterministic component. Schemes such as this could therefore be interesting in high-speed wireless LAN and wireless loop applications. Moreover, the use of additional antenna diversity (space/polarization) with power control will provide even more performance gains. Similar ideas can also be applied in slotted(orthogonal) multiuser systems, in conjunction with channel measurement based

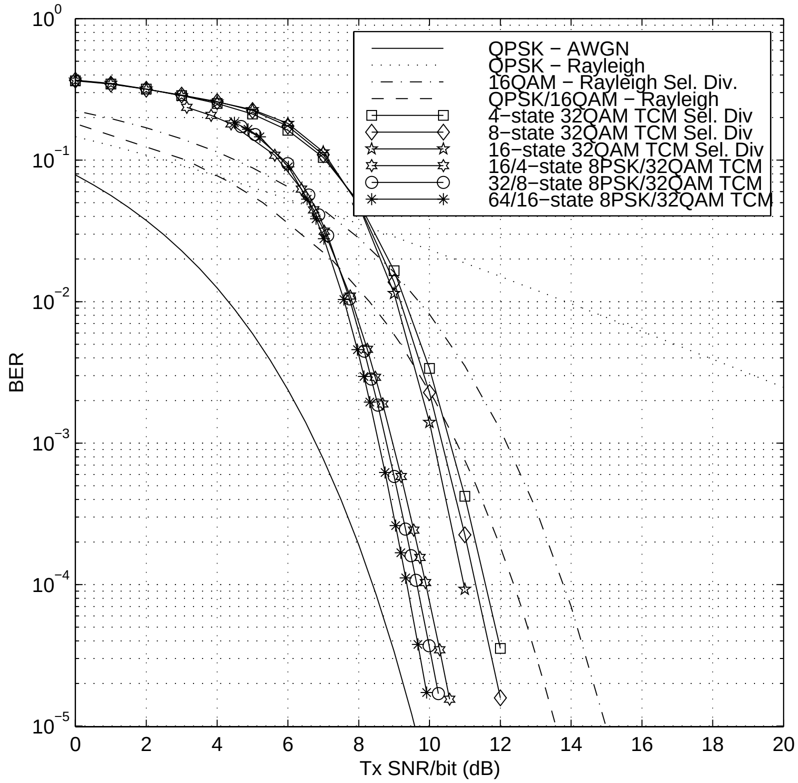

Figure 3: Transmission schemes for $F = 2$ blocks at 2 bits/symbol

allocation schemes. These are topics of our ongoing research.

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