Direct Location Estimation for MIMO Systems in Multipath Environments

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***Abstract*—Location estimation in Multipath Environments (ME) can lead to significantly higher accuracy if the information contained in the Non-Line-of-Sight (NLOS) signal components is exploited thoroughly with the use of multiple antennas at both sides of a communication system and with the aid of appropriate channel modeling. In this contribution, this information is exploited and a Maximum-Likelihood (ML) estimator for the location of the Mobile Terminal is proposed, which is based on the reception of the signal in only one Base Station (BS). The method described herein can be characterized as direct location estimation since, in contrast to the traditional two-step approaches, the solution is formulated in terms of the received signal and not some channel-dependent parameters whose values have been estimated a-priori. The high accuracy of the method is validated using the Cramer-Rao Bound (CRB). Intuitive conclusions are reached through the comparison of different systems and for different propagation environments, which is performed by simulations [1].**

I. Introduction

Multipath and NLOS propagation are considered to be the main sources of inaccuracies for network-based geometrical localization techniques [1], [2]. Moreover hearability, i.e. the ability of reception of the MT's transmitted signal at a sufficient - for localizing - number of BSs, has also been a major concern [3]. The Direct Location Estimation (DLE), which is presented in the following sections, results in a really low positioning error, under any realistic propagation environment, whether that is a multipath, if a LOS signal component exists, or a strictly NLOS environment and in scenarios for which the transmitted signal is received in not more than one BS. It achieves high accuracy by taking advantage of the information contained in all signal components.

Identifiability of a Mobile Terminal's (MT) location in a strictly NLOS environment is feasible if the number of information sources due to the NLOS signal components is larger than the number of the parameters that need to be estimated. In a similar fashion, in a Multipath environment with the location being identifiable solely by using information contained in the LOS component, an improvement in performance is to be expected, when the number of information sources due to the NLOS signal components is larger than the number of the newly introduced nuisance parameters that need to be jointly estimated [4]. Multiple-Input Multiple-Output (MIMO) systems have a tremendous advantage over other systems in meeting this condition due to the fact that in a MIMO communications system the channel matrix depends also on the Angles of Arrival (AOA) and the Angles of Departure (AOD), among other parameters and these two sets of parameters contain information about the MT's position.

In order to express the AOA and the AOD, along with other channel-dependent parameters like the delays and the Doppler shifts, as a function of the MT's coordinates, an appropriate geometrical representation of the propagation environment is required. To that end, we based our method on the Single-Bounce Model (SBM) which has been employed in Localization techniques in [5]. It is due to this widely acceptable channel model, that we were able to create the mapping needed to implement the DLE method. DLE was initially proposed by the authors of [6], [7], who also showed the superiority of its performance at the low and moderate SNR regime and/or with short data records. Since in our analysis we consider a dynamic channel, which varies rapidly due to the movement of the MT and since in wireless communications high SNR is not always guaranteed, DLE could serve as an alternative approach that can still perform close to the CRB.

Notation: Throughout the paper, upper case and lower case boldface symbols will represent matrices and column vectors respectively. $(\cdot)^t$ will denote the transpose, $(\cdot)^*$ the conjugate and $(\cdot)^\dagger$ the conjugate transpose of any vector or matrix. For a $M \times N$ matrix $\mathbf{A} = [\mathbf{a}_1, \ldots, \mathbf{a}_N]$, $vec(\mathbf{A}) = [\mathbf{a}_1^t, \ldots, \mathbf{a}_N^t]^t$ is a vector of length MN. For a square $M \times M$ matrix $\mathbf{A}$, $diag(\mathbf{A})$ is a $M \times 1$ vector composed from its diagonal entries $a_{ii}, 1 \leq i \leq M$, while for a $M \times 1$ vector $\mathbf{a} = [a_1, \ldots, a_M]^t$, $diag(\mathbf{a})$ is an $M \times M$ diagonal matrix with $\mathbf{a}$'s entries along it's main diagonal. $[\mathbf{A}]_{(k:l,m:n)}$ is a submatrix $\mathbf{A}$ containing the common elements of rows $k, \ldots, l$ and columns $m, \ldots, n$. The symbols $\otimes$, $\boxtimes$ and $\odot$ denote the Kronecker, Khatri-Rao (column-wise Kronecker) and Hadamard product respectively. Finally we have used the indicator function, defined as:

$$\mathbf{1}_{\mathbb{A}}(i') \triangleq \begin{cases} 1, & i' \in \mathbb{A} \\ 0, & i' \notin \mathbb{A} \end{cases} \tag{1}$$

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II. Channel Model

A. Geometrical Representation

The SBM can describe accurately the NLOS paths of a multipath propagation environment. It is a valuable tool for localization techniques since it leads to simple equations through which one can express channel-dependent parameters, such as the AOA, the AOD, the delays and the Doppler shifts, as a function of the MT's and the scatterer's coordinates. Albeit simple, it enjoys a wide applicability, beacuse in a wireless propagation environment, the more bounces of a signal, the larger the attenuation will be, not only because the scatterer absorbs some of the energy but also because more bounces usually implies a longer path length. Thus if a limited number of NLOS signal components with non-negligible energy arrive at the receiver, it is reasonable to assume that they have bounced only once.

With respect to figure 1, assuming linear movement of the MT for the very short period of measurements and using the subscript li for the parameters at time instant t_l, $1 \leq l < N_t$ and corresponding to path (or scatterer) s_i, $1 \leq i \leq N_s$, the parameters of the SBM are given by:

$$\phi_{li} = \begin{cases} \tan^{-1} \frac{y_{s_i} - (y_1 + \upsilon_y dt_{l1})}{x_{s_i} - (x_1 + \upsilon_x dt_{l1})} & , \frac{y_{s_i} - (y_1 + \upsilon_y dt_{l1})}{x_{s_i} - (x_1 + \upsilon_x dt_{l0})} > 0 \\ \pi + \tan^{-1} \frac{y_{s_i} - (y_1 + \upsilon_y dt_{l1})}{x_{s_i} - (x_1 + \upsilon_x dt_{l1})} & , \frac{y_{s_i} - (y_1 + \upsilon_y dt_{l1})}{x_{s_i} - (x_1 + \upsilon_x dt_{l1})} < 0 \end{cases} \tag{2}$$

$$\psi_{li} = \psi_i = \begin{cases} \tan^{-1} \frac{y_{s_i} - y_{BS}}{x_{s_i} - x_{BS}} & , \frac{y_{s_i} - y_{BS}}{x_{s_i} - x_{BS}} > 0 \\ \pi + \tan^{-1} \frac{y_{s_i} - y_{BS}}{x_{s_i} - x_{BS}} & , \frac{y_{s_i} - y_{BS}}{x_{s_i} - x_{BS}} < 0 \end{cases} \tag{3}$$

$$\begin{aligned} \tau_{li} = & \frac{1}{c} \sqrt{(y_{s_i} - (y_1 + \upsilon_y dt_{l1}))^2 + (x_{s_i} - (x_1 + \upsilon_x dt_{l1}))^2} \\ & + \sqrt{(y_{s_i} - y_{BS})^2 + (x_{s_i} - x_{BS})^2} \end{aligned} \tag{4}$$

$$f_{d,li} = \frac{f_c}{c} \frac{\upsilon_x (x_{s_i} - (x_1 + \upsilon_x dt_{l1})) + \upsilon_y (y_{s_i} - (y_1 + \upsilon_y dt_{l1}))}{\sqrt{(y_{s_i} - (y_1 + \upsilon_y dt_{l1}))^2 + (x_{s_i} - (x_1 + \upsilon_x dt_{l1}))^2}} \tag{5}$$

where f_c is the carrier frequency, c is the speed of light and $dt_{l1} = t_l - t_1$ is the difference between two time instances. Equations (2),(4) and (5) also hold for the LOS path, if we set $i = 0$ and we use the subscript s_0 for the BS so that $y_{s_0} = y_{BS}$ and $x_{s_0} = x_{BS}$. The AOD at time t_l for the LOS path is just

$$\psi_{l0} = \pi + \phi_{l0}. \tag{6}$$

B. Input-Output Relationship

The input-output relationship of a $n_r \times n_t$ MIMO-OFDM system for a time-variant (due to the MT movement), frequency selective channel is:

$$\mathbf{Y}(f_k, t_l) = \mathbf{H}(f_k, t_l)\mathbf{X}(f_k, t_l) + \mathbf{N}(f_k, t_l) \tag{7}$$

where $\mathbf{X}(f_k, t_l)$ is the $n_t \times N$ transmitted signal matrix, N is the number of OFDM symbols, $\mathbf{Y}(f_k, t_l)$ is the $n_r \times N$ received signal matrix and $\mathbf{N}(f_k, t_l)$ is the $n_r \times N$ noise matrix, all at frequency f_k, $\forall 1 \leq k \leq N_f$ and time t_l. Throughout the rest of the analysis, the dependency on frequency and time will be denoted by the subscript kl for

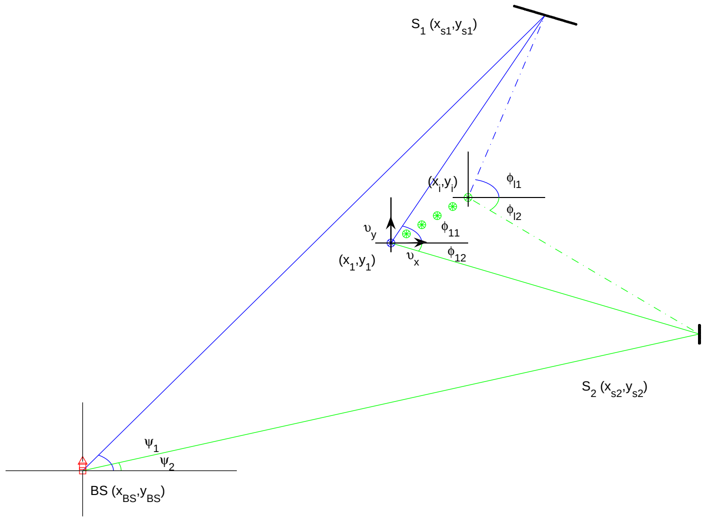

Fig. 1. Single Bounce model

the sake of simplicity. The input-output relationship can be equivalently written in vectorized form, as follows:

$$\mathbf{y}_{kl} = (\mathbf{X}_{kl}^t \otimes \mathbf{I}_{\mathbf{n_r}})\mathbf{h}_{kl} + \mathbf{n}_{kl} \tag{8}$$

where $\mathbf{h}_{kl} = vec(\mathbf{H}_{kl})$ and $\mathbf{n}_{kl} = vec(\mathbf{N}_{kl})$. For a NLOS environment that can be accurately described by the single bounce model, the channel matrix $\mathbf{H}_{NL,kl}$ is given by[2] [8], [9]:

$$\begin{aligned} \mathbf{H}_{NL,kl} = & \frac{1}{\sqrt{P_{tot}}} \sum_{i=1}^{N_s} \sqrt{P_i} \gamma_i e^{j2\pi f_{d,li} t_l} \mathbf{a}_R(\phi_{li}) \mathbf{a}_T^t(\psi_i) \\ & H_{TR,k} e^{-j2\pi f_k \tau_{li}} \\ & = \mathbf{A}_{R,l}(\mathbf{\Gamma} \odot \mathbf{D}_{kl})\mathbf{A}_{T,l}^t = \mathbf{A}_{R,l}\mathbf{\Gamma}\mathbf{D}_{kl}\mathbf{A}_{T,l}^t \end{aligned} \tag{9}$$

If the signal propagates through a strictly NLOS environment, $\mathbf{H}_{kl} = \mathbf{H}_{NL,kl}$. However, if a LOS component exists, then

$$\mathbf{H}_{kl} = \mathbf{H}_{NL,kl} + \mathbf{H}_{L,kl} \tag{10}$$

where the LOS component is

$$\begin{aligned} \mathbf{H}_{L,kl} = & \frac{\sqrt{P_0}}{\sqrt{P_{tot}}} e^{j\theta} e^{j2\pi f_{d,l0} t_l} \mathbf{a}_R(\phi_{l0}) \mathbf{a}_T^t(\psi_0) H_{TR,k} e^{-j2\pi f_k \tau_{l0}} \\ & = e^{j\theta} d_{0,kl} \mathbf{a}_{R,0} \mathbf{a}_{T,0}^t. \end{aligned} \tag{11}$$

The corresponding vectors are

$$\mathbf{h}_{NL,kl} = vec(\mathbf{H}_{NL,kl}) = (\mathbf{A}_{T,l}^t \boxtimes \mathbf{A}_{R,l})\mathbf{D}_{kl}\gamma = \mathbf{Q}_{kl}\gamma \tag{12}$$

$$\mathbf{h}_{L,kl} = vec(\mathbf{H}_{L,kl}) = e^{j\theta} d_{0,kl} (\mathbf{a}_{T,l0}^t \otimes \mathbf{I}_{n_r}) \mathbf{a}_{R,l0}. \tag{13}$$

In the above equations we have introduced the power of the paths $P_{li} = \tau_{li}^{-2}$, along with the normalization constant P_{tot} which contains all the common to the different powers, constant terms. We further introduced the complex Gaussian-distributed amplitudes $\gamma_i \sim CN(0, 1)$, the unknown phase

[2] The proposed channel matrix representation is also valid for any NLOS environment where each AOA is linked with one AOD but not necessarily via a single scatterer.

shift (due to phase noise) of the LOS path $\theta \sim U[0,2\pi]$ and the $n_r \times 1$ and $n_t \times 1$ array responses $\mathbf{a}_R(\boldsymbol{\phi}_{li})$ and $\mathbf{a}_T(\psi_i)$ of the receiver and the transmitter respectively, for the signal component with AOA ϕ_{li} and AOD ψ_i. Also $H_{TR,k}e^{-j2\pi f_k \tau_{li}} = FT\{h_{TR}(\tau-\tau_{li})\}$ is the transfer function (Fourier Transform of the delayed impulse response) of the cascade of the filters at the transmitter's and receiver's front end. Based on these variables we defined:

$$\mathbf{A}_{R,l} \triangleq [\mathbf{a}_R(\phi_{l1}),\ldots,\mathbf{a}_R(\phi_{lN_s})] \tag{14}$$
$$\mathbf{A}_{T,l} \triangleq [\mathbf{a}_T(\psi_{l1}),\ldots,\mathbf{a}_T(\psi_{lN_s})] \tag{15}$$
$$\boldsymbol{\Gamma} \triangleq diag(\boldsymbol{\gamma}) \triangleq diag([\gamma_1,\ldots,\gamma_{N_s}]) \tag{16}$$
$$\mathbf{D}_{kl} \triangleq \frac{1}{\sqrt{P_{tot}}} H_{TR,k} diag(\mathbf{d}_{kl}) \tag{17}$$

where

$$\mathbf{d}_{kl} \triangleq [\sqrt{P_1}e^{j2\pi(f_{d,l1}t_l - f_k\tau_{l1})},\ldots,\sqrt{P_{N_s}}e^{j2\pi(f_{d,lN_s}t_l - f_k\tau_{lN_s})}]. \tag{18}$$

III. ML ESTIMATION OF SPEED AND INITIAL POSITION

Localization of a moving MT can be performed with the aid of a mobility model if its position along with its speed are jointly estimated at some reference time instants (a subset of all time samples). In contrast to the traditional approaches which are based on static snapshots, this approach adds one more dimension to the localization procedure, namely the (variation in) time. Thus, although two more unknown parameters (the speed components) need to be estimated, this new dimension offers a lot of information about the MT's position. Denote by $\mathbf{p}_{int} = [x_1, y_1, \upsilon_x, \upsilon_y]^t$ the parameters that we are interested in estimating, namely the MT's coordinates x_1 and y_1 at the reference time instant t_1 and its speed components υ_x and υ_y. The received signal vectors $\mathbf{y}_{kl}$ can be expressed as a function of the entries of $\mathbf{p}_{int}$ and thus these parameters can be estimated directly. However, $\mathbf{y}_{kl}$ depend also on the position of the scatterers. Moreover if a LOS component exists, $\mathbf{y}_{kl}$ depends on the unknown phase shift θ. Thus we need to estimate $\mathbf{p}_{int}$ in the presence of nuisance parameters which compose the vector $\mathbf{p}_{nuis} = [x_{s1}, y_{s1},\ldots,x_{sN_s}, y_{sN_s},\theta]^t$, i.e. our goal becomes to estimate the $(2N_s+5)\times 1$ vector:

$$\mathbf{p} = [\mathbf{p}_{int}^t, \mathbf{p}_{nuis}^t]^t. \tag{19}$$

The coordinates of the MT and the scatterers can be treated as deterministic unknowns. Furthermore, under the Bayesian framework [10] and having the principle of Maximum Entropy as a guiding rule, we can assign uniform probability density to the speed direction of the MT and the phase θ. Thus, the use of these priors will lead to no improvement in the accuracy of the estimation method, and a Maximum a-Posteriori (MAP) estimator becomes equivalent to a ML estimator, which is implemented below. To formulate the ML estimation problem more precisely, let us introduce the following vectors and matrices:

$$\mathbf{y} = [\mathbf{y}_{11}^t,\ldots,\mathbf{y}_{N_fN_t}^t]^t \tag{20}$$
$$\mathbf{h}_L = [\mathbf{h}_{L,11}^t,\ldots,\mathbf{h}_{L,N_fN_t}^t]^t \tag{21}$$
$$\mathbf{m_y} = \mathfrak{X}\mathbf{h}_L \tag{22}$$
$$\mathbf{Q} = [\mathbf{Q}_{11}^t,\ldots,\mathbf{Q}_{N_fN_t}^t]^t \tag{23}$$
$$\mathbf{V} = \mathfrak{X}\mathbf{Q} \tag{24}$$
$$\mathbf{C_{y|p}} = \mathbf{V}\mathbf{V}^\dagger + \sigma^2\mathbf{I} \tag{25}$$

where the block matrix $\mathfrak{X}$ is constructed as follows:

$$\mathfrak{X} = \begin{bmatrix} (\mathbf{X}_{11}^t \otimes \mathbf{I_{n_r}}) & \mathbf{0} & \cdots \\ \mathbf{0} & \ddots & \mathbf{0} \\ \cdots & \mathbf{0} & (\mathbf{X}_{N_fN_t}^t \otimes \mathbf{I_{n_r}}) \end{bmatrix} \tag{26}$$

The ML estimate of $\mathbf{p}$, denoted as $\widehat{\mathbf{p}}$, is given by maximizing the log-likelihood

$$\widehat{\mathbf{p}} = \underset{\mathbf{p}}{\operatorname{argmax}}\{\mathcal{L}\} \ . \tag{27}$$

which is defined as

$$\begin{aligned}\mathcal{L} \triangleq &\mathcal{L}(\mathbf{y}|\mathbf{p}) = \ln(f(\mathbf{y}|\mathbf{p})) \\ &= -\ln(\det(\mathbf{C_{y|p}})) - (\mathbf{y}-\mathbf{m_y})^\dagger \mathbf{C_{y|p}}^{-1}(\mathbf{y}-\mathbf{m_y}).\end{aligned} \tag{28}$$

IV. CRAMER-RAO BOUND

According to the (CRB) for an unbiased estimator $\widehat{\mathbf{p}}$ of $\mathbf{p}$, the correlation matrix of the parameter estimation errors $\widetilde{\mathbf{p}}$ is bounded below by the inverse of the Fisher Information Matrix (FIM) $\mathbf{J}$ as shown below[3]:

$$R_{\widetilde{\mathbf{p}}\widetilde{\mathbf{p}}} = E\{(\widehat{\mathbf{p}}-\mathbf{p})(\widehat{\mathbf{p}}-\mathbf{p})^t\} \geq \mathbf{J}^{-1} \tag{29}$$

The i', j' entry of the FIM is given by [11, eq.(8.34)]:

$$\begin{aligned}J_{i'j'} =& tr\Big\{\mathbf{C_{y|p}}^{-1}\frac{\partial \mathbf{C_{y|p}}}{\partial p_{i'}}\mathbf{C_{y|p}}^{-1}\frac{\partial \mathbf{C_{y|p}}}{\partial p_{j'}}\Big\} \\ &+ 2Re\Big\{\frac{\partial \mathbf{m_y}^\dagger}{\partial p_{i'}}\mathbf{C_{y|p}}^{-1}\frac{\partial \mathbf{m_y}}{\partial p_{j'}}\Big\}.\end{aligned} \tag{30}$$

The partial derivative of the conditional covariance matrix is

$$\frac{\partial \mathbf{C_{y|p}}}{\partial p_{i'}} = \mathfrak{X}\left(\frac{\partial \mathbf{Q}}{\partial p_{i'}}\mathbf{Q}^\dagger + \mathbf{Q}\frac{\partial \mathbf{Q}^\dagger}{\partial p_{i'}}\right)\mathfrak{X}^\dagger \tag{31}$$

and $\frac{\partial \mathbf{Q}}{\partial p_{i'}}$ is constructed by concatenating the following submatrices:

$$\begin{aligned}\frac{\partial \mathbf{Q}_{kl}}{\partial p_{i'}} =& (\frac{\partial \mathbf{A}_{T,l}}{\partial p_{i'}} \boxtimes \mathbf{A}_{R,l} + \mathbf{A}_{T,l} \boxtimes \frac{\partial \mathbf{A}_{R,l}}{\partial p_{i'}})\mathbf{D}_{kl} \\ &+ (\mathbf{A}_{T,l} \boxtimes \mathbf{A}_{R,l})\frac{\partial \mathbf{D}_{kl}}{\partial p_{i'}}.\end{aligned} \tag{32}$$

The partial derivatives of $\mathbf{A}_{T,l}$ and $\mathbf{A}_{R,l}$ can be found in [9] while the partial derivative of $\mathbf{D}_{kl}$ is given at the bottom of the next page. Similarly, the partial derivative of the conditional mean is

$$\frac{\partial \mathbf{m_y}}{\partial p_{i'}} = \mathfrak{X}\frac{\partial \mathbf{h}_L}{\partial p_{i'}} \tag{34}$$

[3]For matrices $\mathbf{A}$ and $\mathbf{B}$, $\mathbf{A} \geq \mathbf{B}$ means that $\mathbf{A}-\mathbf{B}$ is non-negative definite.

TABLE I
BS, MT AND SCATTERERS' COORDINATES

(x_{BS}, y_{BS})	(x_1, y_1)	(x_{s1}, y_{s1})	(x_{s2}, y_{s2})	(x_{s3}, y_{s3})
$(0,0)$m	$(30,20)$m	$(20,30)$m	$(5,5)$m	$(40,15)$m

and $\frac{\partial \mathbf{h}_L}{\partial p_{i'}}$ is constructed by concatenating the subvectors given by eq. (35). The partial derivatives of the AOA, AODs, delays and Doppler shifts with respect to the entries of $\mathbf{p}$ have been derived in our previous work and can be found in the appendix of [12]. Deriving $\frac{\partial \mathbf{a}_T(\psi_i)}{\partial \psi_i}$ and $\frac{\partial \mathbf{a}_R(\phi_{li})}{\partial \phi_{li}}$ is trivial once the geometry of the antenna arrays is known.

V. NUMERICAL EXAMPLES

In this section we compute and we plot the CRB for three different cases: A LOS environment (information from NLOS signal components is either not available or not used in the estimation process), a Multipath environment with 2 NLOS and a LOS path and a strictly NLOS environment with 3 paths. The power normalization constant ensures that the channel's energy remains the same independently of the case or the number of available paths. The coordinates of the BS, the MT and the scatterers considered, correspond to a picocell and are given in table I. The magnitude of the speed of the MT is $|v| = 1.5$m/sec (average walking speed) and we average the results derived for 20 different directions of the speed, drawn independently from a uniform distribution with support region $[0, 2\pi]$. The $N_t = 40$ time samples are uniformly spaced and $t_{tot} = t_{N_t} - t_1$ is 100msec. Also $N_f = 2$ and $f_c = 1.9GHz$. The transmitted signal is the training matrix $\mathbf{X}_{kl} = \mathbf{I}_{n_t}$, $\forall k, l$. The array response of the receiver's ULA to signal component i arriving at time l, is

$$\mathbf{a}_R(\phi_{li}) = [1, e^{j2\pi \frac{f_c}{c} d \sin(\phi_{li})}, \ldots, e^{j2\pi \frac{f_c}{c} d_r (n_r - 1)\sin(\phi_{li})}]^t \tag{36}$$

and its partial derivative with respect to ϕ_{li} is

$$\frac{\partial \mathbf{a}_R}{\partial \phi_{li}} = j2\pi \frac{f_c}{c} d_r \cos(\phi_{li}) [0, 1, \ldots, (n_r - 1)]^t \odot \mathbf{a}_R(\phi_{li}) \tag{37}$$

where d_r is the distance between two adjacent antenna elements. Replacing d_r with d_t, ϕ with ψ and n_r with n_t, we get the transmitter's array response (and the corresponding derivative). In our simulations we considered $d_r = d_t = \lambda/2$. In figures 2 and 3 we plot the position and speed root mean square error (RMSE) respectively, versus the received SNR for a 2×2 MIMO system. The SNR is

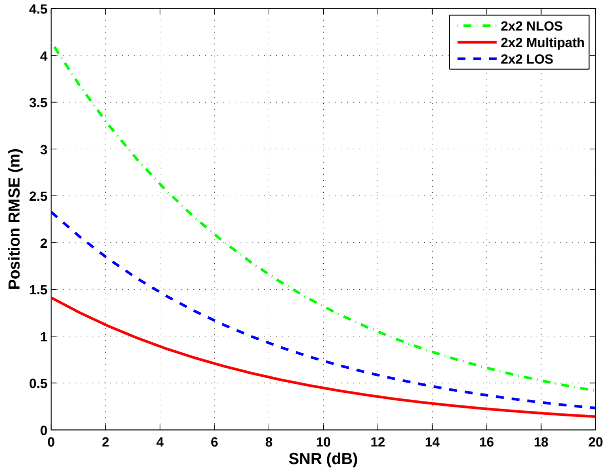

Fig. 2. Position RMSE, various environments

$$SNR = 10 \log_{10} \left(\frac{E\{tr(\mathbf{H}\mathbf{X}\mathbf{X}^\dagger \mathbf{H}^\dagger)\}}{E\{tr(\mathbf{N}\mathbf{N}^\dagger)\}} \right) = 10 \log_{10} \left(\frac{1}{\sigma^2} \right) \tag{38}$$

where $\mathbf{H} = [\mathbf{H}_{11}, \ldots, \mathbf{H}_{N_f N_t}]$, $\mathbf{X} = [\mathbf{X}_{11}^t, \ldots, \mathbf{X}_{N_f N_t}^t]^t$ and $\mathbf{N} = [\mathbf{N}_{11}, \ldots, \mathbf{N}_{N_f N_t}]$. The position and speed RMSE are defined as:

$$RMSE_{\tilde{x}_1, \tilde{y}_1} = \sqrt{\sigma^2_{\tilde{x}_1} + \sigma^2_{\tilde{y}_1}} = \sqrt{tr([\mathbf{J}^{-1}]_{(1:2,1:2)})} \tag{39}$$

$$RMSE_{\tilde{v}_x, \tilde{v}_y} = \sqrt{\sigma^2_{\tilde{v}_x} + \sigma^2_{\tilde{v}_y}} = \sqrt{tr([\mathbf{J}^{-1}]_{(3:4,3:4)})} \tag{40}$$

It can be noticed that the estimation error is very small even for a strictly NLOS environment. Moreover, if the NLOS signal components are considered along with the LOS component, the position RMSE is significantly reduced (e.g. 40% at $10dB$) and speed estimation becomes feasible. In figure 4 the effect of increasing the number of antennas on position accuracy is depicted, for the Multipath environment only. For MISO system, $RMSE_{\tilde{x}_1, \tilde{y}_1} < 1m$ for $SNR > 11dB$, while a 2×2 system can achieve the same accuracy with an SNR of $3dB$. The effect is similar for the other two environments, however position is not identifiable for a MISO system in a NLOS environment.

$$\frac{\partial \mathbf{D}_{kl}}{\partial p_{i'}} = \mathbf{1}_{\{1,\ldots,4,2i+3,2i+4\}}(i') \left[-\frac{1}{\tau_{li}} \frac{\partial \tau_{li}}{\partial p_{i'}} + j2\pi \left(t_l \frac{\partial f_{d,li}}{\partial p_{i'}} - f_k \frac{\partial \tau_{li}}{\partial p_{i'}} \right) \right] \mathbf{D}_{kl} \tag{33}$$

$$\begin{aligned} \frac{\partial \mathbf{h}_{L,kl}}{\partial p_{i'}} =& \mathbf{1}_{\{2Ns+5\}}(i') j\theta \mathbf{h}_{L,kl} + \mathbf{1}_{\{1,\ldots,4\}}(i') \Bigg[\left(\left(-\frac{1}{\tau_{l0}} \frac{\partial \tau_{l0}}{\partial p_{i'}} + j2\pi \left(t_l \frac{\partial f_{d,l0}}{\partial p_{i'}} - f_k \frac{\partial \tau_{l0}}{\partial p_{i'}} \right) \right) \mathbf{h}_{L,kl} \right. \\ &+ \left(\frac{\partial \mathbf{a}_T(\psi_{l0})^t}{\partial \psi_{l0}} \otimes \mathbf{I}_{n_r} \right) \mathbf{a}_R(\phi_{l0}) \frac{\partial \psi_{l0}}{\partial p_{i'}} + (\mathbf{a}^t_{T,l0} \otimes \mathbf{I}_{n_r}) \frac{\partial \mathbf{a}_R(\phi_{l0})}{\partial \phi_{l0}} \frac{\partial \phi_{l0}}{\partial p_{i'}} \Bigg]. \end{aligned} \tag{35}$$

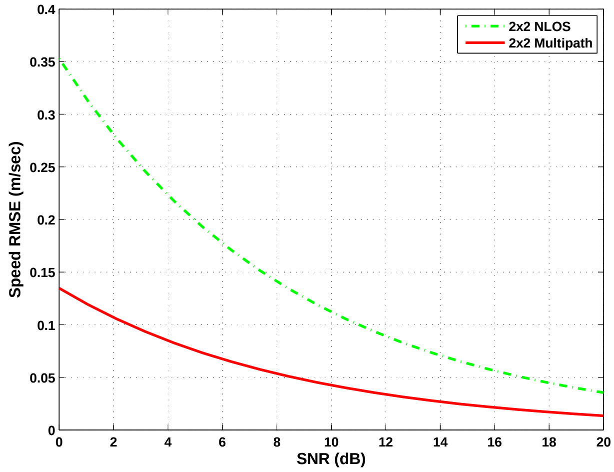

Fig. 3. Speed RMSE, various environments

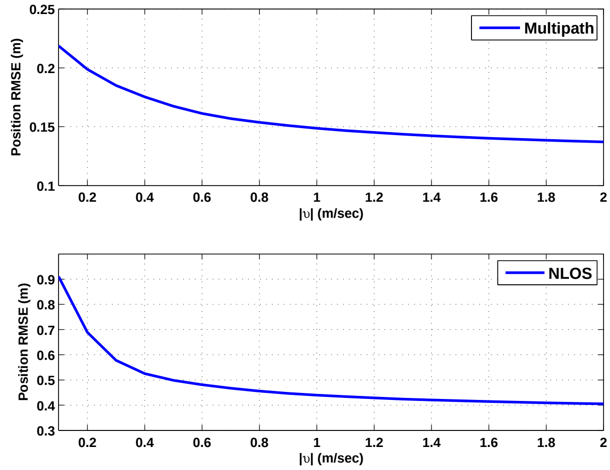

Fig. 5. Position RMSE vs Speed

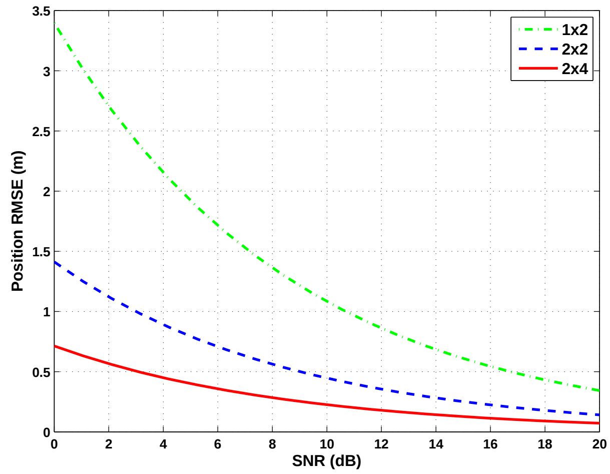

Fig. 4. Position RMSE, various systems

Finally in figure 5, the position's RMSE as a function of the MT's speed is plotted. It can be seen that the movement of the MT has a huge impact on localization accuracy, especially for the NLOS environment, where the error is reduced by more than 50% when the speed is increased to $2m/sec$, largely independently of the direction of movement.

VI. Conclusions

A ML solution for estimating the location of a MT directly from the received signal, under any realistic propagation environment, has been proposed. It is based on appropriate channel modeling, in terms of both geometrical representation of the paths through which the signal propagates and statistical description of the channel's impulse response. The movement of the MT is taken into account with the aid of a simple mobility model and therefore the estimation process is based on consecutive measurements over a small period of time and not on a static snapshot. The proposed method does not require the reception of the transmitted signal in more than 1 BS, however it requires at least 2 antennas at both sides of communication (i.e. a MIMO system) to achieve high accuracy. Performance simulations indicate that in a multipath environment, the improvement in accuracy is significant, if the information contained in NLOS signal components is exploited and that even in a strictly NLOS environment the estimation error is very small.

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