Diversity Order of Linear Equalizers for Block Transmission in Fading Channels

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***Abstract*—We address diversity order of linear equalization for block transmission in fading channels. It is known that zero-padded (ZP) block transmission allows LE to achieve full multipath diversity present in frequency selective channels. We show here that, in a dual fashion, LE can achieve full Doppler-diversity in time-selective channels when guard bands are inserted in the transmit signal. For these transmission schemes we derive an upper bound for the orthogonal deficiency [1] of the effective channel matrix at the receiver to prove that LE can exploit full diversity present in the channel. We also analyze the performance of LE with linear precoded transmission in doubly selective channels and extend this proof to show that LE can also achieve maximal joint multipath-Doppler diversity offered by doubly selective channels.**

I. Introduction

Fading channels pose a major challenge to reliable communications particularly over wireless channels. Equalization at receiver that optimally exploits the inherent diversity in fading channels is a convenient counter-measure against fading channels. Frequency selective fading provides multipath diversity due to the presence of multiple independently fading components. In block transmission systems, when the channel coherence time is shorter than the transmit block length, temporal variations of the channel give rise to time-selectivity. However, this same time-selectivity of the channel also provides Doppler diversity [2] which can be exploited by the receiver. Linear Equalization (LE) is a low-complexity albeit sub-optimal alternative to optimal ML equalization. Recent research has concentrated on quantifying the performance of diversity order of LE in fading channels. While the diversity order of LE for transmission over frequency selective channels has been studied in [3] [4], diversity order of LE in time-selective and doubly selective channels is less understood. In [5], the authors used Complex-Exponential Basis Expansion Model (CE-BEM) [6] with $Q+1$ bases to model the doubly selective channel of memory L, the authors showed that by employing linear precoded block transmission, the maximum diversity in the channel is upper bounded by $(Q+1)(L+1)$ and can be achieved when maximum-likelihood decoding is used at the receiver. However, ML incurs a huge computational complexity therefore it is of interest to investigate diversity order achieved by linear equalization for block transmission over doubly selective channels. In this paper, we study the performance of MMSE-ZF linear equalizers for block transmission over fading channels. For time-selective channels, we show that LE can achieve full Doppler diversity when appropriate guard-bands are inserted into the transmit symbol in much the same way as zero-symbols are padded in ZP-only transmission to enable LE to achieve full multipath diversity. We then study the performance of LE for precoded transmission in doubly selective channel and show that LE also achieve maximal diversity offered by doubly selective channels with the same precoder that enables MLE to achieve multiplicative multipath-Doppler diversity.

II. Signal Model

In Fig. 1 we show the block diagram of the transmission model for block transmission over fading channels.

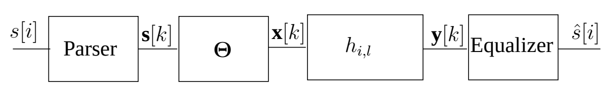

Fig. 1. Block diagram of transmission model.

At the transmitter, complex data symbols $s[i]$ are first parsed into N-length blocks.The n-th symbol in the k-th block is given by $[\mathbf{s}[k]]_n = s[kN+n]$ with $n \in [0, 1, ..., N-1]$. Each block $\mathbf{s}[k]$ is precoded by a $M \times N$ matrix $\boldsymbol{\Theta}$ where $M \geq N$ and the resultant block $\mathbf{x}[k]$ is transmitted over the block fading channel. In the signal model, we consider the general case of doubly-selective channels of order L. Frequency-selective-only and time-selective-only channels can be represented as 1-D cases of doubly selective channels. It is well known that the temporal variation of the channel taps in doubly selective channels with a finite Doppler spread can be captured by finite Fourier bases. We therefore use CE-BEM [6] with $Q+1$ basis functions to model the time variation of each tap in a block duration. The basis coefficients remain constant for the block duration but are allowed to vary with every block. The time-varying channel for each block transmission is thus completely described by the $Q+1$ Fourier bases and $(Q+1)(L+1)$ coefficients. In general Q is chosen such that $Q \geq 2\lceil f_{max} M T_s \rceil$ where $1/T_s$ is the sampling frequency and f_{max} is the Doppler spread of the channel. The coefficients themselves are assumed to be zero-mean complex i.i.d Gaussian random variables. This is a reasonable assumption for a rich scattering environment with non-line-of-sight reception. Using i as the discrete time (sample) index, we can represent the l-th tap of the channel in the k-th block

$$h_{i,l} = \sum_{q=0}^{Q} h_q(k,l) e^{j2\pi f_q i}, \quad (1)$$

$l \in [0, L]$, $f_q = (q - Q/2)/M$. The corresponding receive signal is formed by collecting M samples at the receiver to form $\mathbf{y}[k] = [y(kM+0), y(kM+1), \ldots, y(kM+M-1)]^T$. When $M \geq L$, this block transmission system can be represented in matrix-vector notation as [5]

$$\mathbf{y}[k] = \mathbf{H}[k;0]\boldsymbol{\Theta}\mathbf{s}[k] + \mathbf{H}[k;1]\boldsymbol{\Theta}\mathbf{s}[k-1] + \mathbf{v}[k], \quad (2)$$

where $\mathbf{v}[k]$ is a AWGN vector whose entries have zero-mean and variance σ_v^2 and is defined in the same way as $\mathbf{y}[k]$. $\mathbf{H}[k;0]$ and $\mathbf{H}[k;1]$ are $M \times M$ matrices whose entries are given by $[\mathbf{H}[k;t]]_{r,s} = h_{(kM+r,tM+r-s)}$ with $t \in [0,1], r,s \in [0,...,M-1]$. Defining $\mathbf{D}[f_q]$ as a diagonal matrix whose diagonal entries are given by $[\mathbf{D}[f_q]]_{m,m} = e^{j2\pi f_q m}, m \in [0,1,...,M-1]$, and further defining $[\mathbf{H}_q[k;t]]_{r,s} = h_q(k, tM+r-s)$ as Toeplitz matrices formed of BEM coefficients, it is straightforward to represent Eq. (2) as

$$\mathbf{y}[k] = \sum_{t=0}^{1}\sum_{q=0}^{Q} \mathbf{D}[f_q]\mathbf{H}_q[k;t]\boldsymbol{\Theta}\mathbf{s}[k-t] + \mathbf{v}[k], \quad (3)$$

III. Diversity order of linear equalizers

A. Frequency selective channel

Consider the case of zero-padded block transmission of time-domain symbol vector $\mathbf{s}[k]$ in a frequency selective channel of order L. Such a scheme involves padding $\mathbf{s}[k]$ with $M - N \geq L$ zero symbols before transmission over the frequency selective channel. In other words, the precoding matrix $\boldsymbol{\Theta} = [\mathbf{I}_N \ \mathbf{0}_{N\times(M-N)}]^T$. The frequency selective channel can be represented as a special case of doubly selective channel when $Q = 0$. In order to simplify notation, we therefore drop the superscript q in the received signal representation and rewrite Eq (3) as

$$\mathbf{y}[k] = \mathbf{H}[k;0]\boldsymbol{\Theta}\mathbf{s}[k] + \mathbf{H}[k;1]\boldsymbol{\Theta}\mathbf{s}[k-1] + \mathbf{v}[k], \quad (4)$$

Due to the delay spread of the channel, the received block experiences inter-block-interference. This is represented by the second term on the RHS of Eq (4). Note that $\mathbf{H}[k;1]$ is a strictly upper-triangular matrix with non-zero elements in only the last $M-L$ columns of the matrix. Zero-padding has the effect of setting inter-block-interference to zero and the received signal can be expressed as

$$\mathbf{y}[k] = \widetilde{\mathbf{H}}[k;0]\mathbf{s}[k] + \mathbf{v}[k], \quad (5)$$

where $\widetilde{\mathbf{H}}[k;0]$ is a $M \times N$ Toeplitz matrix with $[h_0(k,0), h_0(k,1), ..., h_0(k,L), \mathbf{0}_{1\times M-L-1}]^T$ as its first column. When the received block is represented by an input-output relationship as in Eq (5), it was shown in [4] that MMSE-ZF receiver achieves has diversity order $L+1$. The linear estimate for the symbols of the k-th received block is then given by the MMSE-ZF equalizer

$$\text{MMSE-ZF} = (\boldsymbol{\mathcal{H}}^H[k]\boldsymbol{\mathcal{H}}[k])^{-1}\boldsymbol{\mathcal{H}}^H[k], \quad (6)$$

where $\boldsymbol{\mathcal{H}}[k] = \widetilde{\mathbf{H}}[k;0]$ is the effective channel matrix seen at the receiver due to zero-padding (in general, precoding) at the transmitter.

1) Diversity order of LE: In [1] the authors introduce a metric namely the orthogonality deficiency of the equivalent channel matrix $od(\boldsymbol{\mathcal{H}}[k])$ at the receiver and prove that LE can achieve the same diversity as ML equalization (MLE) when $od(\boldsymbol{\mathcal{H}}[k]) < 1$. For the case of ZP transmission in frequency selective channels $\boldsymbol{\mathcal{H}}[k] = \widetilde{\mathbf{H}}[k;0]$ is a $M \times N$ Toeplitz channel matrix. In this section we provide an upper bound to $od(\boldsymbol{\mathcal{H}}[k])$ in terms of the infinite order prediction error variance of the spectrum $|\mathbf{h}(f)|^2$ where $\mathbf{h}(f) = \mathbf{h}(e^{j2\pi f})$ and $\mathbf{h} = [h_0(k,0), h_0(k,1), ..., h_0(k,L)]^T$ and prove that $od(\boldsymbol{\mathcal{H}})$ is bounded strictly below 1. We start by noting that $(\det(\boldsymbol{\mathcal{H}}^H[k]\boldsymbol{\mathcal{H}}[k]))^{1/N}$ is a decreasing function of N and

$$\lim_{N\to\infty}(\det(\boldsymbol{\mathcal{H}}^H[k]\boldsymbol{\mathcal{H}}[k]))^{1/N} = \sigma_\infty^2, \quad (7)$$

where σ_∞^2 is the infinite order prediction error variance of $|\mathbf{h}(f)|^2$. Due to the fact that the minimum phase filter coefficients are bounded, it was shown in [7] that

$$\sigma_\infty^2 \geq \frac{1}{c_L}\|\mathbf{h}\|_2^2, \quad c_L = \sum_{l=0}^{L}\binom{l}{L}^2, \quad (8)$$

Since $\text{diag}(\boldsymbol{\mathcal{H}}^H[k]\boldsymbol{\mathcal{H}}[k]) = \|\mathbf{h}\|_2^2\,\mathbf{I}_N$ we have

$$od(\boldsymbol{\mathcal{H}}[k]) = 1 - \frac{\det(\boldsymbol{\mathcal{H}}^H[k]\boldsymbol{\mathcal{H}}[k])}{\det(\text{diag}(\boldsymbol{\mathcal{H}}^H[k]\boldsymbol{\mathcal{H}}[k]))} < 1 - \left(\frac{1}{c_L}\right)^N, \quad (9)$$

which concludes our proof.

B. Time selective channel

We now consider the case of block transmission in time-selective-only channels. The symbol vector $\mathbf{s}[k]$ is now defined in the frequency domain. The time-selective channel is modeled using BEM by setting $L = 0$. The time-variation of the single channel-tap is then captured by $Q+1$ BEM coefficients $h_q(k,0)$. Since there is only a single channel tap in time-domain, in the following, we drop the tap-index in order to simplify the notation. Furthermore, the channel has no delay-spread and therefore does not produce inter-block interference. Consequently, we represent the received block as

$$\mathbf{y}[k] = \sum_{q=0}^{Q} h_q(k)\mathbf{D}[f_q]\boldsymbol{\Theta}\mathbf{s}[k] + \mathbf{v}[k], \quad (10)$$

In a dual fashion to the case of frequency selective channel we propose to insert $M - N \geq Q$ guard-symbols in the frequency-domain symbol vector This is accomplished by setting $[\boldsymbol{\Theta}]_{m,n} = e^{j2\pi mn/M}$ with $m \in [0, 1..., M-1]$ and $n \in [0, 1, ..., N-1]$. By subjecting $\mathbf{y}[k]$ to a M-point DFT at the receiver, the corresponding frequency-domain channel can be represented as

$$\widetilde{\mathbf{y}}[k] = \mathbf{H}_f[k]\mathbf{s}[k] + \mathbf{F}\mathbf{v}[k], \tag{11}$$

Where $\mathbf{F}$ represents the standard DFT matrix and $\mathbf{H}_f[k]$ is the equivalent frequency domain channel which is a $M \times N$ Toeplitz matrix with $[h_0(k), h_1(k), ..., h_Q(k), \mathbf{0}_{1\times M-Q-1}]^T$ as its first column. Note that similar to the case of ZP transmission in frequency selective channel where the effective channel matrix is a Toeplitz matrix formed by the time-domain channel coefficients, in this case, the frequency domain channel matrix is a Toeplitz matrix formed by the frequency domain channel coefficients that contribute to the time-variation of the channel tap. Thus, this scheme can be viewed as a dual of ZP-only transmission for time-selective channels. The MMSE-ZF equalizer is given by Eq (6) where

$$\boldsymbol{\mathcal{H}}[k] = \mathbf{H}_f[k], \tag{12}$$

Eq (11) has a similar input-output relationship as in Eq (5) and it is easily shown that

$$od(\boldsymbol{\mathcal{H}}[k]) < 1 - \left(\frac{1}{c_Q}\right)^N, \quad c_Q = \sum_{q=0}^{Q} \binom{q}{Q}^2, \tag{13}$$

C. Doubly selective channels

We now look at the case of block transmission in doubly selective channels. The channel is assumed to be of order L and the time-variation of each channel tap within a block is captured by $Q+1$ complex-exponential basis functions. The k-th receive block is then represented as in Eq (3) which we reproduce here for clarity.

$$\mathbf{y}[k] = \sum_{t=0}^{1}\sum_{q=0}^{Q} \mathbf{D}[f_q]\mathbf{H}_q[k;t]\boldsymbol{\Theta}\mathbf{s}[k-t] + \mathbf{v}[k],$$

The precoding matrix $\boldsymbol{\Theta}$ that we consider here is given by

$$\boldsymbol{\Theta} = \mathbf{F}_{P+Q}^H \mathbf{T}_1 \otimes \mathbf{T}_2, \tag{14}$$

where $\mathbf{F}_{P+Q}$ is a $(P+Q)$-point DFT matrix, $\mathbf{T}_1 = [\mathbf{I}_P \ \mathbf{0}_{P\times Q}]^T$, $\mathbf{T}_2 = [\mathbf{I}_K \ \mathbf{0}_{K\times L}]^T$. P and K are chosen such that $M = (P+Q)(K+L)$ and $N = PK$. This precoder was proposed in [5] and was shown to enable diversity order of $(Q+1)(L+1)$ for ML receivers in doubly selective channels. The operation of $\boldsymbol{\Theta}$ on $\mathbf{s}[k]$ is explained as follows. First, the N-length block is parsed into P blocks of K symbols. Next, L zero-pads are appended to each of these P blocks in an intermediate step to form P blocks of $K+L$ symbols. Next a set of Q zero-blocks of length $K+L$ are appended to this intermediate block vector to form P-length vector $\widetilde{\mathbf{x}}[k]$ consisting of $P+Q$ blocks of length $K+L$. A block IFFT operation is now performed on $\widetilde{\mathbf{x}}[k]$ to form the precoded transmit symbol vector $\mathbf{x}[k]$ which is transmitted over the doubly selective channel. The above series of operations are compactly represented in the following equations

$$\widetilde{\mathbf{x}}[k] = (\mathbf{T}_1 \otimes \mathbf{T}_2)\mathbf{s}[k], \tag{15}$$

$$\mathbf{x}[k] = (\mathbf{F}_{P+Q}^H \otimes \mathbf{I}_{K+L})\widetilde{\mathbf{x}}[k] = \boldsymbol{\Theta}\mathbf{s}[k], \tag{16}$$

Fig. 2 provides a more insight into subtleties of the precoding operation. The PK-length symbol vector is defined in the frequency domain. These symbols are first re-ordered, and then Q guard symbols are inserted in each block. The IFFT operation transforms these zero-padded blocks into the time-domain where a further L zero-pads are inserted to the symbol vector in the transformed (time) domain.

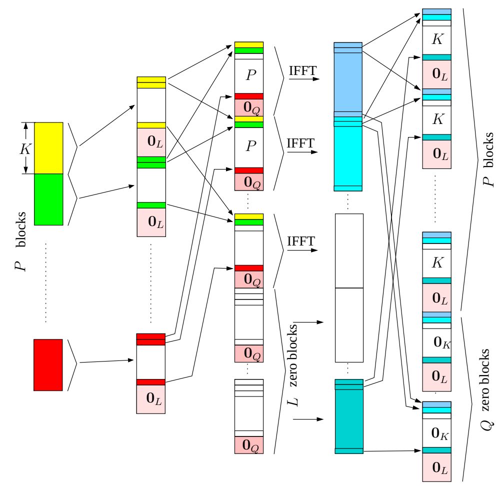

Fig. 2. Precoding operation.

Due to the presence of the zero-padding matrix $\mathbf{T}_2$, it can be easily shown that the inter-block-interference component in the received signal is zero, i.e., $\mathbf{H}_q[k;1]\boldsymbol{\Theta}\mathbf{s}[k-1] = 0$. As a result, the received block can now be represented as

$$\mathbf{y}[k] = \sum_{q=0}^{Q} \mathbf{D}[f_q]\mathbf{H}_q[k;0]\boldsymbol{\Theta}\mathbf{s}[k] + \mathbf{v}[k], \tag{17}$$

Using standard Kronecker product identities, one can show that

$$\mathbf{H}_q[k;0]\boldsymbol{\Theta} = \mathbf{F}_{P+Q}^H \mathbf{T}_1 \otimes \widetilde{\mathbf{H}}_q[k;0]\mathbf{T}_2, \tag{18}$$

where $\widetilde{\mathbf{H}}_q[k;0]$ is a $K+L \times K+L$ Toeplitz matrix formed by the first $K+L$ rows and columns of $\mathbf{H}_q[k;0]$. Eq.(17) can then be re-written as

$$\mathbf{y}[k] = \sum_{q=0}^{Q} \mathbf{D}[f_q]\left(\mathbf{F}_{P+Q}^H \mathbf{T}_1 \otimes \widetilde{\mathbf{H}}_q[k;0]\mathbf{T}_2\right)\mathbf{s}[k] + \mathbf{v}[k], \tag{19}$$

Further insight into the effect of the precoder on the channel is possible by observing that

$$\mathbf{D}[f_q] = \mathbf{D}_{P+Q}[f_q(K+L)] \otimes \mathbf{D}_{K+L}[f_q], \tag{20}$$

Eq (20) represents $\mathbf{D}[f_q]$ as Kronecker product of time-variation over two scales. $\mathbf{D}_{P+Q}[f_q(K+L)]$ is a diagonal matrix of size $P+Q$ that represents time-variation at a coarse scale (complex-exponentials sampled at sub-sampling interval of $(K+L)T_s$ and $\mathbf{D}_{K+L}[f_q]$ is a diagonal matrix of size $K+L$ that represents the time-variation over a finer grid corresponding to the sampling period T_s. Using Eq (20) and standard matrix identities, we can decompose the received

$$\mathbf{y}[k] = \sum_{q=0}^{Q}\Big((\mathbf{D}_{P+Q}[f_q(K+L)]\mathbf{F}_{P+Q}^H\mathbf{T}_1)\otimes(\mathbf{D}_{K+L}[f_q]\widetilde{\mathbf{H}}_q[k;0]\mathbf{T}_2)\Big)\,\mathbf{s}[k]+\mathbf{v}[k], \quad (21)$$

$$\mathbf{y}[k] = (\mathbf{F}_{P+Q}^H\otimes\mathbf{I}_{K+L})\sum_{q=0}^{Q}\Big((\mathbf{J}_{P+Q}[q]\mathbf{T}_1)\otimes(\mathbf{D}_{K+L}[f_q]\widetilde{\mathbf{H}}_q[k;0]\mathbf{T}_2)\Big)\,\mathbf{s}[k]+\mathbf{v}[k], \quad (22)$$

signal as in Eq (21) where $\mathbf{J}[q] = J^{(q-Q/2)}$ and J is a circulant matrix with $[0, 1,\ \mathbf{0}_{1\times P+Q-2}]^T$ as the first column. Since the matrix $(\mathbf{F}_{P+Q}^H \otimes \mathbf{I}_{K+L})$ has no effect on the diversity of the doubly selective channel, for the analysis of the diversity order of MMSE-ZF receiver, the effective channel matrix can be represented as

$$\mathbf{H}_{ds}[k] = \sum_{q=0}^{Q}(\mathbf{J}_{P+Q}[q]\mathbf{T}_1)\otimes(\mathbf{D}_{K+L}[f_q]\widetilde{\mathbf{H}}_q[k;0]\mathbf{T}_2), \quad (23)$$

The channel matrix for this case is therefore given by $\boldsymbol{\mathcal{H}}[k] = \mathbf{H}_{ds}[k]$ and is a highly structured matrix. Fig. 3 illustrates the structure of the equivalent channel matrix due to precoding. Here $\bar{\mathbf{H}}_q$ represents the product matrix $\mathbf{D}_{K+L}[f_q]\widetilde{\mathbf{H}}_q[k;0]$ for ease of illustration. In particular, it is a block-Toeplitz matrix with constituent blocks which are in turn formed by the product of a diagonal matrix $\mathbf{D}_{K+L}[f_q]$ and a Toeplitz matrix formed by the corresponding BEM coefficients of the q-th basis function.

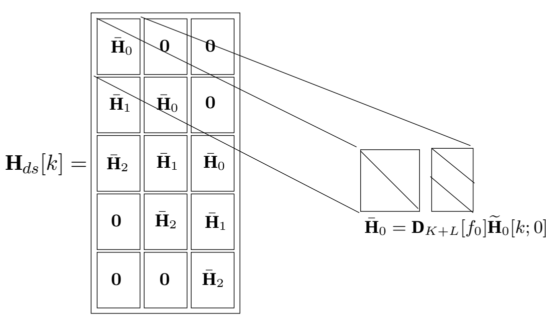

Fig. 3. Equivalent channel matrix for doubly selective channel.

1) Diversity order of LE in doubly selective channels: For the case of doubly selective channel the equivalent channel matrix $\boldsymbol{\mathcal{H}}[k] = \mathbf{H}_{ds}[k]$ is a block Toeplitz matrix. The blocks themselves (cf. $\bar{\mathbf{H}}_q$ in Fig. 3) are Toeplitz to within a multiplication of the diagonal matrix $\mathbf{D}_{K+L}[f_q]$. As both $P \to \infty$ and $K \to \infty$, $\det(\boldsymbol{\mathcal{H}}^H[k]\boldsymbol{\mathcal{H}}[k])$ becomes insensitive to this diagonal multiplication factor. The 2-dimensional (corresponding to the Q(time) and L(delay) dimensions) prediction error variance is the 2-dimensional geometric spectrum average

$$\sigma_\infty^2 = e^{\int_{-1/2}^{-1/2}\int_{-1/2}^{-1/2}\ln|\mathbf{H}(f_1,f_2)|^2 df_1 df_2}, \quad (24)$$

which can be lower bounded by 2-D Matched Filter Bound as

$$\sigma_\infty^2 \geq \frac{1}{c_L c_Q}\int_{-1/2}^{-1/2}\int_{-1/2}^{-1/2}|\mathbf{H}(f_1,f_2)|^2 df_1\, df_2, \quad (25)$$

which leads to

$$od(\boldsymbol{\mathcal{H}}[k]) = 1 - \frac{\det(\boldsymbol{\mathcal{H}}^H[k]\boldsymbol{\mathcal{H}}[k])}{\det(\mathrm{diag}(\boldsymbol{\mathcal{H}}^H[k]\boldsymbol{\mathcal{H}}[k]))} < 1 - \left(\frac{1}{c_L c_Q}\right)^{PK}, \quad (26)$$

Thus, with linear precoding of the form Eq (14), diversity order of LE is $(Q+1)(L+1)$.

IV. Numerical results

In this section we provide simulation results to corroborate our analysis. The diversity order of MMSE-ZF receiver for block transmission is estimated based on the slope of the outage probability curve. Monte-Carlo simulations were carried out for a fixed transmission rate for different SNR points. The decision-point SINR for a fixed arbitrary symbol index n in the k-th symbol block $\mathbf{s}[k]$ was computed as

$$\mathrm{SINR}_n = \frac{\rho}{[\boldsymbol{\mathcal{H}}[k]^H\boldsymbol{\mathcal{H}}[k]]_{n,n}^{-1}}, \quad (27)$$

where $\boldsymbol{\mathcal{H}}[k]$ represents the equivalent frequency-selective, time-selective or doubly selective channel and ρ is the SNR. When the decision point SINR was below the SNR required to support the fixed transmission rate, the channel was declared to be in outage. The slope of the outage probability curve can then be used as an estimate of the diversity order. In addition to this, we compare the slope of the MMSE-ZF receiver to that of the matched filter bound (MFB) which is known to collect all the available diversity in the channel. Fig. 4 shows the diversity order of LE in frequency selective channel when $L = 2$. As expected, ZP enables LE to achieve full multipath diversity of frequency selective channel. Fig. 5 shows the diversity order of LE in time selective channel for the case of $Q = 2$. Observe that, when Q guard frequencies are introduced in the transmit symbol block, LE achieves full Doppler diversity afforded by the time selective channel. This can be seen as a dual of ZP transmission in time selective channels. This is further evidenced by comparison of the slope of the outage probability with that of MFB. Fig. 6 shows the duality of LE in frequency and time selective channel. Note that the slope of the outage probability for LE in frequency selective channel when $Q = 0, L = 2$ is the same as that of LE in time selective channel for the case of $Q = 2, L = 0$. Furthermore both have the same slope as that of MFB in high-SNR regime. In Fig. 7 we plot the performance of in LE for linearly precoded transmission in doubly selective channel with $Q = 2, L = 1$. The outage probability curve exhibits a slope of $(Q+1)(L+1)$ which leads us to conclude that LE achieves full diversity in doubly selective channel when an appropriate diversity enabling precoder is used at the transmitter.

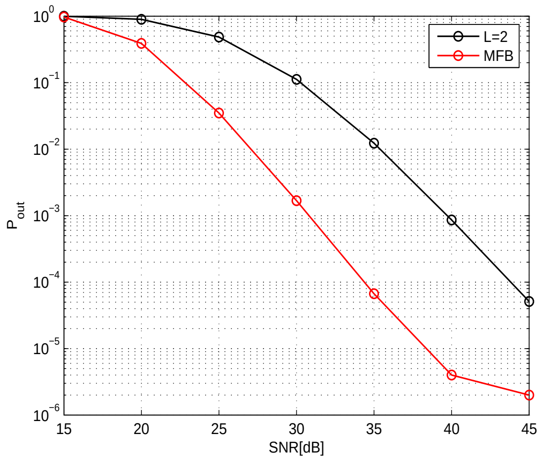

Fig. 4. Diversity order of LE in frequency selective channel.

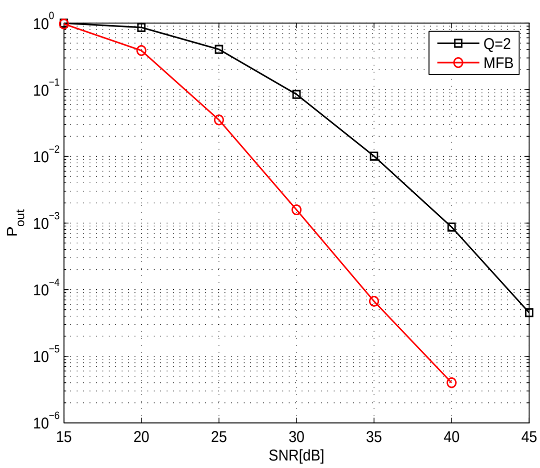

Fig. 5. Diversity order of LE in time selective channel.

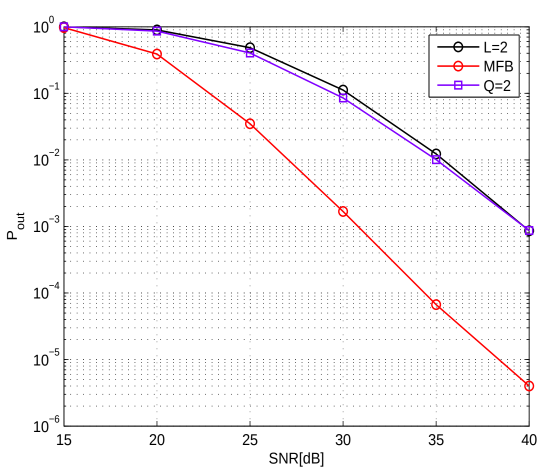

Fig. 6. Duality of LE in time and frequency selective channels.

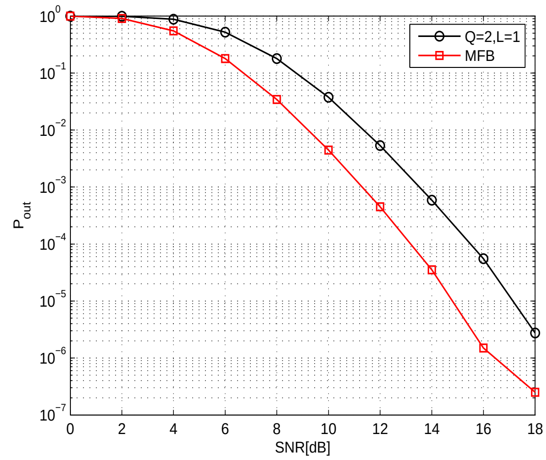

Fig. 7. Diversity order of LE in doubly selective channel.

V. Conclusions

In this contribution we analyzed the diversity order of MMSE-ZF receivers in fading channels. In time-selective channels, we showed that by inserting guard symbols in the transmit block LE can achieve full Doppler diversity of time selective channels. Such a scheme can be viewed as a dual of ZP-only block transmission that enables LE to exploit full multipath diversity of frequency selective channel. For ZP transmission in frequency selective channel we proved that the orthogonal deficiency of the effective channel matrix is bounded strictly below 1 thus allowing LE to achieve full diversity offered by the channel. Further, we analyzed the performance of LE in precoded block transmission in doubly selective channels and showed that it is possible to achieve maximal diversity offered by doubly selective channels by using LE at the receiver.

References

[1] X. Ma and W. Zhang, "Fundamental limits of linear equalizers: Diversity, capacity, and complexity," *Information Theory, IEEE Transactions on*, vol. 54, no. 8, pp. 3442–3456, Aug. 2008.

[2] A. Sayeed and B. Aazhang, "Joint multipath-doppler diversity in mobile wireless communications," *Communications, IEEE Transactions on*, vol. 47, no. 1, pp. 123–132, Jan 1999.

[3] A. Hedayat, A. Nosratinia, and N. Al-Dhahir, "Outage probability and diversity order of linear equalizers in frequency-selective fading channels," *Signals, Systems and Computers, 2004. Conference Record of the Thirty-Eighth Asilomar Conference on*, vol. 2, pp. 2032–2036 Vol.2, Nov. 2004.

[4] C. Tepedelenlioglu and Q. Ma, "On the performance of linear equalizers for block transmission systems," *Global Telecommunications Conference, 2005. GLOBECOM '05. IEEE*, vol. 6, pp. 5 pp.–, Nov.-2 Dec. 2005.

[5] X. Ma and G. Giannakis, "Maximum-diversity transmissions over doubly selective wireless channels," *Information Theory, IEEE Transactions on*, vol. 49, no. 7, pp. 1832–1840, July 2003.

[6] G. Giannakis and C. Tepedelenlioglu, "Basis expansion models and diversity techniques for blind identification and equalization of time-varying channels," *Proceedings of the IEEE*, vol. 86, no. 10, pp. 1969–1986, Oct 1998.

[7] A. Medles and D. Slock, "Decision-feedback equalization achieves full diversity for finite delay spread channels," *Information Theory, 2004. ISIT 2004. Proceedings. International Symposium on*, pp. 99–, June-2 July 2004.