Asymptotic Capacity of Underspread and Overspread Doubly Selective MIMO Channels

Umer Salim
Mobile Communications Department
Eurecom
06560 Sophia Antipolis, France
umer.salim@eurecom.fr

Dirk Slock
Mobile Communications Department
Eurecom
06560 Sophia Antipolis, France
dirk.slock@eurecom.fr

***Abstract*— In this paper, we consider stationary time- and frequency-selective MIMO channels. No channel knowledge neither at the transmitter nor at the receiver is assumed to be available. We investigate the capacity behavior of these doubly selective channels as a function of one of the system parameters, the number of transmit antennas and channel parameters as delay spread, Doppler bandwidth and channel spread factor (the product of the previous two parameters).**

For critically spread channels (channel spread factor of 1), it is widely believed that the dominant term of high-SNR expansion of the capacity is log(log(SNR)) or in other words, the pre-log (the coefficient of log(SNR)) is zero. We provide a very simple scheme showing that for critically spread and mildly overspread channels a non-zero pre-log exists under certain conditions. We specify these conditions in terms of the Doppler bandwidth and the delay spread. We reason that for nearly critically spread channels, MIMO systems exhibit same degrees of freedom as that of a SISO system. At higher channel spread factor (overspread case), the log(SNR) term vanishes and log(log(SNR)) term becomes the dominant capacity term. We specify the range of existence for log(SNR) regime.

I. Introduction

Capacity analysis has been a very rich area of research since Shannon's introduction of the notion of capacity as the unbeatable limit of the data rate possible over a communication channel with probability of error approaching zero. Initially it was assumed that channel is perfectly known at the receiver (channel state information at the receiver (CSIR)) or sometimes even assuming that channel is known at the transmitter (channel state information at the transmitter (CSIT)). But inherently all channels are non-coherent in nature and they need some kind of estimation to get CSIR and then some kind of feedback and/or estimation to have knowledge of CSIT. The area of capacity analysis for non-coherent (no CSIR and no CSIT) fading channels has received considerable attention in recent years.

Initially frequency flat channels with block fading were treated in the no CSIR case. In the standard version of this model [1], the fading remains constant over blocks consisting of T symbol periods, and changes independently from block to block. Capacity bounds are obtained by introducing training segments in an ad hoc fashion. For the standard block fading model, the capacity is shown [1], [2] to grow logarithmically with SNR at high values of SNR, thus $\log(\mathrm{SNR})$ was shown to be the dominant term of capacity. Later Liang and Veeravalli [3] allowed the fading to vary inside the block with a certain correlation matrix characterized by its rank Q and showed for MIMO channels that the capacity pre-log is $\min(n_t, n_r)[1 - \min(n_t, n_r)Q/T]$.

Non-coherent capacity has also been analyzed for flat fading channels with channel fading process taken symbol-by-symbol stationary. In this model, fading is not independent but time selective without any block structure. Surprisingly, this model leads to very different capacity results: contrary to $\log(\mathrm{SNR})$ capacity growth in block fading channels, here the capacity grows only double logarithmically with SNR at high values of SNR [4], [5], [6] when the fading process is non-bandlimited (the Doppler Bandwidth is over the full transmission bandwidth), in this case the channel prediction error is non-zero even if infinite past is known.

For symbol-by-symbol stationary Gaussian fading channels, if the Doppler spectrum is of limited support, then the fading process is called non-regular and the prediction error given the infinite past goes to zero. Lapidoth [7] studied the SISO case for this kind of fading processes showing that the capacity grows logarithmically with SNR and capacity pre-log is the Lebesgue measure of the frequencies where the spectral density of the fading process (Doppler spectrum) has nulls.

Etkin and Tse [8] study the same channel model of bandlimited fading for MIMO systems, they show that pre-log exists even for MIMO systems with no CSIR but they only give a lower bound of the capacity pre-log.

All of the above mentioned studies deal with flat fading channels. We focus our attention on MIMO doubly selective stationary channels. First we characterize the high SNR capacity when these channels are underspread. Then we analyze their behavior in overspread regime where by using a special sub-sampled (zero-padded) input, one can still achieve the pre-log. This behavior finds its analogy with an underspread behavior when one has to optimize (reduce) over the number of active transmit antennas to achieve the optimal pre-log.

The paper organization is as follows. In section II, we give the system model. Section III gives the basis expansion model (BEM) for this channel. We characterize the underspread pre-log and capacity behavior in section IV. In section V, we treat the corresponding overspread channel and give our simple transmission scheme showing the existence of pre-log for overspread channels. In section VI, we relate our novel scheme to a known MIMO behavior. The paper ends with some concluding remarks in section VII.

II. System Model

We consider a multiple-input multiple-output (MIMO) fading channel with n_t transmit and n_r receive antennas, each channel entry has L taps so the time-k output $y[k] \in \mathbb{C}^{n_r}$ is given by

$$y[k] = \sqrt{\frac{\text{SNR}}{n_t}} \sum_{l=0}^{L-1} H[k,l]x[k-l] + z[k] \tag{1}$$

where $x[k] \in \mathbb{C}^{n_t}$ denotes the n_t dimensional time-k channel input, $H[k,l] \in \mathbb{C}^{n_r \times n_t}$ represents the l-th delay FIR (finite impulse response) channel matrix at time k consisting of circularly symmetric complex Gaussian components of zero mean and unit variance, and $z[k] \in \mathbb{C}^{n_r}$ denotes the additive white Gaussian noise vector. Here $\mathbb{C}^n$ denotes the n dimensional complex space.

We assume that the channel matrix is spatially independent and identically distributed (i.i.d.). The channel fading process corresponding to r-th receive antenna, t-th transmit antenna and tap l $\{H^{r,t}[k,l]\}$ is assumed to be stationary, ergodic and bandlimited. They are also independent and identically distributed (i.i.d.) across different taps l. The hypothesis of the bandlimitedness of the fading process is motivated by the physical limitations on the mobile speeds. For a mobile speed v, the maximum Doppler frequency magnitude f_{max} for each path is $f_{max} = v/\lambda_c$ where λ_c is the carrier wavelength. The bandwidth of each fading process will be upper bounded by the two-sided Doppler bandwidth $2f_{max}$. We define the normalized Doppler bandwidth as $B_d = 2f_{max}T_s$ where T_s represents the symbol period, assuming the Doppler spectrum has support between the two extreme Doppler shifts. In general, B_d will denote the support of the Doppler spectrum. The hypothesis of bandlimited Doppler spectrum is an approximation because the Doppler shifts do not remain constant. Similarly, the hypothesis of limited delay spread is an approximation. Limited values for Doppler and delay spreads can be justified at a given working SNR. We define the spread factor (μ) of the channel as $\mu = LB_d$.

The system is normalized so that the channel input has an average power constraint of $\mathbb{E}[||x[k]||^2] \leq n_t$.

The capacity pre-log is normally defined as

$$\text{PreLog} = \lim_{\text{SNR}\to\infty} \frac{C(\text{SNR})}{\log(\text{SNR})} \tag{2}$$

whenever $C(\text{SNR})$ is of order $\log(\text{SNR})$, and the capacity pre-loglog is given by

$$\text{PreLogLog} = \lim_{\text{SNR}\to\infty} \frac{C(\text{SNR})}{\log(\log(\text{SNR}))} \tag{3}$$

whenever $C(\text{SNR})$ is of order $\log(\log(\text{SNR}))$.

III. Representation Using Basis Expansion Model

To get a proper model for the doubly selective channel, we start by considering block transmission with block length N. Continuous transmission results will then be obtained by letting the block size N grow to infinity. Observing a signal over a block can always be thought of as if the block considered is one period of a periodic process, in which case the signal has a Fourier series expansion. This leads to a Basis Expansion Model (BEM) for the time-varying channel coefficients in which the basis functions are complex exponentials with frequencies at the multiples of $1/N$ [9]. As the Doppler spectrum is bandlimited, we shall take the BEM to be correspondingly bandlimited. We should note here that we do not necessarily demand of the BEM to provide an exact description of the channel statistics over the block of length N, as long as the description becomes exact as the block length tends to infinity. The BEM leads to the following representation for the channel coefficients over a block that starts at time zero w.l.o.g.,

$$H^{r,t}[k,l] = \sum_{n=0}^{N_d-1} g^{r,t}[n,l]e^{j2\pi kn/N} \quad , \quad k = 0, 1, \cdots, N-1 \tag{4}$$

where $N_d = \lceil N B_d \rceil$. In the above equation, $g^{r,t}[n,l]$ are independent, uncorrelated, zero mean proper complex Gaussian random variables whose variances are the values of the spectrum of the corresponding fading process at respective frequencies n/N. To avoid inter-block interference and facilitate the description in the frequency-domain, we add a cyclic prefix of length $L-1$ making the total block length to be $N+L-1$. At the receiver the first $L-1$ received samples corresponding to the prefix get neglected and the remaining N outputs, the inputs and the noise get collected in vector form as $\mathbf{y_r} = [y_r[0]\, y_r[1] \cdots y_r[N-1]]^T$, $\mathbf{x_t} = [x_t[0]\, x_t[1] \cdots x_t[N-1]]^T$, $\mathbf{z_r} = [z_r[0] z_r[1] \cdots z_r[N-1]]^T$, leading to the system equation

$$\mathbf{y_r} = \sqrt{\frac{\text{SNR}}{n_t}} \sum_{t=1}^{n_t} \mathbf{H_{r,t}x_t} + \mathbf{z_r} \tag{5}$$

where $\mathbf{H_{r,t}} \in \mathbb{C}^{N\times N}$ is the channel matrix corresponding to t-th transmit and r-th receive antenna over this block and has the circulant structure shown at the top of the next page. If $\mathbf{H_r} = [\mathbf{H_{r,0}H_{r,1}} \cdots \mathbf{H_{r,n_t}}]$ and $\mathbf{x} = [\mathbf{x_1^T x_2^T} \cdots \mathbf{x_{n_t}^T}]^\mathbf{T}$, the signal received at r-th received antenna becomes

$$\mathbf{y_r} = \sqrt{\frac{\text{SNR}}{n_t}} \mathbf{H_r x} + \mathbf{z_r} \tag{6}$$

Now signal from all n_r receive antennas can be combined in a long vector $\mathbf{y} = [\mathbf{y_1^T y_2^T} \cdots \mathbf{y_{n_r}^T}]^\mathbf{T}$ to get

$$\mathbf{y} = \sqrt{\frac{\text{SNR}}{n_t}} \mathbf{Hx} + \mathbf{z} \tag{7}$$

where $\mathbf{H} = [\mathbf{H_1^T H_2^T} \cdots \mathbf{H_{n_r}^T}]^\mathbf{T}$ is of size $Nn_r \times Nn_t$.

We also need a system representation in which the roles of the channel and the input are reversed. Following the same steps as before, input $\mathbf{X}$ over this block length N can be

$$\mathbf{H_{r,t}} = \begin{bmatrix} h^{r,t}[0,0] & & h^{r,t}[0,L{-}1] & \dots & h^{r,t}[0,1] \\ \vdots & h^{r,t}[1,0] & & \ddots & \vdots \\ \vdots & & & & h^{r,t}[L{-}2,L{-}1] \\ h^{r,t}[L{-}1,L{-}1] & \vdots & & & \\ & h^{r,t}[L,L{-}1] & \ddots & & \\ & & \ddots & & \\ & & & & h^{r,t}[N{-}1,0] \end{bmatrix}$$

written as a block diagonal matrix of size $n_r N \times n_t n_r LN$ and all the channel coefficients can be put in a long vector $\mathbf{h}$ of length $n_t n_r LN$, giving us the system equation as

$$\mathbf{y} = \sqrt{\frac{\mathrm{SNR}}{n_t}}\,\mathbf{Xh} + \mathbf{z} \tag{8}$$

Similarly by putting the uncorrelated coefficients of BEM in a long vector $\mathbf{g}$ of length $n_t n_r LN_d$, in the same order that channel coefficients have been put in the long vector $\mathbf{h}$, we can write $\mathbf{h} = \mathbf{F_c g}$ where $\mathbf{F_c} = \mathbf{I_{n_t n_r L}} \otimes \mathbf{F}$, $\otimes$ represents the Kronecker product and $\mathbf{F} \in \mathbb{C}^{N \times N_d}$ is the partial IDFT matrix. With this (8) can be written as

$$\mathbf{y} = \sqrt{\frac{\mathrm{SNR}}{n_t}}\,\mathbf{XF_c g} + \mathbf{z}\ . \tag{9}$$

IV. Underspread Channels

Typically wireless channels are underspread in nature [10], so first of all we study the capacity pre-log for doubly selective MIMO channels when they are underspread (spread factor μ is strictly less than one).

A. Lower Bound of Mutual Information

Using the BEM developed in section III, we derive a lower bound for the mutual information. Due to space limitations, we just give the proof outline. The system input is selected as Gaussian i.i.d. satisfying the average power constraint imposed and the resulting MI is

$$\begin{aligned} I(\mathbf{x^G};\mathbf{y}) &= I(\mathbf{x^G},\mathbf{H};\mathbf{y}) - I(\mathbf{H};\mathbf{y}|\mathbf{x^G}) \\ &= I(\mathbf{x^G};\mathbf{y}|\mathbf{H}) + I(\mathbf{H};\mathbf{y}) - I(\mathbf{H};\mathbf{y}|\mathbf{x^G}) \\ &\geq I(\mathbf{x^G};\mathbf{y}|\mathbf{H}) - I(\mathbf{H};\mathbf{y}|\mathbf{x^G}) \end{aligned} \tag{10}$$

Equalities here follow from the introduction of the channel matrix $\mathbf{H}$ and using the chain rule of mutual information multiple times and the inequality follows from the non-negativity of the mutual information.

First term in the above inequality is the mutual information when the channel is known and so can be evaluated easily using the coherent channel results to

$$\lim_{\mathrm{SNR}\to\infty} I(\mathbf{x^G};\mathbf{y}|\mathbf{H}) = N\min(n_t,n_r)\log(\mathrm{SNR}) + O(1) \tag{11}$$

Now the second mutual information term $I(\mathbf{H};\mathbf{y}|\mathbf{x^G})$ in (10) is the MI due to transmission over a known fictitious channel ($\mathbf{x^G}$) where actual channel ($\mathbf{H}$) plays the role of the input which is of reduced bandwidth. At high SNR, this term can be shown to grow as

$$\lim_{\mathrm{SNR}\to\infty} I(\mathbf{H};\mathbf{y}|\mathbf{x^G}) \leq N n_t n_r \mu \log(\mathrm{SNR}) + O(1) \tag{12}$$

Combining the above results, dividing by N the block length and optimizing over the number of antennas by reducing them to $\min(n_t,n_r)$, we get the proper pre-log result

$$\mathrm{PreLog} \geq \min(n_t,n_r)[1 - \min(n_t,n_r)\mu] \tag{13}$$

B. Upper Bound of Mutual Information

To derive the upper bound of the mutual information for this doubly selective channel, the main point is the intelligent splitting of the mutual information in two parts, in which one term grows with $\log(\mathrm{SNR})$ as for a coherent channel and the other term is shown to have no growth as $\log(\mathrm{SNR})$. Pre-log depends upon what is the minimal number of parameters required to fully estimate the channel, so this number of degrees of freedom are lost and on the rest one can achieve $\log(\mathrm{SNR})$ growth of capacity. We again leave the details of the derivation

$$\mathrm{PreLog} \leq \min(n_t,n_r)[1 - \min(n_t,n_r)\mu] \tag{14}$$

C. The Pre-Log of Underspread Doubly Selective Channel

Based upon the above two bounds on the mutual information of strictly underspread channels, one can conclude that the pre-log is given by

$$\mathrm{PreLog} = \min(n_t,n_r)[1 - \min(n_t,n_r)\mu] \tag{15}$$

It shows that the loss factor in pre-log for a non-coherent MIMO channel is equal to one minus channel spread factor (μ) multiplied by $\min(n_t,n_r)$. The factor of $\min(n_t,n_r)$ bears the interpretation of number of active transmit antennas which should be used to get the capacity pre-log.

D. Large Spread Factor Analysis

Here we treat the case when the channel is still underspread but the inverse of the spread factor $1/\mu$ is comparable to $\min(n_t,n_r)$. Our expression of the pre-log equation (15) shows that the high SNR DOF depend entirely on $\min(n_t,n_r)$ and not on the individual values of n_t and n_r. As $n_t > n_r$ is strictly sub-optimal in the high SNR non-coherent regime so let's take $n_t < n_r$, $1 \leq \acute{n}_t \leq n_t$ and then optimize the pre-log $\acute{n}_t(1-\acute{n}_t\mu)$ over $\acute{n}_t$. We want to analyze what is the optimal value of $\acute{n}_t$ for a fixed large n_r and spread factor μ.

The pre-log $\acute{n}_t(1-\acute{n}_t\mu)$ is a simple parabola, initially pre-log increases with increasing $\acute{n}_t$ reaching its maximum value at $1/(2\mu)$ and starts decreasing onwards becoming zero at $1/\mu$. The explanation is that $\acute{n}_t$ factor represents the number of independent streams which one can multiplex over this system but the coherent reception of this number of streams first requires estimation of the corresponding channel coefficients hence the loss factor also increases with the factor $\acute{n}_t$. Now with large spread factor when the coherence time is very short, using more streams means a greater loss factor which is proportional to the spread factor. But due to very short coherence time, the coherent transmission does not last long enough to compensate that loss factor and to reduce the number of active streams becomes the optimal strategy. Hence if $n_t > 1/(2\mu)$, the active number of transmit antennas should be reduced to $1/(2\mu)$. This discussion indicates that the active number of transmit antennas (streams) in non-coherent MIMO should actually be $\min(n_t, n_r, \frac{1}{2\mu})$ and the pre-log for a non-coherent MIMO system becomes

$$\mathrm{PreLog} = \min(n_t, n_r, \frac{1}{2\mu})[1 - \min(n_t, n_r, \frac{1}{2\mu})\mu] \qquad (16)$$

One very important point to which this pre-log indicates is that when spread factor is sufficiently large (spread factor larger than $1/3$ precisely), the above given pre-log will become $(1-\mu)$, the pre-log of a SISO doubly selective channel. So from the pre-log point of view at these higher spread factors, non-coherent MIMO systems collapse to a SISO or SIMO system.

$$\mathrm{PreLog} = 1 - \mu \qquad \text{for} \quad \mu > 1/3 \qquad (17)$$

V. Overspread Channels

We showed that MIMO doubly selective channels collapse to a SISO channel when spread factor (μ) is greater than $1/3$, which renders the pre-log to $1-\mu$. Now if channel spread increases and reaches to 1 (the so-called critically spread channels) or becomes greater than 1, the pre-log expression dictates that pre-log is zero with $\mu \geq 1$. Below we give a very simple scheme which shows that the $\log(\mathrm{SNR})$ term exists for overspread channels under certain conditions. We describe this scheme in terms of a SISO channels as optimal number of transmit antennas is 1 at these higher spread factors and receive antennas in surplus can only provide diversity gain but add nothing to the pre-log.

A. Transmission Scheme

Our transmission scheme to realize $\log(\mathrm{SNR})$ growth for overspread channels is based upon zero padding. The zero padding is done in such a manner that at the receiver side, each transmitted symbol appears without inter-symbol interference (ISI) for at least one symbol time. So to achieve this one output sample free of ISI, we transmit an input symbol and then do zero padding of $\lfloor L/2 \rfloor$ symbols. That means each information symbol is followed by $\lfloor L/2 \rfloor$ deterministic zeros. Now one may focus attention on the input information symbols transmitted at the transmitter and the ISI free received symbols at the receiver delayed by $(\lfloor L/2 \rfloor + 1)$ symbol intervals. For this scheme $\lfloor L/2 \rfloor$ input symbols are wasted (zero-padded) corresponding to each single information symbol transmitted but the good thing is that the effective channel is frequency flat and each ISI free symbol at the receiver comes multiplied with the same channel tap, the $(\lfloor L/2 \rfloor + 1)$-th tap. This scheme is explained in Figure 1.

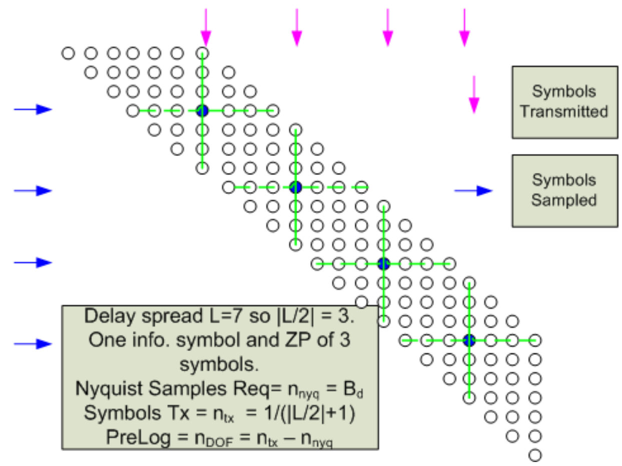

Fig. 1. Transmission Scheme Example (Equiv. Freq. Flat Channel)

Now we need to see what fraction of symbols we are able to transmit in this zero-padded scheme where $\lfloor L/2 \rfloor$ symbols get wasted for each single information symbol. So the fraction of the information symbols is

$$n_{tx} = \frac{1}{\lfloor L/2 \rfloor + 1} \qquad (18)$$

Now keeping in mind that here we are interested in only a single channel tap (which appears with ISI free output symbol) requiring N_d BEM coefficients to be estimated to be fully known over a block length N as we argued in section III. And to estimate a single channel tap, per symbol coefficients required N_d/N is equal to the normalized Doppler bandwidth B_d. We denote this fraction by n_{nyq}, the minimum number of samples required to estimate the channel

$$n_{nyq} = \lim_{N\to\infty} \frac{N_d}{N} = B_d \qquad (19)$$

If we want to estimate the channel by sending pilot symbols, we need to transmit B_d fraction of pilots among the non-zero transmit symbols and then this particular channel tap can be estimated by estimating its BEM coefficients. But in this scheme, the information symbols transmitted is the fraction $1/(\lfloor L/2 \rfloor + 1)$ per symbol. Now there is the possibility that some degrees of freedom (DOF) are left even after estimating this particular channel tap but it will be depending upon the relative values of the channel delay spread L and the normalized Doppler bandwidth B_d.

$$n_{DOF} = n_{tx} - n_{nyq} = \frac{1}{\lfloor \frac{L}{2} \rfloor + 1} - B_d \qquad (20)$$

So we can have coherent transmission albeit with imperfect channel estimate over this fraction n_{DOF} (if this number is non-zero, of course) and so it corresponds to a coherent channel where pre-log exists. Hence pre-log per symbol time is given by

$$\mathrm{PreLog} = n_{DOF} = \frac{1}{\lfloor \frac{L}{2} \rfloor + 1}\left(1 - B_d(\lfloor \frac{L}{2} \rfloor + 1)\right) \quad (21)$$

Formal information theoretic proof for the achievability of the above pre-log for overspread channels has been omitted due to space limitations.

We can find the channel parameter values where the pre-log given by the zero-padded transmission scheme surpasses the pre-log $(1 - LB_d)$ derived in section IV-C. Similarly we can find an upper bound on Doppler bandwidth till when this scheme can give us non-zero pre-log in overspread regime.

$$\frac{\lfloor \frac{L}{2} \rfloor}{(\lfloor \frac{L}{2} \rfloor + 1)(L-1)} \leq B_d \leq \frac{1}{\lfloor \frac{L}{2} \rfloor + 1} \quad (22)$$

The left inequality shows the condition for an underspread channel where the pre-log of this zero-padding scheme takes over the classical pre-log of $(1-LB_d)$ and the right inequality shows the condition under which an overspread channel shows positive pre-log with this scheme. The multiplication of the above inequality with L gives us the corresponding bounds on the channel spread factor.

B. Optimality of Zero Padded Transmission Scheme

In our transmission scheme with zero padding, we transmit one information symbol in each block of $(\lfloor L/2 \rfloor + 1)$ symbols. One can argue if more than 1 symbol is transmitted and zero padding of the same size is done, there might be the possibility of having more DOF and resultantly a higher pre-log factor. We omit the details but we are able to prove that among such kind of ZP schemes with multiple symbols transmitted and ZP of $(\lfloor L/2 \rfloor)$, they don't beat our scheme where one information symbol gets transmitted followed by ZP of $(\lfloor L/2 \rfloor)$ length.

For the channels with very high spread factors (nearly critically spread channels to overspread channels where range was specified in section IV-C), we showed some optimality conditions of this zero padding scheme. Although we don't have a proof for the upper bound of the pre-log for this transient regime but we conjecture that this is the pre-log.

$$\mathrm{PreLog} = \frac{1}{\lfloor \frac{L}{2} \rfloor + 1}\left(1 - B_d(\lfloor \frac{L}{2} \rfloor + 1)\right) \quad (23)$$

VI. Active Tx. Antennas and Zero Padded Input

We showed that the pre-log for MIMO doubly selective channels is given by

$$\mathrm{PreLog} = \min(n_t, n_r, \frac{1}{2\mu})[1 - \min(n_t, n_r, \frac{1}{2\mu})\mu] \quad (24)$$

which indicates that with the increase of channel spread factor, one should turn off more and more transmit antennas to obtain the pre-log. And the reason is that each transmit antenna introduces some channel parameters which need to be known (and hence estimated) for coherent detection of data. Now after spread factor greater than $1/3$, we get only one active transmit antenna giving us the pre-log of $(1 - LB_d)$. The same reasoning makes our zero-padding scheme successful. At spread factors very close to 1, our ZP scheme converts this doubly selective channel into a frequency flat channel of increased Doppler bandwidth. Increase in Doppler bandwidth is $(\lfloor \frac{L}{2} \rfloor + 1)$ but channel parameters get reduced by a factor L (almost double). This difference makes channel estimation possible and guarantees the pre-log.

VII. Concluding Remarks

In this contribution we characterized the capacity pre-log for doubly selective MIMO channels in underspread regime. Then we gave a novel scheme which is able to extract $\log(\mathrm{SNR})$ even from overspread channels under certain channel conditions. We specified the range in terms of delay spread and Doppler bandwidth where ZP scheme is able to achieve non-zero pre-log. We showed that our scheme is analogous to reducing the active transmit antennas in underspread MIMO.

Acknowledgments

Eurecom's research is partially supported by its industrial members: BMW Group Research & Technology, Bouygues Telecom, Cisco, Hitachi, ORANGE, SFR, Sharp, STMicroelectronics, Swisscom, Thales. The research work leading to this paper has also been partially supported by the European Commission under the ICT research network of excellence NewCom++ of the 7th Framework programme and by the French ANR project APOGEE.

References

[1] T. Marzetta and B. Hochwald, "Capacity of a mobile multiple-antenna communications link in rayleigh flat fading," *IEEE Trans. on Information Theory*, vol. 45, pp. 139–157, January 1999.

[2] L. Zheng and D. N. C. Tse, "Communication on the grassmann manifold: A geometric approach to the noncoherent multiple-antenna channel," *IEEE Trans. on Information Theory*, vol. 48, pp. 359–383, February 2002.

[3] Y. Liang and V. V. Veeravalli, "Capacity of noncoherent time-selective rayleigh-fading channels,," *IEEE Trans. on Information Theory*, vol. 50, pp. 3095–3110, December 2004.

[4] A. Lapidoth and S. Moser, "Capacity bounds via duality with applications to multi-antenna systems on flat fading channels," *IEEE Trans. on Information Theory*, vol. 49, pp. 2426–2467, October 2003.

[5] T. Koch and A. Lapidoth, "The fading number and degrees of freedom in non-coherent mimo fading channels: a peace pipe," in *Proc. IEEE International Symposium on Information Theory*, September 2005, vol. 52, pp. 437–453.

[6] A. Lapidoth and S. Moser, "The fading number of single-input multiple-output fading channels with memory," *IEEE Trans. on Information Theory*, vol. 52, pp. 437–453, February 2006.

[7] A. Lapidoth, "On the asymptotic capacity of stationary gaussian fading channels," *IEEE Trans. on Information Theory*, vol. 51, pp. 437–446, February 2005.

[8] R. Etkin and D. N. C. Tse, "Degrees of freedom in some underspread mimo fading channels," *IEEE Trans. on Information Theory*, vol. 52, pp. 1576–1608, April 2006.

[9] M. Guillaud and D.T.M. Slock, "Channel Modeling and Associated Inter-Carrier Interference Equalization for OFDM Systems with High Doppler Spread," in *Proc. Int'l Conf. Acoustics, Speech and Sig. Proc. (ICASSP)*, Hong Kong, Apr. 6-10 2003.

[10] D. Tse and P. Viswanath, *Fundamentals of Wireless Communications*, Cambridge, U.K. Cambridge Univ. Press, 2005.