On Optimum End-to-End Distortion of Spatially Correlated MIMO Systems

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Abstract— [1] **In this paper, we investigate the behaviors of the optimum end-to-end distortion of spatially correlated, multiple-input-multiple-output (MIMO) systems. Assuming Rayleigh fading channel and the transmitter perfectly knows the instantaneous channel rate, we derive an analytic expression of the tight lower bound of the end-to-end mean quadratic distortion at any SNR for transmitting a white thermal noise source, in terms of the spatial correlation matrix, antenna numbers, the ratio of source-bandwidth to channel-bandwidth, the ratio of signal power to noise power (SNR) and the source power. By analyzing the expression, we obtain the SNR exponent and the corresponding factor at the asymptotically high SNR. Also, we show that higher correlation brings higher distortion lower bound, which corresponds to our intuition.**

I. Introduction

End-to-end distortion, *i.e.* the distortion on the recovered source at the receiver, is the primary performance metric in analog source transmission. The relationship between the quadratic end-to-end distortion (mean square error) and the channel capacity is shown by Shannon's inequality [1]. In MIMO systems, this principle remains as well as in SISO systems.

The existence of the SNR exponent in the optimum quadratic distortion at the asymptotically high SNR has been proved in [2]–[4]. In [5]–[7], Caire-Narayanan and Gunduz-Erkip have given the upper bound of the SNR exponent in the mean quadratic distortion at the asymptotically high SNR for joint source-channel coding MIMO systems. It is relevant to the ratio of the source bandwidth W_s to the channel bandwidth W_c (*bandwidth ratio* in our paper).

Inspired by their work, in [8], under the assumption that the transmitter perfectly knows the instantaneous channel rate, we derive the compact analytic expression of the tight lower bound of the mean quadratic distortion for any SNR and the explicit corresponding distortion factor at the asymptotically high SNR. Also, in [8], the same upper bound of the distortion SNR exponent as in [5]–[7] has been derived by a quite different means.

Moreover, Gunduz-Erkip have studied the interleaving effect on the SNR exponent [9] and we have studied the interleaving effect on both of the SNR exponent and the corresponding factor [10].

To our best knowledge, before this paper, all relevant works on optimum end-to-end distortion of MIMO systems, including Caire-Narayanan's, Gunduz-Erkip's and ours, are under the assumption of spatially uncorrelated MIMO channel. An alternative scenario, which is more general, is the spatially correlated case. Intuitively, for a correlated MIMO channel, we would obtain the result that the spatial correlation increases the lower bound of the end-to-end distortion as it decreases the channel capacity.

In [11], Chiani *et al.* give the analytic expression for the moment generating function of channel capacity in both cases of spatially uncorrelated channel and spatially correlated channel. In this paper, on the basis of a part of their mathematical results, we derive the analytic expression of the optimum mean quadratic end-to-end distortion on a thermal noise source and then figure out the SNR exponent and the corresponding factor at the asymptotically high SNR. We also show that higher correlation brings higher distortion lower bound, which demonstrates our intuition.

Throughout the paper, vectors and matrices are indicated by bold, $|\boldsymbol{A}|$ and $\det \boldsymbol{A}$ denote the determinant of matrix $\boldsymbol{A}$, and $\{a_{ij}\}_{i,j=1,\dots,N}$ is an $N \times N$ matrix with elements $a_{ij},\ i,j = 1,...N$. Also, $\mathbb{E}\{\cdot\}$ denotes expectation, and in particular $\mathbb{E}_x\{\cdot\}$ denotes expectation with respect to the random variable x. The superscript † denotes conjugate transpose. *Tight lower bound of* and *optimum* are two phrases exchangeable in this paper.

II. MIMO System Model

Assume a white thermal noise source $\boldsymbol{s}(t)$ is to be transmitted and systems are working on "short" frames due to strict time delay constraint, that is, time-interleaving is impossible to be done and no time diversity can be exploited. The transmitter is supposed to perfectly know the instantaneous channel rate which can be fed back by the receiver as a real scalar. The recovered source at the receiver is denoted by $\widehat{\boldsymbol{s}}(t)$.

Consider a frequency-flat block-fading MIMO channel with N_t inputs and N_r outputs represented by

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} \tag{1}$$

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where $\mathbf{x}$ is an N_t-length column vector denoting transmitted symbols, $\mathbf{y}$ is an N_r-length column vector denoting received symbols, $\mathbf{n}$ is an N_r-length column vector denoting noises and $\mathbf{H}$ is an $N_r \times N_t$ matrix denoting the channel. We assume all elements in $\mathbf{n}$ are zero-mean i.i.d. complex random variables with variance σ_n^2 and all elements in $\mathbf{H}$ are $\mathcal{CN}(0,1)$.

Assume the signals are uncorrelated at the transmit antennas and correlated at the receive antennas. Thus, the correlation matrix $\mathbf{\Sigma} = \mathbb{E}\{\mathbf{H}\mathbf{H}^\dagger\}$.

III. Preliminaries

The mathematic equation and definition below will be used in subsequent derivations and results.

A. The integral of an exponential function

$$\int_0^\infty e^{-px} x^{q-1}(1+ax)^{-\nu} dx = a^{-q}\Gamma(q)\Psi(q, q+1-\nu, p/a),$$
$$\Re\{q\} > 0, \quad \Re\{p\} > 0, \quad \Re\{a\} > 0. \tag{2}$$

It can be seen in [12, pp. 344. 5.].

B. The confluent hypergeometric function $\Psi(a, c; x)$

$$\Psi(a, c; x) = \frac{1}{\Gamma(a)} \int_0^\infty e^{-xt} t^{a-1}(1+t)^{c-a-1} dt, \quad \Re\{a\} > 0. \tag{3}$$

It satisfies

$$x\frac{d^2y}{dx^2} + (c-x)\frac{dy}{dx} - ay = 0. \tag{4}$$

Bateman has given a thorough analysis on $\Psi(a, c; x)$ [13, pp. 257-261]. We will use some of his elegant results as follows.

- If c is not an integer,

$$\Psi(a, c; x) = \frac{\Gamma(1-c)}{\Gamma(a-c+1)}\Phi(a, c; x) + \frac{\Gamma(c-1)}{\Gamma(a)}x^{1-c}\Phi(a-c+1, 2-c; x) \tag{5}$$

  where $\Phi(a, c; x)$ is another confluent hypergeometric function,

$$\Phi(a, c; x) = \sum_{r=0}^{\infty} \frac{(a)_r}{(c)_r}\frac{x^r}{r!}. \tag{6}$$

  Note that $(a)_n = \Gamma(a+n)/\Gamma(a)$.
- if c is a positive integer,

$$\Psi(a, n+1; x) = \frac{(-1)^{n-1}}{n!\Gamma(a-n)}\Bigg\{\Phi(a, n+1; x)\log x + \sum_{r=0}^{\infty} \frac{(a)_r}{(n+1)_r}\left[\psi(a+r) - \psi(1+r) - \psi(1+n+r)\right]\frac{x^r}{r!}\Bigg\} + \frac{(n-1)!}{\Gamma(a)}\sum_{r=0}^{n-1}\frac{(a-n)_r}{(1-n)_r}\frac{x^{r-n}}{r!} \quad n = 0, 1, 2, ... \tag{7}$$

  The last sum is to be omitted if $n = 0$.
-
$$\Psi(a, c; x) = x^{1-c}\Psi(a-c+1, 2-c; x). \tag{8}$$

  Thus, when c is a non-positive integer, we can obtain the form of $\Psi(a, c; x)$ from (7) and (8), which is similar to (7),

$$\Psi(a, c; x) = \frac{(-1)^{-c}}{(1-c)!\Gamma(a)}\Bigg\{\Phi(a+1-c, 2-c; x)x^{1-c}\log x + \sum_{r=0}^{\infty}\frac{(a+1-c)_r}{(2-c)_r}\left[\psi(a+1-c+r) - \psi(1+r) - \psi(2-c+r)\right]\frac{x^{r+1-c}}{r!}\Bigg\} + \frac{\Gamma(1-c)}{\Gamma(a+1-c)}\sum_{r=0}^{-c}\frac{(a)_r}{(c)_r}\frac{x^r}{r!} \tag{9}$$

- When x is small, see Table I on the bottom of this page .

IV. Main Results

The mean end-to-end quadratic distortion

$$ED = \mathbb{E}(D) = \int_0^\infty \left(s(t) - \widehat{s}(t)\right)^2 dt. \tag{10}$$

Theorem 1 (Expected quadratic distortion lower bound): The expected quadratic distortion of spatially correlated systems is tightly lower bounded by

$$ED_{\text{corr}}^{\text{LB}} = \frac{P_s|\mathbf{\Sigma}|^{-N_{\max}} \det \boldsymbol{G}}{\prod_{k=1}^{N_{\min}} \Gamma\left(N_{\max} - k + 1\right) |\boldsymbol{V}_2(\boldsymbol{\sigma})|} \tag{11}$$

where $\boldsymbol{G}$ is an $N_{\min} \times N_{\min}$ matrix with ij-th elements given by

$$g_{ij} = \left(\frac{\rho}{M}\right)^{-d_j}\Gamma(d_j)\Psi\left(d_j, d_j + 1 - \frac{2}{\eta}; \frac{N_t}{\sigma_i\rho}\right). \tag{12}$$

P_s is the source power, ρ is the SNR per receive antenna, η is the bandwidth ratio W_s/W_c, $d_j = N_{\max} - N_{\min} + j$, $N_{\max} = \max\{N_t, N_r\}$, $N_{\min} = \min\{N_t, N_r\}$, and $\boldsymbol{\sigma} = [\sigma_1, \sigma_2, ..., \sigma_{N_{\min}}]$, with $\sigma_1 > \sigma_2 > \cdots \sigma_{N_{\min}} > 0$ denoting the ordered eigenvalues of the correlation matrix $\mathbf{\Sigma}$. $\boldsymbol{V}_2(\boldsymbol{\sigma})$ is a Vandermonde matrix given by

$$\boldsymbol{V}_2(\boldsymbol{\sigma}) \triangleq \boldsymbol{V}_1\left(-\left[\sigma_1^{-1}, \cdots, \sigma_{N_{\min}}^{-1}\right]\right) \tag{13}$$

where the Vandermonde matrix $\boldsymbol{V}_1(\boldsymbol{x})$ is defined by

$$\boldsymbol{V}_1(\boldsymbol{x}) \triangleq \begin{bmatrix} 1 & 1 & \cdots & 1 \\ x_1 & x_2 & \cdots & x_{N_{\min}} \\ \vdots & \vdots & \ddots & \vdots \\ x_1^{N_{\min}-1} & x_2^{N_{\min}-1} & \cdots & x_{N_{\min}}^{N_{\min}-1} \end{bmatrix}. \tag{14}$$

TABLE I
$\Psi(a, c; x)$ FOR SMALL x, REAL c

c	Ψ
$c > 1$	$x^{1-c}\Gamma(c-1)/\Gamma(a) + o\left(x^{1-c}\right)$
$c = 1$	$-\left[\Gamma(a)\right]^{-1}\log x + o\left(\lvert\log x\rvert\right)$
$c < 1$	$\Gamma(1-c)/\Gamma(a-c+1) + o(1)$

Proof: Following the proof of Theorem 1 in [8], the tight lower bound of the mean end-to-end quadratic distortion with respect to $\boldsymbol{H}$,

$$ED_{\text{corr}}^{\text{LB}} = P_s \mathbb{E}_{\boldsymbol{H}}[\det(\boldsymbol{I}_N + \frac{\rho}{N_t}\boldsymbol{H}\boldsymbol{H}^\dagger)^{-\frac{2}{\eta}}]. \tag{15}$$

Observing (15), we can find that it has the same form as the moment generating function of capacity in [11] . Thus, by the mathematical results given by Chiani *et al.* in [11] for the expectation over spatially correlated $\boldsymbol{H}$, we get

$$ED_{\text{corr}}^{\text{LB}} = P_s K_{\boldsymbol{\Sigma}} \det \boldsymbol{G} \tag{16}$$

where $\boldsymbol{G}$ is an $N_{\min} \times N_{\min}$ matrix with ij-th elements given by

$$g_{ij} = \int_0^\infty x^{N_{\max}-N_{\min}+j-1} e^{-x/\sigma_i}(1+\frac{\rho}{N_t}x)^{-\frac{2}{\eta}}dx \tag{17}$$

and

$$K_{\boldsymbol{\Sigma}} = \frac{|\boldsymbol{\Sigma}|^{-N_{\max}}}{\prod_{k=1}^{N_{\min}}\Gamma\left(N_{\max}-k+1\right)|\boldsymbol{V}_2(\boldsymbol{\sigma})|}. \tag{18}$$

By (2), we can write (17) in an analytic form

$$g_{ij} = \left(\frac{\rho}{N_t}\right)^{-d_j}\Gamma(d_j)\Psi\left(d_j, d_j+1-\frac{2}{\eta}; \frac{N_t}{\sigma_i\rho}\right). \tag{19}$$

This concludes the proof of the theorem. ∎

Theorem 2 (Distortion exponent upper bound): At the asymptotically high SNR, there exists an SNR exponent $\Delta_{\text{corr}}^{\text{UB}}$ in the optimum distortion of spatially correlated systems,

$$\begin{aligned}\Delta_{\text{corr}}^{\text{UB}} &= -\lim_{\rho\to\infty}\frac{\log ED_{\text{corr}}^{\text{LB}}}{\log\rho} \\ &= \begin{cases}\sum_{j=1}^{N_{\min}}\min\{\frac{2}{\eta}, d_j\}, & \frac{2}{\eta}\le N_{\max}; \\ \min\{\frac{2}{\eta}, N_{\max}+1\}+\sum_{j=1}^{N_{\min}-1}d_j, & \frac{2}{\eta}>N_{\max}.\end{cases}\end{aligned} \tag{20}$$

Proof: From Theorem 1 and Section III-B, it is easy to see that there exists an SNR exponent in the optimum distortion at the asymptotically high SNR, *i.e.,*

$$\Delta_{\text{corr}}^{UB} = -\lim_{\rho\to\infty}\frac{\log ED_{\text{corr}}^{\text{LB}}}{\log\rho}. \tag{21}$$

We denote $\boldsymbol{G}$ at high SNR by $\widetilde{\boldsymbol{G}}$ where each element $\widetilde{g}_{ij}$ is in the form of $k_{ij}\rho^{-\Delta_{ij}}$. Consequently, at high SNR,

$$\det\widetilde{\boldsymbol{G}} \sim \nu\,\rho^{-\Delta_{\text{corr}}^{\text{UB}}}. \tag{22}$$

Due to the complexity of the function $\Psi(a, c; x)$, we will discuss on $\det\widetilde{\boldsymbol{G}}$ for deriving $\Delta_{\text{corr}}^{\text{UB}}$ in several cases as follows.

i) $d_{N_{\min}} > 2/\eta$, *i.e.,* $2/\eta < N_{\max}$, and $2/\eta$ is not an integer.

In this case, supposing $d_l < 2/\eta < d_{l+1}$, $l \in \mathbb{Z}^+$, according to Table I, we have

$$\Delta_{ij} = \begin{cases} d_j, & 1\le j\le l \\ \frac{2}{\eta}, & l<j\le N_{\min}\end{cases} \tag{23}$$

and

$$k_{ij} = \begin{cases} N_t^{d_j}\Gamma(d_j)\Gamma(\frac{2}{\eta}-d_j)\Gamma^{-1}(\frac{2}{\eta}), & 1\le j\le l \\ N_t^{\frac{2}{\eta}}\sigma_i^{d_j-\frac{2}{\eta}}\Gamma(d_j-\frac{2}{\eta}), & l<j\le N_{\min}.\end{cases} \tag{24}$$

Hence,

$$\Delta_{\text{corr}}^{\text{UB}} = \sum_{i=1}^{N_{\min}}\min\{\frac{2}{\eta}, d_j\} \quad \text{for} \quad \frac{2}{\eta}<N_{\max}, \frac{2}{\eta}\notin\mathbb{Z}. \tag{25}$$

ii) $d_{N_{\min}} > 2/\eta$, $2/\eta$ is an integer.

In this case, supposing $d_l = 2/\eta$, $l \in \mathbb{Z}^+$, according to Table I, we have

$$\Delta_{ij} = \begin{cases} d_j, & 1\le j\le l; \\ \frac{2}{\eta}, & l<j\le N_{\min},\end{cases} \tag{26}$$

and

$$k_{ij} = \begin{cases} N_t^{d_j}\Gamma(d_j)\Gamma(\frac{2}{\eta}-d_j)\Gamma^{-1}(\frac{2}{\eta}), & 1\le j< l; \\ N_t^{d_l}\log\rho, & j=l; \\ N_t^{\frac{2}{\eta}}\sigma_i^{d_j-\frac{2}{\eta}}\Gamma(d_j-\frac{2}{\eta}), & l<j\le N_{\min}.\end{cases} \tag{27}$$

Hence,

$$\Delta_{\text{corr}}^{\text{UB}} = \sum_{i=1}^{N_{\min}}\min\{\frac{2}{\eta}, d_j\} \quad \text{for} \quad \frac{2}{\eta}<N_{\max}, \frac{2}{\eta}\in\mathbb{Z}. \tag{28}$$

iii) $d_{N_{\min}} = 2/\eta$, *i.e.,* $2/\eta = N_{\max}$.

In this case, according to Table I, we have

$$\Delta_{ij} = d_j, \quad 1\le j\le N_{\min}, \tag{29}$$

and

$$k_{ij} = \begin{cases} N_t^{d_j}\Gamma(d_j)\Gamma(\frac{2}{\eta}-d_j)\Gamma^{-1}(\frac{2}{\eta}), & 1\le j< N_{\min}; \\ -N_t^{N_{\max}}\log\frac{N_t}{\sigma_i}, & j=N_{\min}.\end{cases} \tag{30}$$

We remark that $k_{iN_{\min}} = -N_t^{N_{\max}}\log\frac{N_t}{\sigma_i}$ is because the term $N_t^{N_{\max}}\log\rho$ does not contribute to $\det\widetilde{\boldsymbol{G}}$. Hence,

$$\Delta_{\text{corr}}^{\text{UB}} = \sum_{i=1}^{N_{\min}}\min\{\frac{2}{\eta}, d_j\} \quad \text{for} \quad \frac{2}{\eta}=N_{\max}. \tag{31}$$

iv) $d_{N_{\min}} < 2/\eta$, *i.e.,* $2/\eta > N_{\max}$.

This case makes the analysis more complicated as we have to take care of keeping $\widetilde{G}$ as a full rank matrix.

If we consider

$$\widetilde{g}_{iN_{\min}} = N_t^{N_{\max}}\Gamma(N_{\max})\Gamma(\frac{2}{\eta}-N_{\max})/\Gamma(\frac{2}{\eta})\ \rho^{-N_{\max}} \tag{32}$$

as foregoing, then $\widetilde{\boldsymbol{G}}$ would become a singular matrix. Hence, in fact, for $\widetilde{\boldsymbol{G}}$, it should be a lower-order term in the polynomial of $g_{iN_{\min}}$ which involves the row index i into the factor k_{ij} contributes. Thus, we need to look at the polynomial of $g_{iN_{\min}}$ in different cases.
When $2/\eta$ is not an integer, by (5), we can get

$$\Delta_{ij} = \begin{cases} d_j,\ j < N_{\min} \\ \frac{2}{\eta},\ \text{if } j = N_{\min},\ 2/\eta - 1 < N_{\max} < 2/\eta \\ N_{\max}+1,\ \text{if } j = N_{\min},\ N_{\max} < 2/\eta - 1 \end{cases} \tag{33}$$

and

$$k_{ij} = \begin{cases} N_t^{d_j}\Gamma(d_j)\Gamma(\frac{2}{\eta}-d_j)\Gamma^{-1}(\frac{2}{\eta}),\ j < N_{\min} \\ N_t^{\frac{2}{\eta}}\sigma_i^{N_{\max}-\frac{2}{\eta}}\Gamma(N_{\max}-\frac{2}{\eta}), \\ \quad \text{if } j = N_{\min},\ 2/\eta - 1 < N_{\max} < 2/\eta \\ \frac{N_t^{N_{\max}+1}N_{\max}\Gamma(N_{\max})\Gamma(\frac{2}{\eta}-N_{\max})}{\sigma_i(N_{\max}+1-\frac{2}{\eta})\Gamma(\frac{2}{\eta})} \\ \quad \text{if } j = N_{\min},\ N_{\max} < 2/\eta - 1. \end{cases} \tag{34}$$

When $2/\eta$ is an integer, by (9), we can get

$$\Delta_{ij} = \begin{cases} d_j,\ j < N_{\min} \\ N_{\max}+1,\ j = N_{\min} \end{cases} \tag{35}$$

and

$$k_{ij} = \begin{cases} N_t^{d_j}\Gamma(d_j)\Gamma(\frac{2}{\eta}-d_j)\Gamma^{-1}(\frac{2}{\eta}),\ j < N_{\min} \\ -\frac{N_t^{N_{\max}}}{\sigma_i}\log\rho,\ \text{if } \ j = N_{min},\ \frac{2}{\eta} = N_{\max}+1 \\ \frac{N_t^{N_{\max}+1}N_{\max}\Gamma(N_{\max})\Gamma(\frac{2}{\eta}-N_{\max})}{\sigma_i(N_{\max}+1-\frac{2}{\eta})\Gamma(\frac{2}{\eta})} \\ \text{if } \ j = N_{min},\ \frac{2}{\eta} > N_{\max}+1 \end{cases} \tag{36}$$

Hence,

$$\Delta_{\text{corr}}^{\text{UB}} = \min\left\{\frac{2}{\eta}, N_{\max}+1\right\} + \sum_{i=1}^{N_{\min}-1} d_j \quad \text{for} \quad \frac{2}{\eta} > N_{\max}. \tag{37}$$

Combining expressions of $\Delta_{\text{corr}}^{\text{UB}}$ in all cases, we conclude the theorem. ■

Comparing to the distortion SNR exponent upper bound of uncorrelated MIMO systems [5]–[8],

$$\Delta_{\text{uncorr}}^{\text{UB}} = \sum_{k=1}^{N_{\min}} \min\left\{\frac{2}{\eta}, N_{\max}-N_{\min}+2k-1\right\}, \tag{38}$$

we find that when $2/\eta \leq N_{\max}-N_{\min}+2$, $\Delta_{\text{corr}}^{\text{UB}}$ is the same to $\Delta_{\text{uncorr}}^{\text{UB}}$, and when $2/\eta > N_{\max}-N_{\min}+2$, $\Delta_{\text{corr}}^{\text{UB}}$ is smaller to $\Delta_{\text{uncorr}}^{\text{UB}}$. Therefore, when the bandwidth ratio η is not high enough, corresponding to the case of $2/\eta > N_{\max}-N_{\min}+2$, at the asymptotically high SNR, the optimum distortion of a correlated MIMO system has a flatter descendent slope with SNR than that of an uncorrelated MIMO system with same antennas and bandwidth ratio. When the bandwidth ratio is high enough, the two have the same descendent slope.

Theorem 3 (Corresponding distortion factor): Define the corresponding distortion factor μ_{corr}^* for the optimum end-to-end distortion of spatially correlated MIMO systems at the asymptotically high SNR as

$$ED_{\text{corr}}^{\text{LB}} \sim \mu_{\text{corr}}^*\rho^{-\Delta_{\text{corr}}^{\text{UB}}} \tag{39}$$

where

$$\lim_{\rho\to\infty}\frac{\log\mu_{\text{corr}}^*}{\log\rho} = 0. \tag{40}$$

Then

$$\mu_{\text{corr}}^* = \frac{P_s|\boldsymbol{\Sigma}|^{-N_{\max}}\det\boldsymbol{K}}{\prod_{k=1}^{N_{\min}}\Gamma\left(N_{\max}-k+1\right)|\boldsymbol{V}_2(\boldsymbol{\sigma})|} \tag{41}$$

where $\boldsymbol{K}$ is an $N_{\min}\times N_{\min}$ matrix with ijth elements k_{ij}-s which are given in Theorem 2 for all cases.

Proof: The conclusion is straightforwardly from Theorem 2. ■

Note that when $d_1 > 2/\eta$, *i.e.*, $\eta > 2/(N_{\max}-N_{\min}+1)$, which is called *the high-bandwidth-ratio state* in this paper, we can write μ_{corr}^* in a more compact closed-form

$$\mu_{\text{corr}}^* = \frac{P_s|\boldsymbol{\Sigma}|^{-N_{\max}}N_t^{-\frac{2N_{\min}}{\eta}}|\boldsymbol{V}_1(\boldsymbol{\sigma})|\prod_{k=1}^{N_{\min}}\sigma_i^{d_k}\Gamma\left(d_k-\frac{2}{\eta}\right)}{\prod_{k=1}^{N_{\min}}\Gamma\left(d_k\right)|\boldsymbol{V}_2(\boldsymbol{\sigma})|}. \tag{42}$$

V. Simulation and Analysis

The analytical framework we derived is general and valid for correlation matrices $\boldsymbol{\Sigma}$ all of whose eigenvalues are distinct. To give an example, we consider a well-known correlation model: the exponential correlation with $\boldsymbol{\Sigma} = \left\{r^{-i-j}\right\}_{i,j=1,\ldots,N_r}$ and $r\in(0,1)$ [14] as in [11].

Fig. 1 shows lower bounds of the mean quadratic end-to-end distortion for a thermal noise source with power 1 conveyed over a 2×2 MIMO channel under Rayleigh fading. The system is in the high-bandwidth-ratio state, $\eta = 10$. The SNR per receive antenna ranges from 10 dB to 40 dB. The blue lines represent results of Monte Carlo simulations which are carried out by generating 5 000 realizations of $\boldsymbol{H}$ and evaluating (15). The red circles represent the approximate optimum distortion $ED_{\text{corr}}^{\text{LB}*} = \mu_{\text{corr}}^*\rho^{-\Delta_{\text{corr}}^{\text{UB}}}$.

It can be seen that the distortion lower bound when $r = 0.9$ is about 1.5 dB higher than that when $r = 0.1$. It corresponds to our intuition since spatial correlation decreases channel capacity. Comparing the simulation results and the approximate distortion, as we expected, we see that simulation results are approaching approximate optimal distortion with the increase of SNR and overlap approximate optimal distortion when SNR is high enough.

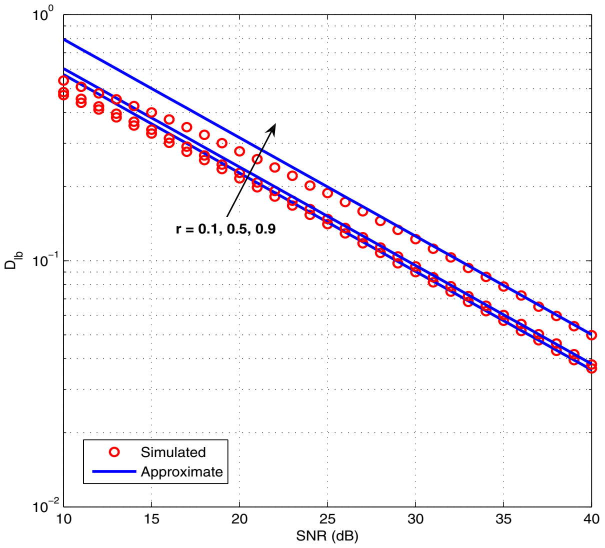

Fig. 1. Tight lower bounds of the mean quadratic distortion for a MIMO system in the high-bandwidth-ratio state conveying a thermal noise source with $N_t = 2$, $N_r = 2$, $P_s = 1$ and $\eta = 10$. Exponential correlation cases with $r = 0.1, 0.5, 0.9$

VI. Conclusion

In this paper, assuming a continuous thermal noise source is transmitted over a spatially correlated MIMO channel under Rayleigh fading, we have derived the compact analytic expression of the tight lower bound on the end-to-end mean quadratic distortion. Stemming from it, we have derived the upper bound on the distortion SNR exponent and the corresponding factor for any bandwidth ratio. Numerical results show that higher correlation improves the distortion lower bound which corresponds to our intuition and simulation results corresponds to approximate lower bounds in closed-form at high SNR.

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