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Spectral efficiency of spectrum-pooling systems

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Abstract: The authors investigate the idea of using cognitive radio to reuse locally unused spectrum to increase the total system capacity. The authors consider a multiband/wideband system in which the primary and cognitive users wish to communicate to different receivers, subject to mutual interference and assume that each user knows only his/her channel and the unused spectrum through adequate sensing. The basic idea under the proposed scheme is based on the notion of spectrum pooling. The idea is quite simple; a cognitive radio will listen to the channel and, if sensed idle, will transmit during the voids. It turns out that, although its simplicity, the proposed scheme showed very interesting features with respect to the spectral efficiency and the maximum number of possible pairwise cognitive communications. We impose the constraint that users successively transmit over available bands through selfish water filling. For the first time, our study has quantified the asymptotic (with respect to the band) achievable gain of using spectrum pooling in terms of spectral efficiency compared with classical radio systems. The authors then derive the total spectral efficiency as well as the maximum number of possible pairwise communications of such a spectrum-pooling system.

1 Introduction

The recent boom in personal wireless technologies has led to an increasing demand in terms of spectrum resources. To combat this overcrowding, the FCC has recently recommended [1] that significantly greater spectral efficiency could be realised by deploying wireless devices that can coexist with the licenced (primary) users, generating minimal interference while taking advantage of the available resources. The current approach for spectrum sharing is regulated so that wireless systems are assigned fixed spectrum allocations, operating frequencies and bandwidths, with constraints on power emission that limit their range. Therefore most communication systems are designed in order to achieve the best possible spectrum efficiency within the assigned bandwidth using sophisticated modulation, coding, multiple antennas and other techniques.

On the other hand, the discrepancy between spectrum allocation and use suggests that this spectrum shortage could be overcome by allowing more flexible usage of a spectrum. Flexibility would mean that radios could find and adapt to any immediate local spectrum availability. A new class of radios that is able to reliably sense the spectral environment over a wide bandwidth, detect the presence/absence of legacy users (primary users) and use the spectrum only if the communication does not interfere with primary users is defined by the term cognitive radio (CR) [2]. CRs have been proposed as a mean to implement efficient reuse of the licenced spectrum. The key feature of cognitive radios is their ability to recognise their communication environment and independently adapt the parameters of their communication scheme to maximise the quality of service (QoS) for the secondary (unlicensed) users while minimising the interference to the primary users.

The basic idea within the paper is based on spectrum pooling. The notion of spectrum pooling was first mentioned in [3]. It basically represents the idea of merging spectral ranges from different spectrum owners (military, trunked radio etc.) into a common pool. It also reflects the need for a completely new way of spectrum allocation as proposed in [4]. The goal of spectrum pooling is to enhance spectral efficiency by overlaying a new mobile radio system on an existing one without requiring any changes to the actual licenced system.

Another technique that has been increasingly popular is time division duplexing (TDD) in which the same carrier is used for both links in different time slots. One property of such systems is that, since the same frequency is used, the channel characteristics are nearly the same in both links, provided the channel does not change too rapidly.

Motivated by the desire for an effective and practical scheme, our study treats the problem of spectrum pooling from sensing to achievable performance. We consider an asynchronous TDD communication scenario in which the primary and cognitive users wish to communicate to different receivers, subject to mutual interference in a heterogeneous network where devices operates in a wideband/multiband context. However, contrary to the work addressed in [5], in this contribution, we impose as a first step that only one user can simultaneously transmit over the same sub-band using successive water filling. Especially OFDM-based WLANs such as IEEE802.11a and HIPERLAN/2 are suitable for an overlay system such as spectrum-pooling as they allow a very flexible frequency management on a carrier-by-carrier basis. We examine the total spectral efficiency of the spectrum-pooling system and show that the overall system spectral efficiency can be considerably enhanced by considering cognitive communications with respect to the traditional system (without cognition). In particular, it is of major interest, in this context, to quantify the spectral efficiency gain in order to show the interest behind using spectrum-pooling terminals with respect to classical systems (without cognition). In fact, although spectrum pooling has spurred great interest and excitement, many of the fundamental theoretical questions on the limits of such technologies remain unanswered. The merits of our approach lie in the simplicity of the proposed scheme and, at the same time, its efficiency. Results showed very interesting performance in terms of the number of cognitive users allowed to transmit as well as the system spectral efficiency gain we obtain. Such an accurate and simple system modelling presents a key to understand the actual benefits brought by spectrum-pooling technology.

The rest of the paper is organised as follows: In Section 2, we describe the channel model. In Section 3, we describe the spectrum-pooling protocol. In Section 4, we address the problem of sensing. Section 5 details the spectral efficiency analysis adopted throughout this paper when the number of sub-bands is limited. In Section 6, we investigate the asymptotic performance of such a system in terms of spectral efficiency. Performance evaluation is provided in Section 7 and Section 8 concludes the paper.

2 Channel model

The baseband discrete-frequency model at the receiver $\mathfrak{R}_l$ (see Fig. 1) is

$$y^i_{\mathfrak{R}_l} = h^i_l \sqrt{P^i_l(h^i_l)} S^i_l + n^i_l, \quad \text{for} \quad i = 1, \ldots, N \quad \text{and} \quad l = 1, \ldots, L \tag{1}$$

where

- h^i_l: is the block fading process of user l on the sub-band i,
- S^i_l: is the symbol transmitted by user l on the sub-band i,
- $P^i_l(h^i_l)$: is the power control (throughout the paper, we will find it convenient to denote by P^i_l the power allocation policy of user l on sub-band i, rather than $P^i_l(h^i_l)$) of user l on the sub-band i,
- n^i_l: is the additive Gaussian noise at the ith sub-band.

We further assume that the channel h_l stays constant over each block-fading length (i.e. coherent communication). The assumption of coherent reception is reasonable if the fading is slow in the sense that the receiver is able to track the channel variations. We statistically model the channel gains h_l to be i.i.d distributed over the L Rayleigh fading coefficients and $\mathbb{E}\{|h_l|^2\} = 1$ for $l = 1, \ldots, L$. The additive Gaussian noise n_l at the receiver is i.i.d circularly symmetric and $n_l \sim \mathcal{CN}(0, N_0)$ for $l = 1, \ldots, L$.

3 Spectrum-pooling protocol

We consider an asynchronous TDD communication scenario in which the primary and cognitive users wish to communicate to different receivers, subject to mutual interference. The basic idea under the proposed protocol is quite simple: the cognitive users listen to the wireless channel and determine, either in time or frequency, which part of the spectrum is unused. Then, they successively adapt their signal to fill detected voids in the spectrum

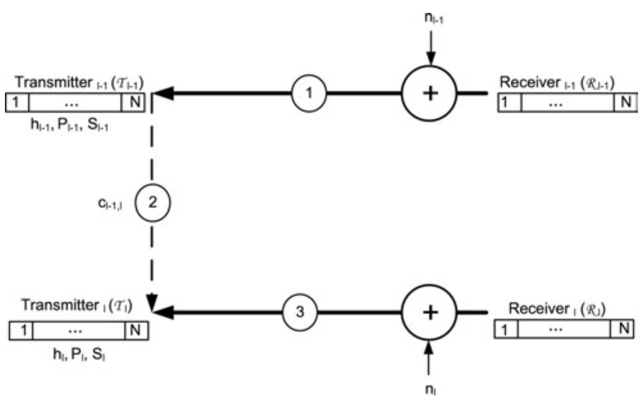

Figure 1 *Cognitive radio channel in a wideband/multiband context with N sub-bands*

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domain. Each transmitter $\mathcal{T}_l$ for $l = 1, \ldots, L$ estimates the pilot sequence of the receiver $\mathcal{R}_l$ in order to determine the channel gain h_l (see links (1) and (3) in Fig. 1). Notice here that since we are in a TDD mode, when we estimate the channel in one way, we can also know it the other way. Thus, each user l is assumed to know only his/her own channel gain h_l and the statistical properties of the other links (probability distribution). We further assume that the channel does not change from the instant of estimation to the instant of transmission.

A particularly noteworthy target in this context, when we employ a 'listen-before-talk' strategy, is to reliably detect the sub-bands that are currently accessed by a specified user in order to be spared from the coming users transmission. This knowledge can be obtained from two manners. In a centralised mode where the proposed system would require information from a third party (i.e. central database maintained by regulator or another authorised entity) to schedule users coming. Alternatively, an extra signalling channel is dedicated to perform the collision detection so that cognitive users will not transmit at the same moment. Specifically, the primary users come first in the system and estimate their channel gain. The second user comes in the system randomly, for instance in a Poisson process manner, and estimates his channel link. Such an assumption could be further justified by the fact that in an asynchronous context, the probability that two users decide to transmit at the same moment is negligible as the number of users is limited. Thus, within this setting, the primary users are assumed not to be aware of the cognitive users. They communicate with their receiver in an *ad hoc* manner, while a set of spectrum-pooling transmitters that are able to reliably sense the spectral environment over a wide bandwidth, and decide to communicate with their respective receivers only if the communication does not interfere with the primary users. Accordingly, under our opportunistic approach, a device transmits over a certain sub-band only when no other user does. Such an assumption is motivated by the fact that when $\mathcal{R}_l$ sends its pilot sequence to $\mathcal{T}_l$; it will not interfere with $\mathcal{T}_{l-1}$ for $l = 2, \ldots, L$. The sensing operation will be discussed in the next section. Throughout the paper, we will adopt this framework to analyse the achievable performance of such a system in terms of spectral efficiency as well as the maximum number of possible pairwise communication within this scenario. Such an accurate and simple system modelling presents a key to understand the actual benefits brought by spectrum-pooling technology. In fact, although cognitive radios have spurred great interest and excitement in industry, many of the fundamental theoretical questions on the limits of such technologies remain unanswered.

Moreover, in order to characterise the achievable performance limit of such systems, three capacity measures can be found in the literature. A comprehensive review of these concepts can be found in [6]. The relevant performance metric of the proposed protocol is the instantaneous capacity per sub-band in bits per second per hertz, also called spectral efficiency, namely [7]

$$C_l = \frac{1}{N}\sum_{i=1}^{N}\log_2\left(1 + \frac{P_l^i|h_l^i|^2}{N_0}\right), \quad l = 1, \ldots, L \tag{2}$$

The sum here is done over the stationary instantaneous distribution of the fading channel on each user l. The instantaneous capacity determines the maximum achievable rate over all fading states without a delay constraint. In this work, we allocate transmit powers for each user (over a total power budget constraint) in order to maximise each user's transmission rate. In fact, when channel state information is made available at the transmitters, users know their own channel gains and thus they will adapt their transmission strategy relative to this knowledge. The corresponding optimum power allocation is the well-known water filling allocation [8] expressed by

$$P_l^i = \left(\frac{1}{\gamma_0} - \frac{N_0}{|h_l^i|^2}\right)^+ \tag{3}$$

[this is given in a simple form of $(x)^+ = \max(0, x)$] where γ_0 is the Lagrange's multiplier satisfying the average power constraint per sub-band

$$\frac{1}{N}\sum_{i=1}^{N} P_l^i = \overline{P} \tag{4}$$

Without loss of generality, throughout the paper, we take $\overline{P} = 1$.

Notice that, although a water filling power allocation strategy is adopted in this analysis, we emphasise that this is not a restriction of the proposed protocol. In fact, as mentioned before, one important task when implementing spectrum pooling is that cognitive users operate on the idle sub-bands of the licenced system delivering a binary channel assignment as shown in Fig. 2. Hence, our study is valid for any binary power control without resorting to the restriction assumption of successive water filling.

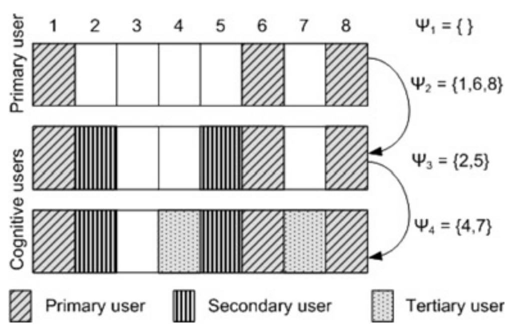

Figure 2 *One primary user and two cognitive users in a spectrum-pooling system with eight sub-bands*

For clarity sake, let us take the following example with $N = 8$ sub-bands. As shown in Fig. 2, the primary user is always prioritised above cognitive users by enjoying the entire band while cognitive users adapt their signal to fill detected voids with respect to their order of priority. As a first step, the primary users maximise their rate according to their channel process. As mentioned before in (3), only user with a channel gain h^i above a certain threshold equal to $\gamma_0 \cdot N_0$ transmits on the sub-band i (Ψ_2). User 2, comes in the system randomly, senses the spectrum and decides to transmit only on sub-bands sensed idle. Thus, following user 2's fading gains, user 2 adapts his/her signal to fill these voids in the spectrum domain in a complementary fashion (Ψ_3). Similarly, user 3 will sense the remaining sub-bands from users 1 and 2 and decides to transmit during the remaining voids (Ψ_4).

4 Sensing issue

So far, we have focused on pairwise communications between transmitters and receivers (see links 1 and 3 in Fig. 1). Let us now investigate the inter-transmitter communications (link 2 in Fig. 1) in order to analyse the problem of sensing. To this effect, let us assume the baseband discrete-time model within a coherence time period T when each user l for $l = 2, \ldots, L$ has N sub-bands as described in Fig. 1

$$y_l^i(k) = c_{l-1,l}^i(k)\sqrt{P_{l-1}^i(h_{l-1}^i)}S_{l-1}^i(k) + n_{l-1}^i(k) \tag{5}$$

where $c_{l-1,l}^i(k)$ is the block-fading process from user $l-1$ to user l on the ith sub-band, at time k. We further assume that $0 \leq k \ll \beta T$ and $\beta < 1$, that is the coherence time is sufficiently large so that the channel stays constant for samples and jumps to a new independent value (block-fading model).

The proposed sensing techniques hinge on the assumption that all devices operate under a unique standard so that they know the pilot sequence used by the other users.

As stated above, in this work, the spectrum-pooling behaviour is assumed to allow only one user to simultaneously transmit over the same sub-band. The received signal at user l can therefore be written as (see link 2 in Fig. 1)

$$y_l^i(k) = \begin{cases} \underbrace{c_{l-1,l}^i(k)\sqrt{P_{l-1}^i}S_{l-1}^i(k)}_{\text{signal}} + \underbrace{n_{l-1}^i(k)}_{\text{noise}}, & \text{if } P_{l-1}^i \neq 0 \\ n_{l-1}^i(k), & \text{otherwise} \end{cases} \tag{6}$$

By assuming that βT is an integer equal to M and by making βT sufficiently large, the mean received power over the detection duration at receiver $\mathcal{R}_l$ is

$$\lim_{M\to\infty} \frac{1}{M}\sum_{k=1}^{M} |y_l^i(k)|^2 = \begin{cases} |c_{l-1,l}^i|^2 P_{l-1}^i + N_0, & \text{if } P_{l-1}^i \neq 0 \\ N_0, & \text{otherwise} \end{cases} \tag{7}$$

Accordingly, in order to determine which part of the spectrum is unused, cognitive user has just to detect the received power and compare it to the noise power N_0. However, in addition to the fact that it supposes that $M \to \infty$ (i.e. infinite time coherence period), the proposed method would be not efficient at low SNR-regime (Fig. 3). In fact, the quality of such a technique is strongly degraded with the reduction in the precision of the noise threshold [9, 10]. The principal difficulty of this detection is to obtain a good estimation of the noise variance. In the setting of spectrum-pooling mechanism, we would need a channel sensing method that continuously senses the channel. Thus, the channel sensing should be performed with a very high probability of correct detection (to assure very low probability of interference with the primary system). Weiss *et al.* [11] proposed a distributed spectrum-pooling protocol where all the nodes participate in channel sensing so that all cognitive users perform detection. Moreover, formulas for the calculation of the detection and false alarm probability in a spectrum-pooling system have been derived in [12] for the general case of an arbitrary primary system's covariance matrix.

5 Spectral efficiency analysis

Let us first define the set of the number of sub-bands sensed occupied by user l by

$$\Psi_l = \{i \in \{1, \ldots, N\};\ P_{l-1}^i \neq 0\} \tag{8}$$

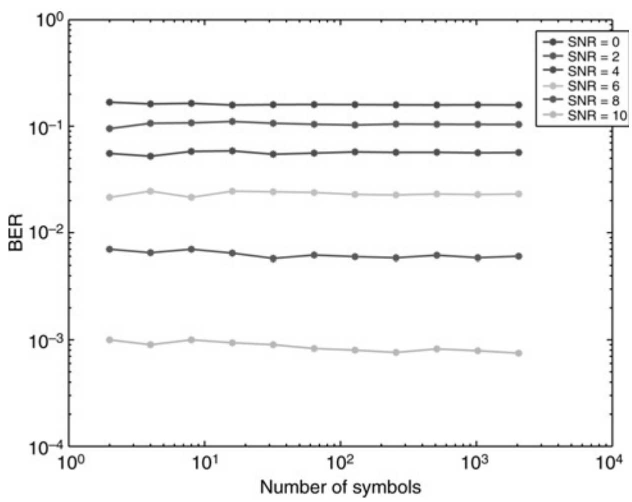

Figure 3 *BER against number of symbols (M) in decibel for BPSK in AWGN using power detection where SNRs are in decibel*

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where Ψ_l obeys to the following properties

$$\begin{cases} \Psi_1 = \emptyset, \\ \bigcup_{l=1}^{L+1} \Psi_l \subseteq \{1, \ldots, N\}, \\ \bigcap_{l=1}^{L+1} \Psi_l = \emptyset \end{cases} \quad (9)$$

The spectral efficiency per sub-band of user l, given a number of sub-bands N, is

$$C_{l,N} = \frac{1}{\text{card}(\Omega_l)} \sum_{i \in \Omega_l} \log_2\left(1 + \frac{P_l^i |h_l^i|^2}{N_0}\right) \text{bits/s/Hz} \quad (10)$$

where Ω_l represents the set of the remaining idle sub-bands sensed by user l, namely

$$\Omega_l = \left\{ i \in \{1, \ldots, N\} \bigcap \overline{\bigcup_{k=1\ldots l} \Psi_k} \right\} \quad (11)$$

For a given number of sub-bands N, the optimal power allocation which maximises the transmission rate of user l is the solution to the following optimisation problem

$$\max_{P_l^1, \ldots, P_l^{\text{card}(\Omega_l)}} C_{l,N}, \quad \text{for } l = 1, \ldots, L$$

subject to the average power constraint per sub-band

$$\begin{cases} \frac{1}{\text{card}(\Omega_l)} \sum_{i \in \Omega_l} P_l^i = 1, \\ P_l^i \geq 0, \end{cases} \quad (12)$$

The resulting optimal power control policy is given by (3). Notice that the maximum number of users L allowed by such a system must satisfy the condition that $\text{card}(\Omega_L) \neq 0$.

Let us now derive the spectral efficiency of such a system. The spectral efficiency per band of user l is given by

$$\Phi_{l,N} = \frac{1}{N} \cdot \sum_{i \in \Omega_l} \log_2\left(1 + \frac{P_l^i |h_l^i|^2}{N_0}\right) \quad (13)$$

By multiplying and dividing (13) by $\text{card}(\Omega_l)$, we obtain

$$\Phi_{l,N} = \frac{\text{card}(\Omega_l)}{N} \cdot C_{l,N}, \quad \text{for } l = 1, \ldots, L \quad (14)$$

(Notice that since the primary user enjoys the entire bandwidth, we have $\text{card}(\Omega_1) = N$.) As expected, when $l = 1$, the spectral efficiency without cognition is equal to the primary user spectral efficiency $C_{1,N}$. We define $\Delta_{1,N}$ as the band factor gain of user l for N sub-bands, namely

$$\Delta_{l,N} \triangleq \frac{\text{card}(\Omega_l)}{N}, \quad \text{for } l = 1, \ldots, L \quad (15)$$

In other words, the band factor gain represents the fraction of the band unoccupied at user l. The spectral efficiency per band of user l can therefore be expressed by

$$\Phi_{l,N} = \Delta_{1,N} . C_{1,N}, \quad \text{for } l = 1, \ldots, L \quad (16)$$

and the sum spectral efficiency of a system with N sub-bands per user is given by

$$\Phi_{\text{sum},N} = \sum_{l=1}^{L} \Phi_{l,N} \quad (17)$$

6 Asymptotic performance

Let us now study the achievable performance when devices operate in a wide-band context (i.e. $N \to \infty$). The spectral efficiency of user l for a large number of sub-bands in (10) becomes

$$C_{l,\infty} = \int_0^\infty \log_2\left(1 + \frac{P_l(t) \cdot t}{N_0}\right) \cdot f(t) \text{d}t, \quad \text{for } l = 1, \ldots, L \quad (18)$$

where P_l is subject to the average constraint

$$\int_0^\infty P_l(t) \cdot f(t) \text{d}t = 1 \quad (19)$$

Although this is not a restriction to our approach, from now on we assume that the channel gains are i.i.d Rayleigh distributed. However, all theoretical results as well as the methodology adopted in this paper can be translated immediately into results for any other probability distribution function of the channel model. In this way, the term $f(t)$ in (18) will be replaced by the appropriate probability distribution function. The spectral efficiency of user l for i.i.d Rayleigh fading is given by

$$C_{l,\infty} = \int_0^\infty \log_2\left(1 + \frac{P_l(t) \cdot t}{N_0}\right) \cdot \text{e}^{-t} \text{d}t, \quad \text{for } l = 1, \ldots, L \quad (20)$$

where P_l is subject to the average constraint

$$\int_0^\infty P_l(t) \cdot \text{e}^{-t} \text{d}t = 1 \quad (21)$$

and γ_0 is the Lagrange's multiplier satisfying

$$\frac{1}{\gamma_0} \int_{\gamma_0 \cdot N_0}^{+\infty} \text{e}^{-t} \text{d}t - N_0 \cdot E_i(\gamma_0 \cdot N_0) = 1 \quad (22)$$

($E_i(x)$ is the exponential integral function defined as $E_i(x) = \int_x^{+\infty} \text{e}^{-t}/t \text{d}t$.) Numerical root finding is needed to determine different values of γ_0. Our numerical results, in Section 7, show that γ_0 increases as N_0 decreases, and γ_0 always lies in the interval $[0, 1]$. On the other hand, an asymptotic expansion of (22) in [13] shows that at very high SNR-regime, $\gamma_0 \to 1$.

Moreover, the spectral efficiency of user l can be computed for $l = 1, \ldots, L$ as follows

$$\begin{aligned} C_{l,\infty} &= \int_0^{\infty} \log_2\left(1 + \frac{P_l(t)\cdot t}{N_0}\right)\cdot \mathrm{e}^{-t}\mathrm{d}t \\ &= \int_{\gamma_0 N_0}^{\infty} \log_2\left(1 + \frac{(1/\gamma_0 - N_0/t)\cdot t}{N_0}\right)\cdot \mathrm{e}^{-t}\mathrm{d}t \\ &= \int_{\gamma_0 N_0}^{\infty} \log_2\left(\frac{t}{\gamma_0\cdot N_0}\right)\cdot \mathrm{e}^{-t}\mathrm{d}t \\ &= \frac{1}{\ln(2)}\cdot E_i(\gamma_0\cdot N_0) \end{aligned} \tag{23}$$

In order to characterise the achievable performance of such system in terms of spectral efficiency, we define the spectral efficiency within the frequency bandwidth W as [14]

$$C_{l,\infty}(W) = \frac{1}{W}\int_{-W/2}^{W/2} \log_2\left(1 + \frac{P_l(f).|H_l(f)|^2}{N_0}\right)\mathrm{d}f \tag{24}$$

By identifying (20) with (24), we obtain a characterisation of the frequency variation f as function of the channel gains t, namely

$$f = -W\cdot \mathrm{e}^{-t} + \frac{W}{2} \tag{25}$$

Similar to our approach in the previous section, we define the band factor gain Δ_∞ as the fraction of the band sensed idle from user l to user $l+1$ over the total bandwidth W for an infinite number of sub-bands

$$\Delta_\infty \triangleq \frac{\Delta f}{W} \tag{26}$$

where Δf represents the frequency interval where the fading gain in (25) is below a certain threshold equal to $\gamma_0\cdot N_0$. By deriving the appropriate vacant band Δf when $t \in [0, \gamma_0\cdot N_0]$ in (25), we obtain

$$\Delta_\infty = 1 - \exp(-\gamma_0\cdot N_0) \tag{27}$$

Accordingly, the asymptotic spectral efficiency of user l is given by

$$\Phi_{l,\infty} = \Delta_\infty\cdot C_{l,\infty}, \quad \text{for } l = 1, \ldots, L \tag{28}$$

Similar to the case where the number of sub-bands is fixed, when $l = 1$, the spectral efficiency without cognition is equal to the primary user spectral efficiency $C_{1,\infty}$. In particular, it is of major interest to quantify the spectral efficiency gain Δ_∞ in order to show the interest behind using spectrum-pooling terminals with respect to classical systems (without cognition). To do so, following the same procedure and going from user 2 to L, we obtain the expression of the asymptotic spectral efficiency as function of $C_{1,\infty}$

$$\Phi_{l,\infty} = \Delta_{l,\infty}\cdot C_{1,\infty}, \quad \text{for } l = 1, \ldots, L \tag{29}$$

The overall asymptotic sum spectral efficiency for a system with L users is therefore

$$\begin{aligned} \Phi_{\text{sum},\infty} &= \sum_{l=1}^{L} \Phi_{l,\infty} \\ &= \sum_{k=0}^{L-1} \Delta_\infty^k C_{1,\infty} \\ &= \underbrace{\frac{1-\Delta_\infty^L}{1-\Delta_\infty}}_{\geq 1}\cdot C_{1,\infty} \end{aligned} \tag{30}$$

Thus, the sum spectral efficiency obtained by considering cognitive communications is greater than or equal to the spectral efficiency without cognition $C_{1,\infty}$. Such a result, rather intuitive, justifies the increasing interest behind using CR terminals in future wireless communication systems since the sum spectral efficiency of such systems performs always better than classical communication systems (without cognition).

On the other hand, by substituting $C_{1,\infty}$ by its expression in (23), we obtain the final expression of the achievable sum spectral efficiency in such a system

$$\Phi_{\text{sum},\infty} = \frac{1}{\ln(2)}\cdot\frac{1-\Delta_\infty^L}{1-\Delta_\infty}\cdot E_i(\gamma_0\cdot N_0) \tag{31}$$

This result is very interesting as, by only knowing the statistics of the channel gains (through γ_0) and the SNR (through N_0), one can derive the achievable spectral efficiency as well as the potential gain resulting from using spectrum pooling.

7 Performance evaluation

In order to validate our approach in Section 6 we compare the theoretical expression of the sum spectral efficiency in (31) with the expression in (17). We model L i.i.d Rayleigh channels (one for each user) and assume perfect sensing of the idle sub-bands. Our numerical result in Fig. 4 tends to validate the asymptotic analysis we adopt throughout the paper. It clearly shows that the sum spectral efficiency in (17) matches (31) even for a moderate number of sub-bands N (from $N = 16$).

Moreover, since the maximum number of users is not theoretically limited, we will consider only L that satisfies the condition that $\text{card}(\Omega_L) \neq 0$, otherwise, the Lth spectral efficiency would be negligible. Fig. 5 characterises the maximum number of users L as function of the received signal energy per information bit E_b/N_0 for different number of sub-bands N. As expected, we remark

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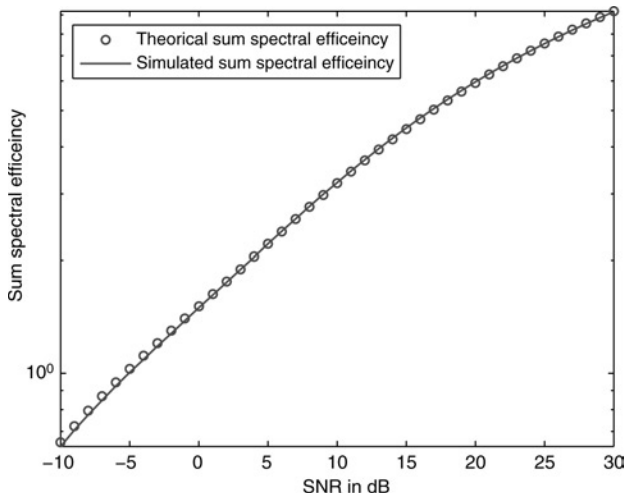

Figure 4 *Comparison between theoretical expression of the sum spectral efficiency in (31) and simulated one in (17) for L = 5 and N = 16*

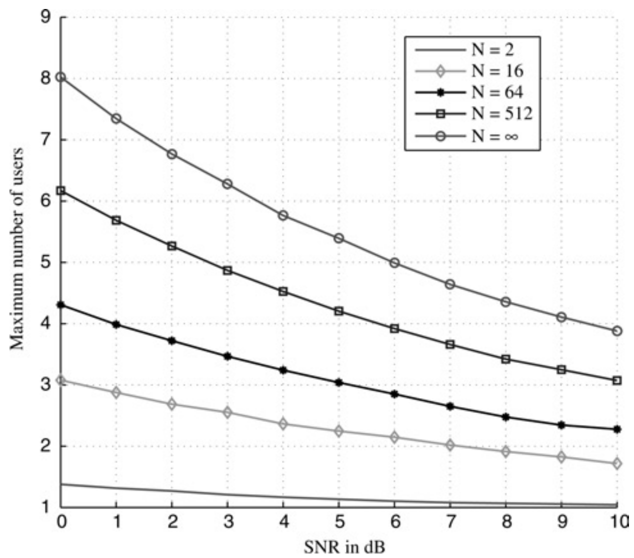

Figure 5 *Maximum number of users for different number of sub-bands (N)*

that the maximum number of users allowed to transmit increases with the number of sub-bands especially at low E_b/N_0 region. Furthermore, the maximum number of cognitive users ranges from one to eight. As an example, the proposed scheme, although its simplicity, allows up to four cognitive users to benefit from the licenced spectrum at 8 dB for $N = 2048$ sub-bands.

In [15], we analysed the different configurations of the sum spectral efficiency for a system with five users as function of the SNR. We showed that at low SNR region, the spectral efficiency is significantly increased with respect to the traditional system without cognition while, at high SNR regime, the maximum sum spectral efficiency reaches $C_{1,\infty}$. In this paper however, we focus on the sum spectral efficiency gains as function of E_b/N_0. In fact, the E_b/N_0 against spectral efficiency characteristic is of primary importance in the study of the behaviour of the required power in the wideband limit (where the spectral efficiency is small). The key idea behind doing so is to find the best tradeoff between transmitted energy per information bit and spectral efficiency [14]. It is also useful for the sake of comparing results obtained for different configurations to represent the fundamental limits in terms of received energy per information bit rather than the SNR. By replacing the SNR in (23) by its equivalent expression in terms of E_b/N_0, the spectral efficiency of the primary user becomes

$$C_{1,\infty} = \frac{1}{\ln(2)} \cdot E_i\left(\frac{\gamma_0}{E_b/N_0 \cdot C_{1,\infty}}\right) \tag{32}$$

In such a case, the explicit solution of the spectral efficiency against E_b/N_0 is not feasible. In Fig. 6, we plot the sum spectral efficiency gains (with respect to the configuration where only the primary user enjoys the entire band) as function of E_b/N_0 where solutions are given by the implicit equation in (32). The goal here is rather to quantify the spectrum pooling spectral efficiency gain from user to user. Simulation results were obtained through dichotomic algorithms in Fig. 6. We found out that the maximum spectral efficiency gain cannot exceed the range of 60% for a configuration with one primary user and four cognitive users. Notice that, as E_b/N_0 increases, all the configurations tend towards the configuration where only the primary user enjoys the entire band. This can be justified by the fact that, at high E_b/N_0 regime, the water-level $1/\gamma_0$ is becoming greater than the quantity $N_0/|h|^2$ and more power is poured within each sub-band (3).

To proceed further with the analysis, we resort to performance comparison of the proposed scheme with respect to a traditional system where no cognition is used. As far as sum spectral efficiency comparison is concerned,

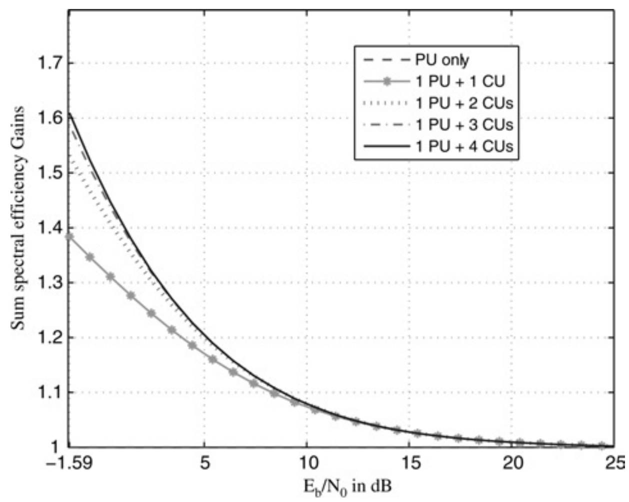

Figure 6 *Sum spectral efficiency gains of the system with one primary user and four CUs*

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this can be conducted by considering the two following configurations:

- The non-cognitive radio configuration (NCR): where the primary user enjoys the entire bandwidth following an average power constraint per sub-band given by

$$\frac{1}{N}\sum_{i=1}^{N} P_l^i = L \cdot \overline{P} \qquad (33)$$

where L is the maximum number of users at each SNR (as shown in Fig. 5). The primary users can accordingly distribute $(N \cdot L \cdot \overline{P})$ over the N sub-bands in order to maximise their capacity,

- The cognitive configuration: where $(L-1)$ cognitive users coexist with the primary user while sharing the N sub-bands available. Each user has to maximise his/her capacity with respect to the average power constraint per band of $(\mathrm{card}(\Omega_l) \cdot \overline{P})$ as in (12).

Fig. 7 validates the expectation from the analysis in (30). It clearly shows that the spectrum-pooling strategy performs always better than traditional communication system using the same spectral resources because of the multi-user diversity gain. In particular, the spectrum-pooling system achieves 1 bit/s hertz more than the NCR system.

Let us now focus on the band factor gains expressions. So far, we have quantified the spectral efficiency gains of different configurations with five users. Let us now investigate how the simulated spectral efficiency gain (with a finite N) converges to the theoretical one (when N is assumed to be infinite).

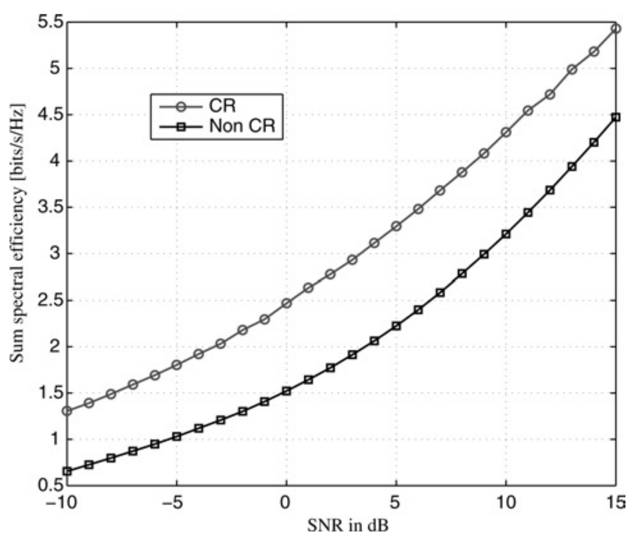

Figure 7 *Sum spectral efficiency of a system using CR and a traditional system (non-CR) for N = 512*

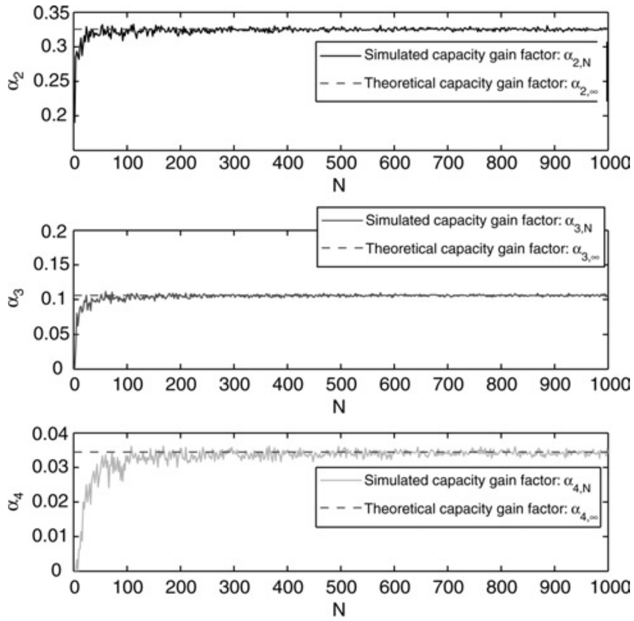

Figure 8 *Convergence of band factor gains at SNR = 0 dB*

Let us first write the spectral efficiency of each user l as follows

$$\Phi_{l,\infty} = \alpha_{l,\infty} \cdot C_{1,\infty}, \quad \text{for } l = 1, \ldots, L \qquad (34)$$

where

$$\alpha_{l,\infty} = \Delta_{\infty}^{l-1}, \quad \text{for } l = 1, \ldots, L \qquad (35)$$

Note here that $\alpha_{l,\infty}$ represents the band factor gain from the primary user to user l. In Fig. 8, numerical simulation is carried out by considering a system with four cognitive users. We compared simulated values of $\alpha_{l,N}$ based on (14) to theoretical values in (35) for each user l and for SNR = 10 dB. We remark that as N increases, the simulated band factor gain tends to $\alpha_{l,\infty}$. Moreover, simulation results show that $\alpha_{2,N}$ converges more rapidly to the associated theoretical gain factor value than for user 3 or user 4.

8 Conclusion

In this work, we have considered a new strategy called spectrum pooling enabling public access to the new spectral ranges without sacrificing the transmission quality of the actual license owners. For the first time, our analysis has quantified the achievable gain of using spectrum pooling with respect to classical radio devices. We found out that though its simplicity, the proposed scheme is effective to provide a higher spectral efficiency gain than the classical scheme does. We further obtained a characterisation of the achievable spectral efficiency as well as the maximum number of possible pairwise communications within such a scenario. Simulation results validate our theoretical claims and offer insights into how much one can gain from spectrum pooling in terms of spectral efficiency. As a future work, it is of major interest to

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generalise the problem to limited feedback in order to characterise the sum spectral efficiency gain of such cognitive protocols with respect to the proposed scenario. It would be further interesting to measure the throughput of the proposed protocol given a realistic primary system model (e.g. ethernet traffic) compared with an OFDM/TDD overlay cognitive radio system.

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