DIVERSITY-MULTIPLEXING TRADEOFF OF SIMPLIFIED RECEIVERS FOR FREQUENCY-SELECTIVE MIMO CHANNELS

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ABSTRACT

Since the introduction of the Diversity-Multiplexing Tradeoff (DMT) by Zheng and Tse for ML reception in frequency-flat MIMO channels, some results have been obtained also for the DMT of frequency-selective MIMO channels and for the DMT of suboptimal receivers such as linear (LEs) and decision-feedback equalizers (DFEs) for frequency-selective SIMO channels or frequency-flat MIMO channels. In this paper we extend these results to the case of linear receivers for frequency-selective MIMO channels. We consider infinite-length and FIR equalizers in standard single-carrier systems, and unconstrained equalizers in cyclic prefix systems. For linear equalizers, the diversity gain suffers significantly in the absence of any Channel State Information at the Transmitter (CSIT), since only a part of the receive spatial diversity gets exploited (the transmit spatial and frequency-selectivity diversities are lost). It is shown that some improvement can be obtained by varying the number of streams transmitted. However, the introduction of simple antenna subset selection CSIT is shown to provide for substantial boosts in the resulting DMT (partial recovery of transmit spatial diversity).

1. INTRODUCTION

Consider a linear modulation scheme and single-carrier transmission over a Multiple Input Multiple Output (MIMO) linear channel with additive white noise. The multiple inputs and outputs will be mainly thought of as corresponding to multiple antennas. After a receive (Rx[1]) filter (possibly noise whitening), we sample the received signal to obtain a discrete-time system at symbol rate[2]. When stacking the samples corresponding to multiple Rx antennas in column vectors, the discrete-time communication system is described by

$$\underbrace{\mathbf{y}_k}_{n_r\times 1} = \underbrace{\mathbf{h}[q]}_{n_r\times n_s}\,\underbrace{\mathbf{a}_k}_{n_s\times 1} + \underbrace{\mathbf{v}_k}_{n_r\times 1} = \sum_{i=1}^{n_s}\underbrace{\mathbf{h}_i[q]}_{n_r\times 1}\,\underbrace{a_{i,k}}_{1\times 1} + \underbrace{\mathbf{v}_k}_{n_r\times 1} \qquad (1)$$

where k is the symbol (sample) period index, n_r is the number of Rx antennas, n_s is the number of active Tx antennas (the number of Tx symbol streams), $1 \le n_s \le n_t$ where n_t is the total number of Tx antennas available (normally $n_s = n_t$ unless antenna selection CSIT is employed). The noise power spectral density matrix is $S_{\mathbf{vv}}(z) = \sigma_v^2 I_{n_r}$ and also the transmit symbol vector sequence is assumed to be spatiotemporally white: $S_{\mathbf{aa}}(z) = \sigma_a^2 I_{n_s}$ q^{-1} is the unit sample delay operator: $q^{-1}a_k = a_{k-1}$, and $\mathbf{h}[z] = \sum_{i=0}^{L}\mathbf{h}_i z^{-i} = [\mathbf{h}_1[z]\cdots\mathbf{h}_{n_s}[z]]$ is the MIMO channel transfer function in the z domain. The channel delay spread is L symbol periods. In the Fourier domain we get the vector transfer function $\mathbf{h}(f) = \mathbf{h}[e^{j2\pi f}]$. The Tx antenna index i in $\mathbf{h}_i[z]$, the SIMO transfer function from Tx antenna i, might possibly be defined after some reordering of the Tx antennas, in the case of selection CSIT.

We introduce the vectors containing the SIMO impulse response coefficients[3] $\mathbf{h}_i = [\mathbf{h}_{i,0}^T\cdots\mathbf{h}_{i,L}^T]^T$ and the overall coefficient vector $\underline{\mathbf{h}} = [\mathbf{h}_1^T\cdots\mathbf{h}_{n_t}^T]^T$. Assume the energy normalization $\mathrm{tr}\{R_{\underline{\mathbf{h}}\underline{\mathbf{h}}}\} = n_r$ with $R_{\underline{\mathbf{h}}\underline{\mathbf{h}}} = \mathrm{E}\{\underline{\mathbf{h}}\underline{\mathbf{h}}^H\}$. By default we shall assume the i.i.d. complex Gaussian channel model: $\underline{\mathbf{h}} \sim \mathcal{CN}(0, \frac{1}{(L+1)n_t} I_{n_r n_t(L+1)})$ so that spatio-temporal diversity of order $n_r n_t(L+1)$ is available (which is the case from the moment $R_{\underline{\mathbf{h}}\underline{\mathbf{h}}}$ is nonsingular). The average per Rx antenna SNR is $\rho = \frac{\sigma_a^2}{\sigma_v^2}$. In this paper we consider full channel state information at the Rx (CSIR) and usually none (otherwise antenna selection) at the Tx (CSIT).

Whereas in non-fading channels, probability of error P_e decreases exponentially with SNR, for a given symbol constellation, in fading channels the probability of error taking channel statistics into account behaves as $P_e \sim \rho^{-d}$ for large SNR ρ, where d is the diversity order. On the other hand, at high SNR the channel capacity increases with SNR as $\log\rho$, which can be achieved with adaptive modulation and coding (AMC) on the basis of the long-term SNR (slow feedback), not to be confused with the instantaneous SINR (fast feedback).. In [1] it was shown however that both benefits at high SNR cannot be attained simultaneously and a compromise has to be accepted: the "diversity-multiplexing tradeoff" (DMT). In [1] the frequency-flat MIMO channel was considered. These results were extended to the frequency-selective SISO channel in [2] and the frequency-selective MIMO channel in [3], see also [4],[5]. In [6], it was shown for the frequency-selective SIMO channel that a Zero Forcing (ZF) or Minimum Mean Squared Error (MMSE) Decision-Feedback Equalizer (DFE) with unconstrained feedforward filter allows to attain the optimum diversity and similar results for the MIMO frequency-flat channel case, with a linear MIMO prefilter and a MMSE MIMO DFE appears in

[1] In this paper, "Rx" stands for "receive" or "receiver" or "reception" etc., and similarly for "Tx" and "transmit", ...

[2] In the case of additional oversampling with integer factor w.r.t. the symbol rate, the Rx dimension would get multiplied by the oversampling factor.

[3] In this paper, $.^*$, $.^T$, and $.^H$ denote complex conjugate, transpose and Hermitian (complex conjugate) transpose respectively, and $\mathbf{h}^\dagger[z] = \mathbf{h}^H[1/z^*]$ denotes the paraconjugate (matched filter). Note that $\mathbf{h}^\dagger[e^{j2\pi f}] = \mathbf{h}^H(f)$.

[7]. These last results confirm the interpretation of the DFE as canonical Rx [8].

In practice also the Linear Equalizer (LE) is often used since its settings are easier to compute and there is no error propagation. Also in practice, for both LE and DFE, only a limited degree of non-causality (delay) can be used and the filters are usually of finite length (FIR). Analytical investigations into the diversity for SISO with LEs are much more recent, see [9],[10] for linearly precoded OFDM and [11] for Single-Carrier with Cyclic Prefix (SC-CP). Earlier on the mean LE SINR for broadband SIMO was investigated in [12]. The use of the DFE appears in [13] (FIR) and [14],[15] (SC-CP) where in the last two references diversity behavior is investigated through simulations. The DMT for various forms of LE and DFE with SIMO channels is investigated in [16]. The DMT for frequency-flat MIMO with LEs has been derived in [17], see also [18] for the diversity in the fixed rate case (no AMC).

(Tx) antenna selection allows to reduce the number of RF chains and has been treated in a number of references (see references in the references to be mentioned). The frequency-flat MIMO optimal DMT with Tx and Rx antenna selection has been presented in [19], see also [20] for the diversity at fixed rate with antenna selection at Tx side only. Here, we shall consider the effect of Tx antenna subset selection, not only to reduce the number of RF chains, but furthermore to improve performance and simplify space-time coding design.

To describe transmission over frequency-selective channels with time-invariant filters and frequency-domain formulas implies the use of infinite block lengths. This represents of course a strong simplification in the context of time-varying wireless systems. In practice time-invariant filters can be used over finite block lengths if guard intervals or cyclic prefixes (CPs) are introduced. Also, in the CP case, IIR or non-causality aspects do not pose any problem.

2. SINR OF OPTIMAL AND INFINITE-LENGTH NON-CAUSAL SUBOPTIMAL RECEIVERS

We get for the Matched Filter Bound (MFB) of stream i

$$\mathrm{MFB}_i = \rho\ \|\mathbf{h}_i\|^2\,,\ \|\mathbf{h}_i\|^2 = \sum_{k=0}^{L}\|\mathbf{h}_{i,k}\|_2^2 = \int_{-0.5}^{0.5}\|\mathbf{h}_i(f)\|_2^2\,df$$

where e.g. $\|\mathbf{h}_{i,k}\|_2^2 = \mathbf{h}_{i,k}^H\mathbf{h}_{i,k}$. The MFB corresponds to Maximum Ratio Combining (MRC) of all energy in the spatio-temporal channel. The MFB is a close approximation for the performance of Maximum Likelihood Sequence Detection (MLSD). In practice one is often forced to resort to suboptimal Rx's when the delay spread and or the constellation size get large. Two popular classes of suboptimal Rx's are linear and decision-feedback equalizers (LE and DFE). Both types of equalizers are in fact linear estimators of the transmitted symbol sequence, one is based on the received signal only whereas the other is also based on the past detected symbols.

The goal of these suboptimal Rx's is to transform the frequency-selective channel into a frequency-flat channel the performance of which depends on the Signal-to-Interference-plus-Noise Ratios (SINRs) at its outputs. For Mutual Information (C) purposes, the channel-equalizer cascade is treated as an AWGN channel, hence $\mathrm{C} = \sum \mathrm{C}_i = \sum_{i=1}^{n_s}\log(1+\mathrm{SINR}_i)$. In the MIMO multichannel context considered here, a zero-forcing (ZF) LE (or DFE) only exists if $n_r \geq n_s$, and is not unique for $n_r > n_s$ since $\mathbf{h}(f)$ has a non-empty orthogonal complement. Among all the ZF equalizers, there is one that will minimize the noise enhancement (MSE), which hence can be called the MMSE-ZF design. For a DFE, which has a feedforward and a feedback filter, this non-uniqueness already arises for the $n_r = n_s$ case. To simplify notation, we shall henceforth refer to the MMSE-ZF design as the ZF design. Introduce

$$\delta = \begin{cases} 0 & \text{, MMSE-ZF design,} \\ 1 & \text{, MMSE design.} \end{cases} \tag{2}$$

For a MMSE design, we need to introduce the following extended transfer function(s):

$$\underline{\underline{\mathbf{h}}}[z] = \begin{bmatrix} \mathbf{h}[z] \\ \frac{\delta}{\sqrt{\rho}} I_{n_s} \end{bmatrix} = [\underline{\underline{\mathbf{h}}}_1[z] \cdots \underline{\underline{\mathbf{h}}}_{n_s}[z]]\,. \tag{3}$$

The description of the LE requires the following orthogonalized SIMO transfer functions:

$$\underline{\underline{\mathbf{h}}}_i^{'}[z] = \mathrm{P}^{\perp}_{\overline{\underline{\underline{\mathbf{h}}}}_i[z]}\,\underline{\underline{\mathbf{h}}}_i[z] \tag{4}$$

where e.g. $\mathrm{P}^{\perp}_{\mathbf{h}[z]} = I - \mathrm{P}_{\mathbf{h}[z]}$, $\mathrm{P}_{\mathbf{h}[z]} = \mathbf{h}[z](\mathbf{h}^{\dagger}[z]\mathbf{h}[z])^{-1}\mathbf{h}^{\dagger}[z]$, and $\overline{\underline{\underline{\mathbf{h}}}}_i[z]$ is obtained from $\underline{\underline{\mathbf{h}}}[z]$ by removing column i, namely $\underline{\underline{\mathbf{h}}}_i[z]$. For DFEs, the (ordered) Gram-Schmidt orthogonalization is required:

$$\underline{\underline{\mathbf{h}}}_i^{''}[z] = \mathrm{P}^{\perp}_{\underline{\underline{\mathbf{h}}}_{i+1:n_s}[z]}\,\underline{\underline{\mathbf{h}}}_i[z] \tag{5}$$

where $\underline{\underline{\mathbf{h}}}_{i:j}[z] = [\underline{\underline{\mathbf{h}}}_i[z]\ \underline{\underline{\mathbf{h}}}_{i+1}[z]\cdots\underline{\underline{\mathbf{h}}}_j[z]]$. For an optimal DFE, the Tx antennas need to be reordered so that at every stage i, $\|\underline{\underline{\mathbf{h}}}_i^{''}\|$ is maximal over reordering the remaining antennas $i,\ldots,n_s$. Note that in the ZF case ($\delta = 0$), the orthogonalization process is limited to the non-extended transfer functions:

$$\underline{\underline{\mathbf{h}}}^{'}[z] = \begin{bmatrix} \mathbf{h}^{'}[z] \\ 0 \end{bmatrix},\ \underline{\underline{\mathbf{h}}}^{''}[z] = \begin{bmatrix} \mathbf{h}^{''}[z] \\ 0 \end{bmatrix}. \tag{6}$$

For infinite-length non-causal (feedforward) filters, we get the following SINR results

- $\mathrm{MFB}_i = \rho\int_{-\frac{1}{2}}^{\frac{1}{2}}\|\mathbf{h}_i(f)\|_2^2 df = \rho\int_{-\frac{1}{2}}^{\frac{1}{2}}\|\underline{\underline{\mathbf{h}}}_i(f)\|_2^2\,df - \delta$ *arithmetic* average
- $\mathrm{SINR}^{\delta}_{DFE,i} = \rho\ \exp\left[\int_{-\frac{1}{2}}^{\frac{1}{2}}\log(\|\underline{\underline{\mathbf{h}}}_i^{''}(f)\|_2^2)\,df\right] - \delta$ *geometric* average
- $\mathrm{SINR}^{\delta}_{LE,i} = \rho\ \left[\int_{-\frac{1}{2}}^{\frac{1}{2}}(\|\underline{\underline{\mathbf{h}}}_i^{'}(f)\|_2^2)^{-1}\,df\right]^{-1} - \delta$ *harmonic* average

with inequalities

$$\mathrm{SINR}^{\delta}_{LE,i}\ \leq\ \mathrm{SINR}^{\delta}_{DFE,i}\ \leq\ \mathrm{MFB}_i\,,\ \mathrm{SINR}^0_i\ \leq\ \mathrm{SINR}^1_i \tag{7}$$

where the last inequality holds for either LE or DFE, and comparison between DFE and LE or MFB assume the same Tx antenna ordering. For the case of MMSE design, the SINR here corresponds to the SINR computed correctly ($\mathrm{SINR}_i = \frac{\sigma_a^2}{\mathrm{MSE}_i} - 1$) which might be more easily interpreted in terms of Unbiased MMSE (UMMSE) design [8].

3. OUTAGE-RATE TRADEOFF

The SINR_i is random due to its dependence on the random channel $\mathbf{h}$. In [21], it was demonstrated that at high SNR outage only depends on the SINR distribution behavior near zero (this was also observed in [1]). This result is quite immediate. Indeed, let us introduce the normalized SINR γ_i through $\mathrm{SINR}_i = \rho\,\gamma_i$ and consider the dominating term in the cumulative distribution function (cdf) of a γ:

$$\mathrm{Prob}\{\gamma \leq \varepsilon\} = c\,\varepsilon^k \tag{8}$$

for small $\varepsilon > 0$. Then the outage probability for a certain outage threshold α is

$$\mathrm{Prob}\{\mathrm{SINR} \leq \alpha\} = c\left(\frac{\alpha}{\rho}\right)^k \tag{9}$$

from which we see that k is the diversity order.

Now consider outage in terms of outage capacity. Since at high SNR the SINR will tend to be proportional to ρ, the mutual information $C = \log(1+\mathrm{SINR})$ in a single stream will tend to be $\log\rho$. So consider the rate $R = r\log\rho$ (in nats, assuming natural logarithm) where $r \in [0,1]$ is the normalized rate. Then the outage probability at high SNR is

$$\begin{aligned} P_o &= \mathrm{Prob}\{C < R\} = \mathrm{Prob}\{\log(1+SINR) < \log(\rho^r)\} \\ &= \mathrm{Prob}\{\rho\gamma < \rho^r - 1\} = \mathrm{Prob}\{\gamma < \tfrac{1}{\rho^{(1-r)}} - \tfrac{1}{\rho}\} \\ &= \mathrm{Prob}\{\gamma < \tfrac{1}{\rho^{(1-r)}}\}\ ,\ \text{for } r > 0 \\ &= c\,\frac{1}{\rho^{(1-r)k}} \end{aligned} \tag{10}$$

Hence for the SISO system with the SINR considered, we get for $r \in (0,1]$:

$$d(r) = (1-r)\,k \tag{11}$$

where $d(r)$ is the diversity(order)-rate tradeoff. The case $r = 0$ (fixed rate) requires separate investigation.

The mutual information for the frequency-selective MIMO channel with white Gaussian input is $C =$

$$\int_{-\frac{1}{2}}^{\frac{1}{2}} \log\det(I_{n_s} + \rho\mathbf{h}^H(f)\,\mathbf{h}(f))\,df = \sum_{i=1}^{n_s}\log(1+\mathrm{SINR}^{MMSE}_{DFE,i})$$

which reconfirms the canonical character of the MMSE DFE Rx. The optimal diversity-multiplexing (or outage-rate) tradeoff (DMT) for such channel has been shown [3],[4],[5] to be given by the piecewise linear curve that connects the points

$$(d,r) = \left(((L+1)n_r - r)(n_s - r)\,,\, r\right),\ \ r = 0,1,\ldots,n_s\,. \tag{12}$$

In the MIMO case, in which $n_s \leq n_r$ streams get transmitted, the normalized rate $r = \frac{R}{\log\rho}$ (at high SNR ρ) can indeed go up to n_s. Even though the diversity order has been introduced here in terms of outage probability only, it also applies to frame error rate for any spatial multiplexing scheme with non-vanishing determinant (since then the probability of error in the case of no outage decreases exponentially with SNR).

4. OUTAGE ANALYSIS OF SUBOPTIMAL RECEIVER SINRS

A suboptimal Rx transforms the channel-Rx cascade into a set of n_s parallel SISO channels, each characterized by their SINR_i. In the case of a ZF DFE with unconstrained (noncausality, length) filters, these SINRs are independent, but in other cases (especially the LE case) they are dependent as we shall see. A perfect outage of stream i occurs when $\mathrm{SINR}_i = 0$. For the MFB_i this can only occur if $\mathbf{h}_i = 0$. For a suboptimal Rx however (or also the MI), the SINR_i can vanish for any $\mathbf{h}$ on the *Outage Manifold* $\mathscr{M}_i = \{\mathbf{h} : \mathrm{SINR}_i(\mathbf{h}) = 0\}$. At fixed rate R, the diversity order is the codimension of (the tangent subspace of) the outage manifold, assuming this codimension is constant almost everywhere and assuming a channel distribution with finite positive density everywhere (e.g. Gaussian with non-singular convariance matrix). For example, for the MFB_i (which only depends on $\mathbf{h}_i$) the outage manifold is the origin, the codimension of which is the total size of $\mathbf{h}_i$. The codimension is the (minimum) number of complex constraints imposed on the complex elements of $\mathbf{h}$ by putting $\mathrm{SINR}_i(\mathbf{h}) = 0$. Some care has to be excercised with complex numbers. Valid complex constraints (which imply two real constraints) are such that their number becomes an equal number of real constraints if the channel coefficients were to be real. A constraint on a coefficient magnitude however, which is in principle only one real constraint, counts as a valid complex constraint (at least if the channel coefficient distributions are insensitive to phase changes). An actual outage occurs whenever $\mathbf{h}_i$ lies in the *Outage Shell*, a (thin) shell containing the outage manifold. The thickness of this shell shrinks as the rate increases.

In the MIMO case with suboptimal Rxs, two cases can be considered [17], depending on whether the n_s streams are the result of joint encoding ($j-enc$ schemes) or separate encoding ($s-enc$ schemes), with ensuing joint or separate decoding. The UMTS HSDPA PARC scheme is an example of a $s-enc$ scheme, but with streamwise fast feedback of C_i knowledge to the Tx. In absence of any (fast) CSIT, a $s-enc$ scheme will distribute the total rate R evenly (R/n_s) over the n_s streams. The outage probabilities of $j-enc$ and $s-enc$ schemes are respectively

$$\begin{aligned} P_o^{j-enc}(R) &= \mathrm{Prob}\left(\textstyle\sum_{i=1}^{n_s}\log(1+\mathrm{SINR}_i) < R\right) \\ P_o^{s-enc}(R) &= \mathrm{Prob}\left(\textstyle\bigcup_{i=1}^{n_s}\left\{\log(1+\mathrm{SINR}_i) < \frac{R}{n_s}\right\}\right)\,. \end{aligned} \tag{13}$$

The outage manifold for a $s-enc$ scheme is given by $\mathscr{M} = \bigcup_{i=1}^{n_s}\mathscr{M}_i$.

5. LINEAR EQUALIZATION (LE) IN SINGLE CARRIER CYCLIC PREFIX (SC-CP) SYSTEMS

The diversity of LE for SC-CP systems has been studied in [11] for the SISO case with i.i.d. Gaussian channel elements, fixed rate R and block size $N = L+1$. The LE DMT for SIMO SC-CP systems appears in [16]. Consider a block of N symbol periods preceded by a cyclic prefix (CP) of length L (as a result of the CP insertion, actual rates are reduced by a factor $\frac{N}{N+L}$, which is ignored here in what follows). The channel input-output relation over one block can be written as

$$\mathbf{Y} = \mathbf{H}\,\mathbf{A} + \mathbf{V} \tag{14}$$

where $\mathbf{Y} = \mathbf{Y}_k = [\mathbf{y}_k^T\ \mathbf{y}_{k+1}^T \cdots \mathbf{y}_{k+N-1}^T]^T$ etc. and $\mathbf{H}$ is a banded block-circulant matrix (see (13) in [16]). Now apply an N-point DFT (with matrix F_N) to each subchannel received signal, then we get

$$\underbrace{F_{N,n_r}\mathbf{Y}}_{\mathbf{U}} = \underbrace{F_{N,n_r}\mathbf{H}F_{N,n_s}^{-1}}_{\mathscr{H}}\ \underbrace{F_{N,n_s}\mathbf{A}}_{\mathbf{X}} + \underbrace{F_{N,n_r}\mathbf{V}}_{\mathbf{W}} \quad (15)$$

where $F_{N,n} = F_N \otimes I_n$ (Kronecker product: $A \otimes B = [a_{ij}B]$), $\mathscr{H} = \text{blockdiag}\{\mathbf{h}(f_0), \ldots, \mathbf{h}(f_{N-1})\}$ with $\mathbf{h}(f_n)$, the $n_r \times n_s$ channel transfer function at tone n: $f_n = \frac{n}{N}$, at which we have

$$\mathbf{u}_n = \mathbf{h}(f_n)\,\mathbf{x}_n + \mathbf{w}_n\ . \quad (16)$$

The $\mathbf{x}_n$ components are i.i.d. and independent of the i.i.d. $\mathbf{w}_n$ components with $\sigma_x^2 = N\sigma_a^2$, $\sigma_w^2 = N\sigma_v^2$. A ZF ($\delta = 0$) or MMSE ($\delta = 1$) LE produces per tone $\widehat{\mathbf{x}} = (\mathbf{h}^H\mathbf{h} + \frac{\delta}{\rho}I_{n_s})^{-1}\mathbf{h}^H\mathbf{u}$ from which $\widehat{\mathbf{a}}$ with components $\widehat{a}_i$ is obtained after IDFT with

$$\text{SINR}^{\delta}_{CP-LE,i} = \rho\left(\frac{1}{N}\sum_{n=0}^{N-1}\|\underline{\underline{\mathbf{h}}}_i^{'}(f_n)\|^{-2}\right)^{-1} - \delta\ . \quad (17)$$

We first focus on the ZF case:

$$\text{SINR}^{ZF}_{CP-LE,i} = \rho\left(\frac{1}{N}\sum_{n=0}^{N-1}\|\mathbf{h}_i^{'}(f_n)\|^{-2}\right)^{-1}\ . \quad (18)$$

Using the exponential equality notation, $\doteq$ (meaning same high SNR diversity order), and $R = r\log\rho$, then

$$\begin{aligned} P_o^{s-enc}(r) &= \text{Prob}\left(\bigcup_{i=1}^{n_s}\left\{\log(1+\text{SINR}^{ZF}_{CP-LE,i}) < \tfrac{R}{n_s}\right\}\right)\\ &\doteq \text{Prob}\left(\log(1+\text{SINR}^{ZF}_{CP-LE,1}) < \tfrac{R}{n_s}\right)\\ &\doteq \text{Prob}\left(SINR^{ZF}_{CP-LE,1} < \rho^{\frac{r}{n_s}}\right)\\ &\doteq \text{Prob}\left(\rho\,\|\mathbf{h}_1^{'}(0)\|^2 < \rho^{\frac{r}{n_s}}\right) \end{aligned} \quad (19)$$

where the second equality is due to the fact that the $\text{SINR}^{ZF}_{CP-LE,i}$ are identically distributed (regardless of dependence) and the last equality is a property of harmonic averages of identically distributed quantities $\|\mathbf{h}_1^{'}(f_n)\|$. Hence

$$d^{ZF,s-enc}_{CP-LE}(r) = (n_r - n_s + 1)(1 - \frac{r}{n_s})\ ,\quad r \in [0, n_s]\ . \quad (20)$$

Note that the outage manifold $\mathscr{M}^{ZF,s-enc}_{CP-LE}$ is the collection of $\mathbf{h}$ for which $\|\mathbf{h}_i^{'}(f_n)\| = 0$ for any i or n. To dig in more deeply, simplify notation $\mathbf{h}_i^{'}(f_n) \to \mathbf{h}_i^{'}$ (any particular n), introduce the normalized vectors $\widetilde{\mathbf{h}}_i = \mathbf{h}_i/\|\mathbf{h}_i\|$, then $\|\mathbf{h}_i^{'}\|^2 = \|\mathbf{h}_i\|^2\,\|\widetilde{\mathbf{h}}_i^{'}\|^2$ where $\|\mathbf{h}_i\|$ and $\widetilde{\mathbf{h}}_i^{'}$ are independent. In the case $n_s = 2$, then $\|\widetilde{\mathbf{h}}_i^{'}\|^2 = \sin^2\theta = 1 - |\widetilde{\mathbf{h}}_1^H\widetilde{\mathbf{h}}_2|^2$ where θ is the "angle" between $\mathbf{h}_1$ and $\mathbf{h}_2$. The diversity order of $\|\mathbf{h}_i^{'}\|^2$ is the minimum of the diversity orders of $|\mathbf{h}_i|^2$ and $\|\widetilde{\mathbf{h}}_i^{'}\|^2$, and hence is the diversity order of $\|\widetilde{\mathbf{h}}_i^{'}\|^2$ which is $n_r - n_s + 1$. Now, $\|\widetilde{\mathbf{h}}_i^{'}\|^2 = 0$ for any i whenever $\exists\,\mathbf{x} \in C^{n_r}, \|\mathbf{x}\| = 1 : \mathbf{h}\,\mathbf{x} = 0$ where all elements in $\mathbf{x}$ will be non-zero w.p. 1. Hence, the $\|\widetilde{\mathbf{h}}_i^{'}\|^2$ fade simultaneously for all i ! So joint encoding does not help: $d^{ZF,j-enc}_{CP-LE}(r) = d^{ZF,s-enc}_{CP-LE}(r)$, $r \in [0, n_s]$. This subsumes a simpler derivation of the results in [17].

6. OPTIMIZING THE NUMBER OF TX STREAMS n_s

From (20) it is clear that at lower rates r it is beneficial to activate a smaller number n_s of streams. Also, by reducing the number of active streams, the case $n_t > n_r$ can trivially be handled. As a result, one can find from (20) the optimal $n_s(r)$ which varies from $n_s(r) = 1$ for $r = 0$ (giving $d(n_s(r), r) = n_r$ for $r = 0$) to $n_s(r) = \min(n_r, n_t)$ for $r = \min(n_r, n_t)$.

7. ANTENNA SUBSET SELECTION CSIT

By ordering the vector channels $\mathbf{h}_i$ as in [4], the diversity order of a reduced set of the first n_s vector channels gets boosted by a factor $n_t - n_s + 1$ (see [4]). As a result the diversity in the DMT curve of (20) gets multiplied with this factor:

$$d(r) = (n_r - n_s + 1)(n_t - n_s + 1)(1 - \frac{r}{n_s})\ ,\quad r \in [0, n_s]\ . \quad (21)$$

After again optimizing over n_s as a function of r, we get in particular a maximal diversity of $d(0) = n_r n_t$ and r can go up to n_s which itself can go up to $\min(n_r, n_t)$. In this way substantial diversity boosts and a simplified Tx scheme are obtained, especially for the case $n_t > n_r$. For small rates r, the diversity obtained is only lightly reduced compared to the optimal (flat channel) MIMO DMT.

8. OFDM

In order to attain the CP MIMO system with LE DMT in an OFDM approach, no coding across tones is required since the frequency-selectivity diversity does not get exploited with LE.

The use of redundant linear precoding can remedy this completely however! One such instance is zero padding (ZP) for which it has been shown in [9] for SISO systems that a LE allows to attain full diversity. The case of more general redundant linear precoding for SIMO systems is considered in [10]. A linear precoder for SIMO OFDM has a redundancy equal to p if it mixes $N - p$ symbols over the N tones. In [10] it has been shown that as long as the precoding introduces a redundancy greater than the maximum degree of singularity (L) that the channel $\mathscr{H}$ can suffer (without becoming completely zero), then LE allows full diversity (and hence DMT). Indeed, the FIR SIMO channel can show at most zeros at L tones. If more zeros would appear, than that means that the whole channel impulse response is zero.

In the MIMO case, $\det(\mathbf{h}^H(f)\mathbf{h}(f))$ can show at most $n_s L$ zeros. Hence the precoder should introduce at least this much redundancy to allow a LE to enjoy full diversity.

9. NON-CAUSAL INFINITE LENGTH LINEAR EQUALIZER

For the infinite length (ZF) LE case, $\mathscr{M} = \{\mathbf{h} : \exists\,\mathbf{x} \in C^{n_r}, \|\mathbf{x}\| = 1, \exists f \in [0,1) : \mathbf{h}(f)\,\mathbf{x} = 0\}$. In spite of the ambiguity on f, the DMT is again as in (20) (see also [16]).

10. FIR LINEAR EQUALIZATION

Consider now the use of an FIR LE of length N. For MIMO channels, there exist indeed FIR equalizers for FIR channels, due to the Bezout identity, as long as $N \geq \frac{L+1-n_s}{n_r - n_s}$. The LE design is based on a banded block Toeplitz input-output matrix

$\overline{\mathbf{H}}$ which can be obtained by starting from a block circulant $\mathbf{H}$ (as in the CP case) of size $N+L$ and removing the top L block rows. We obtain for a certain equalizer delay and stream

$$\mathrm{SINR}^{\delta}_{FIR-LE} + \delta = \frac{\rho}{e^H(\overline{\mathbf{H}}^H\overline{\mathbf{H}} + \frac{\delta}{\rho})^{-1} e} = \frac{\rho}{\sum_i \frac{1}{\lambda_i + \frac{\delta}{\rho}} |V_{i,d}|^2} \tag{22}$$

where e is a standard unit vector containing all zeroes except for a 1 in the position corresponding to the delay and stream index, and we introduced the SVD $\overline{\mathbf{H}}^H\overline{\mathbf{H}} = V\Lambda V^H = \sum_i \lambda_i V_i V_i^H$. The outage manifold is determined (again) by $\lambda_{min} = 0$. Singularity of $\overline{\mathbf{H}}^H\overline{\mathbf{H}}$ occurs whenever $\overline{\mathbf{H}}$ loses full column rank. This occurs whenever a linear combination of the SIMO channels has subchannels with a zero in common, or $\mathscr{M} = \{\mathbf{h} : \exists \mathbf{x} \in \mathrm{C}^{n_r}, \|\mathbf{x}\| = 1, \exists z_o \in \mathrm{C} : \mathbf{h}[z_0]\,\mathbf{x} = 0\}$ $|z_o| = 1$.

11. LE: THE MMSE CASE

The regularization provided in the MMSE case has no effect as soon as $r > 0$. As a result the MMSE DMTs coincide with those of the corredsponding ZF designs for $r > 0$. For $r = 0$ however, full diversity $n_s n_r(L+1)$ is obtained for appropriate block/FIR dimensions (for detailed investigations see e.g. [18] for the frequency-flat MIMO case and [] for the frequency-selective SISO CP case).

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