Institut Eurécom
Corporate Communications Department
2229, route des Crêtes
B.P. 193
06904 Sophia Antipolis
FRANCE

Research Report RR-08-225

Evolutionary Game for Peer-to-peer Storage Audits

22 July 2008

Nouha Oualha and Yves Roudier[1]

Tel: (+33) 4 93 00 81 00
Fax: (+33) 4 93 00 82 00
Email: {oualha, roudier}@eurecom.fr

1 Eurecom's research is partially supported by its industrial partners: BMW Group Research & Technology – BMW Group Company, Bouygues Telecom, Cisco Systems, France Telecom, Hitachi, SFR , Sharp, STMicroelectronics, Swisscom, Thales

Evolutionary Game for Peer-to-peer Storage Audits

Nouha Oualha and Yves Roudier

Abstract

The paper describes an evolutionary game model of a P2P storage system. By studying the equilibrium point of the game, we demonstrate that an audit-based strategy, where audits are used to decide whether to cooperate with a given peer or not, may win or dominate free-riders peers that are using the system without contributing to it. This desirable equilibrium is reached for certain conditions and system's parameters revealed in the paper.

Keywords: evolutionary game, trust establishment, audit, cooperation, P2P storage.

Contents

1. Introduction

Peer-to-peer (P2P) storage systems allow peers to store their personal data at several storage sites in the network. Storage sites stand for volunteer peers that agree to keep data stored at their disk space in a long term basis. To relinquish storage peers from destroying data, cooperation incentive mechanisms must be put in place at the storage system. Generally, a cooperation incentive mechanism is proven to be effective if it is demonstrated that any rational peer from a system supporting such a mechanism will always choose to cooperate whenever it interacts with another cooperative peer. While for now the mostly employed tool for such proofs is game theory, there is a research trend towards the use of evolutionary dynamics. Typically, an evolutionary game model describes the evolution of strategies within large populations as a result of many local interactions, each involving a small number of randomly selected individuals. An individual plays only once; it plays in a one shot game against another randomly selected player with the goal of maximizing its utility (fitness) in that game. We propose in this paper an evolutionary game model of a cooperative storage system so as to give an inkling of the features and the conditions that cooperation incentives must meet to motivate the choice of an auditing-based strategy over self-interested strategies.

The paper is organized as follows: in section 2, an outline of the P2P storage system is first provided, then in section 3 an evolutionary game model of such system is described and the solution of the model, the evolutionary stable strategy, is derived from the model, finally results are validated with simulation experiments.

2. A peer-to-peer storage system

In a self-organizing storage system, peers are able to store their personal data at other peers' space. These latter, called *holders*, should keep data correctly held until the time of its retrieval by their *owner*. The availability and correctness of stored data may be periodically checked by the owner or some of its delegates, called *verifiers*, in order to ensure the longevity of stored data in the network. However, holders or verifiers may still be able to misbehave in various ways with the aim of optimizing their own resource usage or because of malicious intentions that seek at attacking the storage system or the peers themselves.

Peers' misbehavior may be thwarted through the use of audit-based approaches. With auditing, a peer chooses peers that have proved to be reliable in the past and thus they are expected to honestly behave in the forthcoming interactions. Proofs of peers' well behaving can be constructed from data possession checkings periodically performed by verifiers, as detailed in [3] and [4]. Audits are objective and direct observations of the behavior of the other peers, given that they are constructed based on results of verifications performed by the observer or actual storage of its data (see [5]).

3. Evolutionary game

We propose in this paper an evolutionary game model of a cooperative storage system with which we endeavor to demonstrate that peers using the audit-based strategy will dominate the system.

3.1. Game model

In the proposed system, an owner stores data replicas at r holders. It appoints m verifiers for its data replica that will periodically check storage at holders.

The system is modeled as an evolutionary game [1]: "*an evolutionary game is a dynamic model of strategic interaction with the following characteristics: (a) higher payoff strategies tend over time to* *displace lower payoff strategies; (b) there is inertia; (c) players do not intentionally influence other players' future actions*".

The one-stage game represents an interaction between one data owner, *r* data holders, and *m* verifiers. Thus, the considered game players are an owner, *r* holders, and *m* verifiers. Our game is similar to the game in [2] where players have either the role of the donor or the role of the recipient. The donor can confer a benefit *b* to the recipient, at a cost -*c* to the donor.

Holders and verifiers have the choice between either to cooperate that we call interchangeably donate or defect:

- Cooperation whereby the peer is expected to keep others' data in its memory and to verify data held by other peers on behalf of the owner.
- Defection whereby the peer destroys the data it has accepted to hold, and also does not verify others' data as it has promised to do.

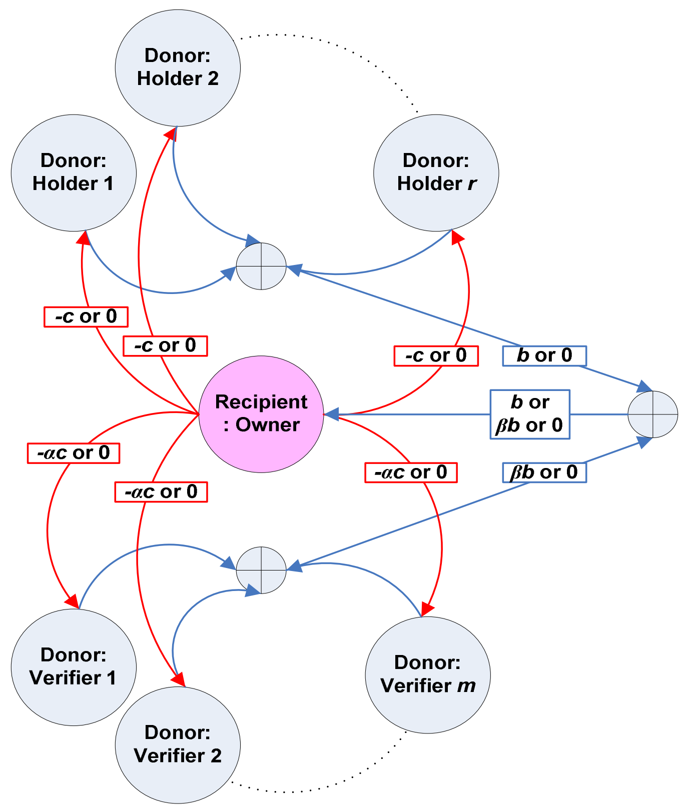

Fig. 1. One-stage game model

In our game, the owner is considered a recipient, the *r* holders and the *m* verifiers are donors. The owner gains *b* if at least one holder donates at a cost $-c$; however if all holders do not donate then the owner gains βb if at least one verifier donates at a cost $-\alpha c$ ($\alpha \leq 1$) for each verifier (Fig. 1). The latter case corresponds to the situation where the cooperative verifier informs the owner of the data destruction, and then the owner may replicate its data elsewhere in the network thus maintaining the security of its data storage. The peers' strategies that we consider for study are:

- Always cooperate (*AllC*): the peer always decides to donate, when in the role of the donor.

- Always defect (*AllD*): the peer never donates in the role of the donor.
- Discriminate (*D*): the discriminator donates under conditions: if the discriminator does not know its co-player, it will always donate; however, if it had previously played with its co-player, it will only donate if its co-player donates in the previous game (it is a Tit-For-Tat strategy that takes into account not only the owner's (the donor) observations, but also those of verifiers).

3.2. Observations

Let's consider a scheme (see Fig. 2) inspired from epidemic models which categorize the population into groups depending on their state [6]. In the scheme, there are two states: "not known" and "known" states. We denote the number of peers that a given peer in average does not know by D at a certain time t and the number of peers that in average it knows by K at time t. Peers that may join the system are peers who were invited by other members with fixed invitation rate λ. We will assume that these newcomers will take the strategy of their inviters (for later use in the evolutionary game). Peers are leaving the system with fixed departure rate of μ. The rate σ designates the frequency of encounter between two peers, one of them being the holder (i.e., probability that a peer knows about the behavior of another peer).

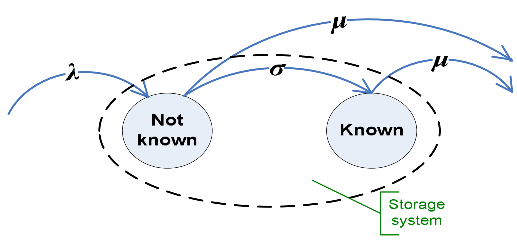

Fig. 2. System dynamics

The dynamics of the number of K and D are given by the following equations:

$$\frac{dD}{dt} = \lambda n - (\sigma + \mu) D$$

$$\frac{dK}{dt} = \sigma D - \mu K = \sigma n - (\sigma + \mu) K$$

We denote the total number of peers in the storage system by n=D+K, thus, we have:

$$\frac{dn}{dt} = (\lambda - \mu) n$$

Let q be the probability that the discriminator knows what a randomly chosen co-player chooses as a holder's strategy in a previous one-stage game with the discriminator (being an owner or verifier). The probability q is equal to K/n, hence we have:

$$\frac{dq}{dt} = \frac{dK/dt}{n} - \frac{Kdn/dt}{n^2}$$

Thus,

$$\frac{dq}{dt} = \sigma - (\sigma + \lambda) q \qquad \textbf{(3.2.1)}$$

At time t=0, the set of peers in the state K is empty. Over time, peers in state D enter the state K with rate σ. A new peer joining the system is attributed to the state D. So, initially q(0)=0. Then, the result of the above differential equation (3.2.1) is:

$$q(t)=\frac{\sigma}{\sigma+\lambda}(1-e^{-(\sigma+\lambda)t})$$

The limit of $q(t)$ when $t \to \infty$ is $\sigma/(\sigma+\lambda)$. If we consider a system without churn (λ=0), the limit becomes 1.

3.3. Fitness

We denote the frequency of (i.e., fraction in the population of peers playing) strategy *AllC* by by x, respectively *AllD* by y, and finally the strategy D by z.

The expected values for the total payoff obtained by the three strategies are denoted by U_{AllC}, U_{AllD} and U_D, and the average payoff in the population by:

$$\overline{U}=x\times U_{AllC}+y\times U_{AllD}+z\times U_D$$

The average payoffs for each strategy are computed below:

$$\begin{aligned}U_{AllC}&=-c-m\alpha c+b(1-y^r)+\beta b(y^r(1-y^m))\\&=-c(1+m\alpha)+b(1-y^r+\beta y^r(1-y^m))\\U_{AllD}&=b(1-(y+qz)^r)+\beta b((y+qz)^r(1-(y+qz)^m)\\&=b(1-(y+qz)^r+\beta(y+qz)^r(1-(y+qz)^m)\\U_D&=-c(1+m\alpha)(1-qy)+b(1-y^r+\beta y^r(1-y^m))\end{aligned}$$

By normalizing U_{AllC} to 0, we obtain:

$$\begin{aligned}U_{AllC}&=0\\U_{AllD}&=b(1-(y+qz)^r+\beta(y+qz)^r(1-(y+qz)^m)\\U_D&=c(1+m\alpha)qy\end{aligned}$$

Such payoffs are called also fitness, and strategies with higher fitness are expected to propagate faster in the population and become more common. This process is called *natural selection*.

3.4. Replicator dynamics

The basic concept of replicator dynamics is that the growth rate of peers taking a strategy is proportional to the fitness acquired by the strategy. Thus, the strategy that yields more fitness than average fitness of the whole system increases, and vice versa. We will use the well known differential replicator equations:

$$\begin{aligned}\frac{dx}{dt}&=x(U_{AllC}-\overline{U})\\\frac{dy}{dt}&=y(U_{AllD}-\overline{U})\\\frac{dz}{dt}&=z(U_D-\overline{U})\end{aligned}\qquad\textbf{(2.4.1)}$$

3.5. Evolutionary stable strategy

A Strategy is said to *invade* a population of strategy players if its fitness when interacting with the other strategy is higher than the fitness of the other strategy when interacting with the same strategy. An evolutionarily stable strategy (ESS) is a strategy such that if all the peers adopt it cannot be invaded by any other strategy.

Case $x \neq 0$, $y=0$, $z \neq 0$: This case corresponds to a fixed point in the replicator dynamics, which means that a mixture of discriminating and altruistic population can coexist and are en equilibrium.

Case $x \neq 0$, $y \neq 0$, $z=0$: In this case, the replicator dynamics of both altruistic and defector populations are:

$$\begin{aligned} \dot{x} &= -xyb(1 - y^r + \beta y^r(1 - y^m)) \leq 0 \\ \dot{y} &= y(1 - y)b(1 - y^r + \beta y^r(1 - y^m)) \geq 0 \end{aligned} \tag{2.5.1}$$

The population of defectors wins the game and the ESS is attained at $x=0$ and $y=1$.

Case $x=0$, $y \neq 0$, $z \neq 0$: There is an equilibrium point for which defectors and discriminators coexist ($x=0$, $y=y_0$, $z=z_0$).

In the case where there is no churn ($\lambda=0$), since the limit of $q(t)$ when $t \to \infty$ is 1, the replicator dynamics for defectors and discriminators are:

$$\begin{aligned} \dot{y} &= -y^2 zc(1 + m\alpha) \leq 0 \\ \dot{z} &= zy(1 - z)c(1 + m\alpha) \geq 0 \end{aligned}$$

The replicator dynamics of defectors is negative when $t \to \infty$. Thus, the frequency of defectors will converge to 0, leaving a population purely composed of discriminators. Then, the evolutionary stationary point is for ($x=0$, $y=y_0=0$, $z=z_0=1$). You may refer to Table .1 for equilibrium values in more cases.

Table .1. Finding the equilibrium for $x=0$, $y \neq 0$, $z \neq 0$.

Conditions	y_0	z_0
$q(t) \xrightarrow[t\to\infty]{} 1$	0	1
$r=1$, $m=0$, $q(t) \xrightarrow[t\to\infty]{} \frac{\sigma}{\sigma+\lambda}$	$\frac{\lambda b}{\sigma c + \lambda b}$	$\frac{\sigma c}{\sigma c + \lambda b}$
$r=0$, $m=1$, $q(t) \xrightarrow[t\to\infty]{} \frac{\sigma}{\sigma+\lambda}$	$\frac{\lambda \beta b}{\sigma c + \lambda \beta b}$	$\frac{\sigma c}{\sigma c + \lambda \beta b}$
$r>>1$, $q(t) \xrightarrow[t\to\infty]{} \frac{\sigma}{\sigma+\lambda}$	$\approx \frac{b(\sigma+\lambda)}{\sigma c(1+m\alpha)}$	$\approx 1 - \frac{b(\sigma+\lambda)}{\sigma c(1+m\alpha)}$

Case $x\neq0$, $y\neq0$, $z\neq0$: There is one stationary point (x=0, y=y_0, z=z_0) during which defectors will exploit and eventually deplete all cooperators. The amount of defectors will first increase, and then converges to the equilibrium where there is coexistence with discriminators. For the model without churn, defectors will first increase by exploiting cooperators before vanishing from the system (Pyrrhic victory for defectors [2]).

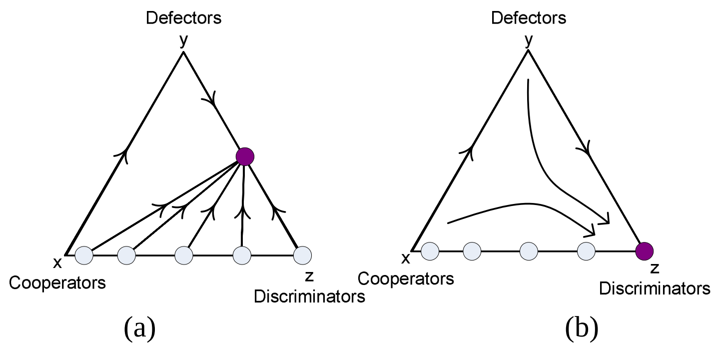

Fig. 3. The replicator dynamics: (a) with churn and (b) without churn.

4. Numerical evaluation

The evolutionary game is simulated with a custom simulator using the differential equations of 2.4.1, and within several scenarios varying storage system's parameters.

Initial frequency of strategies: Fig. 4 shows the frequency of cooperators and defectors over time, and demonstrates that with time cooperators will be eliminated from the system by these defectors. If there were some discriminators in the system, cooperators are still being evicted from the system; however, discriminators and defectors will converge to equilibrium where both coexist (Fig. 5). This equilibrium is not perturbed by the injection of a population of defectors or another population of discriminators, as it is illustrated in Fig. 6 (by varying the initial frequency of z). The figure shows also a little decrease in the frequency of discriminators before converging to the equilibrium. The decrease is due to that fact that discriminators act as cooperators in the beginning of the game since they do not know the behavior of defectors yet.

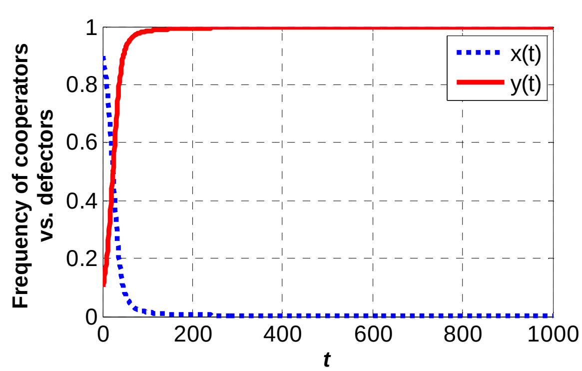

Fig. 4. Frequency of cooperators vs. defectors over time. m=10, r=7, β=0.3, α=0.1, λ=0.3, σ=0.05, b=0.05, c=0.05, x(0)=0.9, y(0)=0.1, and z(0)=0.

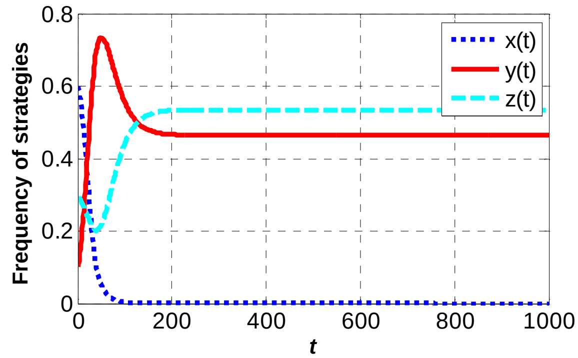

Fig. 5. Frequency of the three strategies over time. m=10, r=10, β=0.3, α=0.01, λ=0.01, σ=0.1, b=0.1, c=0.1, $x(0)$=0.6, $y(0)$=0.1, and $z(0)$=0.3.

Number of verifiers and replicas: Varying the number of data replicas r or the number m of verifiers changes the equilibrium point. Increasing r decreases the equilibrium value of discriminators frequency (see Fig. 7). On the contrary, increasing m quietly make this equilibrium value increase for higher value of m than a certain value (see Fig. 8). Both parameters r and m impact also the probability of encounter σ that increases if m or r increases, and hence the probability q increases which leads in its turn to an increase in the equilibrium value of discriminators frequency (Fig. 9).

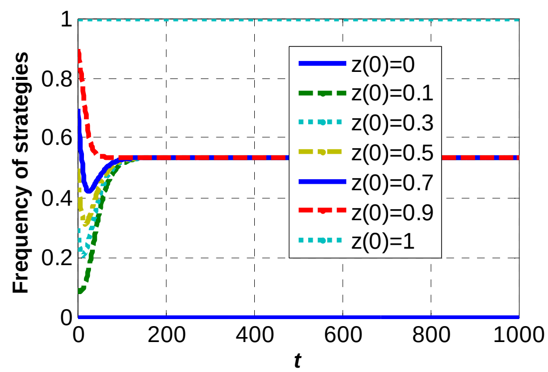

Fig. 6. Frequency of discriminators over time varying $z(0)$. m=10, r=10, β=0.3, α=0.01, λ=0.01, σ=0.1, b=0.1, c=0.1, and $x(0)$=0.

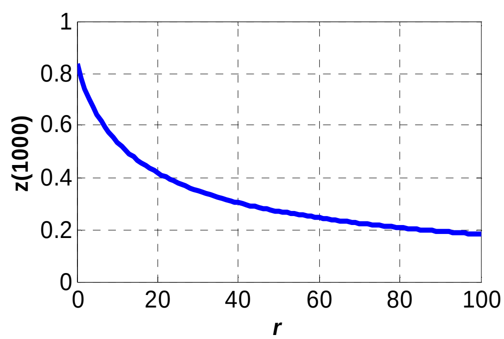

Fig. 7. Frequency of discriminators at equilibrium varying r. m=10, β=0.3, α=0.01, λ=0.01, σ=0.1, b=0.1, c=0.1, $x(0)$=0, $y(0)$=0.5, and $z(0)$=0.5.

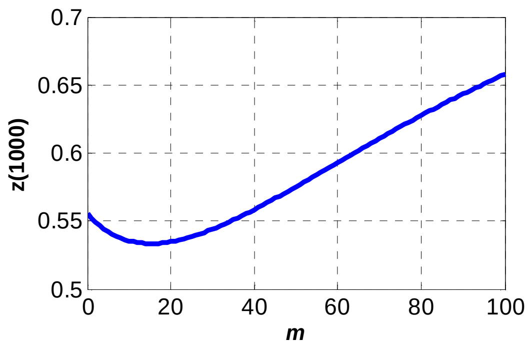

Fig. 8. Frequency of discriminators at equilibrium varying *m*. *r* =10, *β*=0.3, *α*=0.01, *λ*=0.01, *σ*=0.1, *b*=0.1, *c*=0.1, *x*(0)=0, *y*(0)=0.5, and *z*(0)=0.5.

Churn: The arrival rate *λ* of peers effects the probability *q*, and hence the equilibrium point of the game (see Fig. 10). For low churnout (small *λ*), the frequency of discriminators at equilibrium is high; whereas for high churnout (large *λ*) the frequency at equilibrium decreases. For high churnout, peers are not able to get acquainted with all peers since there are always new peers in the system, and defectors may take advantage of the lack of knowledge of discriminators about the system to gain benefit and remain in the game. For a system without churnout (*λ*=0), discriminators win against defectors that are eliminated from the game.

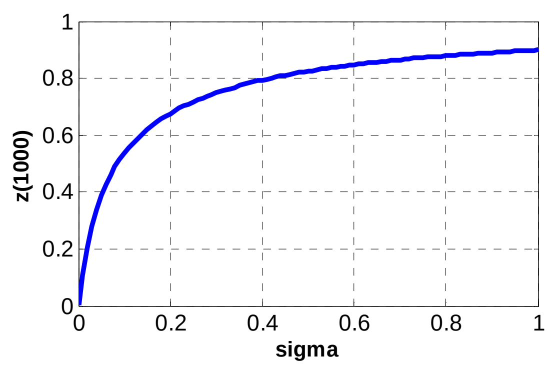

Fig. 9. Frequency of discriminators at equilibrium varying the probability of encounter *σ*. *m*=10, *r*=10, *β*=0.3, *α*=0.01, *λ*=0.01, *b*=0.1, *c*=0.1, *x*(0)=0, *y*(0)=0.5, and *z*(0)=0.5.

Benefit and cost: Fig. 11 depicts the impact of the benefit *b* and the cost *c* on the frequency of discriminators at equilibrium. The figure shows that *b* and *c* have opposite impact on the equilibrium frequency of discriminators: increasing *b* decreases the frequency; whereas, increasing *c* makes it increase. If *c*=0, the replicator dynamics of 2.4.1 converges to a stationary point where discriminators are eliminated from the game and defectors remains (from the dynamics of 2.5.1, if *b*=0, cooperators and defectors converge to a state where both coexist).

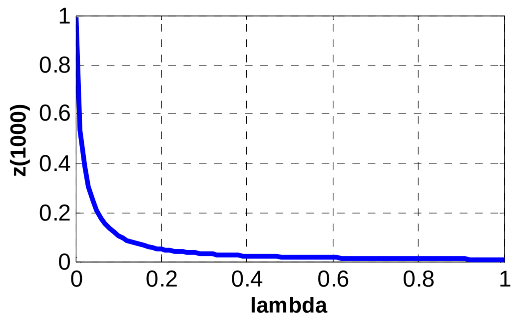

Fig. 10. Frequency of discriminators at equilibrium varying the arrival rate λ. m=10, r=10, β=0.3, α=0.01, σ=0.1, b=0.1, c=0.1, $x(0)$=0, $y(0)$=0.5, and $z(0)$=0.5.

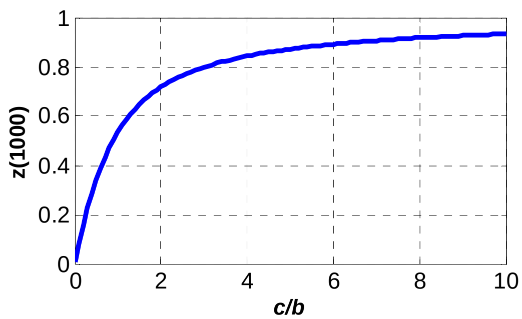

Fig. 11. Frequency of discriminators at equilibrium varying the ratio c/b. m=10, r=10, β=0.3, α=0.01, λ=0.01, σ=0.1, b=0.1, $x(0)$=0, $y(0)$=0.5, and $z(0)$=0.5.

Summary: Simulation results prove that there exist parameters values for which discriminators, who use an audit-based mechanism, may win against defectors who are free-riding. This means that discriminators are not hopeless when confronting defectors, even if these latter may dominate altruists (always cooperate strategy). At the equilibrium point of the game, both discriminators and defectors coexist if there is churn in the system otherwise discriminators will dominate. The initial frequency of discriminators has no effect on the equilibrium point; whereas the replication rate r and the number of verifiers m decreases the frequency of discriminators at the equilibrium for small m. A high cost of the storage and the verification make this frequency increases.

5. Conclusion

In this paper, we validated an audit-based strategy as an evolutionary stable strategy in some conditions and system's parameters within an evolutionary game model of a P2P storage system. The audit-based strategy wins over the free-riding strategy in a closed system. With some particular conditions, the audit-based strategy may coexist with free-riders at a high frequency.

6. References

1. Daniel Friedman, "On economic applications of evolutionary game theory". *Journal of Evolutionary Economics* 8, pp. 15–43.
2. Hannelore Brandt and Karl Sigmund, "The good, the bad and the discriminator--errors in direct and indirect reciprocity", *Journal of Theoretical Biology*, Volume 239, Issue 2, 21 March 2006, Pages 183-194.

3. Nouha Oualha, Melek Önen, and Yves Roudier, “A Security Protocol for Self-Organizing Data Storage”, to appear in *IFIP Sec 2008*.
4. Nouha Oualha, Melek Önen, and Yves Roudier, “A Security Protocol for Self-Organizing Data Storage”, (extended version) Technical Report Nº RR-08-208, EURECOM, January 2008.
5. Nouha Oualha and Yves Roudier, “A reputation system for self-organizing storage”, EURECOM Research report RR-08-222, May 2008.
6. Douglas Samuel Jones and B. D. Sleeman, “Differential Equations and Mathematical Biology”, London: Allen & Unwin, 1983.