Parallel Relay Networks with Phase Fading

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***Abstract*—In this paper, we consider Gaussian parallel relay networks with phase fading where a source node wants to communicate with a destination node with the assistance of two intermediate relay nodes. For this scenario, outer bounds are derived and three achievable schemes are considered. As well as amplify-and-forward (AF) and decode-and-forward (DF) schemes, we also consider a scheme where the relays exploit block quantization and random binning, which we call BQRB relaying. We show that in the broadcast channel limited regime, where received powers at the relay nodes are very small, BQRB outperforms the other schemes with increasing multiple access channel quality. Moreover, it is seen that BQRB achievable rate performance tends to the rate achievable by a point-to-point single-input multiple-output system.**

I. Introduction

In this paper, we consider Gaussian parallel relay networks with phase fading where a source node wants to communicate with a destination node with the assistance of two intermediate relay nodes. The system model is depicted in Figure–1. It is shown in [1] that when the broadcast (BC) link, the link between the source and the relay nodes, is inferior to the multiple-access (MAC) link, the link between the relay nodes to the destination node, decode-and-forward (DF) relaying is suboptimal, while amplify-and-forward (AF) relaying achieves network capacity in the limit of infinite power in the MAC components, at the relay nodes, which is due to the coherent combining effect at the destination node. For this scenario, using source coding tools (reproducing the relay observation with some fidelity) cannot yield better performance. For discrete parallel relay networks, it is shown that when MAC rates (noiseless links between the relay and the destination is assumed there) are high enough to reproduce the relay observations at the destination node, the use of block quantization and random binning (BQRB) techniques can achieve network capacity [1].

With phase fading Gaussian channels the coherent combining effect does not exist. Moreover, knowing that independent signal transmission is the optimal way of communication in the MAC portion, we can apply to the relay observations quantization and random binning tools to yield higher achievable rates than those proposed in [1]. Even though the relay observations are dependent random variables, as such the quantized versions, using random binning independent of the transmitted source signal will result in independent relay bin indexes. We utilize the independence of the signals to improve network achievable rates compared to other schemes.

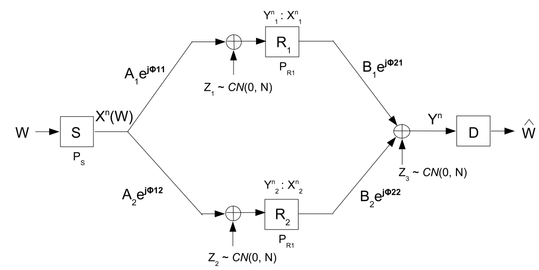

Fig. 1. Parallel Relay Network setup with phase fading.

It is shown that in the BC link limited regime, where received signal powers at the relay nodes are very small, BQRB outperforms the AF and DF relaying with increasing MAC quality. Moreover, it is also seen that BQRB achievable rate performance tends to the rate achievable by a point-to point single-input multiple-output system.

In the following sections we find outer bounds and achievable rates. Then, we will examine how far achievable rates are from the outer bounds. Different outer bounds and achievable rates are examined in [1], [2] where channel is modeled as a simple Gaussian channel. In [3] two-relay half-duplex diamond network is considered where the relays are assumed to have channel side information.

II. Signal and System Model

We assume that our channel is memoryless and undergoes ergodic phase fading and path-loss. The source node and relay nodes have an average power constraint P_s and P_{r_i}, $i = 1, 2$, respectively. In the limit $n \to \infty$, $\frac{1}{n}\sum_{k=1}^{n} \mathrm{E}\{X_k^2\} \leq P_s$ and $\frac{1}{n}\sum_{k=1}^{n} \mathrm{E}\{X_{i,k}^2\} \leq P_{r_i}$ for $i = 1, 2$. The received signals at relay nodes and the destination node, respectively, as follow

$$Y_{i,k} = A_i e^{j\Phi_{1i,k}} X_k + Z_{i,k}, \qquad i = 1, 2 \tag{1}$$

$$Y_k = \sum_{i=1}^{2} B_i e^{j\Phi_{2i,k}} X_{i,k} + Z_{3,k} \tag{2}$$

for $k = 1, 2, \ldots, n$. Here A_i and B_i could be considered as path-loss. $Z_i^n = \{Z_{i,k}; k = 1, 2, \ldots, n\}$, $i = 1, 2, 3$, denote an i.i.d. sequence drawn according to a Gaussian distribution

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with zero mean and variance N representing the channel noise, and $\Phi_{ti}^n = \{\Phi_{ti,k}; k = 1, 2, \ldots, n\}$, for $t = 1, 2$ and $i = 1, 2, 3$ denotes the set of random phases induced by the channel. Φ_{ti}^n are random variables uniformly distributed over $[-\pi; \pi]$, perfectly known to the relevant receiver and unknown to the transmitter, extracted from a jointly stationary and ergodic process. As in point-to-point communication systems, the parallel relay network capacity from the source to the destination is given by the supremum over input distribution of mutual information between transmitted signal sequence, X^n, and received signal sequence, Y^n, that is

$$C_{net} = \sup_{f(X^n)} \frac{1}{n} \mathrm{I}(X^n; Y^n). \quad (3)$$

III. Outer Bounds (Converse Analysis)

In this section, we derive outer bounds for phase fading parallel relay networks. It is assumed that phases are only available to the corresponding receivers, transmitters do not have phase knowledge. We will use $\tilde{Y}^n = (Y^n, \Phi_{11}^n, \Phi_{12}^n, \Phi_{21}^n, \Phi_{22}^n)$, $\tilde{Y}_1^n = (Y_1^n, \Phi_{11}^n)$ and $\tilde{Y}_2^n = (Y_2^n, \Phi_{12}^n)$ for notational convenience. Using the data processing inequality on the two cut-sets corresponding to BC part and MAC part, and knowing that $X \rightarrow (\tilde{Y}_1, \tilde{Y}_2) \rightarrow (X_1, X_2) \rightarrow \tilde{Y}$ forms a Markov chain, we come up with the following outer bound:

$$C_{net} \leq \frac{1}{n} \min\left\{\mathrm{I}(X^n; \tilde{Y}_1^n, \tilde{Y}_2^n), \mathrm{I}(X_1^n, X_2^n; \tilde{Y}^n)\right\} \quad (4)$$

where BC rate is achievable when relay nodes do cooperation to decode transmitted signals, and the rate achievable by MAC part requires transmitted signals to be independent to each other due to the phase fading effect [4], [5]. Defining $\gamma_{r,i} = \frac{A_i^2 P_s}{N}$ and $\gamma_{d,i} = \frac{B_i^2 P_{r_i}}{N}$ for $i = 1, 2$. For a symmetric network, i.e., $A_i = A$, $B_i = B$, $P_{r_i} = P_r$, hence $\gamma_{r,i} = \gamma_r$ and $\gamma_{d,i} = \gamma_d$ for $i = 1, 2$, then network upper bounds become

$$C_{net} \leq \min\left\{\log_2(1 + 2\gamma_r), \log_2(1 + 2\gamma_d)\right\}. \quad (5)$$

IV. Achievable Rates

Here we are going to compare three communication strategies in terms of achievable rates. First, we look at AF relaying strategy followed DF relaying and finally we look at block quantization and random binning (BQRB) at the relay nodes. We are going to limit ourselves to the symmetric case described above.

A. Amplify-and-Forward Relaying Strategy

For simplicity we study symmetric parallel relay networks. Due to the symmetry of the channels, the scaling factors, α_1 and α_2, at the relay nodes are the same, i.e., $\alpha = \alpha_1 = \alpha_2 = \sqrt{\frac{P_r}{A^2 P_s + N}}$. The received signal at the destination node, for $k = 1, 2, \ldots, n$, is given by

$$Y_k = \alpha AB\left(e^{j(\Phi_{11,k} + \Phi_{21,k})} + e^{j(\Phi_{12,k} + \Phi_{22,k})}\right) X_k + Z_k$$

where $Z_k = \alpha B e^{j\Phi_{21,k}} Z_{1,k} + \alpha B e^{j\Phi_{22,k}} Z_{2,k} + Z_{3,k}$. We can express the received SNR as follows

$$\gamma_{AF} = \frac{2\gamma_r\gamma_d(1 + \cos(\Phi_{11} + \Phi_{21} - (\Phi_{12} + \Phi_{22})))}{\gamma_r + 2\gamma_d + 1}$$

and the corresponding AF achievable rate is given by

$$R_{AF} = \mathrm{E}_{\Phi}\left\{\log_2(1 + \gamma_{AF})\right\} \leq \log_2\left(1 + \frac{2\gamma_r\gamma_d}{\gamma_r + 2\gamma_d + 1}\right). \quad (6)$$

Note that the above bound is a result of Jensen's inequality.

B. Decode-and-Forward Relaying Strategy

As we assumed symmetric channel coefficients both in BC and MAC our achievable rate expression is simple. In BC part, transmitted message from the source node can be decoded by both relay nodes if

$$\begin{aligned} R_{DF}^{BC} &\leq \frac{1}{n} \min\{\mathrm{I}(X^n(W); Y_1^n), \mathrm{I}(X^n(W); Y_2^n)\} \\ &\leq \log_2(1 + \gamma_r). \quad (7) \end{aligned}$$

At the relays we send independent signals which can be implemented using a (distributed) space-time code, yielding the following relay-to-destination rate

$$R_{DF}^{MAC} \leq \frac{1}{n} \mathrm{I}\left(X_1^n(W), X_2^n(W); Y^n\right) \leq \log_2(1 + 2\gamma_d). \quad (8)$$

The end-to-end system achievable rate is the minimum of (7) and (8):

$$R_{DF} = \min\left\{\log_2(1 + \gamma_r), \log_2(1 + 2\gamma_d)\right\}. \quad (9)$$

We can see that if $\gamma_r \geq 2\gamma_d$ then DF achievable rate is the capacity of the network. Otherwise, there is a gap between the DF achievable rate and the outer-bound of the network.

C. Block Quantization and Random Binning (BQRB) of the Relay Observations

It is shown in Section-IV-A that for phase fading Gaussian networks using AF relaying cannot achieve the network capacity even for high SNR values at the MAC part. Here we are going to investigate an achievable scheme that mimics multiple-antenna reception performance in the sense that the destination jointly processes the representations of the relay received signals. For the analysis we assume only the receiving nodes has perfect CSI, i.e., each relay node has its corresponding CSI between the source node and itself, and the destination node has full network CSI.

The scheme considered in here is continuous case of the scheme considered in [1]. In addition, we use some fundamental results by Oohama [6] based on joint (weak) typicality among sequences having Markov chain relation. Before continuing, we should indicate that at the destination node there are two decoding steps: the first is channel decoder for the relay bin indexes corresponding to the representations of the relay observations, and the second one is the source message decoding. Note that if there is an error in the first decoding step, we declare an error for the source message. So we need to be careful for choosing the rate pairs (R_1, R_2) for communication between the relay nodes and the destination node.

We follow the same procedure used in [1]. We generate $2^{nR_{ach}}$ input codewords using the input density $f(x)$. We generate approximately $2^{nI(Y_1;V_1)}$ quantization codewords for

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Relay 1 using the marginal density $f(v_1)$. We randomly assign these quantization codewords to 2^{nR_1} bins. We generate 2^{nR_1} input codewords for Relay 1 using the input density $f(x_1)$. Relay 2 proceeds similarly.

Theorem: *For the parallel relay network with Gaussian phase fading memoryless broadcast channel $f(y_1, y_2|x)$ and broadcast channel $f(y|x_1, x_2)$, which is depicted in Figure-1. Choose any probability density function $f(x)$ and any pair of conditional densities $f(v_1|y_1)$ and $f(v_2|y_2)$. We can reliably achieve rate R_{ach}*

$$R_{ach} \quad \leq \quad I(X; V_1, V_2), \tag{10}$$

provided

$$I(Y_1; V_1) - I(V_1; V_2) \quad \leq R_1 \leq \quad I(X_1; Y|X_2), \tag{11}$$

$$I(Y_2; V_2) - I(V_1; V_2) \quad \leq R_2 \leq \quad I(X_2; Y|X_1), \tag{12}$$

$$I(Y_1; V_1) + I(Y_2; V_2) - I(V_1; V_2) \leq R_1 + R_2 \leq I(X_1, X_2; Y) \tag{13}$$

These values are computed with respect to the density $f(x, y_1, y_2, v_1, v_2, x_1, x_2, y) = f(x)\ f(y_1, y_2|x)\ f(v_1|y_1)\ f(v_2|y_2)\ f(x_1|v_1)\ f(x_2|v_2)\ f(y|x_1, x_2)$.

Proof: For now, we choose the decoder's typicality measure, ϵ, and the integer block length , n, arbitrarily. We will later set ϵ sufficiently small and n sufficiently large to make the probability of message error as small as possible.

Randomly generate $2^{nR_{ach}}$ input codewords of block length n. Generate each symbol of each codeword independently according to $f(x)$. Denote these input codewords by $X^n(w)$, $w = 1, 2, \ldots, 2^{nR_{ach}}$, and denote the randomly chosen input codebook by $\mathcal{C} = \{\cup_{w=1}^{2^{nR_{ach}}} X^n(w)\}$. For a particular codebook choice, denote the input codewords by $x^n(w)$, $w = 1, 2, \ldots, 2^{nR_{ach}}$, and denote the input codebook by $c = \{\cup_{w=1}^{2^{nR_{ach}}} x^n(w)\}$.

Set $\delta > 0$ as arbitrary parameter. Randomly generate $2^{n(I(Y_1;V_1)+\delta)}$ Relay 1 quantization codewords of block length n. Generate each symbol of each codeword independently according to $f(v_1)$. Denote these Relay 1 quantization codewords by $V_1^n(j), j = 1, 2, \ldots, 2^{n(I(Y_1;V_1)+\delta)}$. For a particular quantization codebook choice, denote the quantization codebook by $c_{q1} = \{\cup_j v_1^n(j)\}$. Next, having randomly generated the Relay 1 quantization codebook, randomly and uniformly assign each Relay 1 quantization codeword, v_1^n, to one of $B_1 \in \{1, 2, \ldots, 2^{nR_1}\}$ bins. Denote the randomly chosen bin assignments by the function $B_1 = \varphi_1(v_1^n(j))$. Generate 2^{nR_1} independent codewords $X_1^n(B_1)$, $B_1 \in \{1, 2, \ldots, 2^{nR_1}\}$, of length n, generating each element i.i.d. $\sim \prod_{i=1}^n f(x_{1i})$. Relay 2 proceeds similarly. All the codebooks and bin assignments are reveal to the destination.

Encoding: The sender transmits the codeword $x^n(w)$ corresponding to message w. After receiving Y_1^n, Relay 1 searches for any quantization codeword $v_1^n(j_1) \in c_{q1}$ such that $(Y_1^n, v_1^n(j_1)) \in \mathcal{A}_\epsilon^{(n)}$, where $\mathcal{A}_\epsilon^{(n)}$ represents the jointly typical set. If one or more such quantization codewords exist, Relay 1 chooses one of them randomly (uniformly amongst the candidates) and sends $X_1^n(B_1)$ where $B_1 = \varphi_1(v_1^n(j_1))$. Otherwise, Relay 1 declares an error (e.g., sends $B_1 \equiv 0$). In this case, for notational convenience, we define the quantization codeword $v_1^n = \emptyset$. And Relay 2 proceeds similarly.

Decoding: At the decoder, we first try to decode the bin indexes sent by the relay nodes. If we are able to decode the bin indexes correctly, then we continue the following decoding step in which we try to decode the source message. After receiving Y^n, the destination chooses the pair $(\hat{B}_1, \hat{B}_2)$ such that $(x_1^n(\hat{B}_1), x_2^n(\hat{B}_2), y^n) \in \mathcal{A}_\epsilon^{(n)}$ if such a pair $(\hat{B}_1, \hat{B}_2)$ exists and unique; otherwise, an error is declared.

Suppose in the first decoding step we correctly decoded the bin indexes sent by the relay nodes, $(\hat{B}_1, \hat{B}_2) = (B_1, B_2)$. Then, if either of these bin numbers equal the relay error message 0, the decoder declares an error. This indicates that one or both of the relays failed the quantization step. Otherwise, the decoder declares message w was sent if it is the unique message with a pair of quantization codewords, v_1^n and v_2^n, such that $\varphi_1(v_1^n) = \hat{B}_1$, $\varphi_2(v_2^n) = \hat{B}_2$, and $(x^n(w), v_1^n, v_2^n) \in \mathcal{A}_\epsilon^{(n)}$. If no message or if more than one message satisfies this criterion, the decoder declares an error.

Average Probability of Decoding Error: We compute the average probability of message error by averaging over the choice of input codewords, the choice of relay quantization codebooks, the quantization codeword bin assignments, and the broadcast channel outcomes. We denote the average probability of error by $\mathrm{P}_e^{(n)}$.

Before averaging over the input codebook ensemble, the input codeword corresponding to the first message is denoted $X^n(1)$. During operation, the relays receive the pair of observations (Y_1^n, Y_2^n). When computing the average probability of message error, we will consider the input codebook without the first codeword. Define $\mathcal{C}_{-1} = \{\cup_{w=2}^{2^{nR_{ach}}} X^n(w)\}$. Finally, we adopt the conventional set notation $\mathcal{C}_{q1} \backslash V_1^n = \mathcal{C}_{q1} - V_1^n$ and $\mathcal{C}_{q2} \backslash V_2^n = \mathcal{C}_{q2} - V_2^n$.

We define following events:
E_{mac} : Event of error in MAC, i.e., $(\hat{B}_1, \hat{B}_2) \neq (B_1, B_2)$,
E_s : Event of error in decoding the source message, i.e., $\hat{W} \neq W$.

Using these definitions, we can write the average overall probability of message error as

$$\begin{aligned} \mathrm{P}_e^{(n)} &= \Pr(E_{mac}) \Pr(E_s|E_{mac}) + \Pr(E_{mac}^c) \Pr(E_s|E_{mac}^c) \\ &\leq \Pr(E_{mac}) + \Pr(E_s|E_{mac}^c). \end{aligned}$$

First, consider the communication between the relay nodes and the destination node. We want to reliably estimate the transmitted bin indexes at the destination node in order to proceed to decode the source message. Due to random assignment of bin indexes at the relay nodes, the relay signals, X_1 and X_2, are uncorrelated. Note that from [7, Theorem 15.3.1], with the upper bounds in the theorem satisfied, $\Pr(E_{mac}) \to 0$ with $n \to \infty$. Assuming we perfectly decode the relay bin indexes, then we can group all error events into the union of six events for $\Pr(E_s|E_{mac}^c)$. They are:

$$E_0 \quad = \quad \left\{ \frac{1}{n} \sum_{i=1}^{n} X_i^2(1) \ > \ P_s \right\}$$

is the event that the source power constraint is violated.

$$E_1 = \left\{(X^n(1), V_1^n, V_2^n) \notin \mathcal{A}_\epsilon^{(n)}(X, V_1, V_2)\right\}$$

is the event that the input codeword and the actual relay quantization codewords are not jointly typical.

$$E_2 = \left\{ \begin{array}{l} (X^n(1), V_1^n, V_2^n) \in \mathcal{A}_\epsilon^{(n)}, \exists x' \in \mathcal{C}_{-1} \\ \text{s.t. } (x', V_1^n, V_2^n) \in \mathcal{A}_\epsilon^{(n)}. \end{array} \right\}$$

is the event that $(X^n(1), V_1^n, V_2^n)$ is jointly typical, and there is an incorrect input codeword, x', such that the triple (x', V_1^n, V_2^n) is jointly typical.

$$E_3 = \left\{ \begin{array}{l} (X^n(1), V_1^n, V_2^n) \in \mathcal{A}_\epsilon^{(n)}, \\ \exists x' \in \mathcal{C}_{-1} \text{ and } v_1' \in \mathcal{C}_{q1} \backslash V_1^n \\ \text{s.t. } \varphi_1(v_1') = \varphi_1(V_1^n) \text{ and } (x', v_1', V_2^n) \in \mathcal{A}_\epsilon^{(n)}. \end{array} \right\}$$

is the event that $(X^n(1), V_1^n, V_2^n)$ is jointly typical, and there is an incorrect input codeword, x', and a different Relay 1 quantization codeword, v_1', assigned to the same bin as the chosen Relay 1 quantization codeword, such that the triple (x', v_1', V_2^n) is jointly typical.

$$E_4 = \left\{ \begin{array}{l} (X^n(1), V_1^n, V_2^n) \in \mathcal{A}_\epsilon^{(n)}, \\ \exists x' \in \mathcal{C}_{-1} \text{ and } v_2' \in \mathcal{C}_{q2} \backslash V_2^n \\ \text{s.t. } \varphi_2(v_2') = \varphi_2(V_2^n) \text{ and } (x', V_1^n, v_2') \in \mathcal{A}_\epsilon^{(n)}. \end{array} \right\}$$

is the event that $(X^n(1), V_1^n, V_2^n)$ is jointly typical, and there is an incorrect input codeword, x', and a different Relay 2 quantization codeword, v_2', assigned to the same bin as the chosen Relay 2 quantization codeword, such that the triple (x', V_1^n, v_2') is jointly typical.

$$E_5 = \left\{ \begin{array}{l} (X^n(1), V_1^n, V_2^n) \in \mathcal{A}_\epsilon^{(n)}, \\ \exists x' \in \mathcal{C}_{-1} \ , \ v_1' \in \mathcal{C}_{q1} \backslash V_1^n \text{ and } v_2' \in \mathcal{C}_{q2} \backslash V_2^n \\ \text{s.t. } \varphi_1(v_1') = \varphi_1(V_1^n) \ , \ \varphi_2(v_2') = \varphi_2(V_2^n) \\ \text{and } (x', v_1', v_2') \in \mathcal{A}_\epsilon^{(n)}. \end{array} \right\}$$

is the event that $(X^n(1), V_1^n, V_2^n)$ is jointly typical, and there is an incorrect input codeword, x', and two different relay quantization codewords, v_1' and v_2', each assigned to the same bin as the chosen relay quantization codewords, such that the triple (x', v_1', v_2') is jointly typical. Using the basic set theory and the union bound,

$$\Pr(E_s | E_{mac}^c) = \Pr\left(\cup_{i=0}^5 E_i\right) \leq \sum_{i=0}^{5} \Pr\left(E_i\right). \quad (14)$$

We will show that each of these six probabilities can be made arbitrarily small as $n \to \infty$. By law of large numbers $\Pr(E_0) \to 0$ as $n \to \infty$. To bound $\Pr(E_1)$ we need to show joint typicality between X, V_1 and V_2. For the signal model we consider, we have the following Markov chain:

$$(V_1, V_2) \to (Y_1, Y_2) \to X \to Y_i \to V_i, \quad i = 1, 2.$$

A generalized Markov lemma that is proved by Oohama in [6] states that the Markov chain implies,

$$\lim_{n\to\infty} \Pr\left((X^n, Y_1^n, Y_2^n, V_1^n, V_2^n) \in \mathcal{A}_\epsilon^{(n)}\right) = 1. \quad (15)$$

From the definition of jointly typical sequences [7, Theorem 15.2.1] if we have an ordered set of jointly distributed random variables being jointly typical, then any subset of this set also implies jointly typicality. Hence (15) implies

$$\lim_{n\to\infty} \Pr\left((X^n, V_1^n, V_2^n) \in \mathcal{A}_\epsilon^{(n)}\right) = 1. \quad (16)$$

As a consequence, $\Pr(E_1) \to 0$ as $\epsilon \to 0$ and $n \to \infty$ [7].

Consider event E_2. By the union bound and the typicality in (16),

$$\begin{aligned} \Pr(E_2) &\leq \int_{\mathcal{A}_\epsilon^{(n)}} f(x^n, v_1^n, v_2^n) \sum_{w=2}^{2^{nR_{ach}}} \\ &\qquad \Pr\left\{(x^n(w), v_1^n, v_2^n) \in \mathcal{A}_\epsilon^{(n)}\right\} dx^n dv_1^n dv_2^n \\ &\leq 2^{-n(I(X;V_1,V_2) - R_{ach} - 3\epsilon)} \Pr\{\mathcal{A}_\epsilon^{(n)}(X, V_1, V_2)\} \\ &\leq 2^{-n(I(X;V_1,V_2) - R_{ach} - 3\epsilon)} \end{aligned}$$

for some $\epsilon > 0$, where $\epsilon \to 0$ as $n \to \infty$. If we have $R_{ach} \leq I(X; V_1, V_2)$, then $\Pr(E_2) \to 0$ as $\epsilon \to 0$ and $n \to \infty$.

Consider event E_3 which means $(X^n(1), V_1^n, V_2^n)$ is jointly typical and there is an incorrect codeword x', and a different Relay 1 quantization codeword, v_1', assigned to the same bin as the chosen Relay 1 quantization codeword such that (x', v_1', V_2^n) is jointly typical. Then the probability corresponding to event E_3 is given by

$$\begin{aligned} \Pr(E_3) &\leq \int_{\mathcal{A}_\epsilon^{(n)}} f(x^n, v_1^n, v_2^n) \sum_{w=2}^{2^{nR_{ach}}} \cdot \sum_{k=2}^{2^{n(I(Y_1;V_1)+\delta)}} \cdot \\ &\qquad \Bigg\{ \int_{\substack{x', v_1' \\ (x', v_1', v_2^n) \in \mathcal{A}_\epsilon^{(n)}}} f(x') f(v_1') \cdot \\ &\qquad \Pr\left\{\varphi_1(v_1') = \varphi_1(v_1^n)\right\} dx' dv_1' \Bigg\} dx^n dv_1^n dv_2^n \\ &\leq 2^{-n(h(X)+h(V_1)-h(X,V_1|V_2)+R_1-I(Y_1;V_1)-R_{ach}-\delta-4\epsilon)} \\ &\leq 2^{-n(I(X;V_1,V_2)-R_{ach}+R_1+I(V_1;V_2)-I(Y_1;V_1)-\delta-4\epsilon)} \end{aligned}$$

for some $\epsilon > 0$, where $\epsilon \to 0$ as $n \to \infty$. With $R_{ach} \leq I(X; V_1, V_2)$, and $R_1 \geq I(Y_1; V_1) - I(V_1; V_2)$, and since we can take $\delta \to 0$, $\Pr(E_3) \to 0$ as $\epsilon \to 0$ and $n \to \infty$.

Similarly for the event E_4 we have

$$\Pr(E_4) \leq 2^{-n(I(X;V_1,V_2)-R_{ach}+R_2+I(V_1;V_2)-I(Y_2;V_2)-\delta-4\epsilon)}$$

for some $\epsilon > 0$, where $\epsilon \to 0$ as $n \to \infty$. With With $R_{ach} \leq I(X; V_1, V_2)$, and $R_2 \geq I(Y_2; V_2) - I(V_1; V_2)$, and since we can take $\delta \to 0$, $\Pr(E_4) \to 0$ as $\epsilon \to 0$ and $n \to \infty$.

Finally, consider event E_5, using the same reasoning as before gives

$$\begin{aligned} \Pr(E_5) &\leq 2^{-n(I(X;V_1,V_2)-R_{ach})} \cdot \\ &\qquad 2^{-n(R_1+R_2+I(V_1;V_2)-I(Y_1;V_1)-I(Y_2;V_2)-5\epsilon-2\delta)} \end{aligned}$$

for some $\epsilon > 0$, where $\epsilon \to 0$ as $n \to \infty$. With $R_{ach} \leq I(X; V_1, V_2)$, and $R_1 + R_2 \geq I(Y_1; V_1) + I(Y_2; V_2) - I(V_1; V_2)$, and since we can take $\delta \to 0$, $\Pr(E_5) \to 0$ as $\epsilon \to 0$ and $n \to \infty$.

For all of six error events if the derived rate conditions holds with $\epsilon \to 0$ as $n \to \infty$, then $\Pr(E_s | E_{mac}^c) \to 0$. Hence, together with rate conditions on MAC, we can make the average probability of message error sufficiently small,

$\mathrm{P}_e^{(n)} \to 0$. By combining the rate requirements into a compact form, we get (10), (11), (12) and (13).

To evaluate the mutual information terms in the lower bounds, we proceed as the following: due to CSI assumption at the receiving nodes, assume the relay nodes first phase compensated the received signals, $Y_i' = e^{-j\Phi_i}Y_i = A_iX + Z_i'$ for $i = 1, 2$. Note that the phase compensation does not effect the statistics of the noise terms. After phase compensation the relay nodes generate the quantized codewords according to the distribution $f(v_i|y_i) \sim \mathcal{CN}(y_i, D_i)$ as in [8], i.e.,

$$V_i = Y_i' + Z_{d,i} = A_iX + Z_i' + Z_{d,i} \quad (17)$$

where $Z_i' = e^{-j\Phi_i}Z_i \sim \mathcal{CN}(0, N)$ and $Z_{d,i} \sim \mathcal{CN}(0, D_i)$ for $i = 1, 2$. With these settings we have

$$R_{ach} \leq \log_2\left(1 + \frac{A_1^2P_s}{N + D_1} + \frac{A_2^2P_s}{N + D_2}\right),$$
$$\log_2\left(\frac{\sigma_{v_1}^2}{D_1}(1-\rho^2)\right) \leq R_1 \leq \log_2(1 + \gamma_{d,1})$$
$$\log_2\left(\frac{\sigma_{v_2}^2}{D_2}(1-\rho^2)\right) \leq R_2 \leq \log_2(1 + \gamma_{d,2})$$
$$\log_2\left(\frac{\sigma_{v_1}^2}{D_1}\frac{\sigma_{v_2}^2}{D_2}(1-\rho^2)\right) \leq R_1 + R_2 \leq \log_2(1 + \gamma_{d,1} + \gamma_{d,2})$$

where ρ is the correlation factor between V_1 and V_2, and $\sigma_{v_i}^2 = A_i^2P_s + N + D_i$, for $i = 1, 2$. Note that here we have lower bounds depending on the variances and correlation coefficient between quantization outputs, V_1 and V_2.

For the symmetric case, $D_i = D$, and $R_i = R$ for $i = 1, 2$, after some algebra we can lower bound D as

$$D \geq \frac{N\left[1 + \gamma_r + \sqrt{(1+\gamma_r)^2 + 4\gamma_r\gamma_d + 2\gamma_d}\right]}{2\gamma_d}. \quad (18)$$

Then we have the following achievable rate

$$R_{ach} \leq \log_2\left(1 + \frac{2A^2P_s}{N + D}\right) \quad (19)$$
$$= \log_2\left(1 + \frac{2\gamma_r}{1 + \frac{1+\gamma_r+\sqrt{(1+\gamma_r)^2+4\gamma_r\gamma_d+2\gamma_d}}{2\gamma_d}}\right) \quad (20)$$

Note that as $\gamma_d \to \infty$, for fixed γ_r, BQRB scheme yields the capacity of the network.

V. Numerical Results

In this section, we compare the performance between the relaying schemes considered in the symmetric case. For this class the corresponding outer bound and achievable rates are (5), (6), (9), and (19). To see the performance of these relaying schemes, we select the broadcast SNR as $\gamma_r = 10$[dB], and give average rates for different multiple-access SNR values, γ_d. The results are depicted in Figure-2. In the plot, we can see that up to $\gamma_d = 7$ [dB] DF relaying scheme achieves the network capacity, exceeding this value DF performance is bounded by BC link. In contrast to the Gaussian case [1], with phase fading, AF does not even converge to DF performance.

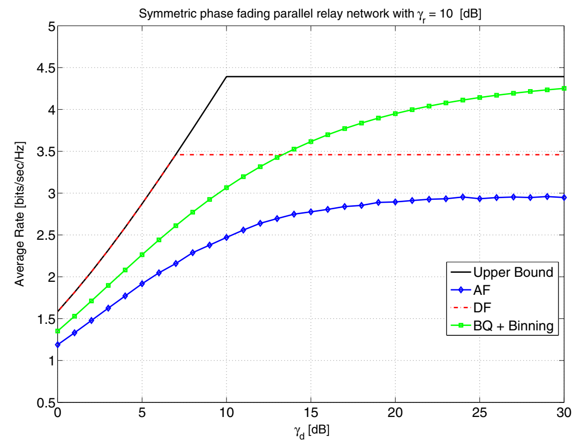

Fig. 2. Symmetric parallel Gaussian phase fading relay network average rates with $\gamma_r = 10$[dB].

On the other hand, the scheme we proposed here converges to the outer bound and always outperforms AF (and DF in the strong relay regime).

VI. Conclusions

In this paper we considered Gaussian phase fading parallel relay networks. An outer bound and achievable rates corresponding to AF, DF and BQRB are given. In contrast to the Gaussian case, with phase fading, the achievable rate in AF relaying is decreased, even below DF relaying. On the other hand, in the BC link limited regime we see that BQRB outperforms the other schemes with increasing MAC quality. BQRB achievable rate performance tends to the rate achievable by a point-to-point single-input multiple-output (SIMO) system.

An interesting extension of BQRB is to consider multiple source node communicating to a single destination node with the assistance of multiple relay nodes. We believe that in this scenario with BQRB relaying, one can achieve multi-input multi-output (MIMO) system rates in a distributed fashion.

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