

DISTRIBUTED TRANSMIT MODE SELECTION FOR MISO BROADCAST CHANNELS WITH LIMITED FEEDBACK: SWITCHING FROM SDMA TO TDMA

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ABSTRACT

We consider a multi-antenna broadcast channel with more single-antenna receivers than transmit antennas and partial channel state information at the transmitter (CSIT). An adaptive transmit mode switching scheme, in which the base station selects between single-user (TDMA) and multiuser transmission (SDMA) is proposed. In particular, we formulate a framework for distributed mode selection and sum-rate optimization with limited feedback. By assuming instantaneous local CSIT and statistical knowledge of non-local information, we derive a distributed algorithm in which each receiver feeds back scalar channel quality information (CQI) as well as its preferred mode decision. Low-complexity variants, which exhibit negligible decrease in sum rate, are also investigated. Performance analysis and simulations show that this scheme provides significant throughput gains for low to moderate number of users and linear capacity growth in the interference-limited region.

1. INTRODUCTION

Multiuser multiple-input, multiple-output (MIMO) communications have attracted significant attention recently, mainly due to the high spectral efficiencies they achieve. In multiuser downlink MIMO systems, a direct capacity gain proportional to the number of transmit antennas M can be achieved by serving multiple users in a space-division multiple access (SDMA) fashion, even with single-antenna receivers [1].

Nevertheless, all these promising results unfortunately come at the critical assumption of sufficient channel state information at transmitter (CSIT). The often impractical assumption of close-to-perfect CSIT, as well as the considerable capacity gap between perfect and no CSIT, have motivated research work on schemes employing partial CSIT [2]. One popular approach to deal with incomplete channel information, often referred to as limited feedback, is to quantize the channel vector (or the precoder) based on a predetermined codebook known at both the BS and the terminals. In this framework, each user feed back quantized information on its channel direction (CDI) and instantaneous channel quality information (CQI) through a finite rate uplink channel [3–5]. The scheduler intelligently selects M spatially separable users with

large channel gains, approaching thus the capacity with full CSIT. Another popular, very low-rate feedback technique is the so-called random beamforming (RBF) [6], where $\mathcal{M} = M$ random orthonormal beamforming vectors are generated and the best user on each beam is scheduled. This scheme is shown to yield the optimal capacity scaling of $M \log \log K$ when the number of users K is asymptotically large,

A major drawback of the above limited feedback systems is that they are interference-limited, thus the sum rate saturates at high signal-to-noise (SNR) and fixed feedback rate since inter-user interference cannot be fully mitigated due to incomplete channel knowledge. Furthermore, contrary to the perfect CSIT case, SDMA transmission does not always outperform single-user transmission under limited feedback. In such settings, adaptive schemes switching from multiuser (SDMA) to single-user (TDMA) transmission are of particular interest, allowing thus the BS to adapt the number of active beams with respect to K and average SNR [5]. In [7], we proposed a low-complexity on/off beam power control, yielding a dual-mode scheme switching from TDMA to full SDMA as a means to render RBF more robust in sparse networks.

To the best of knowledge, in all previous approaches, scheduling and transmit mode decisions are solely performed in a centralized fashion, i.e. at the base station (BS). The mobiles report scalar CQI for user selection and rate allocation. In this work, we take on a different approach. We propose a distributed framework in which the receivers determine and feed back their preferred transmit mode in addition to the corresponding CQI. A key point is that the user's preferred mode is chosen as the one that maximizes the expected system sum-rate and not the user individual rate. The stochastic optimization at the user side relies on instantaneous local information and non-local, statistical information on other users' channels. Based on users' preferred mode and CQIs, the BS selects in turn the group users and the transmit mode, i.e. TDMA or SDMA with $2 \leq \mathcal{M} \leq M$ beams, which maximizes the sum-rate. Our analysis and simulation results show that the distributed multi-mode selection allows switching from SDMA to TDMA as a means to compensate for the capacity ceiling effect and achieve linear sum-rate growth in

the interference-limited region. A simplified switching criterion that does not rely on statistical CSI is also proposed and shown to provide similar performance with the multi-mode variant.

2. SYSTEM MODEL

We consider a multi-antenna Gaussian broadcast channel in which a BS equipped with M transmit antennas communicates with $K \geq M$ single-antenna receivers. The received signal y_k of the k -th user is mathematically described as

$$y_k = \mathbf{h}_k \mathbf{x} + n_k, \quad k = 1, \dots, K \quad (1)$$

where $\mathbf{x} \in \mathbb{C}^{M \times 1}$ is the vector of transmitted signals, subject to a power constraint P , i.e., $\text{Tr}(\mathbf{x}\mathbf{x}^H) \leq P$, where $\text{Tr}(\cdot)$ is the trace operator. The channel gain response of each user k , $\mathbf{h}_k \in \mathbb{C}^{1 \times M}$, has components distributed as $\mathcal{CN}(0, 1)$ (Rayleigh fading). We assume perfect channel knowledge at the receiver, and that n_k is independent and identically distributed (i.i.d.) circularly symmetric additive complex Gaussian noise with zero mean and variance σ^2 , $\forall k$. We consider a frequency non-selective block fading channel and that all users experience the same average SNR.

2.1. Joint Scheduling and Linear Precoding

Let $\mathcal{S} = \{k_i\}_{i=1}^{\mathcal{M}}$ be the set of selected users that are assigned non-zero rate, with cardinality $|\mathcal{S}| = \mathcal{M}$, $1 \leq \mathcal{M} \leq M$. We restrict ourselves to linear precoding and assume equal power allocation over users. Therefore, the BS multiplies the (normalized) data symbol for each user k , s_k , by a beamforming vector \mathbf{w}_k so that the transmitted signal is given by

$$\mathbf{x} = \sum_{k \in \mathcal{S}} \sqrt{\rho} \mathbf{w}_k s_k \quad (2)$$

where $\rho = P/\mathcal{M}$. Let $\mathcal{V} = \{\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_N\}$ be a codebook containing N unit-norm vectors $\mathbf{v}_i \in \mathbb{C}^M$, for $i = 1, \dots, N$, known to both the receivers and the transmitter. In this paper, for ease of exposition, we assume $N = M$ and $\mathbf{w}_i = \mathbf{v}_i$, i.e. the codevectors (beamformers) form an orthonormal basis and are generated in a pseudo-random manner at each time slot. In other words, we focus on codebook-based orthogonal precoding, which is similar in some sense to random unitary beamforming [6]. However, our generic adaptive mode switching framework can be easily extended to most state-of-art codebook-based transmission schemes, such as non-unitary beamforming (e.g. zero-forcing beamforming) and multiuser limited feedback systems [4,5]. The received signal for user $k \in \mathcal{S}$ in beam m is given by

$$y_k = (\sqrt{\rho} \mathbf{h}_k \mathbf{w}_m) s_k + \sum_{j \in \mathcal{S}, j \neq m} (\sqrt{\rho} \mathbf{h}_k \mathbf{w}_j) s_j + n_k \quad (3)$$

3. DISTRIBUTED TRANSMIT MODE SELECTION

In this section, we motivate and present the distributed sum-rate maximization framework under imperfect channel knowledge. Therein, each user, based on instantaneous local CSIT and statistical information, decides and feeds back its preferred transmit mode from a system throughput point of view. These distributed user decisions assist the BS in determining dynamically the system sum-rate maximizing transmit mode.

3.1. Design Challenges and Motivation

As stated before, in MIMO broadcast channels with partial CSIT, it is not guaranteed that multiuser transmission (SDMA) always outperforms single-user transmission (TDMA). There are several contexts in which it is beneficial from a capacity point of view to softly transit to TDMA by switching off beams and communicating with $\mathcal{M} < M$ users. For instance, multiuser MIMO systems with fixed and limited rate feedback become interference-limited and the sum rate exhibits a ceiling effect, saturating to a constant value, even for arbitrary large but finite N and K . Additionally, in sparse networks with low to moderate number of users, schemes allowing for adaptive and soft transition from SDMA to TDMA are of particular interest [5,7]. However, efficient beam selection techniques often require either feedback of M beam gains $|\mathbf{h}_k \mathbf{w}_m|^2$, $m = 1, \dots, M$ or two-stage decision approaches [7]. Obviously, the first method results in a non-negligible feedback load increase, whereas the second may result in excess protocol delays, outdated CSIT and signaling overhead, especially in high mobility systems.

The problem we address here is how to derive a one-step, adaptive SDMA/TDMA switching mechanism without considerably increasing the feedback rate of conventional schemes, i.e. one scalar CQI value plus few bits for beam indices. In contrast to existing approaches where scheduling is performed solely at the BS, the key idea here is that users can perform some sort of transmit mode pre-selection and pre-scheduling, assisting thus the transmitter at its final decision. The intuition is that the receivers have more flexibility in estimating their signal-to-interference-plus-noise ratios (SINRs) for different values of \mathcal{M} and potentially the inter-user interference, since they have more refined CSIT on their own channels. Note that in existing schemes, the reported scalar CQI/SINR is calculated assuming a priori that a certain number of beams \mathcal{M} will be used. However, the challenge is how the system sum rate can be estimated at the user side, prior to scheduling, since users have neither information on the other users' channels nor of the scheduling decisions. The target is to find a means to predict sufficiently at each receiver side the achievable sum-rate, so that users may decide on the capacity-maximizing system mode. A key difference with existing approaches is that the objective function at the user side is the system sum rate and not the individual user rate. Thus,

the preferred user mode is the one that maximizes the system throughput and not the individual rate.

3.2. Algorithm description

In our scheme, each mobile calculates and feeds back its preferred transmission mode, denoted as μ_u ($1 \leq \mu_u \leq M$) in addition to scalar CQI in the form of SINR/SNR along with the preferred beam index. The user-preferred mode is defined as the one that maximizes the sum rate. The system channel information comprises two classes: local information of which we can have instantaneous knowledge, and non-local information of which, we assume only statistical knowledge is available. Based on this information, the user objective function, i.e. system throughput, is calculated and expected sum-rate maximizing mode is fed back. The BS finds in turn the best system operating mode, μ_{BS} (single-user or multiuser transmission), and selects \mathcal{M} users to communicate with. In what follows, we formulate the distributed sum-rate maximization problem under local and statistical information.

3.2.1. Mode selection at the receiver side

We define the SINR of user k on beam m using \mathcal{M} beams, $\text{SINR}_{k,m}^{\mathcal{M}}$, as

$$\text{SINR}_{k,m}^{\mathcal{M}} = \frac{|\mathbf{h}_k \mathbf{w}_m|^2}{\mathcal{I}(\mathcal{M}) + \sigma^2/\rho} \quad k \in \mathcal{S} \quad (4)$$

where $\mathcal{I}(\mathcal{M})$ is the inter-user interference function, non-linear in general, which depends among others on \mathcal{M} . This function has the following two properties: in single-user transmission $\mathcal{I}(1) = 0$, and in full SDMA we have $\mathcal{I}(M) = \sum_{j \neq m} |\mathbf{h}_k \mathbf{w}_j|^2 = \|\mathbf{h}_k\|^2 \sin^2 \angle(\mathbf{h}_k, \mathbf{w}_m)$. The worst-case inter-user interference, resulting in SINR lower bound, is given by $\mathcal{I}(\mathcal{M}) = \max_{j \neq m} |\mathbf{h}_k \mathbf{w}_j|^2$. Actually, we can choose to estimate the multiuser interference at the receiver by using several functions, $\mathcal{I}(\mathcal{M}) = f(|\mathbf{h}_k \mathbf{w}_j|^2)$, $j \neq m$, where $f(x)$ may be equal to $\max(x)$, $\min(x)$, the harmonic mean of x .

Each user k determines its preferred beam m_k as

$$m_k = \arg \max_{1 \leq m \leq \mathcal{M}} \text{SINR}_{k,m}^{\mathcal{M}} \quad (5)$$

As stated before, we assume that each user k has instantaneous knowledge of local CSIT, $\mathcal{G}_k^{\text{local}}$. The set of complete channel information is denoted by $\mathcal{G} = \{\mathbf{h}_k\}, \forall k$, thus the non-local information at the k -th user side is represented by $\tilde{\mathcal{G}}_k = \mathcal{G} - \mathcal{G}_k^{\text{local}}$. Assuming statistical knowledge of $\tilde{\mathcal{G}}_k$, the expected sum-rate capacity calculated by the k -th receiver is given by

$$\mathcal{R}_k(\mathcal{M}) = \mathbb{E}_{\tilde{\mathcal{G}}_k} \left\{ \sum_{j \in \mathcal{S}} \log_2 \left(1 + \text{SINR}_{j,m_j}^{\mathcal{M}} \right) \right\}$$

$$= \log_2 \left(1 + \text{SINR}_{k,m_k}^{\mathcal{M}} \right) + \mathbb{E}_{\tilde{\mathcal{G}}_k} \left\{ \sum_{j \in \mathcal{S}-k} \log_2 \left(1 + \text{SINR}_{j,m_j}^{\mathcal{M}} \right) \right\} \quad (6)$$

Thus, each user k estimates the system sum rate as the sum of its instantaneous rate with the expected sum-rate of the other $\mathcal{M}-1$ users, based on the statistics of $\text{SINR}_{j,m_j}^{\mathcal{M}}$. Its preferred mode, μ_k^u , is calculated as

$$\mu_k^u = \arg \max_{1 \leq \mu \leq M} \mathcal{R}_k(\mathcal{M}) \quad (7)$$

Each user feeds back to the BS its preferred mode μ_k^u in addition to the corresponding SINR_{k,m_k} and beam index m_k . Note that the expected sum-rate in (6) can be calculated either in closed-form or by lookup tables, since statistical CSIT can be acquired a priori during a calibration phase.

3.2.2. Mode selection at the transmitter side

Let $\mathcal{S}_m, m = 1, \dots, M$ be the set of users with preferred beam m , and $\mathcal{S}_m^{\mu_k^u} = \{k : k \in \mathcal{S}_m, 1 \leq \mu_k^u \leq M\}$ be the set of users that selects beam m and reports back preferred mode equal to μ_k^u . For example, \mathcal{S}_m^1 is the group of users selecting single-user transmit mode (TDMA), while \mathcal{S}_m^M is the set of multiuser mode users with M beams (full SDMA). Among these sets, the BS selects the one that provides the highest capacity. The corresponding system transmit mode μ_{BS} is given by

$$\mu_{BS} = \arg \max_{1 \leq \mu_k^u \leq M} \sum_{m=1}^M \mathcal{R}(\mathcal{S}_m^{\mu_k^u}) \quad (8)$$

For instance, if $\mu_{BS} = 1$, the system operates with single-user transmission, meaning that the BS selects the user with the largest CQI value among all sets $\mathcal{S}_m^1, \forall m$.

3.3. Low-complexity SDMA/TDMA switching criterion

We provide here a simplified mode selection criterion that does not rely on statistical information. Each receiver assumes that all the other selected users would exhibit the same SINR as it does, implying that each of the \mathcal{M} selected users has equal contribution to the system sum rate. In that case, user k estimates the sum rate $\hat{\mathcal{R}}_k(\mathcal{M})$ when \mathcal{M} beams are used as

$$\hat{\mathcal{R}}_k(\mathcal{M}) = \mathcal{M} \log_2 \left(1 + \max_{1 \leq m \leq \mathcal{M}} \text{SINR}_{k,m}^{\mathcal{M}} \right) \quad (9)$$

Interestingly, although this seems contradictory to multiuser diversity and the fact that users exhibit different peaks at each instant, simulation results verify the validity of the simplified criterion. Each user k selects its preferred mode as $\mu_k^u = \arg \max_{\mathcal{M}} \hat{\mathcal{R}}_k(\mathcal{M})$. We should note that if a linear interference scaling is assumed for simplicity, i.e. $\mathcal{I}(\mathcal{M}) = \frac{\mathcal{M}-1}{\mathcal{M}-1} \|\mathbf{h}_k\|^2 \sin^2 \angle(\mathbf{h}_k, \mathbf{w}_m)$, the above maximization degenerates into a binary solution: $\mu_k^u = 1$ and $\mu_k^u = M$ [8].

4. DISTRIBUTED BINARY MODE SCHEME

Motivated by the above result, we propose a simple variant of the multi-mode scheme, coined as *binary mode selection*, in which the system operates either in TDMA ($\mu_{BS} = \mathcal{M} = 1$) or full SDMA mode ($\mu_{BS} = \mathcal{M} = M$).

At the receiver side, under binary mode selection, the k -th user chooses its preferred transmit mode by taking the following binary decision:

$$\mu_k^u = \begin{cases} 1 & \text{if } \Delta\mathcal{R} \geq 0 \\ M & \text{if } \Delta\mathcal{R} < 0 \end{cases} \quad (10)$$

where $\Delta\mathcal{R} = \mathcal{R}_{TDMA} - \mathcal{R}_{SDMA}$. The single-user rate is given by

$$\mathcal{R}_{TDMA} = \log_2 \left(1 + P \max_{1 \leq m \leq M} |\mathbf{h}_k \mathbf{w}_m|^2 \right) \quad (11)$$

and \mathcal{R}_{SDMA} is calculated by (6). The expected sum rate of $M - 1$ users (statistical information in (6)) in Rayleigh fading can be calculated in closed form as follows:

Lemma 1: For any values of P , M , and K , the expected sum rate of SDMA with M beams is given by

$$\mathbb{E}\{\mathcal{R}_{SDMA}\} = \frac{M}{\log 2} \sum_{k=1}^K \binom{K}{k} (-1)^{k+1} e^{\frac{k\sigma^2}{2\rho}} \left(\frac{k\sigma^2}{\rho} \right)^{\frac{(M-1)k-1}{2}} \quad (12)$$

$$\mathcal{W}_{\frac{k(1-M)-1}{2}, \frac{k(1-M)}{2}} \left(\frac{k\sigma^2}{\rho} \right) \approx \frac{M}{\log 2} \frac{\rho H_K}{(M-1)\rho + \sigma^2} \quad (12)$$

where $\mathcal{W}_{k,m}(z)$ is the Whittaker function and $H_K = \sum_{k=1}^K \frac{1}{k}$ is

the K -th harmonic number.

Proof - See [9]

5. PERFORMANCE ANALYSIS

In this section, we analyze the sum-rate performance of binary mode scheme, for ease of exposition, providing capacity scaling laws at high SNR ($P \rightarrow \infty$) and in the large number of user regime ($K \rightarrow \infty$).

5.1. TDMA mode selection probability

We first evaluate the probability that a user and the transmitter select single-user transmission mode (TDMA).

5.1.1. At the receiver side

Lemma 2: The probability that a user reports back to the BS TDMA mode transmission, $\mathbb{P}\{\mu_k^u = 1\}$, is given by

$$\mathbb{P}\{\mu_k^u = 1\} = 1 - \frac{M e^{-\zeta/\rho}}{(1 + \zeta)^{M-1}} \quad (13)$$

where $\zeta = 2^{\beta/M} - 1$ and $\beta = \log_2 \left(1 + \max_{1 \leq m \leq M} |\mathbf{h}_k \mathbf{w}_m|^2 \right)$

Proof - See [8]

Corollary 1 [8]: At high SNR and K fixed, we have that

$$\mathbb{P}\{\mu_k^u = 1\} \rightarrow 1 \quad (14)$$

5.1.2. At the transmitter side

Deriving the probability that the BS selects single-user transmission mode, we can show that

Lemma 3: At high SNR and K fixed, we have that

$$\mathbb{P}\{\mu_{BS} = 1\} \rightarrow 1 \quad (15)$$

and for $K \rightarrow \infty$ and P fixed

$$\mathbb{P}\{\mu_{BS} = 1\} \rightarrow 0 \quad (16)$$

Proof - See [8]

Therefore, in the interference-limited region, the system operates in single-user mode since it provides higher capacity than SDMA transmission. However, in the high number of users regime, SDMA outperforms single-user transmission and the system switches to full SDMA mode.

5.2. Sum-rate evaluation

The expected sum rate of the proposed scheme is given by:

$$\mathcal{R}_p = \mathbb{P}\{\mu_{BS} = 1\} \mathcal{R}_{TDMA} + (1 - \mathbb{P}\{\mu_{BS} = 1\}) \mathcal{R}_{SDMA}$$

where \mathcal{R}_{TDMA} and \mathcal{R}_{SDMA} are the expected sum rates of single-user and multi-user transmission respectively.

Theorem 1: The multiplexing gain of the proposed decentralized adaptive mode scheme is

$$\lim_{P \rightarrow \infty} \frac{\mathcal{R}_p}{\log P} = 1 \quad (17)$$

Proof - See [8]

Note that the multiplexing gain of multi-beam RBF [6] in the interference-limited region ($P \rightarrow \infty$) converges to zero, whereas the multiplexing gain of single-beam RBF equals one.

Theorem 2: In the large number of user regime and fixed P , the sum rate of the proposed adaptive mode switching scheme scales as

$$\lim_{K \rightarrow \infty} \frac{\mathcal{R}_p}{M \log \log K} = 1 \quad (18)$$

Proof - See [8]

6. NUMERICAL RESULTS

In this section, we evaluate the sum-rate performance of the distributed multi-mode scheme through Monte Carlo simulations. A system with $M = 3$ transmit antennas is considered and conventional single-beam (TDMA) and multi-beam (SDMA) random beamforming [6] are plotted for comparison. In Figure 1, we present the sum rate of the proposed

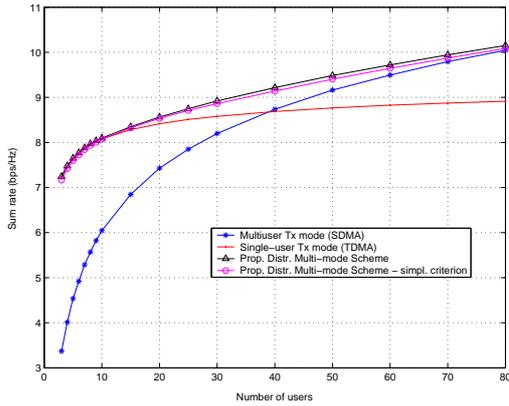


Fig. 1. Sum rate versus the number of users for distributed multi-mode scheme with $M = 3$ and average SNR = 20dB.

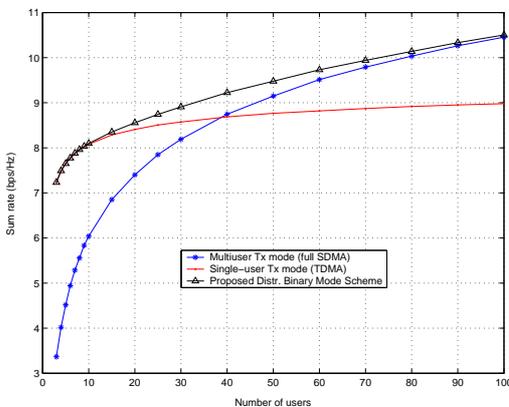


Fig. 2. Sum rate versus the number of users for binary mode selection with $M = 3$ and average SNR = 20dB.

multi-mode mechanism versus the number of active users K at SNR = 20dB. Significant gains compared to full SDMA transmission are observed, especially for low to moderate number of users as expected. Furthermore, the simplified switching criterion, although not relying on statistical information, results in negligible sum rate loss compared to the statistical multi-mode scheme. In Figure 2, we plot the sum rate of binary mode scheme versus the number of users at SNR = 20 dB. The scheme gradually switches from TDMA mode at low K to full SDMA transmission with M beams. Note also that binary decisions result in similar sum rate performance as the multi-mode scheme, in which intermediate SDMA transmission using $M < M$ beams is allowed. Figure 3 shows a sum-rate comparison as a function of the average SNR for $K = 30$ users, illustrating that binary mode scheme prevents the system from becoming interference-limited. Distributed mode adaptation allows us to switch off beams, thus keeping linear and unbounded capacity growth in the interference-limited regime and converging to single-user transmission (TDMA) at high SNR, as predicted from our theoretical analysis.

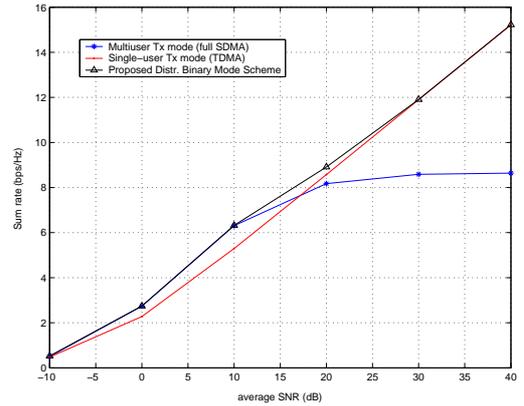


Fig. 3. Sum rate versus average SNR for binary mode selection with $M = 3$ and $K = 30$ users.

7. CONCLUSIONS

We proposed an adaptive SDMA/TDMA mode switching framework, in which the BS selects the sum-rate maximizing transmit mode using imperfect CSIT. For that, the users calculate and feed back their preferred system transmit mode based on local CSIT and statistical information for the other users. We show the capacity gains of this distributed multi-mode mechanism and analyze its performance, demonstrating the linear and unbounded capacity growth at high SNR.

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