Downlink Distributed Binary Power Allocation for Cognitive Radio Networks

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***Abstract*— We consider the downlink of a cognitive radio network consisting of multiple secondary transmitter and receiver communicating simultaneously in the presence of one primary user. The key idea within this paper is to combine multi-user diversity gains with spectral sharing techniques through inter-system coordination, in order to maximize the secondary user sum rate while maintaining a guaranteed quality of service (QoS) to a primary user. We first present a distributed power allocation algorithm that maximizes the capacity of the cognitive ratio network. The algorithm is simple to implement, since a secondary user can decide to either transmit data or stay silent over the channel coherence time depending on a specified threshold without affecting the primary users QoS. Then, we analyze performance of such an algorithm in terms of number of cognitive users able to transmit while minimizing interference to guarantee QoS for the primary user. Simulation results carried out based on a realistic network setting showed promising results.**

I. INTRODUCTION

Cognitive radio offers the opportunity to improve the system throughput in a network. The concept of cognitive radio allows secondary users (SU) to detect the primary user (PU) and adapt their transmission rate accordingly [1]. In current cognitive radio protocol proposals, the secondary users device listens to the wireless channel and determines, either in time or frequency, which part of the spectrum is unused. It then adapts its signal to fill this void in the spectrum domain. Thus, a SU device transmits over a certain time or frequency band only when no other user does, like in [2]. In the same context, it was shown in [3] how we can improve the overall system spectral efficiency compared to classical approaches by considering *spectrum pooling* scenario. The contribution of some recent studies [4] and [5] has however also extended cognitive protocols to allow the SU to transmit *simultaneously* with the PU in the same frequency band. This is exactly the setup in this work, where the cognitive radio behavior is generalized to allow secondary users to transmit simultaneously with PU *as long as the level of interference to primary users remains within an acceptable range*. A key example of multi-user resource allocation is that of power control, which serves as means for both battery savings at the mobile, and interference management. In this work, we will focus on *binary* power control since it has the advantage of leading towards simpler or even distributed power control algorithms [6]. In [7], it was also shown that the *optimal* power control, with respect to the sum rate, is always binary for a two-cell network as well as in the low signal-to-interference ratio (SINR) regime for an $N-$cell (link) network. In the general case when the number of cells (links) > 2, it was also demonstrated by extensive computer simulations that a restriction to binary power levels yields only a negligible capacity loss [8].

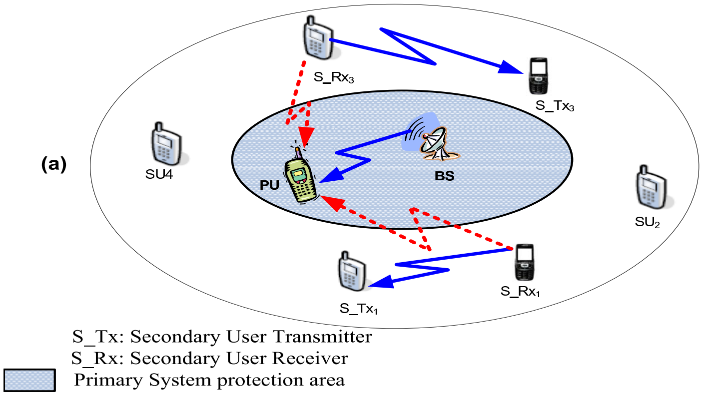

Fig. 1. The downlink of a Cognitive Radio Network with one primary user (PU) and $M = 4$ secondary transmitters.

A particularly noteworthy question in the context of cognitive radio, when we seek to optimize the sum system capacity, is to guarantee a QoS to PUs. There are a large number of proposals for all communication layers treating the increase of restrictions to spectrum utilization [9], but the QoS issue still has not been clearly defined. In addition, it is unclear how secondary system opportunism, is compatible with the support of QoS for both cognitive radio systems and primary systems. The FCC proposed the concept of "*interference temperature*" as a way to have unlicensed transmitters share licensed bands without causing harmful interference. Rather than merely regulate transmitter power at fixed levels, as in the past, the scheme would have governed transmitter power on a variable basis calculated to limit the energy at victim receivers, where interference actually occurs. As a practical matter, however, the FCC abandoned the interference temperature concept recently [10] due to the fact that it is not a workable concept and would result in increased interference in the frequency bands where they were to be used. In this contribution, we will propose a different way to efficiently protect primary systems from SU interference, based on outage probability [11] for the downlink communication. In what follows, we adopt this setting and consider a cognitive radio environment (CRN) in which primary and secondary users attempt to communicate, subject to mutual interference. Our goal is to maximize the total SU throughput under interference and noise impairments, and short term (minimum and peak) power constraints, while preserving the QoS of the primary system. In particular, it is of interest to determine, in a *distributed* manner, the optimal noise/interference threshold above which SUs can decide to transmit without affecting the primary users' QoS. In fact, in a realistic network, centralized system coordination is hard to implement, especially in fast fading environments and in particular if there is no fixed infrastructure for SUs, i.e., no back-haul network over which overhead can be transmitted between users.

The rest of the paper is organized as follows. The next section describes the downlink cognitive radio network. In Section III, the proposed distributed power control algorithm is investigated in both the high and low SINR regimes, respectively, including primary users' QoS issues. Simulation results are provided in Section IV and Section V concludes the paper.

II. THE COGNITIVE RADIO NETWORK

A. The System Model

Consider the downlink of a CRN that consists of a primary user, a base station, and M pairs of secondary users randomly distributed over the system. The channel gains are i.i.d random variable. Throughout the paper, we will use the following notation:

- the index of SUs j lies between 1 and M,
- $h_{bs,n}$ denotes the channel gain from the BS indexed by bs to the desired user n,
- $h_{j,n}$ denotes the channel gain from SU j to the desired user n,
- the transmit power of the base station (BS) is p_{BS},
- the data destined from SU j is transmitted with power p_j.

In the coverage area of the primary system, there is an *interference boundary* within which no SUs can communicate in an ad-hoc manner. Thus, as can be seen in Figure 1, for the impairment experienced by the primary system to be as small as possible, a SU must be able to detect very reliably whether it is far enough away from a primary base station, i.e., in the area of possible cognitive radio operation. The expression of the PU instantaneous capacity is

$$C_{pu} = \log_2\left(1 + \frac{p_{BS} \mid h_{bs,pu} \mid^2}{\sum_{j=1}^{M} p_j \mid h_{j,pu} \mid^2 + \sigma^2}\right) \qquad (1)$$

where σ^2 is the ambient noise variance. On the other hand, by making SUs access the primary system spectrum, the j^{th} SU experiences interference from the PU and all neighboring co-channel SU links that transmit on the same band. Accordingly, the j^{th} SU instantaneous capacity is given by:

$$C_j = \log_2\left(1 + \mathrm{SINR}_j\right); \quad \text{for} \quad j = 1, ..., M \qquad (2)$$

where

$$\mathrm{SINR}_j = \frac{p_j \mid h_{j,j} \mid^2}{\sum_{\substack{k=1 \\ k \neq j}}^{M} p_j \mid h_{k,j} \mid^2 + p_{BS} \mid h_{bs,j} \mid^2 + \sigma^2} \qquad (3)$$

SUs need to recognize their communication environment and adapt the parameters of their communication scheme in order to maximize the per-user cognitive capacity, expressed as[1]

$$C_{sum} = \frac{1}{\tilde{M}} \sum_{j=1}^{\tilde{M}} C_j \,, \qquad (4)$$

while minimizing the interference to the primary users, in a *distributed* fashion. Moreover, we assume that the coherence time is sufficiently large so that the channel stays constant over each scheduling period length. We also assume that SUs know the channel state information (CSI) of their own links, but have no information on the channel conditions of other SUs. No interference cancelation capability is considered. Power control is used for SUs both in an effort to preserve power and to limit interference and fading effects.

B. The Cognitive Radio protocol

Under this scheme, we allow SUs to transmit simultaneously with the PU as long as the interference from the SUs to the PU that transmits on the same band remains within an acceptable range. Specifically, we impose that SUs may transmit simultaneously with the PU as long as the PU in question does not have his QoS affected in terms of outage probability. We consider that PUs operate at a desired rate (depending on their respective QoS demands). From a practical point of view, the outage probability as well as the requested rate can be broadcasted, before the start of the communication, by the primary system base station and is used as a preamble for the PU to get informed which data rate is requested. This preamble can also be overheard by SUs who can then learn about these outage values.

One basic assumption throughout this paper is that a SU can vary its transmit power, under short term (minimum and peak) power constraints, in order to maximize the cognitive capacity, while maintaining a QoS guarantee to the primary user. The idea of the binary on/off power control is simple, as well as yielding quasi-optimal results in a number of cases [8]. As such, it constitutes a promising tool to making spectrum sharing a reality. Besides complexity reduction, an important additional benefit of binary power control is to allow distributed optimization.

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[1] The sum here is made over the $\tilde{M}$ SUs allowed to transmit.

III. BINARY POWER CONTROL ALGORITHM

Our goal within this work is to determine, under the assumption that the PU is oblivious to the presence of the cognitive users, what would be the cognitive system capacity (which can also be viewed as the total increase in system capacity due to the SUs' activity) and, at the same time, the maximum number of cognitive communication links allowed in such a system. We present a distributed algorithm for power allocation in the sense that it requires a SU to decide *distributively* to either transmit data or stay silent over the channel coherence time depending on a specified threshold. The optimization problem can therefore be expressed as follows:

$$\text{Find } \{p_1*, ..., p_M*\} = \arg \max_{p_1,\dots,p_M} C_{sum} \tag{5}$$

subject to:

$$\begin{cases} 0 \leq p_j \leq P_{max}, \quad \text{for} \quad j = 1, ..., M \\ \\ P_{out} = Prob\{C_{pu} \leq R_{pu} \mid R_{pu}, q\} \leq q, \end{cases} \tag{6}$$

where R_{pu} is the PU transmitted data rate. The key idea within the proposed iterative algorithm is, as in [6], is to subsequently limit p_j to $\{0, P_{max}\}$, i.e., to switch "off" transmission in SUs' links which do not contribute enough capacity to outweigh the interference degradation caused by them to the rest of the network. Though other SUs stay silent, they may be active during the next iteration for the PU in question. We propose an adaptation of the distributed algorithm which allows a subset of controlled size $\tilde{M}$ of the total number of SUs M to transmit simultaneously on the same sub-band. It turns out necessary to limit the number of SUs interfering with the primary user so as to guarantee a QoS for the primary system. Let Ψ be the set of indices of all presently active SUs. A SU should be deactivated if this action results in an increase in the cognitive capacity of SUs or if its transmission violates the PU outage constraint.

A. At high SINR regime

The CRN described in the previous subsection can be modeled by interference channels, due to the fact that SUs employ the same spectral resource in each link, giving rise to an interference-limited system. At high SINR regime, in all "on" SU, and assuming an interference-limited system, we obtain after simple manipulations[2]

$$\text{SINR}_m < \frac{\prod\limits_{\substack{j\in\Psi \\ j\neq m}} \left(p_{BS} \mid h_{bs,j} \mid^2 + \sum\limits_{\substack{k\in\Psi \\ k\neq j}} p_k \mid h_{k,j} \mid^2 \right)}{\prod\limits_{\substack{j\in\Psi \\ j\neq m}} \left(p_{BS} \mid h_{bs,j} \mid^2 + \sum\limits_{\substack{k\in\Psi \\ k\neq j\neq m}} p_k \mid h_{k,j} \mid^2 \right)}$$

$$\Downarrow \tag{7}$$

$$\frac{p_m \mid h_{m,m} \mid^2}{p_{BS} \mid h_{bs,m} \mid^2 + \sum\limits_{\substack{k\in\Psi \\ k\neq m}} p_k \mid h_{k,m} \mid^2} < \frac{\prod\limits_{\substack{j\in\Psi \\ j\neq m}} \sum\limits_{\substack{k\in\Psi\cup\{bs\} \\ k\neq j}} p_k \mid h_{k,j} \mid^2}{\prod\limits_{\substack{j\in\Psi \\ j\neq m}} \sum\limits_{\substack{k\in\Psi\cup\{bs\} \\ k\neq j\neq m}} p_k \mid h_{k,j} \mid^2} \tag{8}$$

Suppose that devices operate in a dense network, i.e. a large number of SUs is distributed over a restricted geometrical area. It was shown in [12] that, based on the observation that interference to any user in a large dense network is only weakly dependent on the user's position, we can approximate the interference term in (8) by an average interference gain, (denoted by G^2) which is independent of the user location, multiplied by the total transmit power of active interferers:

$$\sum_{j=1}^{M} p_j \mid h_{n,j} \mid^2 \simeq G^2 \sum_{j=1}^{M} p_j \,, \quad \text{for all} \quad n \tag{9}$$

where G^2 is a constant depending only on the average amplitude of the SU channel gain. Accordingly, condition (8) becomes[3]

$$\frac{p_m \mid h_{m,m} \mid^2}{\sum\limits_{\substack{k\in\Psi\cup\{bs\} \\ k\neq m}} p_k \mid h_{k,m} \mid^2} < \frac{\prod\limits_{\substack{j\in\Psi \\ j\neq m}} G^2 \sum\limits_{\substack{k\in\Psi\cup\{bs\} \\ k\neq j}} p_k}{\prod\limits_{\substack{j\in\Psi \\ j\neq m}} G^2 \sum\limits_{\substack{k\in\Psi\cup\{bs\} \\ k\neq j\neq m}} p_k} \tag{10}$$

As all "on" SUs transmit with P_{max} and denoting by $\tilde{M} = card\{\Psi\}$, the m^{th} SU will be active only if

$$\frac{\mid h_{m,m} \mid^2}{\sum\limits_{\substack{k\in\Psi\cup\{bs\} \\ k\neq m}} \mid h_{k,m} \mid^2} > \left(\frac{\tilde{M} + K - 1}{\tilde{M} + K - 2} \right)^{\tilde{M}-1} \tag{11}$$

The number of SUs increases, we get (as in [6])

$$\lim_{\tilde{M}\to\infty} \left(\frac{\tilde{M} + K - 1}{\tilde{M} + K - 2} \right)^{\tilde{M}-1} = \left(\tfrac{\tilde{M}+K-1}{\tilde{M}+K-2} \right)^{\tilde{M}+K-2} \cdot \left(\tfrac{\tilde{M}+K-1}{\tilde{M}+K-2} \right)^{1-K}$$

$$= e = 2.718281...$$

Thus, for a large network size, a SU will be active if the user signal-to-interference ratio of the scheduled user is more than e, namely

$$\text{SIR}_m > e \tag{12}$$

B. At low SINR regime

The restriction to binary power levels yields in general only a negligible capacity loss. As stated before, it was shown in [8] that at low-SINR regime, i.e., where the approximation $\ln(1+x) \simeq x$ holds with good accuracy, binary power control is in fact always optimal. In the low SINR regime and after simple manipulations, the m^{th} SU will now be active if

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[2] Due to the lack of space, we will not present all analytical derivations in this paper. The reader is referred to the journal version for additional information.

[3] We suppose that $\frac{P_{BS}}{P_{max}} = K$.

$$\mathrm{SINR}_m < \frac{\sum\limits_{\substack{j\in\Psi \\ j\neq m}} p_j \mid h_{j,j} \mid^2}{P_{max}G^2(\tilde{M}+K-2)+\sigma^2} \tag{13}$$

$$\simeq \frac{P_{max}G^2(\tilde{M}-1)}{P_{max}G^2(\tilde{M}+K-2)+\sigma^2}$$

where we use the same dense average network assumptions as in (9). Suppose, as in the high SINR regime, that we are in an interference-limited context. This would suggest that $\sigma^2 \ll P_{max}G(\tilde{M}+K-2)$ in the right hand side of (13). As the number of SUs increases, we get

$$\lim_{\tilde{M}\to\infty}\left(\frac{\tilde{M}-1}{\tilde{M}+K-2}\right)=1 \tag{14}$$

Thus, a SU will be active if the user SIR ratio of the scheduled user is more than 1:

$$\mathrm{SIR}_m > 1 \tag{15}$$

We thus confirm, as intuition would expect, that SUs under better SINR conditions would transmit only above a higher threshold than in the low-SINR regime.

C. Primary system QoS issues

Spectrum utilization can be improved by making SUs access spectrum opportunities by the PU. Secondary devices would attempt to coexist with the primary user, such that the presence of secondary devices goes unnoticed. Secondary devices would then access spectrum opportunistically, when they determine that doing so would not adversely affect primary user QoS. This approach allows cognitive radios to support and guarantee QoS for the PU, while sharing spectrum. In the current study, we adopt a QoS guarantee to the PU by means of an outage constraint. The notion of *information outage probability* defined as the probability that the capacity of the user is below the transmitted code rate [11]. In the proposed framework, the outage probability can be expressed as:

$$P_{out} \triangleq Prob\{C_{pu} \leq R_{pu}\} \leq q, \tag{16}$$

We introduce the PU pathloss gain profile estimate G^2_{pu} based on the following decomposition:

$$h_{pu,pu} \triangleq G^2_{pu} * h'_{bs,pu} \tag{17}$$

where $h'_{pu,pu}$ represents the normalized channel impulse response tap. This gives us the following PU outage probability expression:

$$P_{out} = Prob\left\{ \log_2\left(1+\frac{p_{BS}G^2_{pu} \mid h_{bs,pu}\mid^2}{\sum\limits_{j=1}^{\tilde{M}} p_j \mid h_{j,pu}\mid^2 + \sigma^2}\right) \leq R_{pu} \right\} \leq q$$

$$\simeq Prob\left\{ \mid h_{pu,pu}\mid^2 \leq \left(2^{R_{pu}}-1\right)\left(\frac{\tilde{M}G^2_{su}P_{max}+\sigma^2}{G^2_{pu}p_{BS}}\right)\right\} \leq q$$

From now on, we assume that the channel gains are i.i.d rayleigh distributed[4].

$$P_{out} \simeq \int_0^{\left(2^{R_{pu}}-1\right)\left(\frac{\tilde{M}G^2_{su}P_{max}+\sigma^2}{G^2_{pu}p_{BS}}\right)} \exp(-t)dt \leq q$$

Finally, we get the following outage constraint:

$$P_{out} \simeq 1-\exp\left[-\left(2^{R_{pu}}-1\right)\left(\frac{\tilde{M}G^2_{su}P_{max}+\sigma^2}{G^2_{pu}p_{BS}}\right)\right] \leq q \tag{18}$$

and the maximum number of "on" SU that transmit with P_{max} is given by

$$0 \leq \tilde{M} \leq \frac{-\log(1-q)}{(2^{R_{pu}}-1)}.\frac{G^2_{pu}p_{BS}}{G^2_{su}P_{max}} - \frac{\sigma^2}{G^2_{su}P_{max}} \tag{19}$$

By writing $\mathrm{SNR} = \frac{G^2_{su}P_{max}}{\sigma^2}$, equation (19) can be expressed as:

$$0 \leq \tilde{M} \leq \frac{-\log(1-q)}{(2^{R_{pu}}-1)}.\frac{G^2_{pu}p_{BS}}{G^2_{su}P_{max}} - \frac{1}{\mathrm{SNR}} \tag{20}$$

The left hand side in (20) prevents from obtaining a negative number of users when the SNR decreases significantly. The formula in (20) points out that that the number of SUs allowed to transmit increases as their SNR increases.

IV. NUMERICAL RESULTS

In order to analyze the performance of the proposed strategy in terms of achievable rates and number of active SUs, we consider a cognitive radio network as described in Fig. 1 with one PU and M secondary users attempting to communicate during a downlink transmission, subject to mutual interference. Specifically, a hexagonal cellular system functioning at 1.8 GHz with a primary cell of radius $R = 1000$ meters and a primary protection area of radius $R_p = 600$ meters is considered. Secondary transmitters may communicate with their respective receivers of distances $d < R_p$ from the BS. Channel gains are based on the COST-231 path loss model [13] including log-normal shadowing with standard deviation of 10 dB, plus fast-fading assumed to be i.i.d. circularly symmetric with distribution $\mathcal{CN}(0,1)$. The peak power constraint is given by $P_{max} = 1$ Watt and $K = 10$.

Figure 2 captures the number of active SUs for the downlink for different rates and outage probability. As expected, it is shown that increasing the target data rate, less SUs are allowed to transmit. As an exemple, 9 SUs are allowed to transmit at a rate equal to 0.1 bits/s/Hz and a target outage probability $q = 1\%$ while only 7 SUs are active at a rate equal to 0.5 bits/s/Hz and for the same outage probability. Although not shown here due to lack of space, we also remark that, asymptotically, i.e., as the number of SUs goes large, the number of active SUs keeps constant due to the influence of interference impairments on the PU's QoS. This tends to

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[4] However, such an approach can be immediately translated into results for any other channel model by replacing by the appropriate probability distribution function.

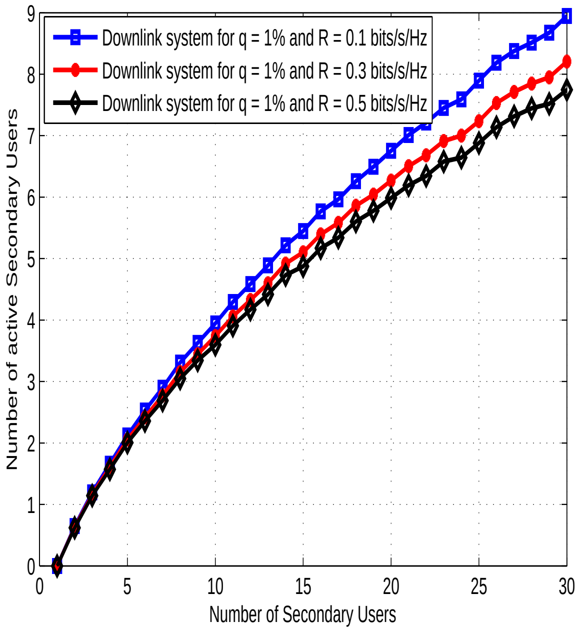

Fig. 2. Number of active secondary users vs. number of SUs for different rates and outage probability.

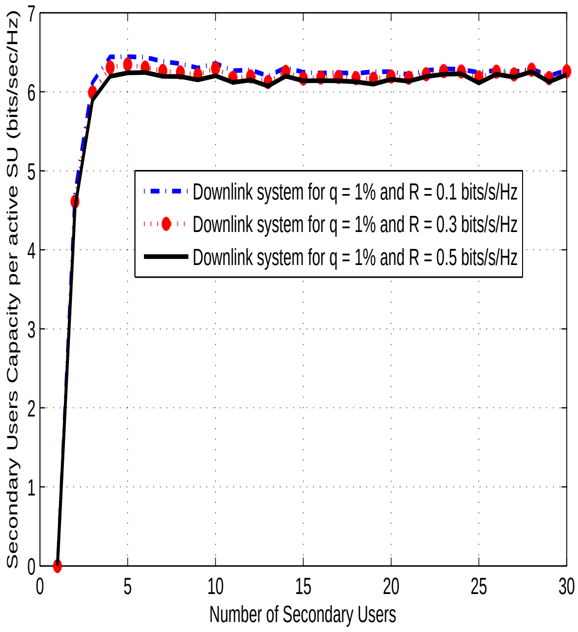

Fig. 3. Sum secondary user capacity per user vs. number of SUs for different rates and outage probability.

confirm the intuition from formula (20) where the number of active SUs is always upper-bounded by $\tilde{M}$. Additional analysis results as well as the impacts of the system topology on the asymptotic performance are analyzed in the journal paper [14]. Figure 3 depicts the sum secondary user capacity per user. It is clear that increasing the number of SUs yields significantly increase in capacity because the increase in degree of freedom more than compensates for the decrease in SINR due to interference. However, reaching a certain number of SUs, the sum SU capacity per user slightly decreases as the number of SUs increases. Notice here that, as the primary cell radius R and the primary protection area radius R_p decrease, the sum secondary user capacity per user becomes more sensitive to the interference impairments leading to a significant decrease in the sum secondary rate [14].

V. CONCLUSION

In this paper, we have explored the idea of combining multi-user diversity gains with spectral sharing techniques to maximize the secondary user sum rate while maintaining a QoS to a primary user. We first derived a distributed power allocation algorithm under a cognitive capacity maximization criterion and minimum and peak power constraints. Then, we investigated the QoS issues from an outage point of view and showed that the proposed approach allows cognitive radios to support and guarantee QoS for the primary user, while sharing spectrum. Simulation results were carried out based on a realistic network setting. As a future work, it would be interesting to investigate the performance of the proposed strategy considering a distributed scheduling approach.

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