Spectrum sharing in multiple-antenna channels: A distributed cooperative game theoretic approach

invited paper

Zuleita Ka Ming Ho and David Gesbert
Eurecom, Sophia-Antipolis, France
hokm, gesbert@eurecom.fr

I. Abstract

We consider a cognitive radio scenario in which two (or more) operators providing services in the same area wish to share the same licensed band of spectrum. This scenario differs from the classical cognitive setup with a primary and a secondary operators, as both operators here are instead on an equal footing. The operators face the choice of competition or cooperation in the way they choose their transmission parameters (here beamforming vectors) to communicate with their respective users. We build on interesting recently published work [5] [6] analyzing the gains of cooperation in this context and propose novel techniques for beamforming within this interference channel. The proposed techniques outperform classical non-cooperative game solutions and mimic known cooperative game solutions while introducing a distributed aspect for the algorithm.

II. Introduction

Interference mitigation is a central problem in cognitive radio. We consider here the problem of independent operators sharing one identical band in the same geographical area, thus creating interference to one another. This context is different from the traditional cognitive setup where some hierarchy is respected between a primary operator and a secondary operator sharing the band. In our scenario, there is no hierarchy as both operators seek to maximize the rate experienced by their own users by the choice of transmission parameters, at the cost of creating and receiving interference to/from the other party. In our paper, we consider multiple-antenna transmitters and the choice of a transmission parameter (from the base to the terminal) is limited to the choice of a beamforming vector, subject to a transmit power constraint. However other type of transmission parameters could also be considered such as power level, modulation/coding, subcarrier assignement etc.

As the the operators (and their users) have the selfish goal of maximizing their own performance, this results in a conflict situation between the transmitter-receiver pairs. Game theory has been a popular tool for years for analysing optimization problems with conflicting objectives by independent players [7] [8]. The classical way of analyzing performance optimization in this scenario is through so called non-cooperative games [7] [10] . The optimal operation point is then known to be the Nash equilibrium, defined to be the set of transmission strategies, such that unilateral deviation from it by one of the player cannot result in an increase of his/her uitility (here, rate). However, in the context of spectrum sharing, the NE can often be seen as a worst case scenario, over which improvements can be made by using concepts of bargaining and learning [11].

A game where some form of trust is established between players for the sake of maximizing their utilities jointly is referred to as a cooperative game. Such games have been brought up recently in the wireless networking literature [1] [2] [3]. The trust is exploited to allow bargaining between the players. The optimal point of operation was analyzed in the game theory literature and referred to as the Nash bargaining (NB) point. The application of cooperative games and NB theory to the spectrum sharing problem was undertaken in [4] [5] [6] [9]. The advantage of the NB point over the classical NE is that it is possible to operate closer to the Pareto boundary of the rate region. The disadvantage of it is that the computation of the NB-achieving strategies require a full exchange of channel state information between the users, which is not possible in practice, especially in the cognitive context where different users belong to different operators. The practicality thus decreases with the growth of the network. Clearly there is a trade-off between cooperation and ability to implement interference mitigation algorithms in a distributed manner. This paper aims at exploring this trade-off by proposing semi-distributed techniques that exploit cooperation in the spectrum sharing context.

More specifically, we propose an iterative beamforming algorithm where each transmitter updates its transmission vector as function of a single bit of feedback provided by their terminal. The bit of feedback allow the transmitters to implicitly *learn* about the channel from the other transmitters, thus improving their own strategy, while maintaining semi-distributedness. In one version of the algorithm, the feedback is exploited in order to adjust the transmit beamforming vector as a linear combination of the NE solution and the so-called zero forcing solution.

Although mainly heuristic in nature, this algorithm finds some theoretical justification in the recently published literature. [5] and [6] showed that all beamforming strategies resulting in rate points lying on the boundary of the rate region (so-called Pareto boundary) are composed from a linear combination of the zero-forcing solution and maximum-ratio-combining solution. But in principle, the construction of such beamformer with an algorithm other than exhaustive search or some other centralized technique, remains an open problem. Our solution thus serves as an practical alternative solution to exploit cooperation.

III. System model

A. Channel model

We consider a set of M operators, each featuring the downlink communication between a base station equipped with N_t antenna and one receiver node, all sharing the same frequency band in a given geographical area. Note that here, whether the band is licensed or unlicensed is irrelevant as we ignore any other external source of interference.

Each receiver node i, $1 \leq i \leq M$, has a single receive antenna and therefore sees a $1 \times MN_t$ MISO channel. The MISO channel between receiver node i and transmitter node k is a complex vector $\bar{H}_{ki} \in \mathcal{C}^{1\times N_t}$.

$$\bar{H}_{ki} = \sqrt{PL(d_{ki})Y} H_{ki} \tag{1}$$

where $PL(d_{ki})$ is the path loss of d_{ki}, distance between transmitter k and receiver i. Y is the linear scale shadowing (which in dB is gaussian distributed with zero mean and shadowing variance σ_y^2). H_{ki} is a complex gaussian vector with zero mean and unit variance.The symbol s_i has unit energy $E|s_i|^2 = 1$. The channel vector is $\mathbf{h}_{ii} \in \mathcal{C}^{1\times N_t}$ and the beamforming vector is $\mathbf{w}_i \in \mathcal{C}^{N_t \times 1}$. The receive signal of user i is

$$y_i = \mathbf{h}_{ii}\mathbf{w}_i s_i + \sum_{k\neq i}^{M} \mathbf{h}_{ki}\mathbf{w}_k s_k + n_i. \tag{2}$$

where n_i has variance σ_n^2.

1) User rates and network sum rate: The Signal-to-Interference-and-Noise-Ratio (SINR) of user i is

$$\gamma_i = \gamma_i(\mathbf{w}_1, \ldots, \mathbf{w}_M) = \frac{|\mathbf{h}_{ii}\mathbf{w}_i|^2}{\sum_{k\neq i}^{M} |\mathbf{h}_{ki}\mathbf{w}_k|^2 + \sigma_n^2} \tag{3}$$

Assuming perfect codes, the theoretic data rate of user i, r_i, is

$$r_i = \log_2(1+\gamma_i) \tag{4}$$

and the network overall sum rate is therefore

$$R = \sum_{i=1}^{M} r_i \tag{5}$$

2) Local Channel Information: We assume the transmitter nodes have only locally observable information of the channel, to ensure semi distributed-ness. This includes the direct channel $\mathbf{h}_{ii}$ and the interference channel to other receiver nodes $\mathbf{h}_{ik}$; transmitter nodes do not know the direct channel of other users $\mathbf{h}_{kk}$ or the interference channel from others to its receiver $\mathbf{h}_{ki}$. Furthermore, we assume that the receiver is able to measure its local signal to interference and noise ratio (SINR). The goal by the transmitters is to optimize the choice of a beamforming vector based on this limited information, so as to reach "good" points in the achievable rate region. There may be numerous desirable points in the achievable rate region depending on the objective of the system. However in a spectrum sharing scenario with independent operators it is likely that points maximizing a trade-off between sum rate and fairness are desirable, rather than maximizing sum rate alone. It is very interesting to see that cooperative game theory provides an answer (or at least a framework) for this problem. In this framework, the maximization of *individual* rates is the objective (not the sum rate) and cooperation only serves that purpose.

B. Achievable rate region and pareto boundary

We assume that no interference precancellation is allowed and interference from other transmitter nodes are treated as noise. The achievable rate region $\mathcal{R}$ is characterized by a set of all possible rate tuples $\mathbf{r}$ such that each rate element r_i satisfies the power constriant.

$$\mathcal{R} = \left\{\mathbf{r} = (r_1, \ldots, r_M) : |\mathbf{w}_i|^2 \leq 1, 1 \leq i \leq M\right\} \tag{6}$$

The pareto boundary $\mathcal{R}^*$ is simply the boundary of the achievable rate region.

Definition 1: The pareto boundary contains rate tuples $\mathbf{r} = (r_1, \ldots, r_M)$ such that any increment of any user's rate in the tuple would fall outside the achievable rate region defined in equation 6.

C. Particular solutions

In this section, we first discuss some particular solutions for the joint beamforming problem above, as introduced recently in [5]. These solutions are not always good ones, but have the merit of being simple to understand and they bear strong connections with game theory. These solutions are generalized readily from classical single cell MIMO theory.

1) The Zero-Forcing solution: The philosophy behind the Zero-Forcing solution (ZF) is *altruism* in a game theory sense. This means that each transmitter selects a beamforming vector so that no interference is created to other receivers. The ZF beamformer is in the null space of the channel matrix between transmitter i and the remaining receivers. Denote the channel matrix excluding the channel from transmitter i by $H_{-i} \in \mathcal{C}^{N_t \times M-1}$,

$$\mathbf{H}_{-i} = [\mathbf{h}_{i1}^T, \ldots, \mathbf{h}_{i(i-1)}^T, \mathbf{h}_{i(i+1)}^T, \ldots, \mathbf{h}_{iM}^T]. \tag{7}$$

Define the projection matrices onto the column space of H_{-i} [5]

$$\Pi_{H_{-i}} = H_{-i}\left(H_{-i}^H H_{-i}\right)^{-1} H_{-i}^H \tag{8}$$

and the orthogonal complement

$$\Pi_{H_{-i}}^{\perp} = I - H_{-i}\left(H_{-i}^H H_{-i}\right)^{-1} H_{-i}^H. \tag{9}$$

The ZF solution is therefore

$$\mathbf{w}_i^{(ZF)} = \frac{\Pi_{H_{-i}^\dagger}^{\perp} \mathbf{h}_{ii}^\dagger}{|\Pi_{H_{-i}^\dagger}^{\perp} \mathbf{h}_{ii}^\dagger|} \tag{10}$$

where H denotes complex conjugate transpose. Note that if the null space of $H_{-i}^{\dagger}$ has dimension larger than one, the ZF solution is the projection of $\mathbf{h}_{ii}^{\dagger}$ onto the null space such that $|\mathbf{h}_{ii}\mathbf{w}_i^{(ZF)}|$ is the largest.

2) The Maximum-Ratio-Combining solution: The maximum-ratio-combining (MRC) beamformer is employed to maximize the received power selfishly at the user, by alining the direction of the beam and the channel, ignoring the resulting interference generated. In a game theory sense, it forms a *egoistic* solution. Also it can be shown to coincide with the Nash Equilibrium (NE) in a non-cooperative strategic game [5] [6].

The MRC breamformer $\mathbf{w}_i^{(MRC)}$ for base station i is

$$\mathbf{w}_i^{(MRC)} = \frac{\mathbf{h}_{ii}^{\dagger}}{|\mathbf{h}_{ii}|} \tag{11}$$

where $\dagger$ denotes the complex conjugate transpose. Like the ZF solution the computation of the MRC solution only requires channel knoweledge which is locally observable at the base (at least in a TDD mode). The following theorem gives an intriguing characterization of the Pareto optimal beamforming strategies. It will serve a justification for the algorithm proposed later on.

Theorem 1: [5] Any point on the Pareto Boundary is proved to be achieved by beamforming vectors which are the linear combination of the zero-forcing solutions maximum-ratio-combining solutions . $\forall \mathbf{r} = (r_1, \ldots, r_M) \in \mathcal{R}^*, 1 \leq i \leq M$,

$$r_i = \log_2(1 + \gamma_i(\{\mathbf{w}_k\})) \tag{12}$$

where $\mathbf{w}_k = \frac{\alpha_k \mathbf{w}_k^{(ZF)} + (1-\alpha_k)\mathbf{w}_k^{(MRC)}}{|\alpha_k \mathbf{w}_k^{(ZF)} + (1-\alpha_k)\mathbf{w}_k^{(MRC)}|}$ and $0 \leq \alpha_k \leq 1, 1 \leq k \leq M$.

IV. Distributed Algorithms

In this section, we present a distributed bargaining solution (DBS) and we will compare it to the non-cooperative (Nash E) and altrustic solutions above presented. The difficulty is that the optimum cooperative points (on Pareto boundary) are given by a linear combination between the ZF and MRC beamformers where the weights are a function of the complete, centralized CSIT. To preserve semi-distributedness, we introduce the idea of a limited feedback link from each user and its *serving* base. The second novel aspect is the idea of iterative bargaining where the transmitters will simultaneously realize small increments of their beamformer in a directon leading to improvements for all parties involved. Users are expected to monitor their rates and indicate to their serving base whether the bargaining is successful or not (via a single bit of feedback). In the proposed framework, a loss of rate by one of the users will cause this user to cease the cooperation.

A. The DBS algorithm

We provide an iterative algorithm which approaches the pareto boundary by incrementally steering the beamforming vector in each iteration so that *every* transmitter and receiver pair would have a higher transmission rate.

Denote the beamforming vector of transmitter i in iteration j by $\mathbf{w}_i(j)$. Intuitively, it is reasonable to initialize the beamforming vectors $\mathbf{w}_i(0)$ to be the MRC solutions $\mathbf{w}_i^{(MRC)}$ because as users start off with a non-cooperative setting. However they can also initialize in an joint altruistic setting (see later).

The beamforming vector is updated at each iteration j by

$$\mathbf{w}_i(j) = \mathbf{w}_i(j-1) + \delta_w(j) \tag{13}$$

$$\mathbf{w}_i(j) \rightarrow \frac{\mathbf{w}_i(j)}{\|\mathbf{w}_i(j)\|} \tag{14}$$

where $\delta_w(j)$ is computed based on the available (quasi-distributed) CSIT feedback. At each iteration j, each receivers computes its rate $r_i^{(j)} = log_2\left(1 + \gamma_i(\mathbf{w}_1(j), \ldots, \mathbf{w}_M(j))\right)$ using locally available information ,

$$r_i^{(j)} = log_2\left(1 + \frac{|\mathbf{h}_{ii}\mathbf{w}_i(j)|^2}{IP_i(j)}\right) \tag{15}$$

where $IP_i(j)$ is the measured interference and noise power at receiver i at j-th iteration.

$$IP_i(j) = \sum_{k \neq i} |\mathbf{h}_{ki}\mathbf{w}_k|^2 + \sigma_n^2 \tag{16}$$

and reports to its transmitter a single bit to inform the base about its satisfaction: increment of data rates (1) or decrement of data rates (0).

B. Iterative Bargaining

Note that MRC outperform ZF solutions in low SNR region and vice versa in high SNR. To further improve the performance, we need an algorithm that can adapt to the channel realizations. and can distributedly and dynamically operate at a better sum rate point. Therefore, we have the following initialization policy:

$$\mathbf{w}_i(0) = \begin{cases} \mathbf{w}_i^{(ZF)} & \text{if } R^{(ZF)} > R^{(MRC)}; \\ \mathbf{w}_i^{(MRC)} & \text{otherwise.} \end{cases} \tag{17}$$

where $R^{(ZF)} = \sum_{i=1}^{M} log_2(1 + \gamma_i(\mathbf{w}_1^{(ZF)}, \ldots, \mathbf{w}_M^{(ZF)}))$ and $R^{(MRC)} = \sum_{i=1}^{M} log_2(1 + \gamma_i(\mathbf{w}_1^{(MRC)}, \ldots, \mathbf{w}_M^{(MRC)}))$. It is interesting to note that different update mechanics in (13) would result in a different data rates trajectory (data rate improvement curve) and would result in a different converged system sum rate. Here we provide two simple examples of $\delta_{w_i(j)}$ which will be shown later to perform better than the non-cooperative and altruistic solutions. Both these algorithm provide with a trajectory linking the MRC and ZF points in the rate region.

1) Zero-Forcing Increment (ZFI): In ZFI, assuming we start with MRC solution, the beamforming vector is steered towards the ZF solution in each iteration. Intuitively, the transmitters are willing to cooperate by lower the interference level caused to other receivers as long as they get benefits, increment in transmission rates, in return. The beamforming vector of transmitter i in iteration j is updated as

$$\mathbf{w}_i(j+1) = \mathbf{w}_i(j) + \alpha_i \mathbf{w}_i^{(ZF)} \tag{18}$$

where α_i is a preset step size constant. The beamforming vector is then normalized as in equation (14). On the other hand, if we start with ZF solution, the beamformer is steered towards the MRC solution in each iteration.

2) Orthogonal Bases Increment (OBI): In OBI, the beamforming vectors are a linear combination of the ZF solution and the following vector of orthogonal to ZF solution,

$$\mathbf{w}_i^{\perp(ZF)} = \frac{\Pi_{H_{-i}^\dagger}\mathbf{h}_{ii}^\dagger}{|\Pi_{H_{-i}^\dagger}\mathbf{h}_{ii}^\dagger|}. \tag{19}$$

Let the beamformer of transmitter i at iteration j be

$$\mathbf{w}_i(j) = \sqrt{\beta_i(j)}\mathbf{w}_i^{\perp(ZF)} + \sqrt{1-\beta_i(j)}\mathbf{w}_i^{(ZF)}. \tag{20}$$

As illustrated in [5] [6], to achieve pareto optimality, it is sufficient to parameterize $\beta_i(j)$ over $0 \leq \beta_i(j) \leq \tilde{\beta}_i$, where $L1 = |\Pi_{H_{-i}}\mathbf{h}_{ii}|^2$, $L2 = |\Pi_{H_{-i}}^{\perp}\mathbf{h}_{ii}|^2$ and $\tilde{\beta}_i = \frac{L1}{L1+L2}$.

To initialize, the beamformer equals

$$\beta_i(0) = \begin{cases} \tilde{\beta}_i & \text{if } R^{(MRC)} > R^{(ZF)} \\ 0 & \text{Otherwise.} \end{cases} \tag{21}$$

At each iteration $j+1$,

$$\beta_i(j+1) = \begin{cases} \beta_i(j) - \delta_\beta & \text{if } R^{(MRC)} > R^{(ZF)} \\ \beta_i(j) + \delta_\beta & \text{Otherwise.} \end{cases} \tag{22}$$

where δ_β is a predefined constant. The beamformer is then normalized as in equation 14.

C. Stopping Condition

A stop condition is implemented so that the beamformer trajectory is halted as near as possible to the Pareto boundary. The stop condition reflects the sharing policy and many options are available. A reasonable and intuitive stopping condition is that each transmitter would stop cooperating and terminates the algorithm when it encounters a decrement of transmission rate. User i, $1 \leq i \leq M$ would stop cooperating if

$$\frac{|\mathbf{h}_{ii}\mathbf{w}_i(j)|^2}{IP_i(j)} > \frac{|\mathbf{h}_{ii}\mathbf{w}_i(j+1)|^2}{IP_i(j+1)} \tag{23}$$

where $IP_i(j)$ is the measured interference and noise energy in equation (16).

V. Results and Discussion

In this section, we illustrate the dynamics (trajectory in the rate region) and the rate performance of DBS. We choose to plot the sum rate.

A. Dynamics of DBS

In figure 1, the achievable rate region is plotted for two transmitter receiver pairs. The ZF solution and MRC solution are both marked within the achievable rate region. The signal to noise ratio is at 15dB. In this channel realization, neither the ZF or the MRC is reaching the pareto boundary. The trajectory, rate bargained at each iteration, starts at the MRC solution. The solid line is the trajectory with stopping condition which ensures the bargaining stops when one of the transmitters has rate decrement. The dotted line is the trajectory path without stopping condition. As seen, the path eventually goes to the ZF point but there is no guarantee of each transmitters' rates. Note that the beamforming vector is steered towards ZF beamformer. Yet, it stops before reaching the ZF solution because one of the transmitter encounters rate decrement which results in a higher sum rate operating point and close to the pareto boundary. As shown in the figure, the resulting rate is higher than both ZF solution and MRC solution. Note that DBS reaches close to the Pareto boundary in 3 iterations in this realization which means that only 3 bits of feedback are required to improve the performance. With such small amount of overhead, the performance of the system is significantly improved.

B. Performance Comparison of DBS

The sum rate comparison between DBS, ZF and MRC between two transmitter pairs against SNR is illustrated in figure 2. In low SNR, SNR $<$ 9dB, the interference is not strong, MRC outperforms ZF as expected. We see that ZFI and OBI outperform conventional NE MRC solution. In medium and high SNR, ZF outperform MRC because the interference power is stronger than noise power and mitigation of interference improves performance. Because of the adaptive initialization, ZFI and OBI outperform ZF solution.

In figure 3, the sum rate of DBS ZF and MRC schemes are plotted against the distance between two transmitters. The distance is calculated as a multiple of the coverage of the transmitter. As the distance increases, interference becomes weaker. MRC and DBS both improves in performance and DBS outperform MRC. On the other hand, ZF solution did not take into account of the power of interference. Instead, ZF scheme consume all transmitter power to mitigate a weak interference, resulting in a constant performance.

VI. Conclusion

In cognitive radio, unlicensed users are competing for frequency resource selfishly. With a lack of network structure, a distributive interference mitigation algorithm is essential to improve the system performance. We provided two simple distributed bargaining solutions, ZFI and OBI which outperform non-cooperative game theory solutions. Future directions include the investigation of different stopping conditions and different game theory models such as coalitions of transmitter and receiver pairs.

VII. Acknowledgement

We gratefully acknowledge the financial support of ETRI Laboratory, Daejeon, Korea as well as fruitful discussions with them for this project.

References

[1] A. Leshem, E. Zehavi, "Bargaining Over the Interference Channel", IEEE International Symposium on Information Theory, 2006.

[2] S. Mathur, L. Sankaranarayanan and N. B. Mandayam,"Coalitional Games in Gaussian Interference Channels", IEEE International Symposium on Information Theory, 2006.

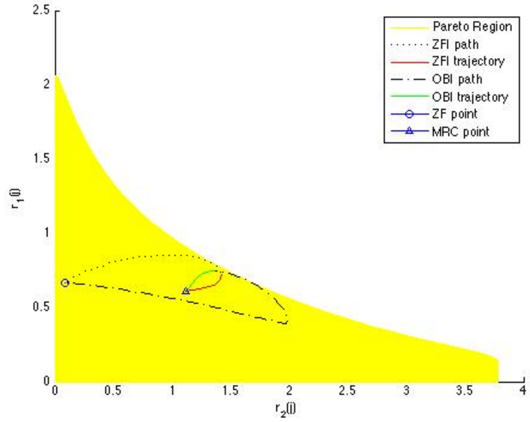

Fig. 1. The trajectory of DBS within the rate region in a 2 users system at system SNR 15dB. The trajectories of both ZFI and OBI stop close to the pareto boundary.

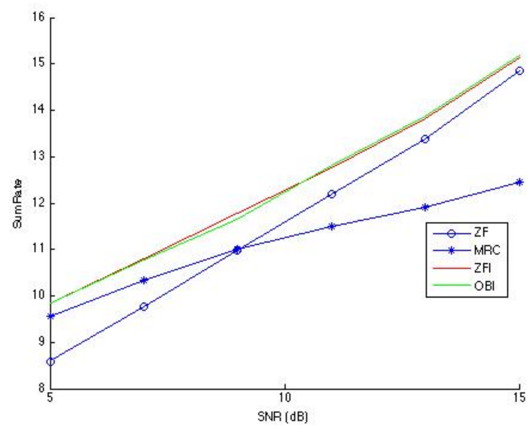

Fig. 2. The performance comparison of DBS against SNR (dB). The two version of DBS, namely ZFI and OBI, outperform the Nash equilibrium (MRC) and the altruistic (ZF) solution.

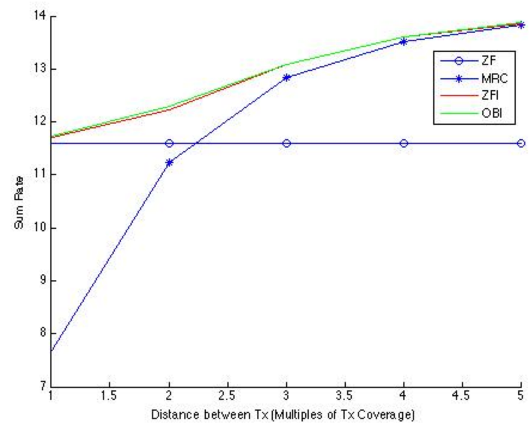

Fig. 3. The performance comparison of DBS against distance between transmitters at system SNR 10dB.

[3] Zhiyao Ma and Zhigang Cao,"Secondary User Cooperation Access Scheme in Opportunistic Cognitive Radio Networks", IEEE Military Communications Conference, 2007.
[4] N. Nie and C. Comaniciu, "Adaptive channel allocation spectrum etiquette for cognitive radio networks",IEEE International Symposium on New Frontiers in Dynamic Spectrum Access Networks, 2005.
[5] E. A. Jorswieck and Erik G. Larsson, "Complete Characterization of Pareto Boundary for the MISO Interference Channel", ICASSP 2008.
[6] E. A. Jorswieck and Erik G. Larsson, " Competition versus Collaboration on the MISO Interference Channel", submitted to
[7] Martin J. Osborne and Arie Rubinstein, *A course in Game Theory*, The MIT Press, Cambridge, Massachusetts, 1994.
[8] Allen B. MacKenzie and L. A. DaSilva, *Game Theory for Wirelss Engineers*, Morgan & Claypool Publishers' series, 2006.
[9] J. E. Suris, L. A. DaSilva, Z. Han and A. B. MacKenzie, " Cooperative Game Theory for Distributed Spectrum Sharing", ICC 2007.
[10] N. Clemens and C. Rose,"Intelligent Power Allocation Strategies in an Unlicensed Spectrum, IEEE SySPAN 2005.
[11] A. Laufer and A. Leshem, "Distributed Coordination of Spectrum and the Prisoner's Dilemma", IEEE DySPAN 2005.