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Receiver Designs for MIMO HSDPA

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***Abstract*—Optimal linear receivers for MIMO HSDPA (as for SISO/SIMO) are symbol-level (deterministic) *multiuser* receivers, known unfortunately to be time-varying in nature and thus prohibitively complex. Traditional less complex alternative is dimensionality-reducing linear chip-equalization followed by further non-linear (interference canceling) or joint detection stages to improve symbol estimates. Well-known versions of former include inter-stream Successive Interference Canceling (SIC) involving all codes while the latter leads to per-code joint spatial maximum-likelihood (ML) receiver. We investigate the class of MIMO HSDPA receivers based upon LMMSE chip-level MIMO (equalizer) front end, and introduce two (one static and the other *time-varying*) models of the resulting spatial channel, a consequence of treating the scrambler as random or deterministic. It is shown that in the random case, statistical properties can be exploited to design MIMO receivers while the deterministic point-of-view leads to another set of reduced-dimensionality linear receivers or interference cancelers.**

I. Introduction

3rd Generation Partnership Project (3GPP) has introduced a variant of Per-Antenna Rate-Control (PARC), namely Dual-Transmit Antenna Array (D-TxAA) for spatial multiplexing (MIMO) [1] in release-7 of UMTS WCDMA. Orthogonal codes of spreading factor 16 are reused across the two streams and the scrambling sequence is also common to both transmit (TX) streams. Like for single-steam HSDPA, in MIMO all (up to 15) spreading codes are allocated to the same user at a certain time. In general, all mobile users served by a base station (BS) feed an SINR-based (or based on some other appropriate measure) Channel Quality Indicator (CQI) back to the BS. In addition, the mobile station also computes (and feeds back) the weighting vector(s) that would ideally provide the best instantaneous rate for the upcoming TX interval. Together, this set of feedbacks translates into a specific transport block size and a specific Modulation and Coding Scheme (MCS) for each mobile user, based upon which the BS is capable of maximizing the downlink throughput for each transmission time-interval.

It is well-understood that MIMO receivers need to deal with inter-code as well as inter-stream interference. One group of interference suppression methods is symbol-level multiuser detection (MUD) where linear or non-linear transformations can be applied to the output of a channel matched filter (RAKE). Linear methods in this category are decorrelating and MMSE MUD both known to deal with inverses of large time-varying code cross-correlation matrices across symbols and thus are impractical. Non-linear MUD methods focus on estimating, reconstructing and subtracting signals of interfering codes. They are in general called interference canceling (IC) methods among which known sub-categories are serial and parallel interference cancelers (SIC/PIC) (see [2] for MUD).

The other group of HSDPA receivers [3] (and references therein) we address recognize that interference arises from loss of orthogonality due to the multipath channel and circumvent this problem by attempting to bring back the orthogonality through a *SINR-maximizing* LMMSE equalizer (or a MMSE-ZF solution). When spatial multiplexing is considered, it is clear that MIMO equalizers have to suppress another spatio-temporal interferer rendering linear equalization less efficient.

A solution of the second group can intuitively be treated as a dimensionality-reduction stage. It may take the form of a general chip-level filter carrying out functions of channel *sparsifier* or indeed a more specific spatio-temporal $\rightarrow$ spatial channel-shortener (e.g., $2N \times 2$ to 2×2 in MIMO HSDPA) [4]. This stage precedes either per-code joint detection of data streams at symbol level [3] or can be followed-up by one of the several possible decision-feedback approaches [4] and [5]. In practice, although the symbol-level spatial channel can now be seen as a per-code spatial mixture to which simplified (per-code) processing can be applied, performance of this approach falls well short of optimal time-varying symbol-level processing (linear and non-linear MUD solutions).

Despite their shortcomings, one may nevertheless point out that complexity/performance equation encourages use of solutions of the second type which are for this reason well-accepted in wireless industry [3]. The paper therefore addresses this class of methods. More precisely, in this paper the first stage is always considered to be MIMO LMMSE chip-level equalizer/correlator. We then focus on the resulting spatial channel model while treating the scrambler as (a) *random* [4] and (b) *deterministic* [6]. We show that random treatment of the scrambler leads to a time-invariant spatial signal model that leads to intuitively pleasing RX solutions. It is shown subsequently that such treatment is sub-optimal and one needs to consider deterministic treatment of the scrambler for any further (spatial) processing which turns out to be time-varying.

II. MIMO Signal Model

For the spatial multiplexing case in MIMO HDSPA, Fig. 1 illustrates the equivalent baseband chip-level downlink signal model. The received signal vector (chip-rate) at the UE can be modeled as

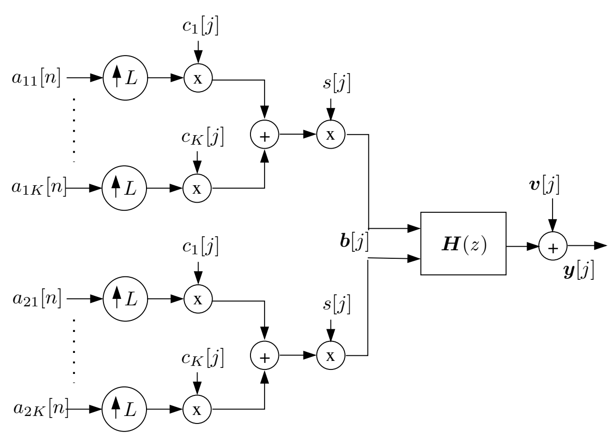

Fig. 1. MIMO signal model with precoding.

$$\underbrace{\boldsymbol{y}[j]}_{2p\times 1} = \underbrace{\boldsymbol{H}(z)}_{2p\times 2}\underbrace{\boldsymbol{x}[j]}_{2\times 1} + \underbrace{\boldsymbol{v}[j]}_{2\times 1}. \quad (1)$$

In this model, j is the chip index, $\boldsymbol{H}(z)$ is the frequency selective MIMO channel the output of which is sampled p times per chip

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and $\boldsymbol{v}[j]$ represents the vector of noise samples that are zero-mean circular Gaussian random variables. The sequence $\boldsymbol{x}[j]$ introduced into the channel is expressed as

$$\boldsymbol{x}[j] = \sum_{k=1}^{K} s[j] c_k[j \mod L] \boldsymbol{a}_k[n] \tag{2}$$

where k is the code index, p is the index of the symbol on code k given by $n = \lfloor \frac{j}{L} \rfloor$, L is the spreading factor ($L = 16$ for HSDPA), The 2×1 symbol vector $\boldsymbol{a}_k[n] = [a_{1k}[n] \;\; a_{2k}[n]]^T$ is a MIMO symbol and represents the n^{th} symbol of the two independent data streams, $\boldsymbol{c}_k = [c_k[0] \ldots c_k[L-1]]^T$, where $\boldsymbol{c}_i^T \cdot \boldsymbol{c}_j = \delta_{ij}$ are unit-norm spreading codes common to the two streams, and $s[j]$ the common scrambling sequence element at chip time j, which is zero-mean *i.i.d.*.

III. MIMO HSDPA RECEIVER STRUCTURES

We concentrate on MIMO receivers based upon dimensionality reducing chip-level equalizer/correlator front-end.

A. Receiver 1: MMSE Chip Equalizer-Correlator

In the spatial multiplexing context, the LMMSE equalization tries not only to suppress all Inter-Chip Interference (ICI) but also all Inter-Stream Interference (ISI). The 2×2 linear FIR MMSE chip-level equalizer tries to obtain chip estimates and is given by the standard expression $\boldsymbol{F} = \boldsymbol{R_{xy}} \boldsymbol{R_{yy}}^{-1}$ (see fig. 2). We can write the equalizer output as the sum of an arbitrarily scaled desired term and an error term

$$\widehat{\boldsymbol{x}}[j] = \boldsymbol{x}[j] - \widetilde{\boldsymbol{x}}[j]. \tag{3}$$

The error $\widetilde{\boldsymbol{x}}[j]$ is a zero-mean complex normal random variable. The error covariance matrix is denoted by $\boldsymbol{R}_{\widetilde{x}\widetilde{x}}$.

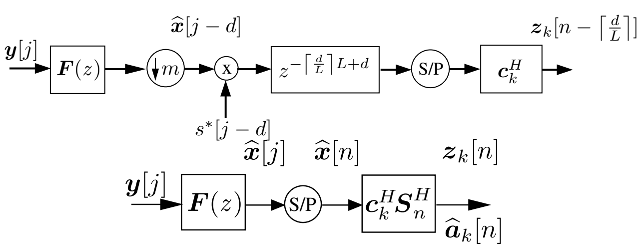

Fig. 2. LMMSE equalizer and correlator. The second figure is a simplified representation used as chip-equalizer /correlator front-end stage for other receiver structures.

As shown in fig. 2, at the output of the equalizer, the estimate of the chip sequence in (3), is obtained after a delay d equal to the equalization delay in chips. After despreading (for the k^{th} code) the 2×1 signal at the symbol level is written as

$$\boldsymbol{z}_k[n] = \boldsymbol{a}_k[n] - \widetilde{\boldsymbol{z}}_k[n] = \boldsymbol{B}\boldsymbol{a}_k[n] - \widetilde{\widetilde{\boldsymbol{z}}}_k[n]. \tag{4}$$

In general, $\boldsymbol{B}$ in this expression is the LMMSE MIMO *joint bias* at output of chip-equalizer/correlator (see appendix A). The significance of MIMO joint-bias is the same as in SISO equalization, and by consequence the quantity $\widetilde{\widetilde{\boldsymbol{z}}}_k[n]$ therefore contains no desired *MIMO symbol* contribution.

1) Estimation of $\boldsymbol{R}_{\widetilde{\widetilde{z}}\widetilde{\widetilde{z}}}$: At this stage we need to determine the SINR at the output of chip-equalizer correlator. Assuming the scrambler to be *i.i.d.*, we can easily calculate the different quantities, i.e., the signal energy and the estimation error variances $\boldsymbol{R}_{\widetilde{\widetilde{z}}\widetilde{\widetilde{z}}}$. These are derived in appendix A.

2) Output SINR: From analysis of $\boldsymbol{R}_{\widetilde{\widetilde{z}}\widetilde{\widetilde{z}}}$, it can be shown that the SINR for the i^{th} stream at the output of the output of the LMMSE chip equalizer/correlator is given by

$$SINR_i = \frac{\sigma_{a_k}^2 |g_{ii}[0]|^2}{\left(\boldsymbol{R}_{\widetilde{\widetilde{z}}\widetilde{\widetilde{z}}}\right)_{ii}}. \tag{5}$$

Taking expectation over the time-varying (random) scrambling sequence as is customary, the bias term can be considered to be constant at the equalizer/correlator output.

Once MIMO joint bias is properly taken into account (see appendix A), the expression for the LMMSE chip equalizer output SINR is exact. The situation is different at the symbol-level where the bias, in practice, varies over time. We will consider this case shortly. The per-code capacity of the i^{th} data stream therefore corresponds to

$$\boldsymbol{C}_i \;\; = \;\; \log\left(\frac{\sigma_a^2}{MMSE_i}\right) \tag{6}$$

3) Deterministic Scrambler: Consider now deterministic treatment of the scrambler. One can express the signal as

$$\boldsymbol{z}_k[n] = \boldsymbol{B}_{n,k}[0]\boldsymbol{a}_k[n] - \widetilde{\widetilde{\boldsymbol{z}}}_k[n], \tag{7}$$

where the time varying MIMO joint-bias $\boldsymbol{B}_{n,k}[0]$ (defined in appendix A) is no longer constant and varies for each symbol. Using the notation from appendix A, the per-user SINR of stream r is given by (8).

B. Receiver 2: MMSE Chip Equalizer-Symbol Level LMMSE

In an alternative receiver structure, the output of the chip-equalizer is fed into a symbol level (spatial) LMMSE filter after the descrambler/correlator block. This is shown in Fig. 3. As discussed in III-A, the output of the correlator is $\boldsymbol{z}_k[n]$ given by (4). $\mathcal{F}_{sp}$ denotes the

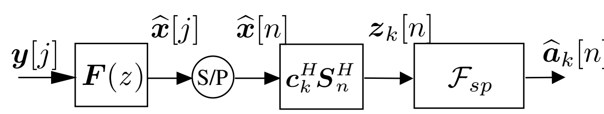

Fig. 3. Chip LMMSE equalizer and correlator followed by symbol-level (spatial) MMSE.

spatial MMSE at the output of which we have a linear estimate of the symbol vector as

$$\widehat{\boldsymbol{a}}_k[n] = \boldsymbol{a}_k[n] - \widetilde{\boldsymbol{a}}_k[n]. \tag{9}$$

The error covariance matrix for the LMMSE estimate of $\boldsymbol{a}_k[n]$ is given by

$$\boldsymbol{R}_{\widetilde{a}\widetilde{a}} \;\; = \;\; \boldsymbol{R}_{aa} - \boldsymbol{R}_{az}\boldsymbol{R}_{zz}^{-1}\boldsymbol{R}_{za} \tag{10}$$

$$= \;\; \sigma_a^2 \boldsymbol{I} - \sigma_a^4 \left(\sigma_a^2 I + \boldsymbol{B}^{-1}\boldsymbol{R}_{\widetilde{\widetilde{z}}\widetilde{\widetilde{z}}}\boldsymbol{B}^{-H}\right)^{-1}. \tag{11}$$

Expressing the above relation in terms of the correlator output covariances, $\boldsymbol{B}\boldsymbol{R}_{\widetilde{\widetilde{z}}\widetilde{\widetilde{z}}}\boldsymbol{B}^{-H}$ and using some algebra leads to the expression

$$R_{\widetilde{a}\widetilde{a}} = \sigma_a^2 \boldsymbol{I} - \sigma_a^4 \left(\sigma_a^2 \boldsymbol{I} + \left(\boldsymbol{R}_{\widetilde{z}\widetilde{z}}^{-1} - \boldsymbol{R}_{zz}^{-1}\right)^{-1}\right)^{-1}. \tag{12}$$

Like the LMMSE chip level equalizer/correlator RX, this translates to a sum-capacity expression similar to the one derived in the previous section.

$$\boldsymbol{C}_1 + \boldsymbol{C}_2 = \log\left(\frac{\sigma_a^4}{\det(\mathrm{diag}(\boldsymbol{R}_{\widetilde{a}\widetilde{a}}))}\right) \tag{13}$$

One may remark that spatial MMSE processing after the equalizer/correlator stage should lead to further suppression of residual interference and lends itself to low-complexity per-code implementation.

$$SINR_{k,r} = \frac{\sigma^2_{a_k}\left(|g_{rr}[0]|^2 + \frac{1}{L^2}\text{tr}\left\{\overline{\boldsymbol{G}}_{rr}[0]\overline{\boldsymbol{G}}^H_{rr}[0]\right\}\right)}{\sigma^2_{a_k}\left(\frac{(K-1)}{L^2}\sum_{s=1}^{2}\text{tr}\left\{\overline{\boldsymbol{G}}_{rs}[0]\overline{\boldsymbol{G}}^H_{rs}[0]\right\} + \frac{K}{L^2}\sum_{i}\sum_{s=1}^{2}\text{tr}\left\{\overline{\boldsymbol{G}}_{rs}[i]\overline{\boldsymbol{G}}^H_{rs}[i]\right\}\right) + \sigma^2_v\|\boldsymbol{f}_r\|^2}. \quad (8)$$

1) Deterministic Scrambler: We consider again the effect of deterministic treatment of scrambler for symbol-level spatial MMSE receiver. In order to claim the quantity $\frac{1}{L^2}\text{tr}\left\{\overline{\boldsymbol{G}}_{rr}[0]\overline{\boldsymbol{G}}^H_{rr}[0]\right\}$ in (8) as part of signal energy, it suffices to put in place time-varying processing at the correlator output, where the n^{th} symbol vector on the k^{th} code, $z_k[n]$ is given by (17). As a result of time-varying symbol level *joint-bias*, the 2×2 MMSE equalizer will now have to be computed for each symbol. This will indeed provide higher gains than the spatial MMSE receiver above which treats the time varying signal contribution as noise.

C. Receiver 3: Chip Level SIC

Indeed a better known SIC receiver detects data symbols from one stream, say stream 1 and re-spreads, re-scrambles, re-channelizes detected data, the contribution of that stream can be subtracted from the received signal. If the two SIMO channel components of the MIMO channel are $\boldsymbol{H}_1$ and $\boldsymbol{H}_2$ with $\widetilde{\boldsymbol{h}}_1$ and $\widetilde{\boldsymbol{h}}_2$ being the corresponding desired response vectors, the second stream can now be detected using a new FIR LMMSE chip-level receiver obtained as

$$\boldsymbol{f}_{sic} = \sigma^2_b \cdot \widetilde{\boldsymbol{h}}^H_2 \boldsymbol{R}^{-1}_{\overline{yy}}, \quad (14)$$

where, from (1),

$$\underbrace{\boldsymbol{y}[j]}_{2p\times 1} = \underbrace{\boldsymbol{H}_2(z)}_{2p\times 1}\underbrace{x_2[j]}_{1\times 1} + \underbrace{v_2[j]}_{2\times 1}. \quad (15)$$

This case, assuming perfect cancellation of stream 1, is analogous to single stream communications and the SINR achieved for stream 2 is much improved. The SINR expressions for this SIC receiver are straightforward and similar to the ones for the MISO LMMSE chip-level equalizer/correlator case (see appendix A). One further consideration in RX 4 is that if stream 1 symbol estimates are obtained at the output of a spatial MMSE, this would also imply spatial processing for stream 2 (since spatial processing by nature is simultaneous). Such treatment increases complexity but may be well worth the effort in terms of SINR gains.

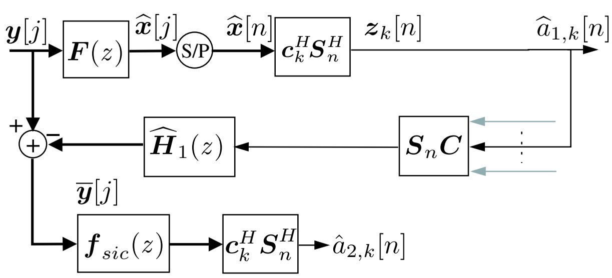

Fig. 4. Chip LMMSE equalizer/correlator followed by spatial MMSE and chip-level SIC for stream 2.

1) Different Types of SIC Receivers: The SIC structure in [4] qualifies as a symbol-level SIC, feeding back symbol decisions to the output of equalizer-correlator. This assumes a time-invariant symbol-level channel $\boldsymbol{B}$ (see appendix A) resulting from treating the scrambler as random. The suboptimalities introduced in all earlier stages, i.e., dimensionality reduction through chip-equalization and reducing the symbol-level channel to its mean value (random scrambler) take their toll; this SIC does not provide significant gains over chip equalizer/correlator solution.

On the other end of the SIC spectrum is the chip-level SIC, discussed here, that is an entirely different solution that considers all stages of the channel-equalizer correlator stage as deterministic and re-creates all components (ISI and MUI) of the stream detected first before subtracting it from the input signal. Subsequently, the second stream can be dealt with in much improved conditions where interference from the first stream is (ideally) entirely suppressed.

D. Receiver 4: Spatial ML Receiver

Another possible receiver structure is shown in Fig. 5 where the chip-equalizer correlator front end is followed up, as before, by a joint detection stage [3] [4]. For a given user code, the 2×1 symbol-level signal is given by (4). This spatial mixture is later processed for joint detection (code-wise ML detection) of the two symbol streams. The ML metric is given as follows.

$$\mathcal{D} = \{\boldsymbol{z}_k[n] - \boldsymbol{a}_k[n]\}^H \boldsymbol{R}^{-1}_{\tilde{\tilde{z}}\tilde{\tilde{z}}}\{\boldsymbol{z}_k[n] - \boldsymbol{a}_k[n]\}.$$

This metric can be solved for $\boldsymbol{a}_k[n]$. It was shown in [4] that joint detection outperforms a symbol-level SIC. For joint detection, the SINR for the i^{th} stream corresponds to the MFB of the spatial channel. The MFB can be interpreted as the SNR of i^{th} stream when it is detected assuming that symbols of the other stream(s) are known. $\boldsymbol{R}_{\tilde{\tilde{z}}\tilde{\tilde{z}}}$ is the *colored* noise variance.

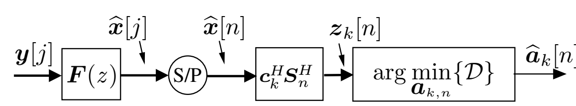

Fig. 5. Chip LMMSE equalizer/correlator followed by spatial MMSE and joint detection.

In treating the scrambler as random the spatial channel ($\boldsymbol{B}$), the ML metrics will deal with a time-invariant channel. A continuous processing matched filter bound can therefore be defined per stream. The i^{th} stream MFB is therefore proportional to the energy in the corresponding SIMO channel. On the contrary, if a deterministic scrambler is assumed, time-variation in the channel must be accounted for in ML metrics. Strictly speaking, the MFB is only defined per symbol as the SINR of the n^{th} symbol considering all other symbols to be known (correctly detected). We can nevertheless argue that deterministic treatment of the scrambler leads to reduced interference variance $\boldsymbol{R}_{\tilde{\tilde{z}}\tilde{\tilde{z}}}$ and increased recoverable signal power that will lead to performance improvement for the ML solution.

IV. Simulation Results

We show here the simulation results and compare the performance of the different receiver structures based on their sum-capacity. For a fixed SNR and over several realizations of a frequency selective $2p\times 2$ MIMO FIR channel $\boldsymbol{H}(z)$, we compute the SINRs of both streams at the output of the receivers to calculate an upperbound on the sum capacity. The channel coefficients are complex valued zero-mean Gaussian of length 20 chips. We simulate here a single-user situation where 15 codes are assigned to the same user. Furthermore, we assume code-reuse across antennas. The length of FIR MIMO equalizers of is comparable to channel delay spread in chips.

The sum-capacity CDF is thus used as a performance measure for all receivers. In Fig. 6 we plot the capacity bounds for two cases. In the first instance, we treat the scrambler as random. The symbol energy for code k is therefore given by the symbol variance for the

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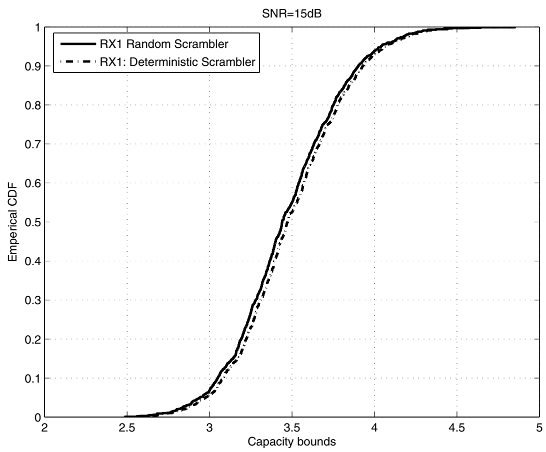

Fig. 6. Sum-capacity at the output of RX 1 with random and deterministic scrambler.

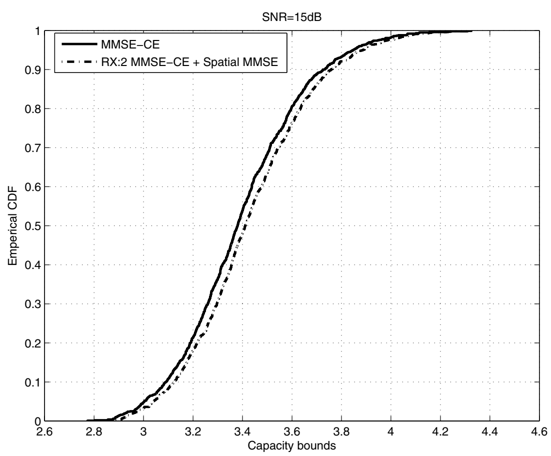

Fig. 7. Upper bounds on the sum-capacity at the output of RX 1 and RX 2 .

code scaled by an arbitrary time-invariant scale factor. In the second case, we treat the scrambler as known (deterministic treatment). In this case, firstly, the signal power now is time-varying at symbol rate. This time varying signal power can be seen as the sum of a "mean" power contribution equal to the signal power when the scrambler is assumed to be random, and time-varying contribution due to deterministic treatment of the scrambler.

Fig. 7 shows distribution of sum-capacity at the output of the MMSE chip-equalizer correlator receiver and that of the spatial MMSE receiver (random scrambler). With an additional processing stage of a very small complexity we are able to see some gain in the achievable rates of the receiver.

The performance of chip-level SIC, is shown in Fig. 8 and in Fig. 9 we plot the capacity bounds for the ML (per-code) receiver. Note that the SINR distribution for the deterministic treatment of the scrambler in Fig. 9 represents the *average* gains and not the true gain. The actual gain will be higher than that seen in Fig. 9.

V. Conclusions

In this contribution, we analyzed several receiver designs for MIMO HSDPA. The MMSE chip equalizer correlator solution can

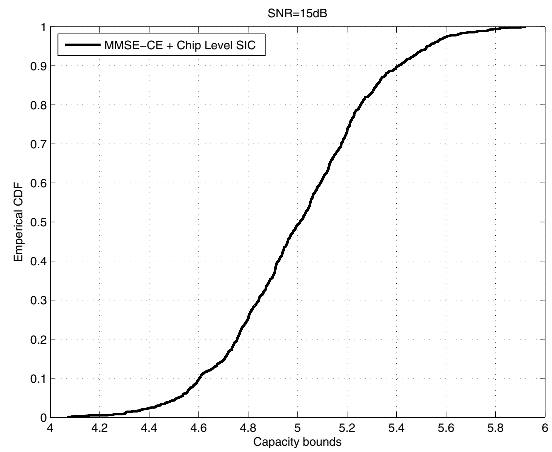

Fig. 8. Sum-capacity at the output of the Chip Level SIC receiver

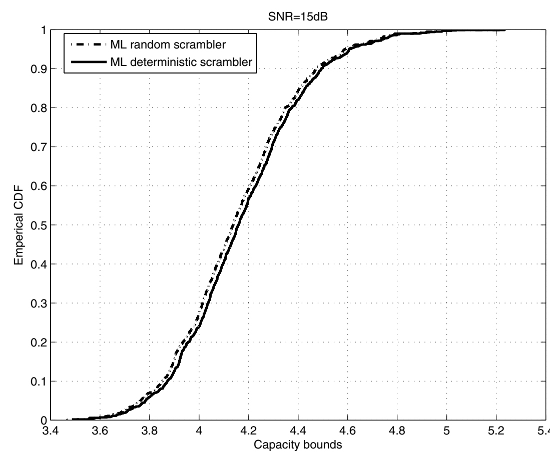

Fig. 9. Sum-capacity at the output of RX 4 with deterministic and random scrambler.

be seen as a dimensionality reduction step in multiuser detection of MIMO CDMA codes. It simplifies the RX solution (either to a spatio-temporal separation or up to a spatial mix after which other RX stages could be employed), but performs significantly worse than the classical linear MU solutions. A further sub-optimality is the treatment of the scrambler as white which, once more a simplifying step, affects performance. We believe that deterministic treatment of the scrambler, leading to time-varying processing after the equalizer/correlator stage offsets some of the performance losses of the dimensionality reduction stage and random scrambler assumption. We consider deterministic treatment of the scrambler and show the additional gains that can be achieved in SINR by exploiting the time-varying signal contribution. For the MIMO single-user case with code reuse across 2 antennas we derived SINR expressions for both deterministic and random treatment of the scrambler for the proposed receiver designs. We presented a comparison of the performance of four distinct receiver structures for MIMO HSDPA all based on the LMMSE chip-level equalizer/correlator as the first processing stage.

Appendix

A. Output Energy of the LMMSE Chip-Equalizer-Correlator

Without loss of generality, we consider linear MMSE estimation of the 2×1 MIMO symbol sequence, $\boldsymbol{a}_k[n]$, of the k^{th} code among K codes (each stream has K codes). Refer to fig. 10, for a vectorized

TX signal model where $\boldsymbol{b}[n]$ is the $2L \times 1$ chip vector defined as $\boldsymbol{b}[n] = [\boldsymbol{b}_0^T[n] \cdots \boldsymbol{b}_{L-1}^T[n]]^T$, where $\boldsymbol{b}_i[n]$ is the i^{th} multi-code (K codes) MIMO (2×1) chip corresponding to the n^{th} MIMO symbol vector, $\boldsymbol{a}[n]$ of size $2K \times 1$. Assuming an oversampling factor of p,

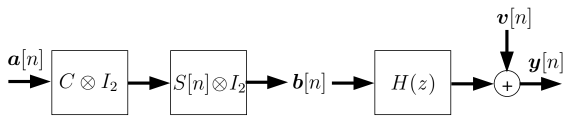

Fig. 10. MIMO TX signal model.

the channel $\boldsymbol{H}(z) = \sum z^{-i}\boldsymbol{H}[i]$ consists of $pL \times L$ matrix taps. If the delay spread is N chips, there are $\lceil N/L \rceil$ *pseudo-circulant* matrix coefficients defined as

$$\boldsymbol{H}[i] = \begin{bmatrix} \boldsymbol{h}[iL] & \boldsymbol{h}[iL+1] & \ldots & \boldsymbol{h}[(i+1)L-1] \\ \boldsymbol{h}[iL-1] & & & \vdots \\ \vdots & & \ddots & \\ \boldsymbol{h}[(i-1)L+1] & \ldots & \ldots & \boldsymbol{h}[iL] \end{bmatrix}$$

with $\boldsymbol{h}[i]$ being the $2p \times 2$ MIMO channel coefficients. The LMMSE equalizer $\boldsymbol{F}(z)$ in fig. 11 can be represented in a similar fashion with $\boldsymbol{f}[i]$, the $2 \times 2p$ equalizer coefficients. The channel equalizer cascade $\boldsymbol{G}(z) = \boldsymbol{F}(z)\boldsymbol{H}(z) = \sum z^{-i}\boldsymbol{G}[i]$ is itself defined similarly with $\boldsymbol{g}[i]$ being the 2×2 elements.

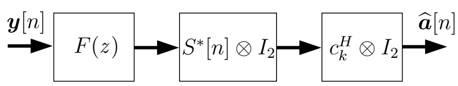

Fig. 11. MIMO RX model.

Let us further define

$$\boldsymbol{G}[0] = \boldsymbol{F}(q)\boldsymbol{H}(q)|_{[0]} = \begin{bmatrix} \boldsymbol{g}[0] & \boldsymbol{g}[1] & \ldots & \boldsymbol{g}[L-1] \\ \boldsymbol{g}[-1] & & & \vdots \\ \vdots & & \ddots & \\ \boldsymbol{g}[-L+1] & \ldots & \ldots & \boldsymbol{g}[0] \end{bmatrix}$$

as the $2L \times 2L$ zeroth MIMO-tap of the channel equalizer cascade. $\overline{\boldsymbol{G}}(z) = \sum_{i \neq 0} z^{-i}\boldsymbol{G}[i]$ represents the MIMO inter-symbol interference (ISI). q^{-1} is the unit-delay operator. We can henceforth write

$$\begin{aligned} \widehat{a}_k[n] = &\left(\boldsymbol{c}_k^H \otimes I_2\right)\left(\boldsymbol{S}^*[n] \otimes I_2\right) \\ &\left\{\boldsymbol{G}(q)\left(\boldsymbol{S}[n] \otimes I_2\right)\left(\boldsymbol{C} \otimes I_2\right)\boldsymbol{a}[n] + \boldsymbol{F}(q)\boldsymbol{v}[n]\right\}. \end{aligned} \quad (16)$$

Defining

$$\boldsymbol{B}_{n,k}(z) = \left(\boldsymbol{c}_k^H \otimes I_2\right)\left(\boldsymbol{S}^*[n] \otimes I_2\right)\boldsymbol{G}(z)\left(\boldsymbol{S}[n] \otimes I_2\right)\left(\boldsymbol{C} \otimes I_2\right)$$

as the symbol-rate channel at time instant n (also a $\overline{\boldsymbol{B}}_{n,k}(z)$ corresponding to $\overline{\boldsymbol{G}}(z)$) , we can write the correlator output as

$$\begin{aligned} \boldsymbol{z}_k[n] = &\overbrace{\boldsymbol{B}_{n,k}[0]\boldsymbol{a}_k[n]}^{kth\ code} + \overbrace{\boldsymbol{B}'_{n,k}[0]\overline{\boldsymbol{a}}[n]}^{other\ codes} + \\ &\underbrace{\sum_i \boldsymbol{B}_{n,k}[i]\boldsymbol{a}[n+i]}_{all\ codes\ other\ symbols} + \underbrace{\boldsymbol{F}(z)\boldsymbol{v}[n]}_{noise}. \end{aligned} \quad (17)$$

In this expression, $\boldsymbol{B}_{n,k}[0]$ is the desired user channel at symbol-time n (time-varying channel), which one can split into a time invariant part $E_n\boldsymbol{B}_{n,k}[0] = \boldsymbol{B}[0] = \boldsymbol{B} \cdot I_L$ (assuming the scrambler to be white), and a time-varying part (if scrambler treated as deterministic). When the scrambler is treated as white, we refer to the 2×2 channel as *spatial channel* or even as *joint MIMO bias* and denote it as $\boldsymbol{g}_0 = \boldsymbol{B}$. As discussed in [6], treating the scrambler as white has the effect of capturing the mean signal energy (corresponding to the $\boldsymbol{g}[0]$ contribution) at the output of the per code MIMO channel while consigning the variance (off-diagonal part in $\boldsymbol{G}[0]$) definitively and irrecoverably to the interference term.

It may be noticed that each element of $\boldsymbol{G}[i]$ is a 2×2 MIMO matrix coefficient. The former can therefore be split into four $L \times L$ SISO submatices $\boldsymbol{G}_{rs}[i]$, for $r, s \in \{1, 2\}$. A corresponding $L \times L$ matrix coefficient $\overline{\boldsymbol{G}}_{rs}[i] = \boldsymbol{G}_{rs}[i] - \boldsymbol{g}_{rs}[i] \cdot I_L$ is also defined and so is $\boldsymbol{g}_{rs}[i]$, the rs^{th} element of the spatial channel $\boldsymbol{g}[i]$.

Taking expectation over the scrambler, we can express the output energy of the receiver as

$$\boldsymbol{R}_{zz} = \boldsymbol{R}_{des} + \boldsymbol{R}_{MUI} + \sum_i \boldsymbol{R}_{i,ISI} + \boldsymbol{F}\boldsymbol{R}_{vv}\boldsymbol{F}^H, \quad (18)$$

where,

$$\boldsymbol{R}_{des} = \begin{bmatrix} |g_{11}[0]|^2 + |g_{12}[0]|^2 & \sum_{s=1}^{2} g_{1s}[0]g_{2s}^*[0] \\ \sum_{s=1}^{2} g_{2s}[0]g_{1s}^*[0] & |g_{21}[0]|^2 + |g_{22}[0]|^2 \end{bmatrix} + \frac{1}{L^2} \cdot \begin{bmatrix} \sum_{s=1}^{2} tr\{\overline{\boldsymbol{G}}_{1s}[0]\overline{\boldsymbol{G}}_{1s}^H[0]\} & \sum_{s=1}^{2} tr\{\overline{\boldsymbol{G}}_{1s}[0]\overline{\boldsymbol{G}}_{2s}^H[0]\} \\ \sum_{s=1}^{2} tr\{\overline{\boldsymbol{G}}_{2s}[0]\overline{\boldsymbol{G}}_{1s}^H[0]\} & \sum_{s=1}^{2} tr\{\overline{\boldsymbol{G}}_{2s}[0]\overline{\boldsymbol{G}}_{2s}^H[0]\} \end{bmatrix},$$

$$\boldsymbol{R}_{MUI} = \frac{K-1}{L^2} \cdot \begin{bmatrix} \sum_{s=1}^{2} tr\{\overline{\boldsymbol{G}}_{1s}[0]\overline{\boldsymbol{G}}_{1s}^H[0]\} & \sum_{s=1}^{2} tr\{\overline{\boldsymbol{G}}_{1s}[0]\overline{\boldsymbol{G}}_{2s}^H[0]\} \\ \sum_{s=1}^{2} tr\{\overline{\boldsymbol{G}}_{2s}[0]\overline{\boldsymbol{G}}_{1s}^H[0]\} & \sum_{s=1}^{2} tr\{\overline{\boldsymbol{G}}_{2s}[0]\overline{\boldsymbol{G}}_{2s}^H[0]\} \end{bmatrix},$$

and, the ISI contribution from the i^{th} symbol can be expressed as

$$\boldsymbol{R}_{i,ISI} = \frac{K}{L^2} \cdot \begin{bmatrix} \sum_{s=1}^{2} tr\{\overline{\boldsymbol{G}}_{1s}[i]\overline{\boldsymbol{G}}_{1s}^H[i]\} & \sum_{s=1}^{2} tr\{\overline{\boldsymbol{G}}_{1s}[i]\overline{\boldsymbol{G}}_{2s}^H[i]\} \\ \sum_{s=1}^{2} tr\{\overline{\boldsymbol{G}}_{2s}[i]\overline{\boldsymbol{G}}_{1s}^H[i]\} & \sum_{s=1}^{2} tr\{\overline{\boldsymbol{G}}_{2s}[i]\overline{\boldsymbol{G}}_{2s}^H[i]\} \end{bmatrix}.$$

In these relations, the $\boldsymbol{R}_{des}$ is composed of two contributions shown above as the sum of two 2×2 matrices. The term scaled by $1/L^2$ is the quantity that ceases being a part of the signal energy contribution and is associated instead with the interference for reasons explained earlier.

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