

BEAMFORMING FOR CORRELATED BROADCAST CHANNELS WITH QUANTIZED CHANNEL STATE INFORMATION

*Ruben de Francisco**, *Claude Simon[†]*, *Dirk T.M. Slock**, and *Geert Leus[†]*

*Eurecom Institute, 2229, Route des Crêtes, 06560 Valbonne Sophia-Antipolis, France

[†]TU Delft, Fac. EEMCS, Mekelweg 4, 2628 CD Delft, The Netherlands

ABSTRACT

The design of channel quantization codebooks for correlated broadcast channels with limited feedback is addressed. A design criterion that effectively exploits the cell statistics is proposed, based on minimizing the average sum-rate distortion in a system with joint linear beamforming and multiuser scheduling. The proposed average distortion function is optimized by generating a set of quantization codebooks through random trials, keeping the codebook that yields the lowest distortion. Comparisons with limited feedback approaches relying on random codebooks are provided, highlighting the importance of matching the codebook design to the cell statistics. Numerical results show a performance gain in scenarios with non-uniform user distributions. Further, we propose a scheme that exploits the limited channel knowledge at the base station to reduce the computational complexity of determining the beamforming vectors and of finding the optimal user set.

1. INTRODUCTION

Multiple-input multiple-output (MIMO) systems can significantly increase the spectral efficiency by exploiting the spatial degrees of freedom created by multiple antennas [1]. In the MIMO broadcast channel, it has recently been proven [2] that the sum capacity is achieved by dirty paper coding (DPC) [3]. However, the applicability of DPC is limited due to its computational complexity and the need for full channel state information (CSI) at the transmitter. As a low complexity alternative, downlink techniques based on Space Division Multiple Access (SDMA) have been proposed that achieve the same asymptotic sum rate as that of DPC, e.g., zero-forcing beamforming [4]. On the other hand, while having full CSI at the receiver can be assumed, this assumption is not reasonable at the transmitter side. Several limited feedback approaches have been considered in point-to-point systems [5, 6], where each user sends to the transmitter the index of a quantized version of its channel vector from a codebook. An extension for MIMO broadcast channels is made in [7], in which each mobile feeds back a finite number of bits regarding its channel realization at the beginning of each block based on a codebook.

Codebook designs for MIMO broadcast channels with limited feedback follow in general simple design criteria, with the purpose of simplifying codebook generation and system analysis. Opportunistic SDMA (OSDMA) has been proposed in [8] as an SDMA extension of opportunistic beamforming [9], in which feedback from

the users to the base station (BS) is conveyed in the form of a beamforming vector index and an individual signal-to-interference-plus-noise ratio (SINR). An extension of OSDMA is proposed in [10], coined as OSDMA with limited feedback (LF-OSDMA), in which the transmitter counts on a codebook containing an arbitrary number of unitary bases. In this approach, the users quantize the channel direction (channel shape) to the closest codeword in the codebook, feeding back the quantization index and the expected SINR. Multiuser scheduling is performed based on the available feedback, using as beamforming matrix the unitary basis in the codebook that maximizes the system sum rate. Other schemes for MIMO broadcast channels propose to use simple Random Vector Quantization (RVQ) [11] for quantizing the user vector channels, such as the approach described in [7]. A simple geometrical framework for codebook design is proposed in [6], which divides the unit sphere in quantization cells with equal surface area. This framework is used for channel direction quantization in [12], where feedback to the base station consists of a quantization index along with a channel quality indicator for user selection. These codebook designs do not take into account either spatial correlations or user distributions present in the system, which could yield better quantization codebooks and in turn better sum-rate performance.

The gains of adaptive cell sectorization have been studied in [13] in the context of CDMA networks and single antenna communications, with the aim of minimizing the total transmit power in the uplink of a system with non-uniform user distribution over the cell. This situation is analogous to a system with multiple transmit antennas in which beamforming is performed, adapting its beams to uneven user distributions. In a scenario with limited feedback available, adaptation of quantization codebooks can be performed instead in order to improve the system performance. In [14], an approach for exploiting long term channel state information in the downlink of multiuser MIMO systems is proposed. A flat-fading multipath channel model is assumed, with no line of sight (NLOS) between the base station and user terminals. Each user can be reached through a finite number of multipath components with a certain mean angle of departure (AoD) from the antenna broadside and angle spread. The mean angles of departure are fixed and thus no user mobility is considered.

In this paper, we highlight the importance of cell statistics for codebook design in MIMO broadcast channels with limited feedback. The average sum rate distortion in a system with joint linear beamforming and multiuser scheduling is minimized, exploiting the information on the macroscopic nature of the underlying channel. A non-geometrical stochastic channel model is considered, in which each user can be reached in different spatial directions and with different angle spread. Based on this model, comparisons with limited feedback approaches relying on random codebooks are provided in order to illustrate the importance of matching the codebook design

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to the cell statistics. As shown through numerical simulations, the proposed approach provides considerable performance gains in scenarios with non-uniform user distributions.

2. SYSTEM DESCRIPTION

We consider a broadcast channel consisting of M antennas at the base station and K single-antenna users in a single cell scenario. Let \mathcal{S} denote an arbitrary set of users with cardinality $|\mathcal{S}| = M$. Given the user set \mathcal{S} scheduled for transmission, the signal received at the k -th user terminal is given by

$$y_k = \mathbf{h}_k \mathbf{w}_k s_k + \sum_{i \in \mathcal{S}, i \neq k} \mathbf{h}_k \mathbf{w}_i s_i + n_k \quad (1)$$

where $\mathbf{h}_k \in \mathbb{C}^{1 \times M}$, $\mathbf{w}_k \in \mathbb{C}^{M \times 1}$, s_k and n_k are the channel vector, the beamforming vector, the transmitted signal, and the additive white Gaussian noise at receiver k , respectively. The first term in the above equation is the useful signal, while the second term corresponds to the interference by the other users. We assume that the variance of the transmitted signal s_k is normalized to one and n_k is independent and identically distributed (i.i.d.) circularly symmetric complex Gaussian with zero mean and variance σ^2 .

The channel is assumed to be perfectly known at the user side. The CSI is transmitted to the base station over a feedback link that is limited to B bits per transmission. Hence, the CSI has to be quantized before it is fed back using a codebook \mathcal{C} with N entries. We assume throughout the paper that $N = 2^B$. Thus, it is possible to feed back every element of the codebook, and the data rate on the feedback link is fully exploited. Strategies that exploit the time correlation of the channel to use larger codebooks with $N > 2^B$ are presented in [15, 16].

The user channels are mapped to the closest codeword in \mathcal{C} , as described by

$$\hat{\mathbf{h}}_k = \arg \min_{\mathbf{c} \in \mathcal{C}} \|\mathbf{h}_k - \mathbf{c}\|^2. \quad (2)$$

Note, that this function also takes the norm of the channel into account unlike the quantization functions used for pure channel direction quantization.

2.1. Linear Beamforming

The beamforming vectors are computed on the basis of the matrix $\hat{\mathbf{H}}$, whose rows are the quantized user channels $\hat{\mathbf{h}}_k$, $k \in \mathcal{S}$. Different linear beamforming techniques may be considered. Commonly applied low-complexity linear beamforming techniques are transmit matched filtering (TxMF) and zero-forcing (ZF) beamforming [4]. Transmit matched filtering uses the normalized columns of $\hat{\mathbf{H}}^H$ as beamforming vectors. Zero-forcing beamforming uses the normalized columns of the pseudo-inverse of $\hat{\mathbf{H}}$.

2.2. User Selection

We consider optimal scheduling throughout the paper, i.e., we do not consider fairness issues between the users. Let \mathcal{Q} be the set of all possible user subsets of cardinality M with disjoint indices in $\{1, \dots, K\}$. The set of users scheduled for transmission at each time slot corresponds to the one that maximizes the estimated sum rate over all possible user sets

$$\hat{\mathcal{S}}^* = \arg \max_{\mathcal{S} \in \mathcal{Q}} \sum_{k \in \mathcal{S}} \log_2(1 + \hat{\text{SINR}}_k). \quad (3)$$

Since the base station has no access to perfect channel state information, the following SINR estimate is computed for the user set \mathcal{S} and k -th user

$$\hat{\text{SINR}}_k = \frac{|\hat{\mathbf{h}}_k \mathbf{w}_k|^2}{\sum_{i \in \mathcal{S}, i \neq k} |\hat{\mathbf{h}}_k \mathbf{w}_i|^2 + \sigma^2} \quad (4)$$

where \mathbf{w}_k denotes the beamforming vector for user k .

3. CHANNEL MODEL

In this section we present the model considered both for the user vector channels and the cell statistics. A non-geometrical stochastic channel is assumed, in which the channel physical parameters are described by probability density functions assuming an underlying geometry. The channel model we propose to use is mainly based on the work in [17], extended to multiuser scenarios. We consider an outdoor environment with NLOS between transmitter and receivers, in which local scatterers, that are randomly distributed around each mobile user, produce a clustering effect. The multipath components (MPC) arrive in clusters in both space and time. For the sake of simplicity, we consider flat fading and hence all paths are assumed to arrive at zero delay. Furthermore, we assume that each user sees MPCs incoming from surrounding scatterers that are grouped into one cluster.

Each user is reached with a different mean angle of departure (AoD) $\bar{\theta}_k$. The AoDs associated to the multipath components are distributed around the mean according to a certain power angular spectrum (PAS), which depends on the spatial distribution of scatterers. In practice, we only consider the azimuth directions (angle of propagation with respect to the antenna array broadside) since the elevation angle spread is generally small compared to the azimuthal angle. Different probability density functions (PDF) are considered in the literature, such as Gaussian, uniform or Laplacian [18]. A summary of the model parameters is given in Table 1.

3.1. User Vector Channels

The signals from the base station (BS) arrive at each user terminal (UT) through a finite number of L paths, which have different AoDs with respect to the antenna array broadside but arrive at the receiver with the same delay. The AoD for the k -th user and l -th path can be expressed as $\theta_{kl} = \bar{\theta}_k + \Delta\theta_{kl}$, where $\bar{\theta}_k$ is the mean AoD for user k and $\Delta\theta_{kl}$ is the angle offset for the l -th multipath component. The multipath components have complex Gaussian distributed gains γ_{kl} with zero mean and unit variance. The channel of user k is given by

$$\mathbf{h}_k = \frac{1}{\sqrt{L}} \sum_{l=1}^L \gamma_{kl} \mathbf{a}(\theta_{kl}) \quad (5)$$

where $\mathbf{a}(\theta_{kl})$ are the steering vectors. An omnidirectional uniform linear array is considered (ULA) although the proposed technique can benefit from any array configuration. The steering vectors $\mathbf{a}(\theta_{kl})$ of a ULA are given by

$$\mathbf{a}(\theta_{kl}) = \left[1, e^{-j2\pi \frac{d \sin \theta_{kl}}{\lambda}}, \dots, e^{-j2\pi \frac{(M-1)d \sin \theta_{kl}}{\lambda}} \right] \quad (6)$$

where λ is the wavelength, and d is the antenna spacing at the BS. The distribution of the angles around the mean AoD is assumed to have a double-sided Laplacian PDF, given by

$$f(\Delta\theta_{kl}) = \frac{1}{\sqrt{2}\sigma_\theta} \exp(-|\sqrt{2}\Delta\theta_{kl}/\sigma_\theta|) \quad (7)$$

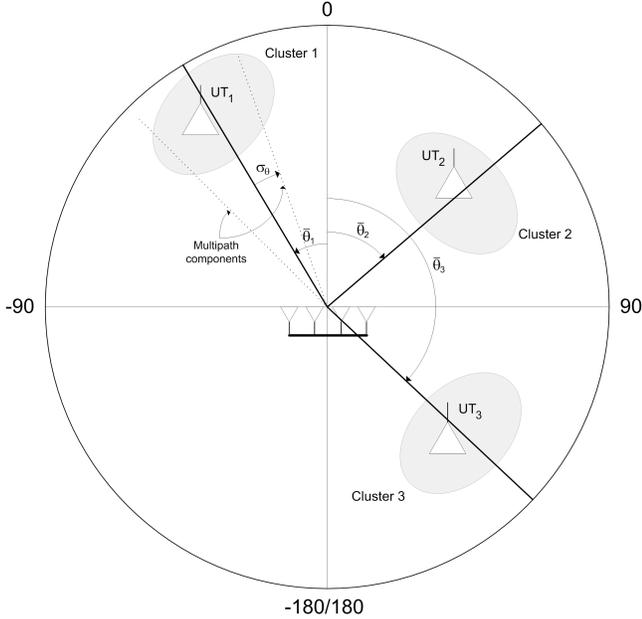


Fig. 1. Broadcast channel model with user terminals (UT) surrounded by local scatterers grouped in clusters, located in different mean angles of departure (AoDs) with respect to uniform linear array (ULA) broadside.

where σ_θ is the angular standard deviation, $\sigma_\theta = \sqrt{E[\Delta\theta_{kl}^2]}$. Under the assumption of using a ULA at the base station, the cross-correlation coefficients of each user's vector channel can be computed in closed form given the PAS, as shown in [19].

3.2. Spatial Cell Statistics

Most papers based on the above mentioned stochastic models assume that mean AoDs are uniformly distributed over all directions. In indoor scenarios, the relative cluster AoD is indeed uniformly distributed over $[0, 2\pi]$, as it has been seen from channel measurements [19], since the location of cluster centers is uniformly distributed over the cell. However, as noted in [19], this is not realistic in outdoor scenarios where the base station is elevated and the mobile stations are often surrounded by local scatterers. In these cases, the mean AoD is very dependent on the macroscopic characteristics of each particular scenario: topology, user distribution, mobility pattern, distribution of scatterers, etc. Hence, the mean AoDs for all users, $\bar{\theta}_k$, do not need to be uniformly distributed over the interval $[0, 2\pi]$. In our model, they are considered to be uniformly distributed over an arbitrary range of angles $\bigcup_i [\bar{\theta}_{\min_i}, \bar{\theta}_{\max_i}]$. A graphical representation of the broadcast channel model is depicted in Fig. 1.

4. CODEBOOK DESIGN

We present in this section the design of the user channel codebook. Compared to existing design approaches [5] we rely on a pure Monte Carlo based approach. This approach allows a wider range of distortion functions than the commonly used generalized Lloyd algorithm, and it also allows to exploit the cell statistics.

As discussed in [20], most techniques relying on limited channel state information consider separate feedback bits (and thus separate

quantization) for channel direction information (CDI) and channel quality information (CQI). Since the amount of feedback is limited, a tradeoff arises between the amount of bits used for CDI quantization, which has an impact on the multiplexing gain, and the amount of bits used for CQI quantization, which has an impact on the multiuser diversity gain achieved from user selection. In this work, we consider joint quantization of CDI and CQI information. Channel quantization is done directly over the user vector channels rather than quantizing the norm and channel direction separately, thus providing better granularity. Hence, since the proposed channel quantization is adapted to the cell statistics, including the average SNR conditions and number of active users, the tradeoff between multiplexing gain and multiuser diversity is implicitly optimized.

The proposed approach consists of designing a channel quantization codebook valid for all users in the cell by minimizing the average sum-rate distortion of the scheduled users. Since scheduling and beamforming are performed jointly at each time slot, the distortion measure needs to account for both jointly. Hence, different linear beamforming techniques will result in different optimized codebooks. This criterion yields quantization codebooks that are statistically matched to the users that maximize the estimated sum rate, which are selected as described in (3). The quantization codebook is optimized during an initial training period, after which the codebook is fixed and broadcasted to the users.

4.1. Design Criterion

The codebook of N codewords, is found by solving the optimization problem

$$\mathcal{C}^* = \arg \min_{\mathcal{C}} E[d(\mathcal{H}, \hat{\mathcal{H}})] \quad (8)$$

where $d(\mathcal{H}, \hat{\mathcal{H}})$ is the distortion measure between the set containing the unquantized user channels $\mathcal{H} = \{\mathbf{h}_1, \dots, \mathbf{h}_K\}$ and the set containing the quantized user channels $\hat{\mathcal{H}} = \{\hat{\mathbf{h}}_1, \dots, \hat{\mathbf{h}}_K\}$.

The distortion measure used throughout the paper is the sum-rate loss due to the channel quantization. The resulting codebook depends on the number of scheduled users M for transmission, the number of active users K in the cell, the used beamforming technique, and the channel statistics. The distortion measure can be described as

$$d(\mathcal{H}, \hat{\mathcal{H}}) = \text{SR}(\mathcal{H}) - \text{SR}(\hat{\mathcal{H}}). \quad (9)$$

The first term in the equation above corresponds to the maximum sum rate that can be achieved with the chosen linear beamforming technique and perfect channel state information, given by

$$\text{SR}(\mathcal{H}) = \max_{\mathcal{S} \in \mathcal{Q}} \sum_{k \in \mathcal{S}} \log_2(1 + \text{SINR}_k). \quad (10)$$

The beamforming vectors and the user set obtained in the case of perfect channel state information are in general different than the ones obtained on the basis of quantized channel information for a given time slot. The second term in (9) corresponds to the actual sum rate achieved by the system. The beamforming vectors are computed on the basis of the quantized channels and the users scheduled for transmission are selected as described in (3). Hence, the achieved sum rate is given by

$$\text{SR}(\hat{\mathcal{H}}) = \sum_{k \in \mathcal{S}^*} \log_2(1 + \text{SINR}_k). \quad (11)$$

Note that, as opposed to the estimated SINR values employed for user selection, the above equation computes the effective SINR experienced by each of the users in the scheduled set $\hat{\mathcal{S}}^*$.

4.2. Codebook Design

We are using a Monte Carlo based codebook design algorithm to generate the channel quantization codebooks. The ability of this algorithm to work with arbitrary distortion functions makes it a prime candidate to solve (8).

The Monte Carlo codebook design algorithm generates random codebooks having the same distribution as the channel. For every one of these random codebooks the average distortion is estimated by averaging over a large number of channel realizations. Finally, the codebook with the lowest average distortion is kept. This codebook minimizes the long term sample average distortion, and thus, provides a good solution to (8).

An alternative procedure consists of using the generalized Lloyd algorithm [21] to iteratively find the optimizing codebook and partition cells. However, the Monte Carlo codebook design avoids convergence to local minima exhibited by Lloyd's algorithm, and thus provides a better performance if the number of tried codebooks is sufficiently high. A codebook design that is more similar to the Monte Carlo based codebook design is random coding [22]. However, random coding just uses N random channel realizations as codebook, and does not allow to optimize an arbitrary distortion function.

4.3. Practical Considerations

The proposed technique for codebook design is expected to perform better in scenarios with strong spatial correlations. Different linear beamforming techniques will yield different performances, since quantization errors affect them differently. For instance, while TxMF and ZF beamforming exhibit similar behavior for a given error variance, optimized unitary beamforming proves to be very robust [23].

Since the statistics of the best M users govern the design, the quantization codebooks may favor certain spatial locations or directions that provide good sum rates, favoring the users in those particular locations. In a system with low mobility and slow variations, this situation may lead to a fairness issue. This behavior may be accentuated when incorporating shadowing and pathloss to the channel model. This effect can be attenuated by performing proportional fair scheduling (PFS), which would yield an average distortion function based on a weighted sum rate, penalizing the users that have already been scheduled.

Instead of simply generating the quantization codebooks during a training period, the base station may slowly adapt the codebook to changes in the environment: changes in traffic and mobility patterns, changes of scatterers, etc. Each time a user enters the system or in case there is a codebook update, the base station would send the updated codebook to the users, which in general changes from cell to cell. In addition, similarly to the work presented in [16] for single-user MIMO communications, the amount of feedback can be reduced by exploiting temporal correlations in the system.

5. LOW-COMPLEXITY BEAMFORMING AND SCHEDULING

The limited channel knowledge at the transmitter side deteriorates the achievable performance of the system, but can also be exploited

to reduce the computational load for beamforming and scheduling. The quantization of the user channels creates equivalence classes between the users. The users whose channels are quantized to the same entry in the codebook are members of the same equivalence class. Thus, the base station only knows which class a user belongs to, but it cannot distinguish between the users in the same class. It is thus sufficient to do the beamforming and the scheduling only based on the representative of the class, i.e., the codeword, instead of based on all the users in the class. We denote a set that consists of M representatives of different classes as a class set. The number of class sets to be considered for beamforming and scheduling $N_{CS} = N^M$ is smaller than the number of user sets $N_{US} = \binom{K}{M}$ for practical system parameters with $N \ll K$. Once the optimal class set is determined, a corresponding user set can be selected by choosing for every class in the class set a corresponding user. The user inside a specific class can be selected randomly or using a fairness constraint.

The complexity of determining the beamforming vectors and the class sets can be further reduced using a lookup table that stores for all the class sets the corresponding sum rate estimates and the beamforming vectors. We assume that this lookup table is sorted based on the estimated sum rate of the class sets, where the first entry contains the class set with the highest estimated sum rate. After the base station received the feedback from all the users, it checks if it has a matching user for every entry in the first class set. If not, then the base station does the same check for the following class sets in the lookup table until it finds a class set that has for every class in the class set an active user. The advantage of using precalculated beamforming vectors stored in a look-up table is that computational more complex beamforming schemes can be used, e.g., Dirty Paper Coding. Note that TxMF is a special case since there the different beamforming vectors are independent of the other users that are scheduled for transmission. Hence, the storage of only N beamforming vectors is sufficient.

The storage requirements for the lookup table can be reduced by storing only the most probable class sets. The probability that the first class set is selected increases with the number of users in the cell. For the event that no class set in the lookup table is selected, a user set can still be calculated using a low-complexity scheme, e.g., TxMF beamforming.

6. SIMULATION RESULTS

We compare the performance of linear beamforming with quantized CSI feedback to LF-OSDMA. The used linear beamforming strategies are ZF and TxMF. We assume a 2-GHz system with an antenna spacing at the base station of $d = 0.4\lambda \approx 15$ cm. Each user channel is modelled with $L = 10$ multipath components. The mean AoD of the different users is uniformly distributed over the interval $[60^\circ, 120^\circ]$, and the angular spread is fixed to $\sigma_\theta = 30^\circ$. We assume single-antenna users and a base station with $M = 2$ antennas. The data rate on the feedback link is limited to 3 bits/transmission. In order to make a fair comparison between the schemes, the SINR feedback of the LF-OSDMA algorithm is also quantized. Thus, the LF-OSDMA algorithm has to share the available 3 bits between the CDI, i.e., the index of the preferred beamforming vector, and the CQI, i.e., the SINR of the preferred beamforming vector. We simulate the performance of all possible CDI/CQI bit allocations, and finally select the allocation that results in the highest sum rate. The codebook to quantize the scalar CQI is designed with the generalized Lloyd algorithm [22], using the mean square error as distortion function. The performance of the different random codebooks, i.e., their resulting average sum rate, is estimated through averaging over

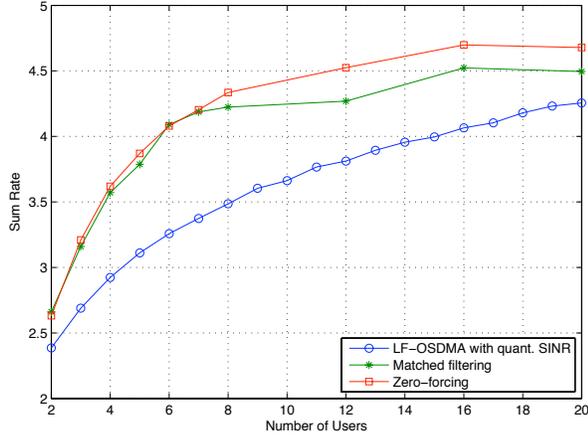


Fig. 2. Sum rate for the correlated channel model for different numbers of users. ($M = 2$, $\text{SNR}=10$ dB)

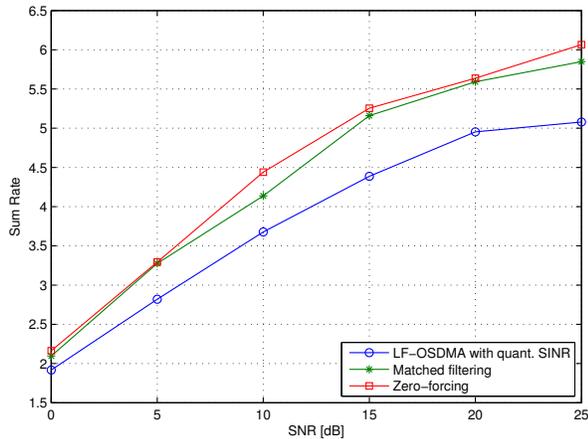


Fig. 3. Sum rate for the correlated channel model for different SNR's. ($M = 2$, $K = 10$)

the instantaneous sum rate of 10 000 channel realizations.

Fig. 2 depicts the performance for different numbers of users with a fixed SNR of 10 dB. We see that ZF and TxMF with quantized CSI outperform LF-OSDMA with quantized SINR feedback. The same result can be seen in Fig. 3 for different SNR values and $K = 10$ users. We see how the sum rate of the different schemes saturates at high SNR, where the performance is limited by the quantization error.

7. CONCLUSION

The problem of designing channel quantization codebooks for correlated broadcast channels with limited feedback has been addressed for systems where joint linear beamforming and multiuser scheduling is performed. The numerical results provided have shown the benefits of using quantization codebooks optimized according to the cell statistics. The generated codebooks perform well in scenarios with reduced angular spread and effective range of mean angles of departure. This makes the proposed approach particularly interesting in outdoor systems with spatial correlation and nonuniform user distribution.

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