Amplify-and-Forward Capacity with Transmit Beamforming for MIMO Multiple-Relay Channels

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***Abstract*—In this paper, we consider a MIMO wireless relay network where communication between a source and a destination node is assisted by multiple relay nodes using the Amplify-and-Forward (AF) strategy. We consider a system where the source node multiplexes the information into parallel streams and uses disjoint subsets of relays to assist the transmission of each stream. Furthermore, we assume that the source and the destination have the same number of antennas and that each transmit antenna is virtually paired to a different destination antenna. We consider source node channel side information (CSI) availability of just the source-relay and source-destination links, and propose a source beamforming method based on maximizing the Sum of Signal-to-Leakage Ratios (SSLR) for all of the data streams. This method maximizes the sum of signal strengths at the set of relay nodes assigned to that stream while suppressing the interference to other relay nodes. Maximizing the SSLR reduces the propagation of interference terms in the AF strategy and results in significant performance gains.**

I. Introduction

Recently, cooperative diversity has captured significant attention in the research community due to its provision of spatial diversity gain for the wireless network in a distributed fashion. Simply stated, in cooperative communications, the source node cooperates with one or more partners to transmit its information to the destination node. This scheme provides a virtual transmit antenna array which results in a more reliable transmission towards the destination node compared to transmitting the stream alone, without any cooperation [2], [3]. To increase the channel capacity, several cooperation modes involving relay nodes have been proposed in the literature [2], [4]. Among them, amplify-and-forward (AF) and decode-and-forward (DF) are two of the fundamental relaying modes. In this paper, we only consider the AF relaying mode. In this mode, the relay nodes simply amplify the received signal according to a power constraint and forward the amplified version of the signal to the destination node.

A significant body of work has appeared in the literature on cooperative communications. Distributed space-time code design and information-theoretic performance limits for single antenna fading relay channels (with a finite number of nodes) have recently been studied in [2], [4], [5], [6]. Capacity results for relay MIMO channels with a finite number of relays can be found in [5], [7], [8]. For the AF relaying mode, ergodic and outage capacities of the fading relay channel was investigated in [6], [9] when each node was equipped with a single antenna. A general framework on the capacity of MIMO relay channel has been proposed in [7]. In [10], the impact of cooperation of multiple relays using the AF strategy on the capacity of rank-deficient MIMO channels has been studied with no CSI available at the transmitter.

In this paper, we investigate a wireless network where the source node multiplexes the information to a number of parallel streams and assigns each stream to a disjoint subset of available relays for cooperation. A multi-user beamforming (MU-BF) method is proposed via which all available spatial degrees of freedom is utilized. As such, each stream is multiplied by a corresponding BF vector so that the average Signal-to-Interference-plus-Noise Ratios (SINR) for all relay nodes are maximized. It has been shown in [11] that MU-MIMO downlink system capacity can be improved by a BF algorithm which is based on maximizing the Signal-to-Leakage Ratio (SLR) for each data stream intended for different receivers. It should be noted here that this objective function imposes no restrictions on the number of transmit antennas and thus, is very convenient for use in wireless networks.

Here, building on the work presented in [1] we propose a MU-BF scheme for the network under investigation. We select the transmit BF vectors so as to maximize, for each data stream, the ratio of the sum of that steam's power at intended relays to the sum of that stream's power at non-intended relays. This way, we not only mitigate the interference observed by each relay node but also aim to prevent the propagation of the interference terms to the destination node. It is seen that with the proposed MU-BF scheme, the AF mode potentially benefits from transmit array gains. In this paper, we provide capacity expressions as a function of the number of available relay nodes and observe that increasing the number of cooperating relays results in array gains. This is because, with more relay nodes cooperating, each data stream is relayed over an increased set of relays, and consequently an increase in the average SINR is observed at the destination node.

II. Protocol Description and System Model

A wireless network where a single source node, $(\mathcal{S})$, transmits data to a single destination node, $(\mathcal{D})$, with the cooperation of K relay nodes, $\mathcal{R}_k, k \in \{1, 2, \ldots, K\}$, is assumed. All nodes are assumed to be equipped with multiple antennas. The source and destination nodes are assumed to have N and the relay nodes are assumed to have M antennas, respectively. We assume that the available channel is divided into two orthogonal sub-channels in the time domain. The source node communicates with the relay nodes as well as the destination node in the first time slot. In the second time slot, only the relay nodes communicate with the destination node.

We assume that the channels, $\mathcal{S} \rightarrow \mathcal{D}$, $\mathcal{S} \rightarrow \mathcal{R}_k$ and $\mathcal{R}_k \rightarrow \mathcal{D}$, for $k \in \{1, 2, \ldots, K\}$, represented by the channel matrices, $\mathbf{H}_D$, $\mathbf{H}_k$ and $\mathbf{G}_k$, respectively, are Rayleigh block fading channels where the channels remain constant for the duration of two consecutive time slots. The channels are assumed to be independent each other and between two consecutive time slots. The system performance is investigated in Section-IV for two scenarios: (i). perfect forward and backward CSI availability at the relay nodes and full CSI availability at the destination, and (ii). additionally, perfect CSI availability of $\mathcal{S} \rightarrow \mathcal{D}$ and $\mathcal{S} \rightarrow \mathcal{R}_k$ channels at the source node. The proposed MU-BF requires CSI availability at the source node.

The system model considered in this paper is illustrated in Fig.1. In the first time slot, the source node transmits the symbol vector $\mathbf{x} \in \mathbf{C}^{N\times 1}$ with the covariance matrix† $\mathbf{Q}_s = \mathcal{E}[\mathbf{x}\mathbf{x}^H]$ satisfying $E_s = tr(\mathbf{Q}_s)$. In the second time slot, the cooperating relays transmit the symbol vectors $\mathbf{t}_k \in \mathbf{C}^{M\times 1}$ with covariance matrices $\mathbf{Q}_{r_k} = \mathcal{E}[\mathbf{t}_k\mathbf{t}_k^H]$ satisfying $E_{r_k} = tr(\mathbf{Q}_{r_k})$.

A. Signal Models at Destination and Relay Nodes

We consider a wireless network where K relays, located randomly and independently in a fixed area, are available for the communication of a source node with a destination node. Unlike [1], we assume that a direct link between the source node and the destination node is available. As common with all cooperative diversity woth in the literature, we assume perfect synchronization between all nodes during transmissions and receptions.

In the first time slot, the input-output relations for $\mathcal{S} \rightarrow \mathcal{D}$ and $\mathcal{S} \rightarrow \mathcal{R}_k$ links are given by, respectively,

$$\begin{aligned} \mathbf{y}_1 &= \mathbf{H}_D\mathbf{x} + \mathbf{n}_{d,1}, \\ \mathbf{r}_k &= \mathbf{H}_k\mathbf{x} + \mathbf{n}_k \end{aligned} \qquad (1)$$

where $\mathbf{y}_1$ and $\mathbf{r}_k$ denote the $N \times 1$ and $M \times 1$ received vector signals, $\mathbf{H}_D = [\mathbf{h}_{d,1}\mathbf{h}_{d,2}\ldots\mathbf{h}_{d,N}]^T$ denotes the $N \times N$ matrix for the $\mathcal{S} \rightarrow \mathcal{D}$ link. $\mathbf{H}_k = [\mathbf{h}_{k,1}\mathbf{h}_{k,2}\ldots\mathbf{h}_{k,N}]$ is the $M \times N$ random channel matrix corresponding to the $\mathcal{S} \rightarrow \mathcal{R}_k$ link, consisting of i.i.d. $\mathcal{CN}(0, 1)$ entries, $\mathbf{x} = [x_1^T x_2^T \ldots x_N^T]^T$ is assume to be a zero mean $N \times 1$ circularly symmetric complex Gaussian transmit signal vector satisfying $\mathcal{E}[\mathbf{x}\mathbf{x}^H] = (E_s/N)\mathbf{I}_N$, and $\mathbf{n}_{d,i}$, $\mathbf{n}_k$ are $N\times 1$ and $M\times 1$ spatio-temporally white circularly symmetric complex Gaussian noise vector sequences with the covariance matrices $\mathcal{E}[\mathbf{n}_{d,i}\mathbf{n}_{d,i}^H] = \sigma_n^2\mathbf{I}_N$ for $i = \{1, 2\}$, and $\mathcal{E}[\mathbf{n}_k\mathbf{n}_k^H] = \sigma_{r_k}^2\mathbf{I}_M$ for $k \in \{1, 2, \ldots, K\}$.

†Throughout the paper, the superscripts ^{T}, * and ^{H} stand for transposition, element-wise conjugate and conjugate transposition, respectively

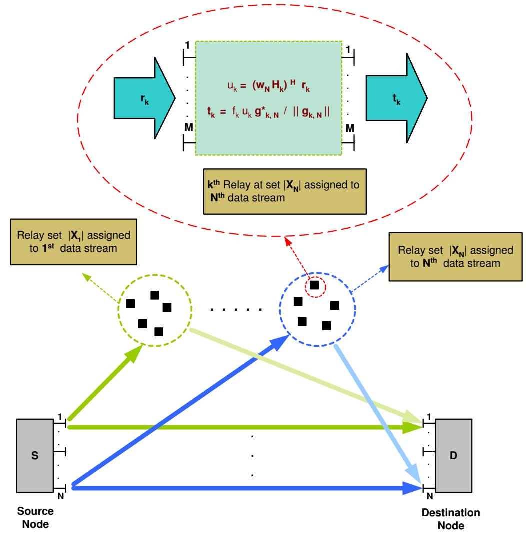

Fig. 1. MIMO multiple relay scenario for the proposed AF mode transmission where each relay set is assigned to only one data stream for assistance.

In the second time slot, all of the cooperating relays transmit and the source remains idle. Each relay processes its received vector signal $\mathbf{r}_k$ to produce the $M \times 1$ vector signal $\mathbf{t}_k$, which is then transmitted to the destination. The $N \times 1$ vector signal, received at the destination node given by,

$$\mathbf{y}_2 = \sum_{k=1}^{K} \mathbf{G}_k\mathbf{t}_k + \mathbf{n}_{d,2} \qquad (2)$$

where $\mathbf{G}_k = [\mathbf{g}_{k,1}\mathbf{g}_{k,2}\ldots\mathbf{g}_{k,N}]^T$ is the corresponding $N \times M$ channel matrix for the $\mathcal{R}_k \rightarrow \mathcal{D}$ link with i.i.d. $\mathcal{CN}(0, 1)$ entries.

B. The Proposed AF Mode for Relay Networks

In this paper, we investigate a wireless network where the source node multiplexes the information to be sent into parallel streams. Each stream is virtually paired with one of the antennas at the destination node. Assisting relays are partitioned disjointly so that each subset is concerned with relaying only one of the streams to the destination node. As seen in Fig. 1, we assume that a direct link between the source and the destination nodes exists. The set of relays assigned to the i^{th} data stream is denoted by $\mathcal{X}_i$, and the cardinality of this set is denoted by $|\mathcal{X}_i|$. During the first time slot, the source broadcasts data streams and the relays as well as the destination node listen. In the second time slot, the source becomes idle and the relays amplify-and-forward their corresponding data streams. The destination node combines the signals from the source as well as the relay nodes using Maximum Ratio Combining (MRC) and performs independent decoding at each of its antennas, i.e., the receiver antenna of a virtual transmit-receiver antenna pair assumes that the non-intended streams to this antenna are interference and performs decoding accordingly. The relays are assumed to have both forward and backward CSI available to them.

III. MU-BF WITH SSLR MAXIMIZATION

The main purpose in MU-BF is to minimize the interference terms at each cooperating relay node. When the proposed sum of signal-to-leakage ratio (SSLR) based MU-BF is employed, the interference observed at each of the cooperating relays is minimized, resulting in an overall gain in the system performance. This is because, the reduced interference at the relays corresponds to a reduced propagation of this interference by the AF scheme. The performance metric used in this paper is based on [11] where BF vectors are formulated to maximize SLR for each data stream, which results in minimizing the interference terms caused by this stream to non-intended relay nodes.

Let us express the transmitted signal by,

$$\mathbf{x} = \sum_{n=1}^{N} \mathbf{w}_n s_n = [\mathbf{w}_1 \mathbf{w}_2 \cdots \mathbf{w}_N] \ \mathbf{s} = \mathbf{W}\mathbf{s} \quad (3)$$

where $\mathbf{s} = [s_1, s_2, \ldots, s_N]^T$ is the vector of parallel data streams with s_n denoting the stream intended for the relay set $\mathcal{X}_n$. s_n is assumed to satisfy $\mathcal{E}[|s_n|^2] = E_n = E_s/N$ for $n \in \{1, 2, \ldots, N\}$. In (3), $\mathbf{w}_n$ represents unit norm $N \times 1$ beamforming vector.

Let us use the $\mathcal{S} \rightarrow \mathcal{D}_l$ notation to represent the link from the source node to the destination node's lth antenna. Then, in the first time slot, the beamforming vector is set so that the lth data stream is successfully received by both a set of pre-defined relays and the lth antenna of the destination. The metric in setting the beamforming vector entries is the maximization of the ratio of the sum of the power at intended relays and the interference power at all other cooperating relays for each data stream. We define this metric as the *maximum SSLR* criteria.

The total power of lth data stream at the assigned set of relays, $\mathcal{X}_l$, and at the lth antenna of the destination is given by

$$\sum_{k, \mathcal{R}_k \in \mathcal{X}_l} \|\mathbf{H}_k \mathbf{w}_l\|^2 + |\mathbf{h}_{d,l}^T \mathbf{w}_l|^2 \quad (4)$$

and the sum of interference powers seen by all of the non-intended relays can be expressed as,

$$\sum_{k, \mathcal{R}_k \notin \mathcal{X}_l} \|\mathbf{H}_k \mathbf{w}_l\|^2 + \sum_{i=1, i\neq l}^{N} |\mathbf{h}_{d,i}^T \mathbf{w}_l|^2. \quad (5)$$

Then, the SSLR expression for lth data stream is given by,

$$SSLR_l = \frac{\sum_{k, \mathcal{R}_k \in \mathcal{X}_l} \|\mathbf{H}_k \mathbf{w}_l\|^2 + |\mathbf{h}_{d,l}^T \mathbf{w}_l|^2}{\sum_{k, \mathcal{R}_k \notin \mathcal{X}_l} \|\mathbf{H}_k \mathbf{w}_l\|^2 + \sum_{i=1, i\neq l}^{N} |\mathbf{h}_{d,i}^T \mathbf{w}_l|^2}. \quad (6)$$

The problem statement is then to design $\mathbf{w}_l$ such that the SSLR is maximized for every data stream:

$$\mathbf{w}_l^{max} = \arg \max_{\mathbf{w}_l \in C^{N\times 1}} \{SSLR_l\} \quad (7)$$

subject to the constraints, $\|\mathbf{w}_l\|^2 = 1$, $l \in \{1, 2, \ldots, N\}$. We can rewrite the expression in (6) as follows:

$$SSLR_l = \frac{\mathbf{w}_l^H \overline{\mathbf{H}}_l^H \overline{\mathbf{H}}_l \mathbf{w}_l}{\mathbf{w}_l^H \widetilde{\mathbf{H}}_l^H \widetilde{\mathbf{H}}_l \mathbf{w}_l} \quad (8)$$

where $\overline{\mathbf{H}}_l$ and $\widetilde{\mathbf{H}}_l$ are given by

$$\overline{\mathbf{H}}_l = [\mathbf{H}_{\mathbf{u}_l(1)}^H \ \mathbf{H}_{\mathbf{u}_l(2)}^H \ldots \mathbf{H}_{\mathbf{u}_l(|\mathcal{X}_l|)}^H \ \mathbf{h}_{d,l}^*]^H \quad (9)$$

and

$$\widetilde{\mathbf{H}}_l = [\mathbf{H}_{\mathbf{v}_l(1)}^H \ \mathbf{H}_{\mathbf{v}_l(2)}^H \ldots \mathbf{H}_{\mathbf{v}_l(K-|\mathcal{X}_l|)}^H \ \mathbf{h}_{d,1}^* \ldots \mathbf{h}_{d,l-1}^* \ \mathbf{h}_{d,l+1}^* \ldots \mathbf{h}_{d,N}^*]^H \quad (10)$$

and $\mathbf{u}_l$ is a $|\mathcal{X}_l| \times 1$ array that contains the indices of the relays assigned to the lth data stream and $\mathbf{v}_l$ is a $(K - |\mathcal{X}_l|) \times 1$ array that contains the indices of the relays that are in the compliment of the set $\mathcal{X}_l$. It is easy to see that (8) has the form of generalizes eigenvalue problem with the following inequality:

$$\frac{\mathbf{w}_l^H \overline{\mathbf{H}}_l^H \overline{\mathbf{H}}_l \mathbf{w}_l}{\mathbf{w}_l^H \widetilde{\mathbf{H}}_l^H \widetilde{\mathbf{H}}_l \mathbf{w}_l} \leq \lambda_{max} \left(\overline{\mathbf{H}}_l^H \overline{\mathbf{H}}_l, \widetilde{\mathbf{H}}_l^H \widetilde{\mathbf{H}}_l \right) \quad (11)$$

where λ_{max} is the largest generalized eigenvalue of the matrix pair $\overline{\mathbf{H}}_l^H \overline{\mathbf{H}}_l$ and $\widetilde{\mathbf{H}}_l^H \widetilde{\mathbf{H}}_l$. Equality occurs if $\mathbf{w}_l$ is proportional to a generalized eigenvector that corresponds to the largest generalized eigenvalue.

IV. COHERENT AF MODE MULTI-RELAY NETWORK CAPACITY

We now derive the capacity expression for the wireless network under investigation, illustrated in Fig. 1. In the transmitted signal definition given in (3), for the case where there is no CSI available at the source, no beamforming is possible. Therefore, the BF matrix will simply be $\mathbf{W} = \mathbf{I}_N$. On the other hand, when CSI is available at the source node, one can calculate $\mathbf{W}$ using (7). So, the capacity expressions for both cases are the same except for the BF matrix entries.

In first time slot, the received signal terms at the destination and kth relay nodes, given in (1) can be rewritten as

$$\begin{aligned} \mathbf{y}_1 &= \mathbf{H}_D \sum_{i=1}^{N} \mathbf{w}_i s_i + \mathbf{n}_{d,1}, \\ \mathbf{r}_k &= \mathbf{H}_k \sum_{i=1}^{N} \mathbf{w}_i s_i + \mathbf{n}_k = \mathbf{H}_k \mathbf{w}_l s_l + \mathbf{H}_k \sum_{\substack{i=1 \\ i\neq l}}^{N} \mathbf{w}_i s_i + \mathbf{n}_k. \end{aligned} \quad (12)$$

Assuming that the kth relay is assigned to the lth transmit-receive antenna pair, upon the reception of $\mathbf{r}_k$, the relay, using the available backward channel CSI, performs matched filtering to obtain,

$$\begin{aligned} u_k &= \mathbf{w}_l^H \mathbf{H}_k^H \mathbf{r}_k \\ &= \|\mathbf{H}_k \mathbf{w}_l\|^2 s_l + (\mathbf{H}_k \mathbf{w}_l)^H \mathbf{H}_k \sum_{\substack{i=1 \\ i\neq l}}^{N} \mathbf{w}_i s_i + (H_k w_l)^H \mathbf{n}_k \end{aligned} \quad (13)$$

for $k \in \{1, 2, \ldots, K\}$. Using the constraint on the relay transmit power, the received signal is scaled by,

$$f_k = \sqrt{\frac{E_{r_k}}{\|\mathbf{H}_k\mathbf{w}_l\|^4 E_l + \sum_{\substack{i=1\\i\neq l}}^{N} |(\mathbf{H}_k\mathbf{w}_l)^H\mathbf{H}_k\mathbf{w}_i|^2 E_i + \|(\mathbf{H}_k\mathbf{w}_l)^H\|^2\sigma_{r_k}^2}} \tag{14}$$

at each relay node, and assuming forward CSI availability at the relay nodes, the scaled signal is assumed to be beamformed to the destination node by each relay using a unit-norm BF vector corresponding to the $\mathcal{R}_k \rightarrow \mathcal{D}_l$ link. Then, the kth relay's transmitted signal can be written as,

$$\mathbf{t}_k = f_k\, u_k \frac{\mathbf{g}_{k,l}^*}{\|\mathbf{g}_{k,l}\|} \tag{15}$$

The signal received at the ith receiver antenna of the destination node at the first and second time-slots, $y_{1,i}$ and $y_{2,i}$, respectively, can be written as,

$$\begin{aligned} y_{1,i} &= \mathbf{h}_{d,i}^T\mathbf{w}_i s_i + \mathbf{h}_{d,i}^T \sum_{j=1,j\neq i}^{N} \mathbf{w}_j s_j + n_{1,i} \\ y_{2,i} &= \sum_{n=1}^{N}\left(\sum_{k,\mathcal{R}_k\in\mathcal{X}_n} \mathbf{g}_{k,i}^T\mathbf{t}_k\right) + n_{2,i}. \end{aligned} \tag{16}$$

Using (13), (14) and (15), $y_{2,i}$ can be rewritten as,

$$\begin{aligned} y_{2,i} &= \sum_{k,\mathcal{R}_k\in\mathcal{X}_i} \mathbf{g}_{k,i}^T\mathbf{t}_k + \sum_{\substack{n=1\\n\neq i}}^{N} \sum_{k,\mathcal{R}_k\in\mathcal{X}_n} \mathbf{g}_{k,i}^T\mathbf{t}_k + n_{2,i} \\ &= \sum_{k,\mathcal{R}_k\in\mathcal{X}_i} \|\mathbf{g}_{k,i}\| f_k u_k + \sum_{\substack{n=1\\n\neq i}}^{N} \sum_{k,\mathcal{R}_k\in\mathcal{X}_n} \frac{\mathbf{g}_{k,i}^T\mathbf{g}_{k,n}^*}{\|\mathbf{g}_{k,n}\|} f_k u_k + n_{2,i} \\ &= \Bigg(\sum_{k,\mathcal{R}_k\in\mathcal{X}_i} \|\mathbf{g}_{k,i}\| f_k \|\mathbf{H}_k\mathbf{w}_i\|^2 \\ &\quad + \sum_{\substack{n=1\\n\neq i}}^{N} \sum_{k,\mathcal{R}_k\in\mathcal{X}_n} \frac{\mathbf{g}_{k,i}^T\mathbf{g}_{k,n}^*}{\|\mathbf{g}_{k,n}\|} f_k (\mathbf{H}_k\mathbf{w}_n)^H\mathbf{H}_k\mathbf{w}_i\Bigg) s_i \\ &\quad + \sum_{\substack{j=1\\j\neq i}}^{N} \Bigg(\sum_{k,\mathcal{R}_k\in\mathcal{X}_i} \|\mathbf{g}_{k,i}\| f_k (\mathbf{H}_k\mathbf{w}_i)^H\mathbf{H}_k\mathbf{w}_j \\ &\quad + \sum_{k,\mathcal{R}_k\in\mathcal{X}_j} \frac{\mathbf{g}_{k,i}^T\mathbf{g}_{k,j}^*}{\|\mathbf{g}_{k,j}\|} f_k \|\mathbf{H}_k\mathbf{w}_j\|^2 + \\ &\quad \sum_{\substack{m=1\\m\neq\{i,j\}}}^{N} \sum_{k,\mathcal{R}_k\in\mathcal{X}_m} \frac{\mathbf{g}_{k,i}^T\mathbf{g}_{k,m}^*}{\|\mathbf{g}_{k,m}\|} f_k (\mathbf{H}_k\mathbf{w}_m)^H\mathbf{H}_k\mathbf{w}_j\Bigg) s_j + N_{2,i} \end{aligned} \tag{17}$$

which can be expressed compactly as,

$$y_{2,i} = h_i^{sig} s_i + \sum_{\substack{j=1\\j\neq i}}^{N} h_{i,j}^{int} s_j + N_{2,i} \tag{18}$$

where h_i^{sig} denotes the effective scalar channel gain for the data stream transmitted from the ith source node antenna, $h_{i,j}^{int}$ is the effective channel seen by the interfering stream, s_j, and $N_{2,i}$ denotes the effective noise term. This term is given by,

$$\begin{aligned} N_{2,i} &= \sum_{k,\mathcal{R}_k\in\mathcal{X}_i} \|\mathbf{g}_{k,i}\| f_k (\mathbf{H}_k\mathbf{w}_i)^H\mathbf{n}_k \\ &\quad + \sum_{\substack{n=1\\n\neq i}}^{N} \left(\sum_{k,\mathcal{R}_k\in\mathcal{X}_n} \frac{\mathbf{g}_{k,i}^T\mathbf{g}_{k,n}^*}{\|\mathbf{g}_{k,n}\|} f_k (\mathbf{H}_k\mathbf{w}_n)^H\mathbf{n}_k\right) + n_{2,i}. \end{aligned} \tag{19}$$

We assume that all noise vectors have the same covariance matrices, $E[\mathbf{n}_k\mathbf{n}_k^H] = \sigma_r^2\mathbf{I}_M$ for $k \in \{1, 2, \ldots, K\}$. Furthermore, we assume that the received signals at each antenna of destination node during the first and the second time slots are Maximum Ratio Combined (MRC) and therefore, the post-combining SINR is simply the summation of the SINRs at each time slot. The SINRs observed at the first and second time slots, $\Gamma_{1,i}$, and $\Gamma_{2,i}$, respectively, for the ith antenna, can be written as,

$$\Gamma_{1,i} = \frac{|\mathbf{h}_{d,i}^T\mathbf{w}_i|^2 E_i}{\sum_{j=1,j\neq i}^{N} |\mathbf{h}_{d,i}\mathbf{w}_j|^2 E_j + \sigma_n^2} \tag{20}$$

$$\Gamma_{2,i} = \frac{|h_i^{sig}|^2 E_i}{\sum_{\substack{j=1\\j\neq i}}^{N} |h_{i,j}^{int}|^2 E_j + \sigma_r^2\left(\sum_{k,\mathcal{R}_k\in\mathcal{X}_i} a_{k,i} + \sum_{\substack{n=1\\n\neq i}}^{N} \sum_{k,\mathcal{R}_k\in\mathcal{X}_n} b_{k,i,n}\right) + \sigma_n^2} \tag{21}$$

where $a_{k,i}$ and $b_{k,i,n}$ are given by

$$\begin{aligned} a_{k,i} &= f_k^2\|\mathbf{g}_{k,i}\|^2\|\mathbf{H}_k\mathbf{w}_i\|^2 \\ b_{k,i,n} &= \frac{f_k^2\|\mathbf{H}_k\mathbf{w}_n\|^2|\mathbf{g}_{k,i}^T\mathbf{g}_{k,n}^*|^2}{\|\mathbf{g}_{k,n}\|^2}. \end{aligned} \tag{22}$$

Finally, we can write the network capacity of the proposed relay network as follows,

$$C = \frac{1}{2}\sum_{i=1}^{N} \mathcal{E}_{\{\mathbf{H}_D,\{\mathbf{H}_k,\mathbf{G}_k\}_{k=1}^K\}}\Big\{\log_2\left(1 + \Gamma_{1,i} + \Gamma_{2,i}\right)\Big\}. \tag{23}$$

Here, we use the fact that perfect CSI is available at the destination node which results in the noise plus interference contributions in (18) being circularly symmetric complex Gaussian. Having expressed the capacity in compact form for the proposed scheme, it is easy to observe that for both scenarios of source node CSI availability, the only difference is in the BF vectors, $\mathbf{w}_i$ for $i = \{1, 2, \ldots, N\}$ or $\mathbf{W}$ matrix given in (3). $\mathbf{I}_N$ for the case with no CSI availability at the source node. Then, it is straightforward to observe that the capacity expression of (23) boils down to the expression given in [1] for this case.

A. Numerical Results

To observe the performance improvement due to the proposed maximum SSLR based MU-BF at the source node, we conduct numerical calculations where the network capacity is calculated as a function of E_s/σ_n^2 for different number of available relay nodes. We let $N = 2$, $M = 1$ and $\sigma_r^2 = \sigma_n^2$ and $E_s/\sigma_n^2 = E_{r_k}/\sigma_n^2 = 10$dB for $k \in \{1, 2, \ldots, K\}$. For simplicity we assign each data stream the same number of randomly selected relays, which means K is a multiple of N and $|\mathcal{X}_1| = |\mathcal{X}_1| = \ldots = |\mathcal{X}_N| = K/N$. In Fig. 2 and Fig. 3 we plot the capacity as a function of E_s/σ_n^2 for $K = 4$ and $K = 10$ relays, respectively, including the capacity for a network where there is no direct link between the source and the destination nodes, for comparison. From the graphs it can be seen that the proposed MU-BF scheme provides gains over the non-BF (no CSI availability) transmission at all times. However, at high E_s/σ_n^2 values, the relative gains become more pronounced. It can also be seen that increasing the number of cooperating relays results in an increase in the gains observed by the proposed MU-BF scheme.

V. Conclusions

In this paper, we show that with the use of a MU-BF algorithm, a transmit array gain is also achievable with the AF relaying strategy. The MU-BF scheme we consider here aims to maximize the SSLR of each transmitted data stream. Considering the structure of the AF mode, with this scheme, the interference from a data stream to the set of the cooperating relays that are not assigned for that stream is mitigated, and consequently, the propagation of the interference terms through the AF strategy is also mitigated. Thus, additional array gains are possible in relay networks when limited CSI is available at the source node. For this purpose, the CSI for $\mathcal{S} \rightarrow \mathcal{R}_k, \forall k$ and $\mathcal{S} \rightarrow \mathcal{D}$ links are necessary, but there is no need for the CSI of the $\mathcal{R}_k \rightarrow \mathcal{D}$ links.

The coherent AF relaying mode that we consider in this paper asymptotically (in K) turns the network into a point-to-point MIMO link with a multiplexing gain of $N/2$ and a distributed per-stream array gain of K. Additionally, this relaying strategy also orthogonalizes the effective MIMO channel between the source and the destination nodes and hence the multi-stream interference is effectively mitigated without any cooperation between the relay nodes.

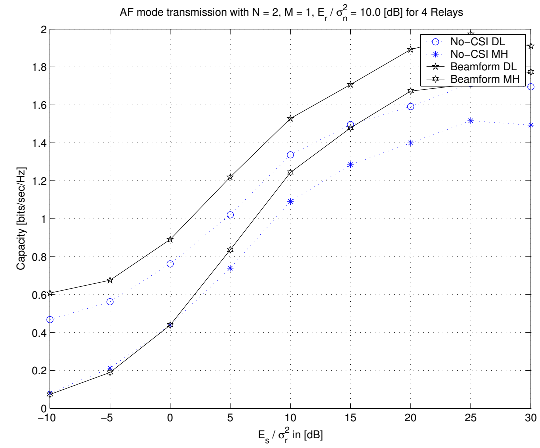

Fig. 2. Network capacities for the proposed scheme when there is a direct link available (DL) and no direct link available (MH) between the source and destination nodes. $N = 2$, $M = 1$ and $E_r/\sigma_n^2 = 10$dB for $K = 4$ relays.

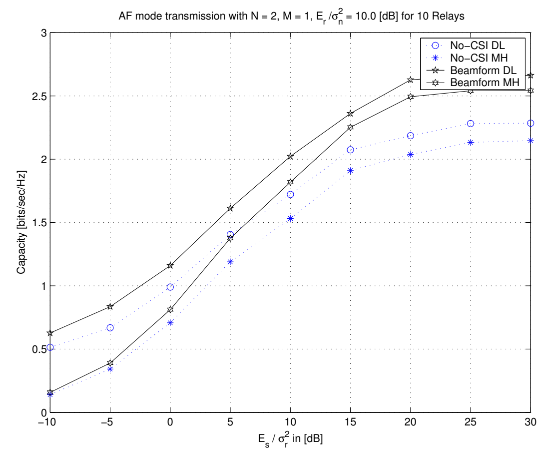

Fig. 3. Network capacities for the proposed scheme when there is a direct link available (DL) and no direct link available (MH) between the source and destination nodes. $N = 2$, $M = 1$ and $E_r/\sigma_n^2 = 10$dB for $K = 10$ relays.

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