Optimal PHY and MAC Protocols for Power-Limited Ad-hoc Networks

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***Abstract*— We consider a power-limited ad-hoc network with coherent radios in a slow-fading environment, and we analyze the performance of different physical layer (PHY) and medium access (MAC) schemes in such a network. Most of the existing analysis assume that PHY is based on the nearest neighbor decoding, which is not optimal in this case. Instead, we focus on the MLE detector. We show that, since some of the interference is mitigated by the MLE detector, the network design paradigm changes significantly: a non-coordinated PHY and MAC, in addition to a low complexity, exhibit better performance than more complex coordinated, currently used PHY and MAC schemes. Our results suggest that most of the complexity should be invested in a receiver design instead of intricate MAC or signaling protocols. We also present a novel algorithm, using Monte-Carlo method, to calculate bounds on the rates that can be achieved with MLE detector. We show that these rates can be achieved with a detector of linear complexity.**

I. Introduction

One characteristic of wireless transmissions in general is that the performance of a link is substantially decreased in presence of interference. In order to control the interference in a network, different physical (PHY) layer and medium access control (MAC) layer schemes are introduced. Wireless LAN and PAN networks are decentralized, ad-hoc wireless networks. Common MAC schemes for decentralized wireless networks are based on mutual exclusion. The most prominent example is CSMA/CA, where a node restrains from transmitting if another node in its vicinity is already transmitting or receiving. Hence, close neighbours exclude each other. A decentralized MAC successfully reduces the interference on active links, but it introduces a significant signaling overhead. Additionally, interference in a wireless network may also be controlled through PHY techniques, such as power control. However, efficient implementation of power control algorithms in a decentralized network also induces significant overhead. In this paper we analyze the optimal cross-layer MAC and PHY design for power-limited wireless networks in coherent regime with slow-fading channels and we show that, due to some specificities of power-limited communication, MAC and PHY design can be simplified to a large extent, without negatively impacting performance.

Power-limited wireless communication [1] means both spectral efficiency and energy-per-bit are relatively low. Nevertheless, interference from concurrent transmissions can still be high (e.g. in a near-far scenario, where an interferer is much closer to the receiver than the corresponding transmitter) and significantly impact the performance of wireless links.

Indeed, several PHY and MAC protocols have been proposed to control interference in power-limited ad-hoc networks [2], [3], [4], [5], [6]. All these works are based on the assumption that when transmission power is small and/or bandwidth is large, the rate is a linear function of signal-to-interference rate at a receiver. The assumption holds when the interference is Gaussian. However, this is not necessarily the case in the power-limited regime.

It has been shown in [1] that the capacity-achieving signaling for power-limited channels is the binary antipodal signaling. Consequently, if interfering links use the binary antipodal signalling, the interference they generate is not Gaussian. It is well known that the Gaussian noise is the worst type of noise. However, as shown in [7], the nearest neighbor decoding in presence of non-Gaussian noise is suboptimal, and achieves the same performance as in the presence of a Guassian noise of equivalent power.

We start by revisiting assumptions on the decoder design, and we consider the optimal detector, instead of the nearest neigbor one. We assume each receiver can estimate only the long-term statistics of the interferer's channel and the signal transmitted by an interferer, but cannot decode the signal. It then uses the optimal, maximum-likelihood estimator (MLE), based on the interference statistics, to mitigate the interference from concurrent transmissions. A detailed description of the detector is given in Section II. Possible implementations of this type of detector have been proposed in [8], [9], [10]. Note that, in contrast to multi-user detection techniques, our MLE detector does not intend to fully decode the interfering signals (see Section II-C for a further discussion).

Since the MLE detector may eliminate the effect of impulsive interference to a certain extent, it is not clear whether complex PHY and MAC interference mitigation protocols are needed anymore. To this end, we develop a mathematical model of the PHY and MAC layers with the MLE detector and use them to assess different performance aspects.

Our first result is a characterization of the capacity of a coherent power-limited radio with interference in a slow-fading channel environment. Although we cannot calculate the maximum achievable rate explicitly, we give a novel lower bound on the achievable rate based on random coding and MLE decoding. Moreover, this bound is achievable using a sub-optimal decoder of linear complexity, as described in Section II-C.

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Our second result is to show that the fully decentralized, non-coordinated PHY and MAC are optimal. We apply previously derived bounds to assess the performance of different PHY and MAC protocols on different network topologies. We show that MLE decoding itself is sufficiently effective in mitigating interference and that no additional medium access or power control protocols are needed. In other words, we show that an uncoordinated MAC and PHY, where each node transmits, with maximum allowed power, whenever it has a packet to transmit, performs better than any exclusion based protocol, even when the signaling overhead is disregarded.

As a special case of previous findings, we show analytically that the performance of a link in presence of a single interferer drops by a constant factor that does not depend on interferer's signal strength, hence power control does not help. We also show that this factor is so small that medium access control cannot significantly improve performance.

Our theoretical results confirm findings from [9]. They suggest that, due to the particularities of antipodal signaling, design complexity should be invested in the physical layer (as in [11], [12], [10], [8]) to mitigate interference, and the MAC should be kept simple. This is in contrast to [2], [3], [4], [5] where, due to the nearest neighbor decoding, a complex and inefficient MAC protocol is needed to mitigate the interference.

The results presented in our paper are based on a theoretical model of a network with coherent radios in power-limited regime (with multi-path), slow-fading channels and MLE-based multi-user decoders. Our model is based on [13], [14], [15], and extends them by introducing interference. We use a combination of analytical and numerical approaches to demonstrate the optimality of a non-coordinate access. Therefore, this paper is a theoretical confirmation of results conjectured in [9]. Extending our results to particular implementations of a detector, such as [10], [9], remains as a future work.

In Section II we present our assumptions on PHY and MAC. We derive bounds on achievable rates of the MLE detector in Section III. We analyze the performance of different PHY and MAC protocols in Section IV and we conclude in Section V.

II. System Assumptions

In this section we present assumptions on channel behavior, receiver and transmitter design.

A. Channel model

We model the communication channel using the following descrete vector channel model

$$\mathbf{r}[n] = u_1[n]A_1\mathbf{h}_1 + \sum_{i=2}^{I} u_i[n]A_i\mathbf{h}_i + \mathbf{z}[n]. \tag{1}$$

where $u_i[n]$ is the n-th transmitted simbol by user i, A_i is the long-term average amplitude of the signal from node i received at node 0, $\mathbf{z}[n]$ are i.i.d. Gaussian samples background noise and $\mathbf{r}[n]$ are the received channel samples for n-th symbol.

Vector $\mathbf{h}_i = (h_{i1}, \ldots, h_{iM})$ models the random fading, and M represents the number of dimensions in the system (e.g. number of frequency carriers). Futhermore, we assume slow-fading , that is the channel fading $\mathbf{h}_i = \mathbf{h}_i[n]$ is constant throughout channel duration. We also assume coherent communication, so receiver 0 disposes of the value of $\mathbf{h}_1$ (but does not dispose of $\mathbf{h}_i$ for $i > 1$). For simplicity of exposition, we assume a real-valued channel ($\mathbf{h}_i \in \mathbb{R}^M$). Analysis can be extended to complex-valued channels.

It is difficult to calculate the capacity of the channel (1) numerically. Instead, in what follows we will present a set of assumptions that will help us to derive a lower and an upper bound on the capacity of the channel, and also give us some insight in how these rates might be achieved with linear complexity.

B. Transmitter Structure

The optimal signaling for a real-valued coherent power-limited channel is BPSK [1]. Each symbol slot n node i transmits one of the symbols from the set $u_i[n] \in \{-1, 1\}$. We assume that $\{u_i[n]\}_n$ are i.i.d. random variables with $P(u_i[n] = 1) = P(u_i[n] = -1) = 1/2$. We also assume that the average power during transmission $P_i = A_i^2$ is upper-bounded by P_i^{MAX}, where the value of P_i^{MAX} is specified by regulations.

The goal of our physical layer is to adapt the the coding rate in order to maximize the performance. We will assume that for any given signal and interference power, node 1 always send to node 0 at the capacity of the channel, that is at the highest rate $R(1 \to 0)$ that gives arbitrarily low bit-error probability.

C. Receiver Structure and Maximum-likelihood Receiver

Suppose the source sends codeword $\mathbf{u}_1$ from a codebook $\mathcal{C} \subseteq \{-1, 1\}^N$ that corresponds to the selected transmission rate $R(1 \to 0)$. The destination receives $\mathbf{R} = \{\mathbf{r}[n]\}_{n=1,\cdots,N}$, as described in (1). We consider a coherent communication where the receiver dispose of an accurate estimate of channel-impulse response $\mathbf{h}_1$. The maximum likelihood (MLE) detector then, knowing statistics of noise and interference, selects the codeword $\hat{\mathbf{u}}_1 \in \mathcal{C}$ that maximizes the likelihood $P(\mathbf{R} \mid \hat{\mathbf{u}}_1, \mathbf{h}_1)$.

This formulation of the MLE detector implies that the receiver has some knowledge about interferers. In particular, it needs to know the number of interferers $I - 1$ and their long-term average channel characteristics A_i. In some cases, this may be feasible in practice. For example, a receiver may focus on estimating only the strongest interferer. It can estimate long-term average channel statistics of this interferer during receiver's idle times, and during reception of a packet it can detect whether the interferer is active or not. Further implementation details are out of scope of this paper.

It is difficult to calculate exactly the capacity of the channel defined by (1). Instead, we shall use two bounds: a simple upper-bound based on the assumption that there are no impulsive interferers (they can all be extracted), and a novel lower-bound for an MLE decoder. In order to derive the lower-bound, we consider a sub-optimal threshold-based MLE decoder, described in Section III-A.1. In addition to being tractable, this threshold-based MLE decoder has linear complexity, as opposed to exponential complexity of MLE decoder. Both bounds are explained in detail in Section III.

D. Medium Access and Power Control

We next discuss PHY and MAC protocols. Let us denote with $\mathcal{L} \subseteq \{1, \cdots, I\}$ the set of links that actively communicate. We further define the an "activation profile" $S \subseteq \mathcal{L}$ as a set of links that are transmitting together. A medium access protocol $(\mathcal{S}, \{\alpha_S\}_{S\in\mathcal{S}}, \{\vec{P}_S\}_{S\in\mathcal{S}})$ is defined by a set of activation profiles $\mathcal{S}$, the fractions of time $\alpha_S \in [0,1]$ each profile $S \in \mathcal{S}$ is active ($\sum_{S\in\mathcal{S}} \alpha_S \le 1$) and the set of transmission powers $\vec{P}_S = (P_{S,l})_{l\in\mathcal{L}}$ used in each profile.

Let us also define $R(l, S, \vec{P}_S)$ to be the rate achieved on link l when links from profile $S \ni l$ are active and use transmission powers $\vec{P}_S$. We can calculate bounds on this rate using the results from Section III. If link l is not active during profile S we say $R(l, S, \vec{P}) = 0$. The average rate of link l in MAC $(\mathcal{S}, \{\alpha_S\}_{S\in\mathcal{S}}, \{\vec{P}_S\}_{S\in\mathcal{S}})$ is $\bar{R}(l) = \sum_{S\in\mathcal{S}} \alpha_S R(l, S, \vec{P}_S)$.

We will further consider two types of MAC and PHY protocols. The first one is the uncoordinated protocol. This is the protocol in which all links are activated at the same time (hence $\mathcal{S} = \{\mathcal{L}\}$), and all transmit with the maximum power. The second one is the exclusion-based MAC, where some links are not allowed to be scheduled together, and where each link can adapt transmission power. In order to implement an exclusion-based protocol, one needs some coordination among nodes. Note that a non-coordinated access can be seen as a special case of an exclusion-based protocol with zero exclusions.

The considered protocol model does a priori not assume any constraints on whether a node can transmit to or receive from several nodes at the same time since this is not fundamental to our analysis. However, it can easily be extended to such cases.

III. Bounds on Achievable Rates

We next derive bounds on the achievable link's physical data rate, given the received signal power and the received powers of interferers. For that matter, we will consider the discrete vector channel model described in (1). We will first derive a novel lower-bound, which is one of the main results of our paper, and then we will present a simple upper-bound.

A. Lower bound

1) Threshold Decoding: Next, we will derive a lower-bound on achievable rates using a practical decoding scheme. We suppose the source sends data in packets of length N, where N is assumed large. Each packet has a coding rate C_R associated with it, yielding error probability of decoding $P(\mathrm{err})$. We consider an upper bound on $P(\mathrm{err})$ using random coding bound technique [16].

Suppose packets are coded using a random codebook $\mathcal{C}, ||\mathcal{C}|| = 2^{NC_R}$. The optimal decoder is the maximum likelihood (MLE) decoder described in Section II-C. However, the performance of the maximum likelihood decoder is hard to analyze. Since we are interested in an upper-bound on the probability of error, we shall consider a simple threshold decoding scheme, based on an arbitrary threshold θ. If the likelihood $P(\mathbf{R}\,|\,\mathbf{u}_1, \mathbf{h}_1) > \theta$ for only one $\mathbf{u}_1 \in \mathcal{C}$, then the decoding is successful. Otherwise, it fails.

2) Performance of Threshold Decoding: We start by giving the short-hand notation, based on (1), that we will be using in the following section. We denote with $\mathbf{H} = \{\mathbf{h}_i\}_{i=1,\cdots,I}$, $\mathbf{H}_{-1} = \{\mathbf{h}_i\}_{i=2,\cdots,I}$ matrices representing channel states. Matrix $\mathbf{R} = \{\mathbf{r}[n]\}_{n=1,\cdots,N}$, $\mathbf{u}_i = \{u_i[n]\}_{n=1,\cdots,N}$ denotes the set of received samples. Transmitted codewords are denoted with $\mathbf{U} = \{\mathbf{u}_i\}_{i=1,\cdots,I}$, $\mathbf{U}_{-1} = \{\mathbf{u}_i\}_{i=2,\cdots,I}$ and $\mathbf{U}[n] = \{u_i[n]\}_{i=1,\cdots,I}$. We will also use a short notation for $P(\mathbf{Y}\,|\,\mathbf{v}, \mathbf{h}) = P(\mathbf{R} = \mathbf{Y}\,|\,\mathbf{u}_1 = \mathbf{v}, \mathbf{h}_1 = \mathbf{h})$ and $P(\mathbf{Y}\,|\,\mathbf{V}, \mathbf{h}) = P(\mathbf{R} = \mathbf{Y}\,|\,\mathbf{U} = \mathbf{V}, \mathbf{h}_1 = \mathbf{h})$.

We first need to choose the threshold θ which will yield good performance. Ideally, $\theta(\mathbf{u}_1, \mathbf{h}_1)$ is a function of the (unknown) transmitted codeword $\mathbf{u}_1$ and the channel-state $\mathbf{h}_1$, and we shall choose it to minimize the probability of false-negative $P(P(\mathbf{R}\,|\,\mathbf{v}, \mathbf{h}_1) < \theta(\mathbf{v}, \mathbf{h}_1)\,|\,\mathbf{u}_1 = \mathbf{v}, \mathbf{h}_1)$. We will show later that the optimal θ does not depend on the choice of $\mathbf{v}, \mathbf{h}_1$.

The noise and the interferences are ergodic processes hence for a large packet size N we have that $P(P(\mathbf{R}\,|\,\mathbf{v}, \mathbf{h}_1) > \theta(\mathbf{v}, \mathbf{h}_1)|\mathbf{u}_1 = \mathbf{v}, \mathbf{h}_1) \to 1$ if

$$\begin{aligned}
\theta(\mathbf{v}, \mathbf{h}_1) &= (1-\epsilon)\mathbb{E}_{\mathbf{R}}(P(\mathbf{R}\,|\,\mathbf{v}, \mathbf{h}_1)\,|\,\mathbf{u}_1 = \mathbf{v}, \mathbf{h}_1) \quad (2)\\
&= (1-\epsilon)\int_{\mathbf{Y}} P(\mathbf{R} = \mathbf{Y}\,|\,\mathbf{v}, \mathbf{h}_1)^2 d\mathbf{Y} \quad (3)
\end{aligned}$$

for any $\epsilon > 0$. We will choose $\theta(\mathbf{v}, \mathbf{h}_1) = \int_{\mathbf{Y}} P(\mathbf{R} = \mathbf{Y}\,|\,\mathbf{v}, \mathbf{h}_1)^2 d\mathbf{Y}$ (i.e. $\epsilon = 0$), and assume further $P(P(\mathbf{R}\,|\,\mathbf{v}, \mathbf{h}_1) > \theta(\mathbf{v}, \mathbf{h}_1)|\mathbf{u}_1 = \mathbf{v}, \mathbf{h}_1) = 1$. We next show that $\theta(\mathbf{v}, \mathbf{h}_1)$ does not actually depend on $\mathbf{v}, \mathbf{h}_1$, hence we can write $\theta(\mathbf{v}, \mathbf{h}_1) = \theta$.

Let $D(\mathbf{v}, \mathbf{w}) = |\{n : |v[n] - w[n]| \neq 0\}|$ be the distance between vectors $\mathbf{v}$ and $\mathbf{w}$. We will show in the following proposition that the probability of falsely detecting vector $\mathbf{v}$ as $\mathbf{w}$ depends only on this distance.

Proposition 1: The following integral

$$p(D(\mathbf{v}-\mathbf{w}), \mathbf{h}_1) = \int_{\mathbf{Y}} P(\mathbf{R} = \mathbf{Y}\,|\,\mathbf{v}, \mathbf{h}_1) P(\mathbf{R} = \mathbf{Y}\,|\,\mathbf{w}, \mathbf{h}_1) d\mathbf{Y}$$

depends only on $D(\mathbf{v} - \mathbf{w})$ and $\mathbf{h}_1$. Also, $\theta(\mathbf{v}, \mathbf{h}_1)$ depends neither on $\mathbf{v}$ nor on $\mathbf{h}_1$.

Proof: Let us denote with $\mathbf{Q}[n] = \sum_{i=2}^{I} u_i[n] A_i[n] \mathbf{h}_i[n] + \mathbf{z}[n]$. Then, $P(\mathbf{R} = \mathbf{Y}\,|\,\mathbf{v}, \mathbf{h}_1) = P(\bigcup_{n=1\cdots N} \mathbf{Q}[n] = \mathbf{Y}[n] - v[n] A_1 \mathbf{h}_1)$ and

$$\begin{aligned}
&\int_{\mathbf{Y}} P(\mathbf{R} = \mathbf{Y}\,|\,\mathbf{v}, \mathbf{h}_1) P(\mathbf{R} = \mathbf{Y}\,|\,\mathbf{w}, \mathbf{h}_1) d\mathbf{Y} = \\
&\qquad \int_{\mathbf{y}} P\left(\bigcup_{n=1\cdots N} \mathbf{Q}[n] = \mathbf{Y}[n]\right) \times \\
&\times \quad P\left(\bigcup_{n=1\cdots N} \mathbf{Q}[n] = \mathbf{Y}[n] - (w[n] - v[n]) A_1 \mathbf{h}_1\right) d\mathbf{y}.
\end{aligned}$$

The distribution of the vector $\{\mathbf{Q}[n]\}_{n=1\cdots N}$ is by definition symmetric and invariant to a permutation of its elements, hence the value of the integral depends only on $D(\mathbf{v}, \mathbf{w})$. Furthermore, if $||\mathbf{v} - \mathbf{w}|| = 0$, as in (3), then the integral does not depend on $\mathbf{h}_1$ either. ■

Now we are interested in the probability of error of decoding a random transmitted codeword. We consider a random codebook $\mathcal{C}$ with a distribution as described in Section II-B, and from there select a random codeword $\mathbf{v}$ to transmit. Note that $P(\mathbf{v} = \boldsymbol{\omega} \,|\, \mathcal{C} = C) = 2^{-C_R N} 1\{\boldsymbol{\omega} \in C\}$ since by definition all the codewords from $\mathcal{C}$ are equiprobable. The probability of error can be bounded by the union bound as

$$\begin{aligned}
&P(\mathrm{err}|\mathbf{h}_1) \leq \\
&\leq \mathbb{E}_{\mathcal{C}, \mathbf{v}\in\mathcal{C}}\left[\sum_{\boldsymbol{\omega}_w \in \mathcal{C}, \boldsymbol{\omega}_w \neq \mathbf{v}} P(P(\mathbf{r} \,|\, \boldsymbol{\omega}_w, \mathbf{h}_1) > \theta \,|\, \mathbf{v}, \mathbf{h}_1)\right] \\
&= \sum_{C, \boldsymbol{\omega}_v, \boldsymbol{\omega}_w \neq \boldsymbol{\omega}_v} P(\mathcal{C} = C) P(\mathbf{v} = \boldsymbol{\omega}_v \,|\, \mathcal{C} = C) \\
&\quad 1\{\boldsymbol{\omega}_w \in C\} P(P(\mathbf{r} \,|\, \boldsymbol{\omega}_w, \mathbf{h}_1) > \theta \,|\, \boldsymbol{\omega}_v, \mathbf{h}_1) \\
&= \sum_{C, \boldsymbol{\omega}_v, \boldsymbol{\omega}_w \neq \boldsymbol{\omega}_v} P(\mathcal{C} = C) P(\mathbf{v} = \boldsymbol{\omega}_v \,|\, \mathcal{C} = C)\ 2^{C_l N} \\
&\quad P(\mathbf{v} = \boldsymbol{\omega}_w \,|\, \mathcal{C} = C) P(P(\mathbf{r} \,|\, \boldsymbol{\omega}_w, \mathbf{h}_1) > \theta \,|\, \boldsymbol{\omega}_v, \mathbf{h}_1) \\
&= 2^{C_l N} \mathbb{E}_{\mathbf{v}, \mathbf{w}, \mathbf{v}\neq\mathbf{w}}\left[P(P(\mathbf{r} \,|\, \mathbf{w}, \mathbf{h}_1) > \theta \,|\, \mathbf{v}, \mathbf{h}_1)\right] \qquad (4)
\end{aligned}$$

where $\mathbf{v}, \mathbf{w}$ are two randomly chosen codewords from a random codebook.

Next, using Markov inequality, we bound

$$\begin{aligned}
&P(P(\mathbf{R} \,|\, \mathbf{w}, \mathbf{h}_1) > \theta \,|\, \mathbf{u}_1 = \mathbf{v}, \mathbf{h}_1) \leq \qquad (5) \\
&\leq \frac{1}{\theta} \mathbb{E}_{\mathbf{R}}\left[P(\mathbf{R} \,|\, \mathbf{w}, \mathbf{h}_1) \,|\, \mathbf{v}, \mathbf{h}_1\right] \qquad (6) \\
&= \frac{p(D(\mathbf{v} - \mathbf{w}), \mathbf{h}_1)}{\theta} \qquad (7)
\end{aligned}$$

where the last equation follows from Proposition 1. Note that, since $\mathbf{v}$ and $\mathbf{w}$ are random codewords, (7) is a random variable as well. The Markov bound is the best bound we can use knowing only the mean of a random variable, and numerical results in Section IV show that the bound is useful for performance evaluation of the channel.

Since we have $P(u_i[n] = 1) = P(u_i[n] = -1) = 1/2$, we can easily express

$$P(D(\mathbf{u} - \mathbf{v}) = d) = \binom{N}{d} 2^{-N}. \qquad (8)$$

Next, let

$$e_d = (\underbrace{1, \ldots, 1}_{d}, \underbrace{-1, \ldots, -1}_{N-d}).$$

Clearly, $D(\mathbf{e}_d - \mathbf{e}_0) = d$. Then from Proposition 1, (4), (7) and (8) we have

$$\begin{aligned}
p(d, \mathbf{h}_1) &= \mathbb{E}_{\mathbf{U}_{-1}, \mathbf{V}_{-1}}\Big[\int_{\mathbf{Y}} P(\mathbf{R} = \mathbf{Y} \,|\, [\mathbf{e}_d, \mathbf{U}_{-1}], \mathbf{h}_1) \\
&\qquad P(\mathbf{R} = \mathbf{Y} \,|\, [\mathbf{e}_0, \mathbf{V}_{-1}], \mathbf{h}_1) d\mathbf{Y}\Big], \qquad (9) \\
P(\mathrm{err}) &\leq \frac{2^{C_R N}}{\theta} \sum_{d=1}^{N} \binom{N}{d} 2^{-N} p(d, \mathbf{h}_1) \qquad (10)
\end{aligned}$$

where $\mathbf{U}_{-1} = \{u_i\}_{i=2,\cdots,I}, \mathbf{V}_{-1} = \{v_i\}_{i=2,\cdots,I}$ are random codewords transmitted by interferers. We can express $\int_{\mathbf{Y}} P(\mathbf{R} = \mathbf{Y} \,|\, [\mathbf{e}_d, \mathbf{U}_{-1}], \mathbf{h}_1) P(\mathbf{R} = \mathbf{Y} \,|\, [\mathbf{e}_0, \mathbf{V}_{-1}], \mathbf{h}_1) d\mathbf{Y}$ in a closed-form, as explained in Appendix, and calculate the mean over interferers' codewords $\mathbf{U}_{-1}, \mathbf{V}_{-1}$ using Monte-Carlo simulations. Since $p(0, \mathbf{h}_1) = \theta$, we can use the same procedure to calculate θ (note that in addition θ does not depend on $\mathbf{h}_1$). Details on Monte-Carlo simulations are given in Section III-C.

From (10) we can obtain a lower-bound C_l on the communication rate C_R. Let us fix an arbitrary small probability of error $P(\mathrm{err}) = \epsilon$. We then have

$$\begin{aligned}
C_R(N) &= -\frac{1}{N} \log_2\left(\sum_{d=1}^{N} \binom{N}{d} 2^{-N} \frac{p(d, \mathbf{h}_1)}{\theta}\right) \\
&\quad + \frac{1}{N} \log_2(\epsilon)
\end{aligned}$$

When $N \to \infty$, the second term vanishes. Hence, we can achieve arbitrarily small $P(\mathrm{err})$ using communication rate

$$\begin{aligned}
C_l &= \liminf_{N\to\infty} C_R(N) \qquad (11) \\
&= \liminf_{N\to\infty} -\frac{1}{N} \log_2\left(\sum_{d=1}^{N} \binom{N}{d} 2^{-N} \frac{p(d, \mathbf{h}_1)}{\theta}\right) \qquad (12) \\
&= \liminf_{N\to\infty} -\frac{1}{N} \log_2\left(\sum_{d=0}^{N} \binom{N}{d} 2^{-N} \frac{p(d, \mathbf{h}_1)}{\theta}\right) \qquad (13)
\end{aligned}$$

Note that limits (12) and (13) are equivalent but the latter is easier to calculate numerically.

B. Upper bound

As an upper bound, we use the information-theoretic capacity of the channel (1) without interference ($I = 1$). Note that in this case our lower bound coincides with a well-known random-coding bound for AWGN channels, which is tight. In other words, we can readily calculate the upper bound using (13) and setting $A_i \to \infty$ for all $i > 1$.

C. Numerical Methods

We calculate θ and $\sum_{d=1}^{N} \binom{N}{d} 2^{-N} p(d, \mathbf{h}_1)$ using Monte-Carlo simulations, averaging over many random samples of $\mathbf{V}_{-1}, \mathbf{U}_{-1}$ and $\mathbf{h}_1$.

In the case of θ, we verify that the samples obtained by Monte-Carlo fit the Gaussian distribution well. This allows us to calculate confidence intervals of the simulation [17], and in all cases the relative confidence intervals are smaller than 10%.

In case of $\sum_{d=1}^{N} \binom{N}{d} 2^{-N} p(d, \mathbf{h}_1)$ the samples are no longer Gaussian, but we verify that a log transform is Gaussian. There is a simple intuitive explanation for this. For very small d the candidate and the transmitted codewords are similar, the probability of error (estimated through $p(d, \mathbf{h}_1)$) is high. However, there are a few such codewords. On the contrary, for large d, $p(d, \mathbf{h}_1)$ is small, but there are a lot of such words.

In all the simulations we find that the relative confidence intervals for the error probability of decoding $P(\text{err})$ are smaller than 10%. We are interested in C_l, given by (13), which is of order of $\log_2(P(\text{err}))$. Since the values of interest of $\log_2(P(\text{err}))$ are smaller than -13, we can see that the relative confidence for C_l is around 5%-10%.

IV. MAC and PHY Protocol Performance Evaluation

A. Network Topologies

In this section we analyze the performance of different PHY and MAC schemes on different network topologies. We will compare the non-coordinated PHY and MAC schemes with the optimal exclusion-based schemes and show that the uncoordinated schemes are equally good.

There are two main difficulties arising in these comparisons. The first one is a choice of the performance metric. Several performance metrics have been considered in evaluation of computer networks, such as maximum throughput, max-min fairness and proportional fairness [18], and we need to decide which one to use.

The second difficulty is the complexity of the underlying optimization problem. Finding the optimal MAC protocol in an arbitrary network is an NP hard problem [19]. In our setting the problem becomes even more complex because calculating bound on channel rates themselves is computationally very expensive, as explained in Section III. Furthermore, the resulting MAC protocol depends to a large extent on the performance metric of choice.

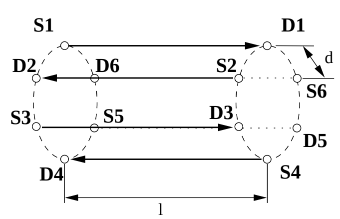

Fig. 1. Cylindric scenario: There are K nodes and I links (in this example $K = 12, I = 6$). S_i sends to D_i. Adjacent links have opposite directions.

This work focuses on the performance of a non-coordinated protocol, and not on discussing the issues of different scheduling techniques and performance metrics. Therefore, we shall consider a class of symmetric cylindric network topologies with K nodes and I links, described in Figure 1, which greatly simplifies the analysis but still gives a broad insight into the performance of non-coordinated protocols. It is easy to verify for the network on Figure 1 that, due to the symmetry, all nodes should have the same rate regardless of the metrics ($\bar{R}(l) = \bar{R}$ for all $l \in \mathcal{L}$). Our goal is to maximize $\bar{R}$.

We can further show that the activation profiles need to be symmetric as well. Let us define the rotation of an activation profile $\rho_i(S) = \{l \,|\, (l+i) \bmod I \in S\}$. We will show that the optimal MAC consists of rotating a single activation profile.

Proposition 2: There exists an activation profile S^* and a set of transmit powers $\{\vec{P}_S^*\}_{S\in\mathcal{S}}$ such that $\mathcal{S} = \{\rho_i(S^*) \,|\, i = 1, \cdots, I\}$, $\alpha_Q = 1/I$ for all $Q \in \mathcal{S}$ and $(\mathcal{S}, \{\alpha_S\}_{S\in\mathcal{S}}, \{\vec{P}_S\}_{S\in\mathcal{S}})$ is the optimal MAC.

Proof: For every link l we have

$$\begin{aligned}
\bar{R} &= \sum_{S\in\mathcal{S}} \alpha_S R(l, S, \vec{P}_S) \\
&= \sum_{i=1,\cdots,I} \frac{1}{I} \sum_{S\in\mathcal{S}} \alpha_S R(l, \rho_i(S), \vec{P}_{\rho_i(S)}) \\
&= \sum_{S\in\mathcal{S}} \alpha_S \sum_{i=1,\cdots,I} \frac{1}{I} R(l, \rho_i(S), \vec{P}_{\rho_i(S)}) \\
&\leq \max_{S\in\mathcal{S}} \sum_{i=1,\cdots,I} \frac{1}{I} R(l, \rho_i(S), \vec{P}_{\rho_i(S)}) \\
&= \sum_{i=1,\cdots,I} \frac{1}{I} R(l, \rho_i(S^*), \vec{P}_{\rho_i(S^*)}).
\end{aligned}$$

where S^* achieves the max in the last inequality. Hence, for any MAC $(\mathcal{S}, \{\alpha_S\}_{S\in\mathcal{S}}, \{\vec{P}_S\}_{S\in\mathcal{S}})$ we can construct a MAC that satisfies the claims of the theorem and performs at least as good. ■

For example, for $K = 12, I = 6$, we might have that the optimal activation profile is $S = \{(S_1, D_1), (S_3, D_3), (S_5, D_5)\}$, meaning that every second link is active. This profile defines an exclusion based protocol that may corresponds to a CSMA/CA based protocol where the carrier can be sensed at distance d. A non-coordinate PHY and MAC are represented by a single user profile $S = \{1, \cdots, I\}$ where all links are active all the time, using the same transmission powers, due to symmetry($P_l = P_k$ for all $l, k \in \mathcal{L}$).

Our goal is to evaluate whether the non-coordinate PHY and MAC are as good as any coordinated, exclusion-based ones. We will use (13) to calculate a lower bound and an upper bound on performance of non-coordinated protocol, and we will show that the two are very close and that no exclusion protocol can perform better.

B. System Parameters

In all the simulations we will assume that the signal transmitted power is $P_{trans} = 0.1$mW. The received signal at distance l is $P_{rcv}(l) = P_{trans}\, b\, l^{-\alpha}$, where we take the signal propagation parameters $b = 10^{-5.5}, \alpha = 3.3$ from measurements [20]. We assume white-noise power is $\sigma_W^2 = 10^{-13}W$. Typical communication range for these system parameters is up to 10m.

We will also consider codeword sizes of $N = 80$, which is sufficiently large to give reliable results and is computationally feasible. We will take $M = 5$ that corresponds to a 5-tap receiver, again the highest computationally feasible.

For channel statistics we use the measurements from [13] which says that the tap energy drops linearly with the tap delay. Approximately 14% of the total energy is in the first 5 taps. We also assume here that taps are independent. In order to avoid additional unnecessary variance in our results, we will assume here that $\mathbf{h}_1$ is fixed to the average measured values. Similar results hold for different values of $\mathbf{h}_1$ (hence will hold for the average channel realizations as well).

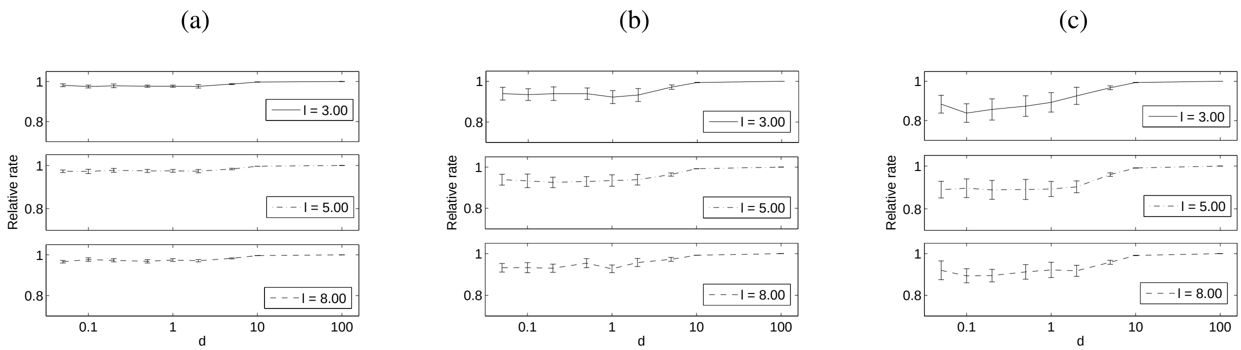

Fig. 2. Cylindric scenario: We consider different numbers of interfering links ((a) $I = 2$, (b) $I = 4$, (c) $I = 6$). We vary d for different l, and plot the ratio between the rate achieved in presence of the interferers vs the rate of a link in isolation (e.g. $d = 100$m).

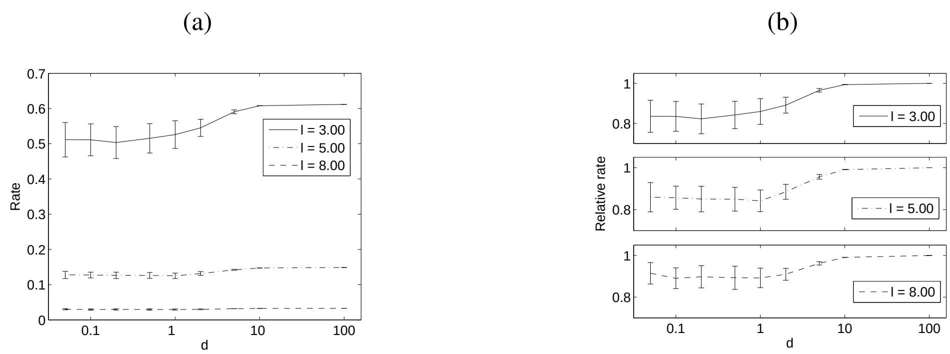

Fig. 3. A special case of cylindric scenario with 7 interfering link ($I = 8$). We vary d for different l, and plot the absolute rates (a), as well as the ratio (b) between the rate achieved in presence of the interferer vs the rate of a link in isolation (e.g. $d = 100$m).

C. Numerical Results

In this section we consider numerical results for cylindric networks, depicted in Figure 1. We first look at the non-coordinate MAC. Again, due to symmetry, we have that the transmitted power $P = P_i$ for all i. We do not vary the transmitted power explicitly in our simulations, as it is equivalent to changing l and d.

We look at the achievable rates for cylinder topologies with 1,3,5 and 7 interferers ($I = 2, 4, 6, 8$). The results are depicted in Figure 2 and Figure 3. We see that when the interferers are sufficiently far away (e.g. $d \geq 10$m), there is no drop in rate. As the interferers approach, a slight drop in rate occurs. This drop is insignificant for the single interferer case ($I = 2$), and can go up to 20% in case of 7 interferers.

In all cases we observe that the performance drop occurs when we decrease d from 10m to 1m. As we continue to decrease d, no further drop occurs. This means that for $d <$ 1m, the interferers' signals are strong enough to be extracted to the same extent.

In order to better explain the above phenomenon, we consider the simple case of a single interferer ($I = 2$) and a single channel sample ($M = 1$), with N large. We then have the following proposition

Proposition 3: Let us consider the channel model given in (1), with $I = 2$ and $M = 1$. Then, if $NA_2^2\sigma_M^2 \gg \sigma_W^2$, the lower bound (13) on feasible communication rate does not depend on A_2.

Proof: We first observe that $T = \sigma_M^2$ is scalar, since $M = 1$. We start from the output distribution of the channel (14), given in the appendix, and we denote $D[n] = y[n] - h_1 A_1 v_1[n]$. We then have

$$P(\mathbf{R} = \mathbf{Y} \,|\, \mathbf{V}, \mathbf{h}_1) = \left(\frac{1}{\sqrt{2\pi\sigma_W^2}}\right)^N \sqrt{\frac{\sigma_W^2}{\sigma_W^2 + \sigma_M^2 A_2^2 N}}$$
$$\exp\left(-\frac{1}{2\sigma_W^2}\sum_{n=1}^{N} D[n]^2\right)$$
$$\exp\left(\frac{1}{2\sigma_W^2}\frac{\sigma_M^2 A_2^2}{\sigma_W^2 + \sigma_M^2 A_2^2 N}\left(\sum_{n=1}^{N} D[n] v_2[n]\right)^2\right).$$

Now sinc $NA_2^2\sigma_M^2 \gg \sigma_W^2$, we have

$$P(\mathbf{R} = \mathbf{Y} \,|\, \mathbf{V}, \mathbf{h}_1) = \left(\frac{1}{\sqrt{2\pi\sigma_W^2}}\right)^N \sqrt{\frac{\sigma_W^2}{\sigma_W^2 + \sigma_M^2 A_2^2 N}}$$
$$\exp\left(-\frac{1}{2\sigma_W^2}\left(\sum_{n=1}^{N} D[n]^2 - \frac{1}{N}\left(\sum_{n=1}^{N} D[n] v_2[n]\right)^2\right)\right).$$

and only the multiplicative factor depends on A_2. However, since in (13) we have ratio $\frac{p(d,\mathbf{h}_1)}{\theta}$ and both $p(d, \mathbf{h}_1)$ and θ have the same multiplicative factor, it cancels out, hence (13) does not depend on A_2. ■

A similar result can be derived for $I = 2$ and arbitrary M; see [21]. We conjecture that it can be generalized for arbitrary I as well, although the derivation becomes extremely complex.

We can conclude from Proposition 3 that the performance of a channel in presence of interferers depends only on their number, and drops by a constant factor regardless of the actual position of interference. It is difficult to characterize this constant factor analytically. Our numerical results suggest that this factor is very small, even when the number of interferers is relatively large. Note that our numerical results illustrate the result of Proposition 3 only for $d < 2\text{m}$ (which ensures that the condition $NA_2^2\sigma_M^2 \gg \sigma_W^2$ holds).

We also give the following proposition about the shape of function $p(\cdot)$ in the special case of a single interferer

Proposition 4: Let us consider the channel model given in (1), with $I = 2$. Then, $p(\cdot)$ has the following form

$$\int_{\mathbf{Y}} P(\mathbf{R} = \mathbf{Y} \,|\, \mathbf{V}, \mathbf{h}_1) P(\mathbf{R} = \mathbf{Y} \,|\, \mathbf{W}, \mathbf{h}_1) d\mathbf{Y} =$$
$$= \quad K \exp\left(\frac{A_1^2}{\sigma_W^2} \mathbf{h}_1^T \mathbf{h}_1 f(\mathbf{V}, \mathbf{W}) \right).$$

where $\mathbf{h}_1^T \mathbf{h}_1 = 1$, K is a constant that does not depend on $\mathbf{V}, \mathbf{W}$, $f(\mathbf{V}, \mathbf{W})$ is a function that depends solely on $\mathbf{V}, \mathbf{W}$ and $\frac{A_1^2}{\sigma_W^2}$ is the signal-to-noise ratio of the received signal. We do not give the proof here as it requires tedious derivations; see [21]. As a consequence we see that (13) does not depend on statistics of channel-impulse response.

It is easy to see that one cannot construct an exclusion-based protocol with this performance. First, consider a protocol that ensures that when link l is active, link $l-1$ and $l+1$ will be silent. The most dense schedule that satisfies this property is based on activation profile $P = \{(S_1, D_1), (S_3, D_3), (S_5, D_5), \cdots\}$ (every second link is active). In this case we can upper-bound the performance of link l by assuming there is no interference at all. However, link l is scheduled at most 50% of the time, and the performance of this MAC is much lower than the performance of the non-coordinated one.

In order to match the rate of non-coordinated MAC in, for example, the case of 8 links, we need to schedule each link at least 80% of a time, which implies approximately 5 interfering links in each slot. However, the drop of performance caused by 5 interfering links is already very similar to what one can achieve with a non-coordinated MAC, hence there will be no benefit of further exclusions.

Furthermore, from Proposition 3 we see that the impact of the interference does not depend on the power of the interfering signal, hence any form of power control will not improve performance. Instead, it is optimal that all links transmit with the maximum power.

V. Conclusions and Future Work

We analyzed the performance of different PHY and MAC protocols for coherent, slow-fading, wide-band channel with MLE detector and random coding. We showed that the impact of the wide-band interference can be diminished to a large extent by the use of the maximum-likelihood decoder. We also showed that the performance drop due to interference depends only on the number of interferences, and not on their actual position (interfering signal strength). Even in a harsh scenario, with 7 near-by interferers, we show that the link rate drops by less than 20%.

We further concluded that there is no need for an exclusion-based MAC protocol as the interference is successfully mitigated by the PHY. The non-coordinate MAC, in which all links transmit regardless of the others, with the maximum powers, is very close to a theoretical optimal, and it largely outperforms already proposed systems [4], [5] based on the nearest neighbor decoder and repetition coding.

We also presented a novel procedure to calculate a lower-bound on achievable rates using random-coding techniques and Monte-Carlo simulations. We showed that, in the case of a single interferer, bound drops by a constant factor, regardless of the interferer's strength.

Our results indicate that when designing a wide-band network, one should not invest in complex MAC protocol, with a lot of signaling overhead, in order to control the interference, as done in [4], [5]. Instead, the complexity should be invested in a multi-user decoder that can successfully mitigate interference with much lower complexity, as proposed in [10].

References

[1] S. Verdu, "Spectral efficiency in the wideband regime," *IEEE Transactions on Information Theory*, vol. 48, no. 6, pp. 1319–1343, June 2002.

[2] R. Cruz and A. Santhanam, "Hierarchical link scheduling and power control in multihop wireless networks," in *Allerton 2002*, September 2002.

[3] B. Radunovic and J. Y. Le Boudec, "Optimal power control, scheduling and routing in UWB networks," *IEEE Journal on Selected Areas in Communications*, vol. 22, no. 7, pp. 1252–1270, September 2004.

[4] F. Cuomo, C. Martello, A. Baiocchi, and C. Fabrizio, "Radio resource sharing for ad hoc networking with UWB," *IEEE Journal on Selected Areas in Communications*, vol. 20, no. 9, pp. 1722–1732, December 2002.

[5] P. Baldi, L. De Nardis, and M.-G. Di Benedetto, "Modeling and optimization of uwb communication networks through a flexible cost function," *IEEE Journal on Selected Areas in Communications*, vol. 20, no. 9, pp. 1733–1744, December 2002.

[6] R. Negi and A. Rajeswaran, "Capacity of power constrained ad-hoc networks," in *INFOCOM 2004*, 2004.

[7] A. Lapidoth, "Nearest neighbor decoding for additive non-gausian noise channels," *IEEE Transactions on Information Theory*, vol. 42, no. 5, pp. 1520–1529, September 1996.

[8] C. Steiner and K. Witrisal, "Multiuser interference modeling and suppression for a multichannel differential IR-UWB system," in *ICUWB*, 2005.

[9] R. Merz, J. Widmer, J.-Y. Le Boudec, and B. Radunovic, "A joint PHY/MAC architecture for low-radiated power TH-UWB wireless ad-hoc networks," *Wireless Communications and Mobile Computing Journal, Special Issue on Ultrawideband (UWB) Communications*, vol. 5, no. 5, pp. 567–580, August 2005.

[10] M. Flury and J.-Y. Le Boudec, "Interference mitigation by statistical interference modeling in an impulse radio UWB receiver," in *IEEE International Conference on Ultra-Wideband (ICUWB 2006)*, 2006.

[11] P. Spasojevic and X. Wang, "Multiuser detection in impulsive noise via slowest descent search," in *IEEE Workshop on Statistical Signal and Array Processing*, 2000.

[12] A. Zoubir and A. Lane-Glover, "Multiuser detection in impulsive noise," in *IEEE Workshop on Statistical Signal Processing*, 2001.

[13] Y. Souilmi and K. Raymond, "Challenges in UWB signaling for ad-hoc networking," in *DIMACS Series*, November 2003, pp. 271–284.

[14] Y. Souilmi and R. Knopp, "On the achievable rates of ultra-wideband systems in multipath fading environments," in *ISIT*, July 2003.

[15] A. Adinoyi and H. Yanikomeroglu, "Practical capacity calculation for time-hopping ultra-wide band multiple-access communications," *IEEE Communications Letters*, vol. 9, no. 7, pp. 601–603, July 2005.

[16] R. Gallager, *Information Theory and Reliable Communication*. Wiley, 1968.

[17] J.-Y. Le Boudec, "Performance evaluation," EPFL," Lecture Notes, 2006. [Online]. Available: http://ica1www.epfl.ch/perfeval/

[18] F. P. Kelly, A. Maulloo, and D. Tan, "Rate control in communication networks: shadow prices, proportional fairness and stability," *Journal of the Operational Research Society*, vol. 49, pp. 237–252, 1998.

[19] G. Sharma, N. Shroff, and R. Mazumdar, "On the complexity of scheduling in wireless networks," in *ACM Mobicom*, 2006.

[20] S. Ghassemzadeh and V. Tarokh, "Uwb path loss characterization in residential environments," in *IEEE Radio Frequency Integrated Circuits (RFIC) Symposium*, June 2003, pp. 501–504.

[21] B. Radunovic, J.-Y. Le Boudec, and R. Knopp, "Optimal PHY and MAC protocols for wide-band ad-hoc networks - derivation of bounds," in *Technical report*, 2007. [Online]. Available: http://research.microsoft.com/users/cambridge/bozidar/Publications/tr_allerton07.pdf

APPENDIX

In the appendix we explain how to calculate the channel output distribution $P(\mathbf{R} = \mathbf{Y} \,|\, \mathbf{V}, \mathbf{h}_1)$ and $\int_{\mathbf{Y}} P(\mathbf{R} = \mathbf{Y} \,|\, \mathbf{V}, \mathbf{h}_1) P(\mathbf{R} = \mathbf{Y} \,|\, \mathbf{W}, \mathbf{h}_1) d\mathbf{Y}$. First, conditional to the channel realization $\mathbf{H}$ and the transmitted symbols $\mathbf{U}$, the channel outputs $R = \{r_m[n]\}$ are Gaussian i.i.d. RV with distribution

$$P(\mathbf{R} = \mathbf{Y} | \mathbf{U}, \mathbf{H}) = \left(\frac{1}{\sqrt{2\pi\sigma_W^2}} \right)^{MN} \times$$
$$\times \quad \exp\left(-\sum_{n=1}^{N} \sum_{m=1}^{M} (y_m[n] - \sum_{i=1}^{I} v_i[n] A_i h_{im})^2 / 2\sigma_W^2 \right).$$

Also, each channel response $\mathbf{h}_i$ is multivariate Gaussian with distribution

$$P(\mathbf{H}_{-1}) = \left(\frac{1}{\sqrt{(2\pi)^M |T|}} \right)^{I-1} \exp\left(-\frac{1}{2} \sum_{i=2}^{I} \mathbf{h}_i^T T^{-1} \mathbf{h}_i \right),$$

Thus, we have

$$P(\mathbf{R} = \mathbf{Y} \,|\, \mathbf{V}, \mathbf{h}_1) = \mathbb{E}_{\mathbf{H}_{-1}}(P(\mathbf{R} = \mathbf{Y} | \mathbf{V}, \mathbf{H})) = \tag{14}$$
$$= \quad \int_{\mathbf{H}_{-1}} \left(\frac{1}{\sqrt{(2\pi)^M |T|}} \right)^{I-1} \left(\frac{1}{\sqrt{2\pi\sigma_W^2}} \right)^{MN} \times$$
$$\times \quad \exp\left(-\sum_{i=2}^{I} \frac{\mathbf{h}_i^T T^{-1} \mathbf{h}_i}{2} \right) \times$$
$$\times \quad \exp\left(-\sum_{n,m} \frac{(y_m[n] - \sum_{i=1}^{I} v_i[n] A_i h_{im})^2}{2\sigma_W^2} \right) d\mathbf{H}_{-1}$$

which is again a multivariate Gaussian and can be expressed in closed form. Similarly, since $P(\mathbf{R} = \mathbf{Y} \,|\, \mathbf{V}, \mathbf{h}_1)$ is exponential, $\int_{\mathbf{Y}} P(\mathbf{R} = \mathbf{Y} \,|\, \mathbf{V}, \mathbf{h}_1) P(\mathbf{R} = \mathbf{Y} \,|\, \mathbf{W}, \mathbf{h}_1) d\mathbf{Y}$ is also exponential and can be calculated explicitly. See [21] for details.