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An Iterative Optimization Method for Unitary Beamforming in MIMO Broadcast Channels

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Abstract— **An iterative optimization method for unitary beamforming in MIMO broadcast channels is proposed, based on successive optimization of Givens rotations. Under the assumption of perfect channel state information at the transmitter (CSIT) and for practical average SNR values, the proposed technique provides higher sum rates than zero forcing (ZF) beamforming while performing close to minimum mean squared error (MMSE) beamforming. Moreover, it is shown to achieve linear sum-rate growth with the number of transmit antennas. Interestingly, the proposed unitary beamforming approach proves to be very robust to channel estimation errors, providing better sum rates than ZF beamforming and even MMSE beamforming as the variance of the estimation error increases. In addition, the proposed technique is presented as a performance reference for evaluation of existing reduced-complexity unitary beamforming techniques, providing numerical results in systems with multiuser scheduling.**

I. INTRODUCTION

Multiple-input multiple-output (MIMO) communication systems have the potential to offer high spectral efficiency as well as link reliability. In multiuser scenarios, space division multiple access (SDMA) can boost the downlink capacity by exploiting the spatial multiplexing capability of multiple transmit antennas at the base station, transmitting to multiple users simultaneously [1]. Since the capacity-achieving dirty paper coding (DPC) approach [2] is difficult to implement, linear beamforming techniques have been proposed that achieve a large portion of DPC capacity with lower complexity. Special attention has been paid to Zero forcing (ZF) beamforming (or channel inversion) techniques in the recent literature [3], extended in [4] to systems with multiuser scheduling. However, a main drawback of this technique is that the sum rate does not scale with the number of antennas. In order to overcome this limitation, minimum mean squared error (MMSE) beamforming (or regularized channel inversion) [5] has been proposed, which maximizes the signal-to-interference-plus-noise ratio (SINR) at each receiver and provides linear capacity growth with the number of antennas.

Unitary beamforming (UBF) techniques have recently become a focus of interest in MIMO broadcast channels, especially in scenarios where the amount of feedback available at the base station is limited. Particularly, random beamforming (RBF)[6] has been proposed as a simple technique that achieves optimal capacity scaling in MIMO broadcast channels. In [7], orthogonal SDMA with limited feedback (LF-OSDMA) is proposed as a transmission technique in which the transmitter counts on a codebook containing an arbitrary number of unitary bases. In this aproach, the users quantize the channel shape (channel direction) to the closest codeword in the codebook, feeding back the quantization index and expected SINR. Multiuser scheduling is performed based on the available feedback, using as beamforming matrix the unitary basis in the codebook that maximizes the system sum rate. An extension to scenarios with a sum feedback rate constraint is provided in [8], coined as orthogonal SDMA with threshold feedback (TF-OSDMA). Codebook-based unitary precoding is a solid candidate for MIMO downlink trasmission in future mobile communication standards, currently under study in 3GPP [9][10][11]. Similarly to the work reported in [7], feedback from the mobile users in the form of a quantization index and channel quality indicator are used for user scheduling and beamforming design. Simple codebooks containing unitary bases have been considered so far, generated either randomly or from phase rotations of a DFT matrix. An advantage of DFT matrices is that multiplication with vectors can be done efficiently in reduced time. In addition, unitary beamforming yields smooth switching between single user point-to-point MIMO operation and multiuser SDMA.

In order to obtain good sum rates, the precoding matrices, quantization codebooks and feedback strategies need to be jointly designed. When constraining the precoding matrices to be unitary, the performance of suboptimal schemes should be evaluated by comparison with optimal unitary beamforming in order to measure the degree of suboptimality introduced. Conversely, limited feedback schemes relying on unitary beamforming should be designed with low complexity and reduced feedback, while approaching the performance of the optimal unitary beamforming solution. However, optimal unitary beamforming in MIMO broadcast channels - in the sense of system sum-rate maximization - is not yet known. Thus, most limited feedback schemes with unitary beamforming use low complexity as main design criterion, evaluating their performances through simulations. Multiuser MIMO schemes based on full channel knowledge at the transmitter and unitary beamforming have been proposed in [12], exhibiting performance gains over ZF beamforming approaches particularly at low SNR. However, the beamforming matrices in [12] are generated by following low-complexity design criteria with the aim of simplifying the scheduling algorithms in scenarios where the number of users is larger than the number of transmit antennas.

In this paper, an iterative optimization method for unitary beamforming in MIMO broadcast channels is proposed, based on successive optimization of Givens rotations. Initially, we consider a system with perfect channel state information at the transmitter (CSIT) side. As we show, the proposed technique provides higher sum rates than ZF beamforming while performing close to MMSE beamforming for practical average signal-to-noise ratio (SNR) values. However, as the average SNR becomes large, the slope of the sum-rate versus SNR curve converges to the one of a system with time-division multiple access (TDMA) that selects the best user, thus incurring in a loss of multiplexing gain. Moreover, it is shown to achieve linear sum-rate growth with the number of transmit antennas. The main advantage of the proposed unitary beamforming aproach is its robustness to channel estimation errors. As shown through numerical simulations, it provides better sum rates than ZF beamforming and even MMSE beamforming as the variance of the estimation error increases. Hence, the proposed beamforming technique can be seen as an interesting alternative to other existing linear beamforming schemes, such as ZF and MMSE. In addition, it provides a performance reference for evaluation of reduced-complexity unitary beamforming techniques. In the last part of this paper, the proposed technique is investigated in MIMO broadcast channels with multiuser scheduling, evaluating the performance of unitary beamforming approaches with limited feedback, namely RBF and LF-OSDMA.

II. SYSTEM MODEL

We consider a multiple antenna broadcast channel consisting of M antennas at the transmitter and $K \geq M$ single-antenna receivers. Given a set of M users scheduled for transmission, the signal received at the k-th mobile is given by

$$y_k = \sqrt{\frac{P}{M}}\mathbf{h}_k^H \mathbf{w}_k s_k + \sqrt{\frac{P}{M}} \sum_{i=1, i\neq k}^{M} \mathbf{h}_k^H \mathbf{w}_i s_i + n_k \quad (1)$$

where $\mathbf{h}_k \in \mathbb{C}^{M\times 1}$, $\mathbf{w}_k \in \mathbb{C}^{M\times 1}$, s_k, n_k and P are the channel vector, beamforming vector, transmitted signal, additive white Gaussian noise at receiver k and transmit power, respectively. The first term in the above equation is the useful signal, while the second term corresponds to the interference. We assume that the channels are i.i.d. block Rayleigh flat fading, the variance of the transmitted signal s_k is normalized to one and n_k is independent and identically distributed (i.i.d.) circularly symmetric complex Gaussian with zero mean and variance σ^2. Hence, the SINR of user k is given by

$$SINR_k = \frac{\frac{P}{M}|\mathbf{h}_k^H \mathbf{w}_k|^2}{\sum_{i=1, i\neq k}^{M} \frac{P}{M}|\mathbf{h}_k^H \mathbf{w}_i|^2 + \sigma^2} \quad (2)$$

A unitary beamforming matrix is considered at the transmitter $\mathbf{W} = [\mathbf{w}_1\ \mathbf{w}_2 \ldots \mathbf{w}_M] \in \mathbb{C}^{M\times M}$ and thus the average transmitted power is equal to P. In order to design the bemforming matrix, perfect knowledge of the user channels at the transmitter is assumed unless otherwise stated.

Imperfect CSIT model

The robustness of the proposed aproach to channel estimation errors is studied through numerical simulations. When imperfect knowledge of the user channel vectors is available at the transmitter side, the estimation error is modeled as an additive spatially white complex gaussian noise. Hence, the channel estimate of user k is given by

$$\widehat{\mathbf{h}}_k = \mathbf{h}_k + \widetilde{\mathbf{h}}_k \quad (3)$$

where $\widetilde{\mathbf{h}}_k$ has a distribution $\mathcal{CN}(0, \sigma_e^2 \mathbf{I})$. Imperfect CSIT can be the result of a combination of channel estimation noise, quantization errors, prediction errors, etc.

Notation: We use bold upper and lower case letters for matrices and column vectors, respectively. $(\cdot)^H$ stands for Hermitian transpose. $\mathbb{E}(\cdot)$ denotes the expectation operator and $tr(\cdot)$ is the trace operator. The notation $\|\mathbf{x}\|$ refers to the Euclidean norm of the vector $\mathbf{x}$ and $\|\mathbf{X}\|_F$ refers to the Frobenius norm of the matrix $\mathbf{X}$, defined as $\|\mathbf{X}\|_F = \sqrt{tr\left(\mathbf{X}\mathbf{X}^H\right)}$. The amplitude and phase of a complex scalar are denoted as $|\cdot|$ and $\angle(\cdot)$, respectively.

III. LINEAR BEAMFORMING IN MULTIUSER MIMO SYSTEMS

In order to compare unitary linear beamforming with commonly applied linear beamforming techniques, we provide a brief description of the most extended approaches: ZF beamforming (or channel inversion) and MMSE beamforming (or regularized channel inversion). Both beamforming matrices are computed on the basis of the concatenated user channels $\mathbf{H} = [\mathbf{h}_1 \ldots \mathbf{h}_M]^H$. The ZF beamformer is computed as follows

$$\mathbf{W}_{ZF} = \frac{1}{\lambda}\mathbf{H}^H(\mathbf{H}\mathbf{H}^H)^{-1} \quad (4)$$

where $\lambda = \frac{1}{\sqrt{P}} tr\left[(\mathbf{H}\mathbf{H}^H)^{-1}\right]$. The MMSE beamformer is given by

$$\mathbf{W}_{MMSE} = \gamma \mathbf{H}^H(\mu\mathbf{I} + \mathbf{H}\mathbf{H}^H)^{-1} \quad (5)$$

where γ is chosen such that $tr\left(\mathbf{W}_{MMSE}\mathbf{W}_{MMSE}^H\right) = P$. By fixing $\mu = \frac{M\sigma^2}{P}$, the resulting SINR with MMSE beamforming is maximized for large K, as shown in [5]. A drawback of channel inversion is that the resulting sum rate does not grow linearly with the number of antennas. This is due to the large spread in the singular values of the channel matrix, as discussed in [5]. By regularizing the channel inversion, the condition of the inverse is improved, enabling linear growth with the number of transmit antennas.

IV. PROBLEM FORMULATION

The optimization criterion considered in our problem is sum rate maximization, constrained to using linear unitary beamforming at the transmitter. Hence, the optimization problem can be formulated as follows

$$\max_{\mathbf{W}} \sum_{k=1}^{M} \log_2 \left(1 + SINR_k\right) \quad s.t. \quad \mathbf{W}^H\mathbf{W} = \mathbf{I} \tag{6}$$

This optimization problem is rather difficult to solve using this formulation, since the problem is nonconvex and the constraints are nonlinear. The problem can be reformulated by exploiting the particularities of the $SINR_k$ expression when unitary beamforming is used, which can be simplified as [13], [14]

$$SINR_k = \frac{\|\mathbf{h}_k\|^2 \rho_k^2}{\|\mathbf{h}_k\|^2 (1-\rho_k^2) + \frac{M\sigma^2}{P}} \tag{7}$$

where ρ_k is the alignment between the k-th user instantaneous normalized channel vector $\overline{\mathbf{h}}_k = \frac{\mathbf{h}_k}{\|\mathbf{h}_k\|}$ (channel direction) and the corresponding beamforming vector $\mathbf{w}_k$, defined as $\rho_k = \left|\overline{\mathbf{h}}_k^H \mathbf{w}_k\right|$. Define the vector $\boldsymbol{\rho} = [\rho_1\, \rho_2 \ldots \rho_M]$. Note that, when subtituting the $SINR_k$ expression shown in (7) into equation (6), the k-th term in the sum of logarithms becomes only a function of the variable ρ_k. The difficulty now lies in determining the feasible set of solutions for $\boldsymbol{\rho}$, i.e. the set of values for which a $\mathbf{W}$ matrix exists given that the user channels are known and fixed. This can be done by incorporating the geometrical structure of the problem into new constraints on $\boldsymbol{\rho}$, which is also a difficult task. Instead, as we show in next section, we propose a simple method to iteratively improve $\boldsymbol{\rho}$, while ensuring its feasibility by algorithm construction.

Another way to simplify the constrained optimization problem in equation (6) is to transform it in an unconstrained problem. Define the initial matrix $\mathbf{W}^0$ as an arbitrary unitary matrix. Let $\mathbf{R}_{mn}$ be the Givens rotation matrix in the $(\mathbf{w}_m, \mathbf{w}_n)$-plane, which performs an orthogonal rotation of the m-th and n-th columns of a unitary matrix while keeping the others fixed, thus preserving unitarity. Assume $n > m$ without loss of generality. The Givens rotation matrix in the $(\mathbf{w}_m, \mathbf{w}_n)$-plane is given by

$$\mathbf{R}_{mn}(\alpha,\delta) = \begin{bmatrix} 1 & \cdots & 0 & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & & \vdots & & \vdots \\ 0 & \cdots & \cos\alpha & \cdots & \sin\alpha e^{j\delta} & \cdots & 0 \\ \vdots & & \vdots & \ddots & \vdots & & \vdots \\ 0 & \cdots & -\sin\alpha e^{-j\delta} & \cdots & \cos\alpha & \cdots & 0 \\ \vdots & & \vdots & & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & 0 & \cdots & 1 \end{bmatrix} \tag{8}$$

where the non trivial entries appear at the intersections of m-th and n-th rows and columns. Hence, any unitary matrix $\mathbf{W}$ can be expressed using the following parameterization

$$\mathbf{W} = \mathbf{W}^0 \prod_{m=1}^{M} \prod_{n=m+1}^{M} \mathbf{R}_{mn} \tag{9}$$

up to a global $e^{j\theta}$ factor. Note that such global factor has no importance for transmision purposes. Each rotation matrix $\mathbf{R}_{mn}$ in (9) is function of 2 rotation parameters, α and δ. Hence, by imposing this structure, the optimization problem in equation (6) becomes unconstrained and it boils down to finding the optimal $2\binom{M}{2}$ rotation parameters of the corresponding $\binom{M}{2}$ rotation matrices. Since the resulting ρ_k values, $k = 1, \ldots, M$, are complicated non-linear functions of the rotation parameters, we propose an iterative algorithm to compute the optimal rotation matrix for a given plane, iterating along different planes until convergence is reached. Hence, the algorithm we propose is based on a divide-and-conquer type of approach. The matrix $\mathbf{W}$ is divided into smaller instances that are solved recursively in order to provide a solution to the optimization problem in (6). However, convergence to a global optimum can not be ensured for an arbitrary channel.

TABLE I
ALGORITHM OUTLINE

Initialization

- Initialize the UBF matrix $\mathbf{W}^0$

i-th iteration step, $i = 1, \ldots, N_{PR}$

- Select an index pair $\{m, n\}$ from $\mathcal{Q}$
- Find optimal rotation parameters for the $(\mathbf{w}_m, \mathbf{w}_n)$-plane
$$\{\alpha^*, \delta^*\} = \arg\min_{\alpha,\delta} F_{mn}(\alpha,\delta)$$
- Update UBF matrix $\mathbf{W}^i = \mathbf{W}^{i-1}\mathbf{R}_{mn}(\alpha^*, \delta^*)$

V. ALGORITHM DESCRIPTION

The proposed unitary beamformer is designed on the basis of the available user channels $\mathbf{h}_k, k = 1, \ldots, M$ and balances the amount of power and interference received by each user. Given an initial unitary beamforming matrix $\mathbf{W}^0$ available at the transmitter, we propose an iterative algorithm which consists of rotating the beamforming matrix by performing successive optimization of Givens rotations until convergence is reached. At the i-th iteration, a refined unitary beamforming matrix is computed by rotating the matrix $\mathbf{W}^{i-1}$ - computed at the previous iteration - in the plane defined by the complex vectors $(\mathbf{w}_m, \mathbf{w}_n)$, performing right multiplication with the rotation matrix defined in equation (8). For each plane rotation, the optimal α^* and δ^* rotation parameters are found. Let $\mathcal{Q}$ be the set of all possible index pairs among the complete index set $\{1, \ldots, M\}$, in which each $\{m, n\}$ index pair satisfies $n > m$. Define N_{PR} as the total number of plane rotations performed by the proposed approach. An outline of the proposed algorithm is provided in Table I.

It can be seen from the structure of the matrix in (8) that rotation in the $(\mathbf{w}_m, \mathbf{w}_n)$-plane does not change the directions of the remaining beamforming vectors. Equivalently, since $SINR_k$ is only function of $\rho_k = \left|\overline{\mathbf{h}}_k^H \mathbf{w}_k\right|$, a rotation in the $(\mathbf{w}_m, \mathbf{w}_n)$-plane only modifies $SINR_m$ and $SINR_n$.

Hence, the optimal rotation parameters are found by solving the following optimization problem

$$\{\alpha^*,\delta^*\}=\arg\max_{\alpha,\delta}\left\{\log_2\left(1+\frac{\|\mathbf{h}_m\|^2\rho_m^2(\alpha,\delta)}{\|\mathbf{h}_m\|^2(1-\rho_m^2(\alpha,\delta))+\frac{M\sigma^2}{P}}\right)\right.$$
$$\left.+\log_2\left(1+\frac{\|\mathbf{h}_n\|^2\rho_n^2(\alpha,\delta)}{\|\mathbf{h}_n\|^2(1-\rho_n^2(\alpha,\delta))+\frac{M\sigma^2}{P}}\right)\right\} \quad (10)$$

where $\rho_m(\alpha,\delta),\rho_n(\alpha,\delta)$ are the modified alignments between channels and beamfoming vectors after rotation , given by

$$\rho_m(\alpha,\delta)=\left|\overline{\mathbf{h}}_m^H\left(\mathbf{w}_m\cos\alpha-\mathbf{w}_n\sin\alpha e^{-j\delta}\right)\right| \quad (11)$$
$$\rho_n(\alpha,\delta)=\left|\overline{\mathbf{h}}_n^H\left(\mathbf{w}_m\sin\alpha e^{j\delta}+\mathbf{w}_n\cos\alpha\right)\right|$$

Defining the following variables

$$r_{mm}=\left|\overline{\mathbf{h}}_m^H\mathbf{w}_m\right| \qquad r_{mn}=\left|\overline{\mathbf{h}}_m^H\mathbf{w}_n\right|$$
$$r_{nm}=\left|\overline{\mathbf{h}}_n^H\mathbf{w}_m\right| \qquad r_{nn}=\left|\overline{\mathbf{h}}_n^H\mathbf{w}_n\right|$$
$$\Delta_{mn}=\angle\overline{\mathbf{h}}_m^H\mathbf{w}_m-\angle\overline{\mathbf{h}}_m^H\mathbf{w}_n \quad \Delta_{nm}=\angle\overline{\mathbf{h}}_n^H\mathbf{w}_n-\angle\overline{\mathbf{h}}_n^H\mathbf{w}_m \quad (12)$$

we have that

$$\rho_m^2(\alpha,\delta)=r_{mm}^2\cos\alpha^2+r_{mn}^2\sin\alpha^2-r_{mm}r_{mn}\cos(\Delta_{mn}+\delta)\sin 2\alpha$$
$$\rho_n^2(\alpha,\delta)=r_{nm}^2\sin\alpha^2+r_{nn}^2\cos\alpha^2+r_{nm}r_{nn}\cos(\delta-\Delta_{nm})\sin 2\alpha \quad (13)$$

Define the parameter $\beta_k=\frac{M\sigma^2}{P\|\mathbf{h}_k\|^2}, k=m,n$. Since the logarithm is a monotonically increasing function, the optimization problem in equation (10) can be transformed into

$$\{\alpha^*,\delta^*\}=\arg\min_{\alpha,\delta}F_{mn}(\alpha,\delta) \quad (14)$$

where the function F_{mn} is defined as follows

$$F_{mn}(\alpha,\delta)=\left(1-\rho_m^2(\alpha,\delta)+\beta_m\right)\left(1-\rho_n^2(\alpha,\delta)+\beta_n\right) \quad (15)$$

The solution is found by equating the gradient of F_{mn} to zero

$$\frac{\partial F_{mn}(\alpha,\delta)}{\partial\alpha}=0 \quad (16)$$
$$\frac{\partial F_{mn}(\alpha,\delta)}{\partial\delta}=0 \quad (17)$$

In order to solve the above equations, we introduce the change of variable $t=\tan\alpha$ to solve equation (16) and $s=\tan\delta/2$ to solve equation (17). After some algebraic manipulations the problem is reduced to finding the roots of polynomials of the form

$$P_\alpha(t)=f_4t^4+f_3t^3+f_2t^2+f_1t+f_0 \quad (18)$$
$$P_\delta(s)=g_4s^4+g_3s^3+g_2s^2+g_1s+g_0 \quad (19)$$

where $f_i,g_i,i=0,\ldots,4$ are real coefficients involving simple arithmetic and trigonometric operations, defined in Appendix I. The roots of these 4-th degree polynomials can be found by solving the respective quartic equations, for which closed form solutions exist [15]. Once the real roots are found, we invert the changes of variable introduced. The roots of P_α correspond to the extremes of the function $F_{mn}(\alpha,\delta)$ for fixed δ, while those of P_δ are the extremes of $F_{mn}(\alpha,\delta)$ for fixed α. Since up to 4 real roots may be found, the function $F_{mn}(\alpha,\delta)$ needs to be evaluated in the obtained roots in order to find the minimizing value α^*. An equivalent operation is performed for obtaining δ^*. Since computing α^* requires a constant value for δ and computing δ^* requires a constant value for α, the optimal values are found iteratively. Hence, α^* is computed initially by considering a certain initial value for δ (e.g. $\delta=0$) and the resulting α^* is kept constant for computation of δ^*. This operation is iterated I_R times until convergence, which in practice occurs after 1 or 2 iterations.

Practical Considerations

Although closed form solutions exist for quartic equations, fast converging algorithms can be applied involving much lower complexity. Since only real roots are seeked, the Quotient-Difference (QD) algorithm can be used to identify the roots followed by a fast converging algorithm like Newton-Raphson (NR) [16]. The initial unitary beamforming matrix $\mathbf{W}^0$ can be generated randomly, although more complex initializations may yield faster convergence. For instance, $\mathbf{W}^0$ can be constrained to have one of its vectors well aligned with the user channel that has the largest channel norm, as proposed in [12] as a suboptimal beamforming approach. In practice, this can be implemented by storing a number of unitary matrices (codebook), selecting the most appropriate for initialization at each slot. For simplicity, in the remainder of the paper, we consider that the proposed algorithm is initialized by choosing $\mathbf{W}^0$ randomly unless stated otherwise. Note that the proposed algorithm provides computational flexibility, since the number of plane rotations N_{PR} can be modified. In the most general case, all possible combinations of plane rotations should be performed, i.e. $\binom{M}{2}$ combinations. Moreover, the order in which these plane rotations are performed has an impact on the convergence. Hence, the total number of plane rotations can be expressed as $N_{PR}=I_T\binom{M}{2}$, where I_T is a natural number.

VI. CONVERGENCE

When optimizing the rotation along the $(\mathbf{w}_m,\mathbf{w}_n)$-plane, the sum of the rates provided by the m-th and n-th beamforming vectors is maximized with respect to the rotation parameters. Thus, defining $SR_{mn}=\log_2(1+SINR_m)+\log_2(1+SINR_n)$, at each plane rotation optimization we have that $SR_{mn}(\alpha^*,\delta^*)\geq SR_{m,n}(\alpha,\delta)$. In addition, as discussed in the previous section, the SINR values associated to the remaining beamforming vectors do not change. Hence, at each iteration the resulting sum rate does not decrease, i.e. $SR(\mathbf{W}^i)\geq SR(\mathbf{W}^{i-1})$. On the other hand, since the transmitted power is finite, the sum rate is bounded from above, which is the objective function that the algorithm tries to maximize. Thus, local convergence is guaranteed in the proposed optimization problem. In practice, given arbitrary channel realizations, simulations have shown that the proposed algorithm always converges to the same beamforming matrix regardless of the algorithm initialization. Thus,

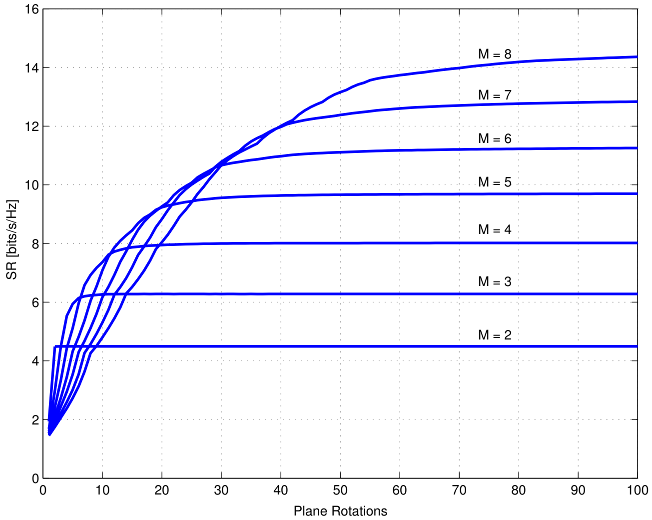

Fig. 1. Sum rate as a function of the number of plane rotations (algorithm iterations) for different number of transmit antennas, $K = M$ users and average $SNR = 10$ dB.

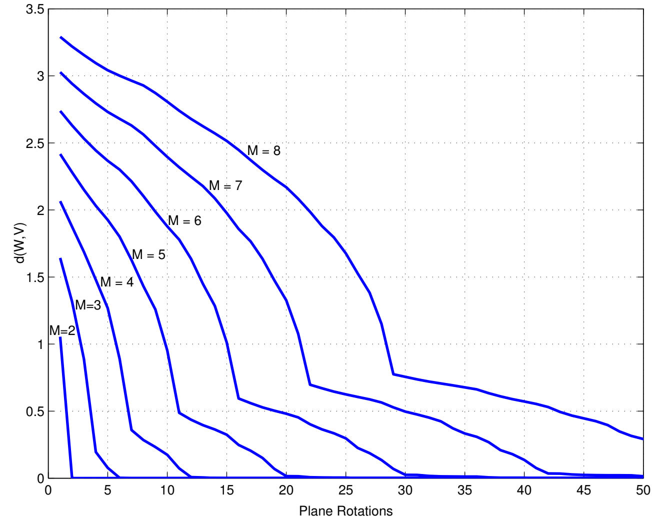

Fig. 2. Convergence of unitary beamforming matrix for different number of transmit antennas.

experiments seem to indicate that the proposed algorithm converges to a global optimum. The convergence behavior of the proposed iterative algorithm is exemplified in Fig. 1 for different number of transmit antennas. In this simulation, Givens rotations are performed in all possible $(\mathbf{w}_m, \mathbf{w}_n)$-planes, and a large number of plane rotations $N_{PR} \to \infty$ is considered.

In order to better illustrate the convergence speed of the proposed algorithm for different number of transmit antennas, we study a simple case in the remainder of this section for which the optimal solution is known. Let $\mathbf{H}$ be the concatenation of the user channels $\mathbf{H} = [\mathbf{h}_1 \ldots \mathbf{h}_M]^H$. Consider a simple channel model in which the concatenated channel can be factorized as $\mathbf{H} = \mathbf{\Lambda V}^H$, where $\mathbf{\Lambda}$ is a diagonal matrix with real entries ordered in descending order and $\mathbf{V}$ is a unitary matrix. This is equivalent to a point-to-point MIMO channel $\overline{\mathbf{H}}$ in which, given its singular value decomposition $\overline{\mathbf{H}} = \mathbf{U \Lambda V}^H$, the receiver filters the received signal with the matrix $\mathbf{U}^H$. If perfect channel state information is available at the transmitter and equal power allocation per beam is assumed, the optimal transmission is known to be $\mathbf{W} = \mathbf{V}$, yielding M virtual parallel channels [17][18]. In order to evaluate the convergence of the proposed algorithm to the optimal solution, we compute the following Frobenius distance at each iteration

$$d(\mathbf{W}, \mathbf{V}) = \left\| \mathbf{W}^H \mathbf{V} - \mathbf{I} \right\|_F \quad (20)$$

Fig. 2 shows the convergence behavior of the proposed algorithm for different number of transmit antennas. In this scenario, the proposed algorithm converges iteratively to the optimal solution. Note that for each value of M there are 2 differentiated regions with different convergence speed. The 1-st part converges faster, which corresponds to the 1-st $\binom{M}{2}$ iterations while the 2-nd part converges slower. This is due to the fact that the order in which plane rotations are performed matters, becoming more important as the size of the unitary beamforming matrix increases.

VII. PERFORMANCE EVALUATION

In this section, we evaluate the performance of the proposed unitary beamforming approach and compare it to other existing approaches. The proposed algorithm is initialized by choosing $\mathbf{W}^0$ randomly. Plane rotations are performed in all possible combinations, resulting in $N_{PR} = I_T \binom{M}{2}$ rotations, with $I_T = 3$. In the simulated scenarios, the algorithm approximately converges for this choice. The MATLAB function *roots* is used to compute the polynomial roots, which involves computing the eigenvalues of the companion matrix for each polynomial. In subsections *A* and *B*, a system with $K = M$ is studied, hence assuming a given set of M users has been scheduled for transmission. While in *A* perfect CSIT is assumed to be available, a system with imperfect CSIT is considered in *B*. In the last subsection, a system with multiuser scheduling is considered, comparing the proposed approach to limited feedback techniques based on unitary beamforming.

A. Case $K = M$, perfect CSIT

The performances of the proposed unitary beamforming technique, ZF beamforming and MMSE beamforming are compared in a system in which perfect CSIT is available, given a set of $K = M$ users scheduled for transmission. In addition, the performance of a system that performs TDMA is also plotted for reference, selecting the user with largest channel norm out of M available users.

Figure 3 shows a performance comparison in terms of sum rate versus number of transmit antennas M, for $SNR = 10$ dB. As expected, the MMSE solution provides linear sum-rate growth with the number of transmit antennas, while ZF beamforming flatens out [5]. The proposed algorithm also

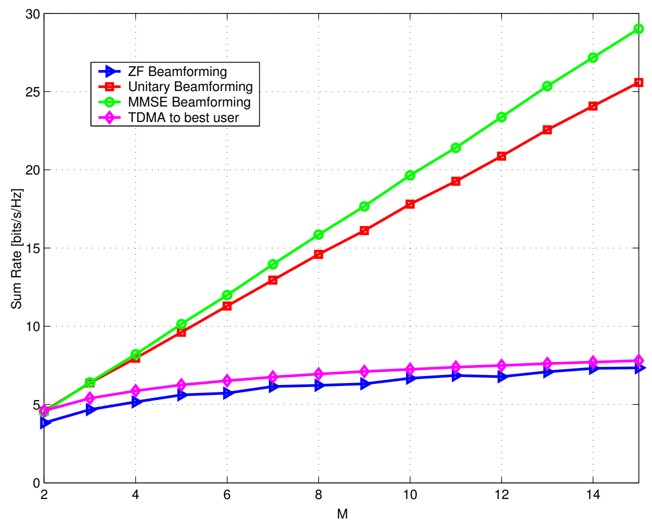

Fig. 3. Sum rate as a function of the number of antennas M for $K = M$ users and average $SNR = 10$ dB.

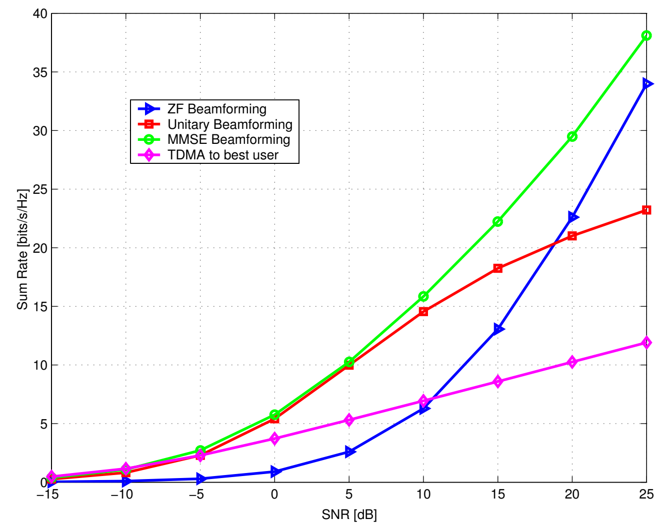

Fig. 4. Sum rate as a function of the average SNR for $M = 8$ transmit antennas and $K = M$ users.

provides linear growth with M, performing close to MMSE beamforming.

In Figure 4, we compare the sum rate as function of the average SNR in a system with $M = 8$ transmit antennas. As the SNR increses, the MMSE solution converges to ZF, removing all multi-user interference. The proposed technique provides considerable gains over ZF in the regular SNR range, performing close to the MMSE solution. On the other hand, the proposed algorithm does not completely eliminate interference, since instead it balances the useful power and undesired interference in the SINR expression. Suboptimal techniques based on unitary beamforming have shown to become interference limited at high SNR, thus providing zero multiplexing gain [6][7]. The multiplexing gain is defined as follows

$$m = \lim_{P\to\infty} \frac{\sum_{k=1}^{M} \mathbb{E}[\log_2(1 + SINR_k)]}{log_2(P)} \tag{21}$$

However, as it can be observed from Figure 4, the multiplexing gain of the proposed scheme converges to the one of TDMA (same slope). A particular case of the proposed approach corresponds to the case in which one of the unitary beamforming vectors is aligned with the channel vector that has largest norm. In that case, at least one of the users does not see any interference from the other users and hence at least $m = 1$ is achieved. Thus, for the proposed approach we obtain

$$m^{UBF} \geq \lim_{P\to\infty} \frac{\mathbb{E}\left[\log_2\left(1 + \frac{P}{M\sigma^2} \max_{i\in 1,\ldots,M} \|\mathbf{h}_i\|^2\right)\right]}{log_2(P)} \tag{22}$$
$$+ \frac{\sum_{k=1,k\neq i}^{M} \mathbb{E}[\log_2(1 + SINR_k)]}{log_2(P)} \geq 1$$

where the first term in the summation corresponds to aligning a unit-norm beamforming vector along the channel direction of the user with largest channel gain and the second term corresponds to the remaining $M - 1$ beamforming vectors. The second inequality in the above equation follows from the fact that if none of the $M - 1$ beamforming vectors in the second term is aligned with the remaining $M - 1$ channels, they exhibit zero multiplexing gain.

B. Case $K = M$, imperfect CSIT

The impact of imperfect channel knowledge at the transmitter in a system with $K = M$ users is investigated. The beamforming matrices are computed on the basis of noisy channel estimates $\widehat{\mathbf{h}}_k$, modeled as described in equation (3), which produces a performance degradation in terms of system sum rate. Figure 5 shows a sum-rate comparison between the proposed approach, ZF beamforming, MMSE beamforming and TDMA as a function of the variance of the channel estimation error, for $M = 4, 8$ antennas and average SNR of 10 dB. The proposed unitary beamforming approach proves to be more robust to CSIT errors than ZF or MMSE beamforming. Indeed, a small error variance suffices for unitary beamforming to outperform MMSE beamforming, even for large number of transmit antennas. However, TDMA provides higher rates in scenarios with reduced number of transmit antennas and very low quality of CSIT.

C. Case $K \geq M$, evaluation of limited feedback approaches

The proposed technique is used in this section as performance reference for evaluation of linear beamforming techniques based on unitary beamforming and limited feedback. A scenario with $K \geq M$ is considered and thus the need for multiuser scheduling arises. For simplicity, exhaustive user search is performed, i.e. the base station evaluates the sum rate of all possible user sets with cardinality M and selects the one that provides higher sum rate. Thus, the user set scheduled for transmission is found as follows

$$\mathcal{S}^* = \arg\max_{\mathcal{S}\in\mathcal{Q}} \sum_{k\in\mathcal{S}} \log_2\left[1 + SINR_k(\mathcal{S})\right] \tag{23}$$

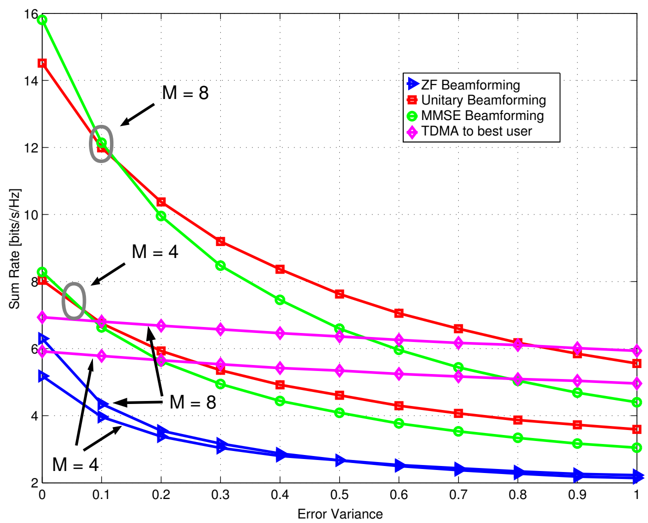

Fig. 5. Sum rate as a function of channel estimation error variance for $M = 4, 8$ transmit antennas, $K = M$ users and average $SNR = 10$ dB.

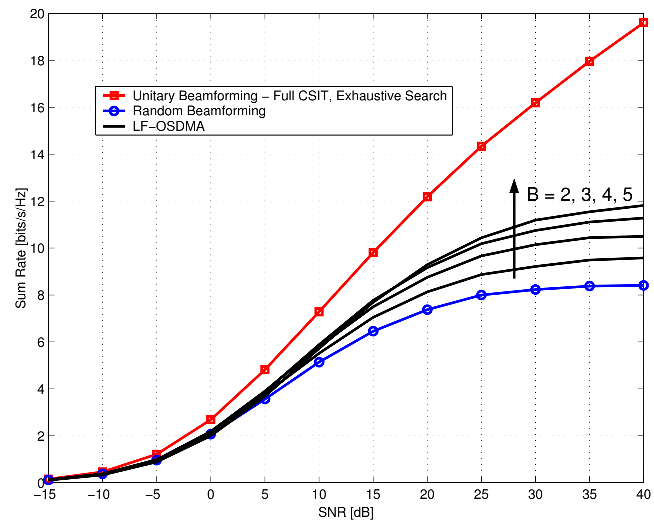

Fig. 6. Sum rate as a function of the average SNR in a system with joint beamforming and user scheduling, $M = 2$ transmit antennas and $K = 10$ users.

where $\mathcal{Q}$ is the set of all possible subsets of cardinality M of disjoint indices among the complete set of user indices $\mathcal{K} = \{1, \cdots, K\}$. The limited feedback approaches we consider are RBF [6] and LF-OSDMA [7]. In RBF, the base station generates a random unitary matrix and the users feed back their preferred beamforming vector along with their SINR. LF-OSDMA can be viewed as an extension of RBF for an arbitrary number of random unitary matrices. A codebook with N random unitary matrices is generated (each with M unit-norm vectors), known both to the base station and mobile users. The users feed back a codeword index using $B = log_2(MN)$ bits together with the expected SINR, which in the case of unitary beamforming can be precisely determined without knowledge of the beamforming vectors intended to other users [13], [14].

In Figure 6, a sum-rate comparison as function of the average SNR is shown in a system with $M = 2$ transmit antennas and $K = 10$ users. As expected, the limited feedback approaches become interference limited at high SNR. Similarly to the work presented in [19] for ZF beamforming, the scaling of the proposed technique can be achieved by limited feedback approaches as long as the amount of feedback bits scales with the SNR.

The sum rate versus number of active users is depicted in Figure 7, in a system with $M = 2$ transmit antennas and average $SNR = 10$ dB. The simulated techniques benefit from multiuser diversity gain, exhibiting optimal sum-rate scaling $M \log\log K$. Note that the performance of RBF, which was shown to achieve the optimal capacity scaling in [6], is a pessimistic lower bound on the performance of LF-OSDMA and the proposed approach. Even though the simulated limited feedback approaches can benefit from multiuser diversity, there is a considerable performance gap when compared with optimized unitary beamforming. Hence, an appropriate criterion to optimize codebook and feedback techniques in systems with unitary beamforming and limited feedback consists of bridging this performance gap.

VIII. CONCLUSIONS

An iterative optimization method for unitary beamforming in MIMO broadcast channels has been proposed, based on successive optimization of Givens rotations. In a scenario with perfect CSIT and for practical average SNR values, the proposed technique provides higher sum rates than ZF beamforming and performs close to MMSE beamforming, achieving linear sum-rate growth with the number of transmit antennas. The proposed unitary beamforming approach exhibits robustness to channel estimation errors, providing better sum rates than ZF beamforming and even MMSE beamforming as the variance of the estimation error increases. In addition, our approach can be used as a performance reference for design and evaluation of limited feedback techniques based on unitary beamforming.

APPENDIX I

This appendix describes a procedure to obtain the coefficients of the polynomials P_α and P_δ of equations (18) and (19), respectively. Although the procedure to obtain these coefficients can be described in different ways, here we present it in a simple and sequential fashion for straightforward software implementation. For each plane rotation, the auxiliary variables defined in Table II are computed and used for the computation of the coefficients of both P_α and P_δ. These auxiliary variables are functions of r_{mm}, r_{mn}, r_{nm}, r_{nn}, Δ_{mn}, Δ_{nm}, which are given in equation (12), and the parameter β_k.

A. Computation of the polynomial coefficients of P_α

The coeffients of P_α are functions of the rotation parameter δ. For clarity of exposition, the following functions are

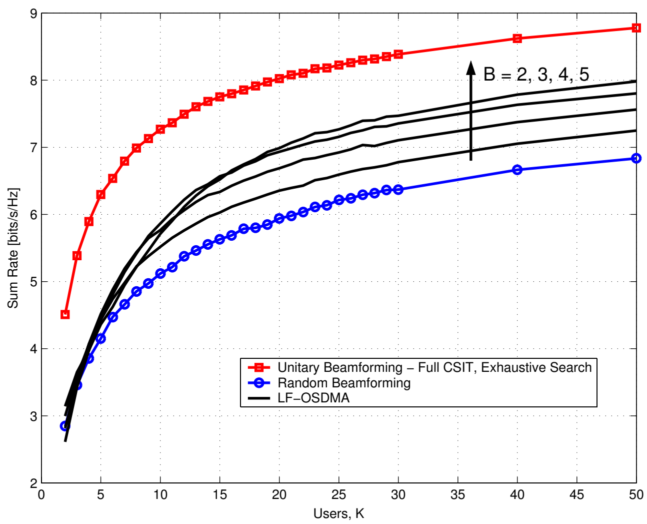

Fig. 7. Sum rate as a function of the number of users in a system with joint beamforming and user scheduling, $M = 2$ transmit antennas and $SNR = 10$ dB.

defined

$$\phi_1(\delta) = d_1 \cos(\Delta_{mn} + \delta)$$
$$\phi_2(\delta) = -d_2 \cos(\delta - \Delta_{nm})$$

The coefficients of the polynomial P_α are given by

$$f_4 = -2\phi_1(\delta)(e_3 - e_4) - 2\phi_2(\delta)(e_2 - e_5)$$
$$f_3 = -2\phi_1(\delta)\phi_2(\delta) + e_1 - e_6$$
$$f_2 = -12\left[\phi_1(\delta)e_4 + \phi_2(\delta)e_5\right]$$
$$f_1 = 2\phi_1(\delta)\phi_2(\delta) - e_1 - e_6$$
$$f_0 = 2\phi_1(\delta)(e_3 + e_4) + 2\phi_2(\delta)(e_2 + e_5)$$

B. Computation of the polynomial coefficients of P_δ

The coeffients of P_δ are functions of the rotation parameter α. The following functions are defined

$$\varphi_1(\alpha) = e_2 \sin 2\alpha + \frac{e_5 \sin 4\alpha}{2}$$
$$\varphi_2(\alpha) = e_3 \sin 2\alpha + \frac{e_4 \sin 4\alpha}{2}$$
$$\varphi_3(\alpha) = \frac{1-\cos 4\alpha}{4}$$

The coefficients of the polynomial P_δ are given by

$$g_4 = -d_8\varphi_1(\alpha) + d_6\varphi_2(\alpha) + d_4\varphi_3(\alpha)$$
$$g_3 = d_7\varphi_1(\alpha) - d_5\varphi_2(\alpha) + d_3\varphi_3(\alpha)$$
$$g_2 = -6d_4\varphi_3(\alpha)$$
$$g_1 = d_7\varphi_1(\alpha) - d_5\varphi_2(\alpha) - d_3\varphi_3(\alpha)$$
$$g_0 = d_8\varphi_1(\alpha) - d_6\varphi_2(\alpha) + d_4\varphi_3(\alpha)$$

TABLE II
AUXILIARY VARIABLES

$b_{mn} = -(r_{mm}^2 + r_{mn}^2)$	$b_{nm} = -(r_{nn}^2 + r_{nm}^2)$
$c_{mn} = -(r_{mm}^2 - r_{mn}^2)$	$c_{nm} = -(r_{nn}^2 - r_{nm}^2)$
$d_1 = 2r_{mm}r_{mn}$	$a_m = 1 + \beta_m$
$d_2 = 2r_{nm}r_{nn}$	$a_n = 1 + \beta_n$
$d_3 = -2d_1d_2\cos(\Delta_{nm} - \Delta_{mn})$	$e_1 = 2c_{mn}c_{nm}$
$d_4 = -\frac{d_1d_2}{2}\sin(\Delta_{nm} - \Delta_{mn})$	$e_2 = \frac{a_m}{2} + \frac{b_{mn}}{4}$
$d_5 = 2d_1\cos\Delta_{mn}$	$e_3 = \frac{a_n}{2} + \frac{b_{nm}}{4}$
$d_6 = d_1\sin\Delta_{mn}$	$e_4 = \frac{c_{nm}}{4}$
$d_7 = 2d_2\cos\Delta_{nm}$	$e_5 = \frac{c_{mn}}{4}$
$d_8 = -d_2\sin\Delta_{nm}$	$e_6 = 4(c_{nm}e_2 + c_{mn}e_3)$

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