Optimal Power Allocation For Cognitive Radio Based on a Virtual Noise Threshold

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ABSTRACT
In this paper[1], we investigate the idea of using *cognitive radio* to reuse locally unused spectrum for communications. We consider a multiband/wideband system with two users in which the primary (licensed) user and the secondary (cognitive) user wish to communicate to the base station, subject to mutual interference. We introduce the notion of the virtual noise-threshold which represents a proxy for the primary user to allow cognitive communications. We determine, under the assumption that the primary user is oblivious to the presence of the cognitive user the acceptable interference level within a given quality of service. We also give an interesting way in the paper to acquire the primary user's side information. The proposed strategy is proved to be the optimal one that achieves the maximum rate for each of the two users under the constraint that the secondary user maintains a guarantee of service to the primary user when cognitive communication is considered.

Keywords
Cognitive radio, interference temperature, power allocation, virtual noise threshold, capacity.

1. INTRODUCTION
The recent boom in personal wireless technologies has led to an increasing demand in terms of spectrum resources. To combat this overcrowding, the Federal Communications Commission (FCC) has been investigating new ways to manage RF resources involving progressive redefinition of rules for accessing to the radio spectrum and posing several tasks in the management and in the sharing strategies for such a precious resource. Within this setting, the FCC has recently recommended [1] that significantly greater spectral efficiency could be realized by considering *cognitive radio* Such a scheme would define at least two classes of spectrum users. The first would be primary users who already possess a license to use a particular frequency. The second would be secondary (cognitive) users consisting of unlicensed users. Primary users would always have full access to the spectrum when they need it. Secondary users could use the spectrum when it would not interfere with the primary user. Cognitive radio systems offer the opportunity to improve spectrum utilization by detecting unoccupied spectrum bands and adapting the transmission to those bands while avoiding the interference to primary users. This novel approach to spectrum access is therefore based on reliable detection of primary users and adaptive transmission over a wide bandwidth. However, there are many challenges across all layers of a cognitive radio system design, from its application to its implementation.

In this work, we consider a TDD-uplink communication scenario in which the primary and the secondary user wish to communicate with the base station (BS), subject to mutual interference in a heterogeneous network where devices operate in a wideband/multiband context. One property of such systems is that, since the same frequency is used, the channel characteristics are nearly the same in both links, provided the channel does not change too rapidly. Under this scheme, we allow the secondary user to transmit simultaneously with the primary user as long as the primary user has not his quality of service (QoS) affected. We impose that the secondary user can transmit simultaneously with the primary user as long as the level of interference with the primary user remains lower than a specified virtual noise threshold. We derive the optimum power allocation for each of the two users in terms of maximizing their own capacity.

This work is motivated by the fact that in a cognitive radio protocol as proposed by the Working Group on Wireless Regional Area Networks (WRANs), the primary user would always have his QoS not affected when cognitive communication is considered. The proposed strategy is proved to be the optimal one that achieves the maximum rate for each of the two users under the constraint that the secondary user maintains a guarantee of service to the primary user when

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cognitive communication is considered.

The remainder of the paper is organized as follows: In section 2, we present the channel model system. Section 3 describes the cognitive radio scenario as well as a simple way to acquire the primary user's side information. Section 4 details the optimal power allocation policy for each user. Simulation results are provided in Section 5 and Section 6 concludes the paper.

2. THE CHANNEL MODEL

The baseband discrete-frequency model for uplink channel with two users as described in fig.(1) is:

$$y_{BS}^i = h_1^i \sqrt{p_1^i} s_1^i + h_2^i \sqrt{p_2^i} s_1^i + n^i, \quad (1)$$

where:

- h_k^i is the block fading process of user k for $k = 1, 2$ on the sub-band i for $i = 1, ..., N$,
- s_k^i is the symbol transmitted by user k on the sub-band i,
- p_k^i is the power control of user k on the sub-band i,
- n^i is the additive gaussian noise at the ith sub-band.

We statistically model the channel h to be i.i.d distributed over the two rayleigh fading coefficients and $\mathbb{E}\left\{\left|h_k^i\right|^2\right\} = 1$. The additive gaussian noise n at the receiver is i.i.d circularly symmetric and $n \sim \mathcal{CN}(0, \sigma^2)$.

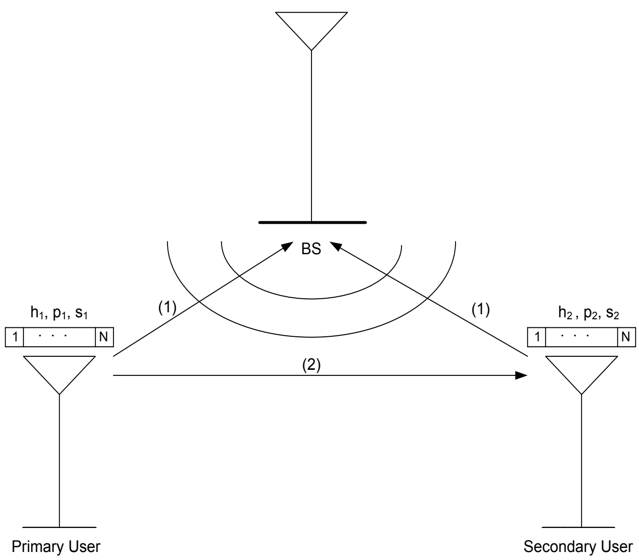

Figure 1: Two-user cognitive radio uplink in a wideband/multiband context.

3. THE COGNITIVE RADIO SCENARIO

Spectrum utilization can be improved by making a secondary user to access a spectrum hole unoccupied by the primary user at the right location and the right time. In current cognitive radio protocol proposals, the device listens to the wireless channel and determines, either in time or frequency, which part of the spectrum is unused. It then adapts its signal to fill this void in the spectrum domain. Thus, a device transmits over a certain time or frequency band only when no other user does like in [4]. In the same context, authors showed in [5] how we can improve the overall system spectral efficiency on classical approaches by considering orthogonal cognitive communications if there is ordering in the transmission. The contribution of some recent studies [6] and [7] has extended the cognitive protocol to allow the cognitive users to transmit simultaneously with the primary users in the same frequency band. This is exactly the question tackled in this work where the cognitive radio behavior is generalized to allow the secondary user to transmit simultaneously with the primary user as long as the level of interference with the primary user remains within an acceptable range.

As in [8], we consider a TDD-uplink communication scenario in which the primary and the cognitive users wish to communicate with the base station (BS), subject to mutual interference in a heterogeneous network where devices operate in a wideband/multiband context. We found out that a cognitive user can vary its transmit power in order to maximize its own capacity while maintaining a guarantee of service to the primary user. However, contrary to the work addressed in [8], in this contribution, it is proposed that the cognitive user is able to gather the instantaneous channel state information (CSI) of the primary user, while the primary user is assumed to only know his proper channel gains. The acquisition of primary user CSI will be discussed later.

Specifically, we consider a communication scenario where devices operates in a wideband/multiband context. In traditional systems, when the primary user considers only the ambient noise level σ^2, he will exploit all the resources by water-filling [9] on this noise level and therefore, leaves no resources for the secondary user to transmit. In our case, the primary user implicity over-estimates the noise level (which he considers as thermal noise plus interference) so as he leaves space for the other user. A key idea behind doing so is that, in any case, the primary user will not necessarily need all that rate. In fact, the primary user will experience σ_v^2, and as long as his capacity is maximized, he does not care about what he leaves more.

As an example, imagine a primary user communicating with the BS, subject to a virtual noise interference σ_v^2 and a secondary user able to reliably sense the spectral environment over a wide bandwidth, detect the presence/absence of the licensed user and use the spectrum to communicate with the BS only if the communication does not interfere with primary users like in [10].

Moreover, one basic assumption throughout this paper is that a cognitive user can vary its transmit power in order to maximize the capacity while maintaining a guarantee of service to the primary user. This implies the following inequality:

$$\log_2\left(1+\frac{p_1^i\left|h_1^i\right|^2}{\sigma_v^2}\right) \le \log_2\left(1+\frac{p_1^i\left|h_1^i\right|^2}{p_2^i \mid h_2^i \mid^2 + \sigma^2}\right)$$

Reliable communication can therefore be achieved when the virtual noise threshold is higher than the cognitive interferer contributes, yielding:

$$\sigma_v^2 \ge p_2^i \mid h_2^i \mid^2 + \sigma^2, \qquad i = 1, ..., N \tag{2}$$

Accordingly, the virtual noise threshold has a double role:

(i) it allows cognitive user to profit from the primary user resources in an opportunistic manner,

(ii) it maintains a guarantee of service to the primary user when cognitive communication is considered.

Throughout the rest of the paper, we will adopt this framework and derive the optimum power allocation policies of each user in terms of maximizing their proper capacity. We also determine what would be the average capacity rate of the primary and the secondary user in this setting.

4. OPTIMAL POWER ALLOCATION POLICIES

In this section, we will study the optimal power allocation policies that achieves the maximum capacity for each user in such a scenario. For a given virtual noise-threshold σ_v^2, the maximum achievable rate that the primary user can obtain over the N sub-bands is given by:

$$C_{1,N} = \frac{1}{N}\sum_{i=1}^{N}\log_2\left(1+\frac{p_1^i \mid h_1^i \mid^2}{\sigma_v^2}\right) \qquad (bits/s/Hz) \tag{3}$$

The optimal power allocation which maximizes the transmission rate at the primary user is solution of the optimization problem of:

$$\max_{p_1^1,\dots,p_1^N} C_{1,N},$$

subject to:

$$\begin{cases} \frac{1}{N}\sum_{i=1}^{N} p_1^i = 1, \\ p_1^i \ge 0, \end{cases} \tag{4}$$

In [9], authors looked at the problem of maximizing instantaneous capacity subject to an average power constraint, and showed that the optimum power allocation follows from Shannon's principle of water-filling, namely [2]:

$$p_1^{i*} = \left(\frac{1}{\gamma_0} - \frac{\sigma_v^2}{\left|h_1^i\right|^2}\right)^+, \qquad i = 1, ..., N \tag{5}$$

Where γ_0 is the Lagrange's multiplier satisfying the average power in (4).

[2] $(x)^+ = \max(0, x)$.

Let us now focus on the secondary user capacity. The expression of the instantaneous capacity is given by:

$$C_{2,N} = \frac{1}{N}\sum_{i=1}^{N}\log_2\left(1+\frac{p_2^i \mid h_2^i \mid^2}{p_1^i \mid h_1^i \mid^2 + \sigma^2}\right) \tag{6}$$

As far as channel estimation is concerned, this can be conducted in three steps: a) Each user estimates the pilot sequence transmitted by the BS in order to determine his channel gain via link (see link (1) in fig. 1). Notice here that since we are in a TDD mode, when the users estimate the channel in one way, they can also know it on the reverse link,

b) In the second frame, the primary user broadcasts a pilot sequence (see link (1) in fig. 1) so that the BS estimates the channel h_1,

c) In a third step, when the primary user sends his information with power p_1, the secondary user can estimate the power knowing the inter-user channel in link (2). In other words, assuming that the two users operate on a unique standard, the secondary user estimates the inter-user channel via link (2) and can accordingly obtain the primary user power p_1.

Thus, the secondary user can gather the primary users' channel gains by reverse engineering. In fact, by only knowing the virtual noise threshold (σ_v^2) and the power allocation policy of the primary user (waterfilling), the cognitive user can obtain the primary users' channel gains following equation in (5).

The secondary user offers the opportunity to improve the sum capacity over the system by reliably detecting primary user activity and adapting his transmission while avoiding the interference to the primary user by satisfying constraint in (2). In fact, the spectrum utilization can be improved by making a secondary user to access to the primary user spectrum at the right location and the sub-band in question. The optimal secondary user power allocation which maximizes the capacity of the system is solution of the optimization problem:

$$\max_{p_2^1,\dots,p_2^N} C_{2,N} \tag{7}$$

subject to:

$$\begin{cases} \frac{1}{N}\sum_{i=1}^{N} p_2^i = 1 \\ p_2^i \ge 0; \qquad \text{for} \quad i = 1, ..., N \\ \sigma_v^2 \ge p_2^i \mid h_2^i \mid^2 + \sigma^2; \qquad \text{for} \quad i = 1, ..., N \end{cases} \tag{8}$$

THEOREM 1. *The optimal secondary user power allocation solution of the problem (7) under the constraints in (8)* *is:*

$$p_2^{i*} = \begin{cases} \left(\frac{1}{\lambda} - \frac{p_1^i \mid h_1^i \mid^2 + \sigma^2}{|h_2^i|^2}\right)^+ ; \text{ if } \lambda > \frac{|h_2^i|^2}{p_1^i \mid h_1^i \mid^2 + \sigma_v^2} \\ \frac{\sigma_v^2 - \sigma^2}{|h_2^i|^2} ; \quad \text{otherwise} \end{cases} \tag{9}$$

Where λ *is the Lagrange's multiplier satisfying the average power constraint in (8).*

PROOF. Due to the lack of space, we will not present these consistencies here and the reader is referred to the reference [11] for additional analysis results. □

Accordingly, the optimal power allocation for this problem was shown to be a mixture of channel inversion and water-filling allocation. Notice here that the proposed strategy prevents to obtain infinite power in extreme fading environments, i.e. for bad fading states h_2^i, the power allocation policy is the waterfilling.

Discussion: Assume σ_v^2 and h_1^i fixed. For good h_2^i the optimal power-control law is the channel inversion policy. On the contrary, for bad values of h_2^i the optimal power-control law is *water-filling* on the inverse of the channel gain. This stands in contrast to the traditional case of channel inversion policy where more power is allocated when the channel is bad than when the channel is good. Notice that in the event of deep fades, i.e. h_2^i tends to be zero, the secondary user power allocation is zero.

5. SIMULATIONS AND RESULTS

In order to analyze the performance of the proposed policies in terms of achievable rates, we model two rayleigh channel models, one for each user. In figure 2, the solid line stands for the sum capacity of a system where the primary user decides to maximize his rate selfishly. In other words, he will water-fill over the ambient noise level σ^2 and no resources will be left for cognitive users. The dashed line represents the sum capacity of the proposed system where the primary user experiences the virtual noise threshold σ_v^2. It is clear that the cognitive system performs always better than for system where no cognition exists.

6. CONCLUSION

We introduced the notion of the virtual noise-threshold in order to maintain a guarantee of service to the primary user when cognitive communication is considered. We also present a simple algorithm so that the secondary user can gather the side information about the primary user. The proposed policies are proved to be the optimal ones that achieve the maximum rate for each of the two users under the constraint that the secondary user maintains a guarantee of service to the primary user when cognitive communication is considered. As a future work, it is of major interest to study the impact of the proposed strategy on the primary user in terms of outage capacity and to characterize this outage with respect to classical communication systems. The related work can also be extended to the two-way channel context.

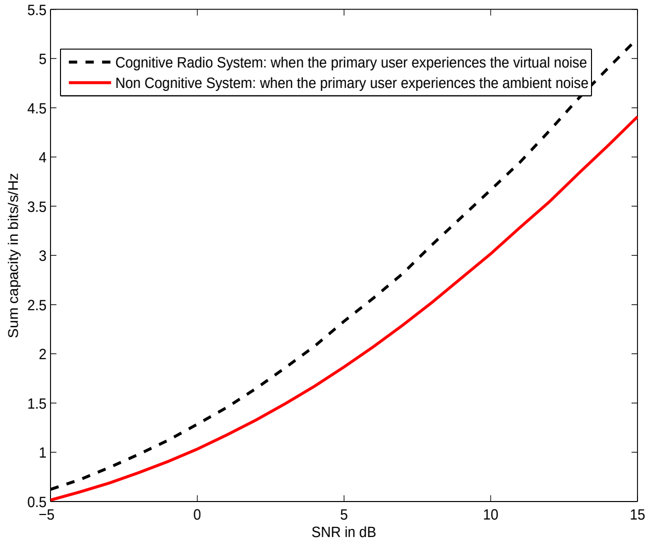

Figure 2: Sum Capacity for $N = 10$.

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