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par

Nicolas Bonneau

Mécanismes Multi-Utilisateurs Centralisés et Decentralisés pour les Communications sans Fil

Contents

Résumé

Avec la taille croissante des systèmes de communication sans fil, de nouveaux défis apparaissent. En particulier, localiser l'intelligence revêt une importance primordiale. Deux vues complémentaires dominent, à savoir le contrôle par une unité centrale de traitement (approche centrée sur le réseau), ou l'administration du réseau par les utilisateurs eux-mêmes (approche centrée sur l'utilisateur).

Cette thèse se concentre sur l'analyse de performance des systèmes de communication sans fil, et introduit des mécanismes pour améliorer leur efficacité, particulièrement dans un contexte distribué. Les travaux sont caractérisés par l'utilisation de méthodologies sophistiquées pour analyser les réseaux sans fil: théorie des matrices aléatoires et théorie des jeux. Le but est de fournir une qualité optimale de service aux utilisateurs, sous des contraintes comme la consommation d'énergie et la connaissance limitée de l'environnement.

Une première partie du travail concerne des applications de la théorie des matrices aléatoires à l'optimisation de grands réseaux cellulaires. Lorsque la taille des réseaux augmente, les interférences entre utilisateurs augmentent, et les simulations impliquent un nombre énorme de paramètres (aléatoires). L'effet de moyennage de la théorie des matrices aléatoires permet d'isoler de manière élégante les paramètres pertinents dans un système asymptotique, quand le facteur d'étalement, le nombre d'antennes ou le nombre de porteuses, et le nombre d'utilisateurs deviennent tous deux très grands à ratio fixé. Même si les résultats sont obtenus asymptotiquement, ils donnent des prévisions très précises du comportement de systèmes de taille finie, comme le montrent les simulations. Cette analyse est destinée à servir d'étalon, en comparaison à l'approche décentralisée, quand les mobiles prennent leurs propres décisions basées sur leur information locale.

Une deuxième partie du travail est consacrée à appliquer des cadres de théorie des jeux aux protocoles distribués. La théorie des jeux fournit une vaste panoplie d'outils pour étudier toutes sortes d'interactions faisant intervenir des joueurs égoïstes qui raisonnent stratégiquement afin de prendre des décisions. Avec l'intérêt croissant pour le déploiement de réseaux s'auto-organisant, il est tentant de considérer des mobiles présentant de telles caractéristiques. Certains domaines de la théorie des jeux, que nous fûmes les premiers à introduire dans un contexte de réseau, sont particulièrement prometteurs. Les jeux corrélés permettent d'inclure un mécanisme de coordination entre les joueurs, tandis que les jeux évolutionnaires fournissent une propriété additionnelle de robustesse des équilibres. Dans le régime asymptotique, les jeux non-atomiques étudient des interactions au sein de populations denses où le comportement d'un individu a un impact négligeable sur le bien-être de la population. De tels cadres permettent de concevoir des protocoles distribués efficaces.

Abstract

With the growing size of wireless communication systems, new challenges are emerging. One of the main topics concerns the actual topology to route efficiently information in the network. In particular, the exact localization of the intelligence is of paramount importance. Two complementary views dominate, either managing through a central controller (network centric) or letting the users themselves administrate the network (user centric).

This thesis focuses on the performance analysis of wireless communication systems, and introducing mechanisms to improve their efficiency, especially in a distributed context. The research is characterized by the novel use of sophisticated methodologies for analyzing wireless networks: random matrix theory and game theory. The purpose is to deliver an optimal quality of service to users, under constraints like energy consumption and limited knowledge of the environment.

A first part of the work concerns applications of random matrix theory and unitary random matrix theory to the optimization of large cellular networks. As multiuser networks grow large, the amount of interfering communications increases, and simulations involve a huge number of (random) parameters. The self-averaging effect of random matrix theory enables to elegantly single out parameters of interest in asymptotic systems, when the number of chips, antennas or carriers and the number of users both grow very large with fixed ratio. Even if the results are obtained in the asymptotic regime, they give very accurate predictions of the system behavior in the finite size case, as shown by simulations. The performance analysis of centralized systems may serve as a benchmark, compared to the user centric case when mobiles take individual decisions based upon their local information.

A second part of the work is devoted to applying game theoretical frameworks to distributed multiuser schemes. Game theory provides a vast array of tools to study all kinds of interactions among selfish players who reason strategically in order to take rational decisions. With the increasing interest in deployment of self-organizing networks, it is very alluring to consider mobiles as independent actors that demonstrate such characteristics. A few subfields of game theory, which we were the first to introduce in a network context, are particularly promising. Correlated games enable to include a coordination mechanism between players, while evolutionary games provide additional properties of robustness of equilibrium strategies. In the asymptotic regime, the relevant framework is non-atomic games, studying interactions of dense populations where the behavior of a single individual has a negligible impact on the welfare of the population as a whole. Such frameworks enable to derive efficient distributed schemes.

Résumé en Français

Introduction

Analyse de Systèmes Multi-Utilisateurs

La publication pionnière de 1948 de Shannon [Sha48] a initié à elle seule la discipline de la théorie de l'information. Dans cet article, parmi beaucoup d'autres définitions et théorèmes fondamentaux, Shannon présente la notion de *capacité* d'un canal en tant que mesure de performance. La capacité détermine le débit réalisable entre deux terminaux qui communiquent à travers un canal bruité. Cette contribution fondamentale fut l'étincelle qui mena à la publication d'une multitude de travaux, comme le prouve l'étude [Ver98]. Presque soixante ans après sa naissance, le cas "simple" d'un utilisateur unique, comme considéré dans [Sha48], peut être considéré comme ayant été traité dans toute son ampleur par la communauté spécialisée dans la théorie de l'information.

Le cas plus complexe de plusieurs utilisateurs partageant le même milieu de communication a été abordé pour la première fois par Shannon dans un article de 1961 [Sha61]. Cette contribution marque la base de la *théorie de l'information à utilisateurs multiples*. Le travail [Sha61] est consacré à l'étude d'un canal bidirectionnel, semblable à la téléphonie, où l'interférence se produit entre les signaux transmis dans les deux directions opposées. La notion conventionnelle de capacité n'est plus appropriée ; néanmoins, une région bidimensionnelle de capacité peut être définie, qui indique l'ensemble de paires réalisables de débit. Malheureusement, aucune expression explicite pour cette région de capacité n'existe, même dans des cas particulièrement simples. Seules des bornes peuvent être dérivées.

Shannon conclut son article par la phrase suivante: "Dans un autre papier, nous discuterons le cas d'un canal partagé par deux terminaux ou plus, uniquement dédiés à la transmission, et un terminal uniquement dédié à la réception, cas pour lequel une solution complète et simple de la région de capacité a été trouvée. "

Ceci définit ce qui est désigné maintenant sous le nom de *canal à accès multiple*. Plusieurs émetteurs communiquent avec un récepteur simple à travers un canal commun. Ceci a engendré une abondance de contributions ; une sélection effectuée par Verdù parmi celles-ci peut être trouvée dans l'étude [Ver98], pp. 11–12. Il est à noter que, contrairement à son annonce, Shannon n'a pas publié plus avant sur le sujet, donc la portée réelle de sa solution reste indéterminée.

De nombreux protocoles pour permettre une mise en application efficace des communications quand plusieurs utilisateurs sont impliqués ont été proposés dans la littérature, et ont également trouvé une mise en place pratique. Les protocoles orthogonaux, tels que l'Accès Multiple à Répartition dans le Temps (TDMA) et l'Accès Multiple par Répartition en Fréquence (FDMA), existaient déjà avant la parution de l'article de Shannon et étaient employés en télégraphie. D'autres, comme l'Accès Multiple par Répartition en Code (CDMA), en étaient encore à leurs balbutiements. Il fut rapidement démontré que les protocoles orthogonaux, tels que TDMA et FDMA, ne permettent généralement pas de réaliser la totalité de la région de capacité du canal à utilisateurs multiples. D'autre part, les protocoles avec un niveau contrôlé de l'interférence entre les utilisateurs, tels que le CDMA, permettent une telle optimisation. Afin de réaliser la totalité de la région de capacité dans le cas de plusieurs terminaux communiquant simultanément avec une station de base unique, un analogue pour utilisateurs multiples de l'algorithme à remplissage d'eau (water-filling) individuel a été proposé dans [TH98].

La frontière de la région de capacité détermine le *débit global* du système. C'est l'une des multiples mesures de performance qui peuvent être optimisées. Par exemple, le *délai* que subissent les communications peut également être considéré, comme il est souligné dans [HT98]. La *complexité* est aussi fréquemment un facteur à prendre en compte, en particulier aux noeuds mobiles, qui disposent seulement d'une connaissance locale du système, ainsi que d'une puissance de calcul et d'un approvisionnement en énergie limité.

En outre, un système à utilisateurs multiples fonctionne rarement en isolation. Au contraire, il fait habituellement partie d'un réseau plus vaste. De plus en plus, les interactions entre plusieurs systèmes à utilisateurs multiples séparés sont prises en considération, ainsi que le démontre la courte étude [SSZ04]. Afin de concevoir une architecture de réseau efficace, beaucoup de possibilités ont été considérées dans toute la littérature. En particulier, la conception de protocoles pour servir les utilisateurs est un vaste sujet d'étude.

Dans cette thèse, nous nous limitons aux protocoles de couche physique tels que l'accès multiple par répartition en code, et nous passons en revue leurs performances dans plusieurs cadres. Notre but est de développer et déployer une infrastructure dans le but de servir les utilisateurs. En conséquence, nous étudions les différences entre systèmes centralisés et décentralisés. Pour cela, nous aurons tout d'abord besoin de quelques définitions.

Définitions Utiles

Cellulaire, Ad-hoc, Hybride

Cellulaire signifie que la surface considérée est divisée en cellules. Dans chaque cellule, les utilisateurs communiquent avec une station de base, i.e., un "super-noeud" qui est habituellement supposé être relié (par l'intermédiaire de lien câblé) à d'autres stations de base, à l'Internet, etc. Les formes des cellules sont déterminées par différentes règles, par exemple leur surface est fixée, ou déterminée selon les zones qui ont le rapport signal à bruit (SNR) maximal. En 2-D, un des modèles le plus souvent considéré est celui de cellules hexagonales régulières, afin de remplir le plan. Il est à noter que ceci est un cas particulier de diagramme de Voronoi, quand les stations de base sont régulièrement distribuées sur le plan. En 1-D, ceci correspondra à des segments de longueur égale sur l'axe réel, qui sera un modèle que nous adopterons. De nos jours, les systèmes cellulaires sont largement déployés, par conséquent l'intérêt de leur analyse est intense, aussi bien en ce qui concerne les aspects théoriques que pratiques. Une vue d'ensemble récente de la littérature consacrée aux réseaux cellulaires est proposée dans l'étude [SSZ04].

Un système s'appelle *ad-hoc* s'il ne bénéficie pas d'une telle infrastructure fixe. Habituellement, les noeuds sont alors supposés communiquer directement les uns avec les autres et sont capables d'auto-organisation. La capacité de tels réseaux a été analysée de manière intensive, commençant par le célèbre article de Gupta et Kumar [GK99], et en notant l'amélioration en débit fournie par la mobilité, aux dépens d'un délai plus important [GT02]. Le compromis entre le débit et le délai a depuis été étudié [GMPS06].

Les *systèmes hybrides* sont également considérés dans la littérature. Les noeuds qui sont loin d'une station de base communiquent entre eux, en mode ad-hoc, jusqu'à ce que la communication arrive à un noeud près d'une station de base. De tels systèmes représentent en quelque sorte un compromis entre les systèmes centralisés et décentralisés.

Systèmes Centralisés et Décentralisés

Un environnement *centralisé* est un environnement dans lequel sont disponibles des informations globales sur tous les noeuds dans le système. Les noeuds sont administrés par une unité centrale de traitement, qui collecte ces informations. Habituellement, les réseaux cellulaires sont supposés bénéficier d'une commande centralisée fournie par la station de base. Un terme équivalent que nous emploierons est *approche centrée sur le réseau*, puisque le réseau est considéré comme une entité globale dans cet environnement.

Au contraire, les réseaux ad-hoc sont généralement perçus comme l'exemple type de systèmes *décentralisés*. Il n'y a aucune unité centrale de traitement, les mobiles administrent le réseau par eux-mêmes. Les algorithmes de communication dans ce cas sont dit *distribués*. Un terme équivalent que nous emploierons est *approche centrée sur l'utilisateur* puisque l'acteur principal dans un tel environnement est l'utilisateur.

Coopération et Compétition

Quand les mobiles sont considérés en tant qu'entités indépendantes administrant leurs transmissions, deux modèles de communication, dérivés de la théorie des jeux, peuvent avoir lieu. Le premier est *coopératif*. Dans ce modèle, les mobiles collaborent avec le but de réaliser un objectif commun, par exemple maximiser le débit global du système.

Au contraire, dans le modèle *non coopératif*, les noeuds sont des individus au comportement égoïstes. Ils ne s'inquiètent pas du bien-être du système dans son ensemble, mais seulement de maximiser leur propre gain. Une solution dans ce cadre, quand aucun mobile ne peut tirer bénéfice en déviant unilatéralement, s'appelle un équilibre de Nash. Généralement, la performance globale à l'équilibre de Nash est inférieure à la performance optimale qui peut être atteinte dans un contexte coopératif. Des propriétés additionnelles de stabilité peuvent être mises en application, par exemple en présentant le concept de biologie mathématique dénommé stratégie évolutionnairement stable.

Coordination

Enfin, *coordination* se rapporte à un domaine particulier de la théorie des jeux. Un jeu corrélé peut avoir lieu entre joueurs coopératifs ou non coopératifs. Il consiste en la présence d'un degré additionnel de liberté aux mobiles par la présence d'un arbitre. L'arbitre est une entité qui peut envoyer des messages aux mobiles ; elle n'a pas besoin de disposer d'intelligence, ni d'une quelconque connaissance du système. Elle envoie simplement des signaux aléatoires, dont les joueurs peuvent tenir compte afin de maximiser conjointement leur utilité.

Une façon intuitive d'illustrer un phénomène de coordination est la suivante. Prenons deux personnes et disons-leur de choisir un entier positif, sans communication entre elles. Si elles choisissent toutes deux le même entier, alors elles reçoivent toutes les deux une récompense. Sinon, aucune d'entre elles ne reçoit quoi que ce soit. Spontanément, les deux personnes choisissent le chiffre 1. C'est le point commun que toutes deux partagent. Ainsi, la coordination peut représenter un juste milieu entre les aspects coopératifs et non-coopératifs d'une situation donnée. Elle permet d'améliorer d'une façon très simple et ne nécessitant pas de calculs complexes ou de transmissions de volumes importants de données la quantité d'information transmise dans un système donné, comme nous le verrons plus tard.

Présentation de la Thèse

Outils Mathématiques

Dans un premier temps, nous donnons une introduction aux outils mathématiques utilisés dans le cadre de cette thése. Ceux-ci proviennent principalement de deux théories mathématiques: la théorie des jeux et la théorie des matrices aléatoires.

L'introduction de la théorie des jeux et de la théorie des matrices aléatoires au sein de la communauté de théorie de l'information est plutôt récente. Dans le chapitre 2, une courte histoire de ces domaines est retracée et de nombreux résultats illustratifs et utiles sont fournis. En particulier, la théorie des jeux est un vaste domaine mathématique, ainsi l'introduction est nécessairement très partielle et limitée à des concepts utilisés dans les communications sans fil et plus particulièrement dans le reste de ce travail. Une application particulière de la théorie des jeux dans le domaine des communications sans fil est illustrée dans le contexte de l'attribution de ressources.

Approche centrée sur le réseau

Dans un second temps, nous accomplissons l'analyse de performance de systèmes centralisés dans plusieurs cadres. Nous nous limitons au protocole CDMA (Accès Multiple par Répartition en Codes). Nous étudions la performance de systèmes CDMA dans quatre cas.

- Le déploiement cellulaire infini en liaison descendante, avec codes orthogonaux.
- Le déploiement cellulaire infini en liaison montante, avec codes aléatoires.
- Le déploiement unicellulaire en liaison montante, avec codes orthogonaux.
- Enfin, le déploiement cellulaire infini en liaison montante, avec codes orthogonaux, est considéré.

Cette analyse de performance est présentée de manière détaillée dans les différentes sections du chapitre 3.

L'analyse de performance de systèmes centralisés nous permet d'obtenir des bornes sur la performance pouvant être obtenue dans le cas de systèmes décentralisés. Ce qui est vraiment intéressant dans le cas de systèmes accomodant un grand nombre d'utilisateurs, c'est de construire des protocoles distribués. Ceux-ci sont hautement désirables à plusieurs titres: chaque utilisateur accomplit les calculs individuellement, à partir de son information locale, sans avoir recours à des transmissions non-informationnelles avec la station de base; les protocoles obtenus s'adaptent naturellement à une mise à l'échelle.

Approche centrée sur l'utilisateur

Ainsi, dans un troisième temps, nous introduisons différents concepts de théorie des jeux dans un contexte de système multi-utilisateurs. Nous nous intéressons cette fois aux protocoles ALOHA, CDMA et OFDMA. Les cadres considérés sont les suivants.

- Nous introduisons la théorie des jeux évolutionnaires pour une grande population de mobiles utilisant un protocole ALOHA.
- Nous introduisons la théorie des jeux corrélés pour une population de mobiles communiquant via un protocole ALOHA à fenêtres d'émission discrètes.
- Nous utilisons la théorie des jeux pour construire un algorithme d'allocation de puissance pour un système CDMA unicellulaire.
- Enfin, nous utilisons la réciprocité du canal pour construire un protocole permettant aux utilisateurs de communiquer avec la station de base sans que celle-ci n'ait de connaissance du canal pour un système OFDMA unicellulaire.

Ces protocoles et leur analyse sont présentés dans les différentes sections du chapitre 4.

Dans ce résumé en français de la thèse, l'accent sera mis sur certains résultats particuliérement illustratifs. Pour plus de détails sur les calculs ainsi que sur l'ensemble des résultats, le lecteur est invité à se référer au corps de la thèse.

Outils Mathématiques

Théorie des Jeux

Pour cette thèse, la théorie des jeux est un domaine mathématique qui occupe une grande importance, et une partie entière de ce rapport est consacrée à donner une introduction aux notions importantes de ce domaine. Nous allons donner les grandes lignes et définitions de la théorie des jeux dans les paragraphes qui suivent.

Bref Historique

Les racines de la théorie des jeux sont basées sur des situations de la vie courante. Une analyse mathématique fut initialement proposée pour étudier des problèmes impliquant plusieurs joueurs, tels que les échecs ou les jeux de cartes. De nombreux travaux ont posé les fondations, notamment pendant les dix-neuvième et vingtième siècles. Néanmoins, il est généralement considéré que le véritable texte fondateur de la théorie des jeux est le livre *Theory of Games and Economic Behavior*, par J. von Neumann and O. Morgenstern, paru en 1944. Ce livre donne un formalisme pour les jeux coopératifs aussi bien que non-coopératifs.

Ces concepts ont ensuite été étendus par J. Nash. En particulier, c'est celui-ci qui démontra l'un des théorèmes les plus importants en théorie des jeux, à savoir l'existence d'un *équilibre* dans les jeux non-coopératifs, dénommé depuis lors équilibre de Nash. Nash étudia également les jeux coopératifs, qui sont généralement considérés comme requérant une analyse plus complexe, étant donné que les joueurs peuvent former des coalitions pour atteindre le plus grand bien-être collectif. Il dériva la solution de marchandage de Nash pour de tels jeux. De nos jours, la théorie des jeux est utilisée dans de nombreux domaines, en économie, en biologie mathématiques, ainsi qu'en communications sans fil et en réseaux.

Généralités

La théorie des jeux fournit une vaste gamme d'outils pour étudier toutes sortes d'interactions parmi des individus prenant des décisions. En particulier, la théorie des jeux non-coopératifs est fondée sur deux prémisses majeures. La première est la *rationalité* des individus impliqués, c'est à dire que les joueurs (égoïstes) prennent leurs décisions en fonction de leurs préférences quant au résultat qui en découle. La seconde est leur capacité de *raisonnement stratégique*, c'est à dire que chaque joueur prend également en compte les préférences des autres joueurs.

Un jeu non-coopératif est défini par trois ensembles.

- Un ensemble de joueurs S^K, qui consiste de K individus;
- Pour chaque joueur $k \in S^K$, un ensemble de stratégies (ou actions) $\mathbb{S}_k$;
- Pour chaque joueur $k \in S^K$, une relation de préférence $\mathbb{S} = \prod_k \mathbb{S}_k$.

En général, la relation de préférence de chaque joueur s'exprime sous la forme d'une fonction $u_k : \mathbb{S} \to \mathbb{R}$, appelée fonction d'utilité. Une discussion approfondie sur ce sujet est fournie dans [MD06].

Ce modèle se situe à un niveau d'abstraction très élevé. Ainsi, il permet de couvrir une très large gamme de situations. L'inconvénient est que sous cette forme, aucun résultat général ne peut être dérivé directement. Les paramêtres doivent être spécifiés pour obtenir des résultats spécifiques à la version du jeu considéré.

Equilibre de Nash

Un vecteur de stratégies est dénoté $\mathbf{p} \in \mathbb{S}$. Pour chaque joueur k, la stratégie de ce joueur est p_k. Le vecteur de stratégies de tous les joueurs excepté le joueur k est dénoté $\mathbf{p}_{(-k)}$. Si le joueur k joue la stratégie q alors que tous les autres joueurs conservent leurs stratégies selon $\mathbf{p}$, le vecteur de stratégies résultant est dénoté $\mathbf{p}_{(-k)}, q$. Avec l'aide de cette notation, il est possible de donner la définition suivante, originellement formulée par J. Nash. Quand chaque joueur a une vision pertinente du jeu et agit rationnellement, alors le concept d'équilibre est appelé *équilibre de Nash*. L'équilibre de Nash d'un jeu stratégique $(S^K, \mathbb{S}, (u_k)_{k \in S^K})$ est un vecteur de stratégies $\mathbf{p}^* \in \mathbb{S}$ tel que

$$\forall k \in S^k,\ \forall p_k \in \mathbb{S}_k,\ u_k(\mathbf{p}^*_{(-k)}, p_k) \leq u_k(\mathbf{p}^*).$$

En d'autres termes, un équilibre de Nash est un profil d'actions tel qu'aucun joueur ne peut tirer un bénéfice en déviant unilatéralement. C'est une meilleure réponse à lui-même. L'équilibre de Nash est un concept utilisé en thérie des jeux non-coopératifs.

Equilibre de Pareto

D'un autre côté, quand les joueurs coopérent, le concept de solution est appelé *équilibre de Pareto*. Un équilibre de Pareto est une solution coopérative dominante: il est impossible d'accroître le bien-être d'un joueur sans décroître celui d'un autre joueur. En général, les équilibres de Nash et de Pareto ne coïncident pas.

Applications aux Télécommunications

Avec l'intérêt croissant pour le déploiement de réseaux à organisation automatique, la théorie des jeux représente un domaine alléchant pour considérer des mobiles en tant qu'acteurs indépendants qui possèdent ces caractéristiques. L'étude [ABA+06] décrit une vaste gamme d'outils de théorie des jeux et leurs applications dans le domaine des réseaux. Un livre dédié à introduire la théorie des jeux aux ingénieurs en télécommunications a été récemment publié [MD06]. Une étude détaillant les applications des jeux non-coopératifs aux communications sans fil est également en cours de parution [AA07]. Ce ne sont que quelques exemples qui illustrent la popularité entourant la théorie des jeux dans le domaine des télécommunications.

Parmi les sous-domaines de la théorie des jeux, certains sont particulièrement prometteurs. Dans cette thèse, nous avons considéré notamment les deux suivants: les jeux évolutionnaires et les jeux corrélés.

Théorie des jeux évolutionnaires

La théorie des jeux évolutionnaires, adaptée de la biologie mathématique, est employée pour décrire et prévoir les propriétés de populations denses dont l'évolution dépend d'un grand nombre d'interactions locales, chacune impliquant un nombre fini d'individus. La théorie des jeux évolutionnaires peut être reliée à Darwin, qui a introduit le concept de sélection naturelle et donc la compétition entre les génotypes et les phénotypes des individus. C'est J. Maynard Smith qui a véritablement défini les jeux évolutionnaires, et en particulier leur solution possible au travers du concept fondamental de *stratégie évolutionnairement stable* (ESS) en 1972 dans [Smi72].

Dans le contexte biologique, l'utilité gagnée par un individu est directement reliée à sa capacité de reproduction. Une façon intuitive de décrire une ESS est la suivante. C'est une stratégie qui est bien adaptée contre elle-même. Ceci exprime le fait que, si des animaux jouant l'ESS prédominent dans la population, ils vont tendre à rencontrer surtout d'autres animaux jouant la même stratégie. Donc, l'ESS doit être une bonne stratégie contre elle-même afin de rester populaire.

Bien que définie dans un contexte biologique, l'ESS est pertinente dans un contexte d'ingénierie [VV00]). En particulier, en ce qui concerne l'accès à un médium commun, nous pouvons nous attendre à ce qu'une technologie qui fournit une meilleure performance gagne plus de parts de marché par rapport à des concurrentes moins performantes. La dynamique de réplication associée aux jeux évolutionnaires est également à l'origine de nombreuses applications prometteuses.

Théorie des Jeux Corrélés

Les jeux corrélés étudient l'impact d'ajouter un mécanisme permettant la coordination entre les joueurs sur les équilibres possibles et les optimisations conjointes que les joueurs peuvent atteindre. La notion d'équilibre corrélé a été introduite par R. Aumann en 1974 dans [Aum74].

L'équilibre corrélé est une généralisation du concept d'équilibre de Nash ; la notion d'équilibre corrélé intervient lorsque les joueurs sont en présence d'un *arbitre* qui peut envoyer des signaux (publics ou privés) aux joueurs. Ces signaux permettent aux joueurs de coordonner leurs actions, et, en particulier, d'accomplir des choix aléatoires communs de stratégies. L'arbitre dont il est question n'est pas une entité intelligente et n'a pas besoin d'avoir la moindre connaissance du système. Tout ce en quoi consiste le travail de l'arbitre est de générer des signaux aléatoires (selon un mécanisme aléatoire connus des joueurs) dont la prise de connaissance peut aider les joueurs à se coordonner entre eux. Une multi-stratégie obtenue en utilisant les signaux est un ensemble de stratégies (une stratégie pour chaque joueur qui dépend sur toute l'information disponible pour ce joueur, y compris le signal qu'il reçoit). C'est un équilibre corrélé si aucun joueur ne bénéficie en déviant de sa partie de la multi-stratégie.

Dans un contexte d'accès de multiples utilisateurs (considérés comme joueurs) à un médium commun, l'introduction d'un degré de liberté supplémentaire par la présence de l'arbitre peut apporter une amélioration de performance avec un surcoût minime, étant donné le peu de contraintes pour définir un arbitre.

Théorie des Matrices Aléatoires

Bref Historique

La théorie des matrices aléatoires a été récemment introduite dans le cadre de la théorie de l'information. Jusqu'à une date récente, des simulations intensives paraissaient être le seul moyen pour optimiser un réseau donné. Cependant, avec l'augmentation de la taille des réseaux, les simulations impliquaient un nombre énorme de paramètres (aléatoires), et les simulations étaient également compliquées par le fait que, dans les systèmes à accès multiples, les communications interfèrent entre elles. En outre, les simulations ne permettaient pas de distinguer aisément les paramètres d'intérêt, étant donné qu'elles dépendaient d'un tel nombre de paramètres.

Finalement, en 1999, Tse [TH99] et Verdú [VS99] ont simultanément introduit la théorie des matrices aléatoires pour analyser des systèmes multi-utilisateurs. Tous deux ont traité le cas de la performance de récepteurs linéaires pour des systèmes CDMA, dans la limite où le nombre d'utilisateurs ainsi que le facteur d'étalement tendent vers l'infini avec un ratio fixé. Dans un tel scénario asymptotique, l'effet de moyennage des grandes matrices aléatoires permet d'obtenir des expressions explicites pour diverses mesures de performance telles que le rapport signal à bruit (SINR) ou la capacité. Ceci permet de distinguer d'une manière élégante des paramètres d'intérêt pour les systèmes dans le régime asymptotique. Même si les résultats sont obtenus dans le régime asymptotique, ils donnent des prévisions très précises du comportement du système dans le cas de taille finie, comme montré par des simulations.

Généralités

Les propriétés des matrices aléatoires ont tout d'abord été étudiées en physique statistique. La question typique est de caractériser la distribution des valeurs propres de familles de matrices aléatoires. Pour une taille finie, la distribution elle-même est le plus souvent aléatoire. En revanche, le vrai intérêt des matrices aléatoires repose dans le fait que, pour de nombreux cas, lorsque les dimensions des matrices tendent vers l'infini avec un ratio fixé, la distribution limite est non-aléatoire et peut même être caractérisée analytiquement. Une autre propriété intéressante des matrices aléatoires est qu'il est possible de dériver des résultats pour des sommes et des produits de matrices aléatoires.

De nos jours, la théorie des matrices aléatoires est utilisée dans de nombreux domaines, incluant l'hypothèse de Riemann, les équations différentielles stochastiques, la physique des matériaux condensés, les systèmes chaotiques, l'analyse des fluctuations boursières, etc. Et bien sûr, la théorie de l'information, en particulier en ce qui concerne l'analyse de performances. La théorie de l'information a même représenté une influence pour la dérivation de certains résultats sur les matrices aléatoires, comme par exemple la démonstration rigoureuse de certaines expressions de rapport signal à bruit utilisées dans la littérature [CM04].

Récemment, un livre recensant les principaux résultats de théorie des matrices aléatoires utilisés dans le domaine des télécommunications a été publié [TV04].

Approche Centrée sur le Réseau

L'approche centrée sur le réseau est exemplifiée par l'analyse de performance de systèmes cellulaires CDMA. Une cellule CDMA consiste en une station de base, couvrant une certaine surface et communiquant avec de multiples utilisateurs, qui sont en pratique des mobiles.

Liaisons descendante et montante

Deux modalités de communication sont possibles: la liaison descendante et la liaison montante. Une liaison descendante a lieu lorsque la station de base envoie simultanément des informations à tous les utilisateurs présents dans la cellule. Dans le cas de la liaison montante, ce sont les utilisateurs qui transmettent simultanément leur information à la station de base. Le cas d'une cellule CDMA unique, opérant sans interférence venant d'en-dehors de la cellule, a été abondamment traité dans la littérature, aussi bien dans le cas de la liaison descendante que de la liaison montante.

Système Cellulaire

Usuellement, les cellules CDMA n'opèrent pas en isolation, mais font partie d'un réseau. Dans ce cas, les communications à l'intérieur de la cellule considérée subiront également de l'interférence en provenance des cellules avoisinantes, c'est l'interférence inter-cellulaire.

Modèle de Communication

Transmission en MC-CDMA

Le modèle de communication que nous considérons est le MC-CDMA, qui est asymptotiquement équivalent au CDMA en séquence directe [Hac04]. Le signal est envoyé au travers d'un médium de communication sans fil, pour lequel nous considérons le cas réaliste d'un canal à trajets multiples, qui est sélectif en fréquence.

Globalement, le signal $\mathbf{y}$ reçu à la réception est donné par

$$\mathbf{y} = (\mathbf{H}\sqrt{\mathbf{P}} \odot \mathbf{W})\mathbf{s} + \mathbf{n},$$

où $\odot$ est le produit de Hadamard, élément par élément.

Dans cette équation,

- $\mathbf{H}$ est la matrice d'atténuation sélective en fréquence, de taille $N \times K$.
- $\sqrt{\mathbf{P}}$ est la racine carrée de la matrice diagonale de perte par rayonnement, de taille $K \times K$.
- $\mathbf{W}$ est la matrice d'étalement, i.e., les codes utilisés, de taille $N \times K$.
- $\mathbf{s}$ est le vecteur de signal transmis, de taille $K \times 1$.
- $\mathbf{n}$ est le vecteur de bruit additif Gaussian blanc, de moyenne nulle et de variance σ^2, de taille $N \times 1$.

Mesures de Performance

La mesure de performance que nous prenons en compte pour l'analyse de performance de ce système est le rapport signal à interférence plus bruit (SINR). Le SINR pour l'utilisateur k est défini comme suit:

$$\mathrm{SINR}_k = \frac{\text{Puissance du signal utile pour l'utilisateur } k}{\text{Puissance du signal utile pour les autres utilisateurs} + \text{Puissance du bruit}}.$$

Obtenir le SINR permet d'obtenir l'efficacité spectrale, qui mesure le nombre de bits par unité de temps, de fréquence et de distance que le système est capable de transmettre. L'efficacité spectrale du système est proportionnelle à la somme sur les utilisateurs du logarithme de $1 + \mathrm{SINR}$.

$$C \propto \sum_{k=1}^{K} \log(1 + \mathrm{SINR}_k).$$

Canal à Trajets Multiples

En ce qui concerne le médium de communication sans fil, pour chaque utilisateur k, nous considérons un canal à trajets multiples. La réponse impulsionnelle du canal est

$$c_k(\tau) = \sum_{p=0}^{L-1} c_{pk}\phi(\tau - \tau_{pk})$$

où $\phi(\cdot)$ est le filtre impulsionnel de transmission.

La transformée de Fourier de c_k après filtrage impulsionnel adapté au récepteur est

$$h_k(f) = \sum_{p=0}^{L-1} c_{pk} e^{-j2\pi f \tau_{pk}} \left|\Psi(f)\right|^2 \text{ où } \Psi(f) = \begin{cases} 1 & \text{si } -\frac{W}{2} \le f \le \frac{W}{2} \\ 0 & \text{sinon.} \end{cases}$$

Pour $x \in [0, \alpha]$, nous définissons le profil de variance de la matrice $\mathbf{H}$ par

$$h(f, x) = h_k(f) \text{ si } \frac{k}{N} \le x < \frac{k+1}{N}.$$

Répartition en Codes

En ce qui concerne l'étalement du signal, nous considérons deux types de codes: codes orthogonaux et codes aléatoires. Usuellement, les codes orthogonaux utilisés en pratique sont des colonnes extraites de matrices de Walsh-Hadamard, Pour pouvoir utiliser des résultats de théorie des matrices aléatoires unitaires, les codes considérés par la suite sont des colonnes extraites de matrices unitaires suivant la distribution de Haar. Une matrice unitaire suit la distribution de Haar si elle est tirée uniformément dans le groupe $\mathcal{U}(N)$ des matrices unitaires. Pour générer une matrice unitaire suivant la distribution de Haar, il suffit d'opérer l'orthogonalisation de Gram-Schmidt d'une matrice aléatoire Gaussienne. Il peut être démontré que les résultats ainsi obtenus sont identiques à ceux obtenus en considérant des codes extraits de matrices de Walsh-Hadamard.

En ce qui concerne les codes aléatoires, ce sont simplement des colonnes extraites d'une matrice aléatoire dont les éléments sont indépendants et identiquement distribués, avec moyenne nulle et variance $\frac{1}{N}$. La distribution particulière des éléments n'a pas d'importance, néanmoins considérer une distribution Gaussienne peut simplifier certains calculs.

Déploiement cellulaire infini, codes orthogonaux, en liaison descendante

Cas unicellulaire

Précédemment, seuls une cellule isolée, ou un réseau avec peu de cellules interférentes ont été considérés. La nouvelle contribution de cette thèse se situe dans l'analyse de réseaux infinis, où la contribution de toutes les cellules interférentes est prise en considération. De tels réseaux sont étudiés dans la liaison descendante aussi bien que dans la liaison montante. L'analyse est basée sur des résultats de théorie des matrices aléatoires et de théorie des matrices unitaires aléatoires, dans le but d'obtenir des expressions analytiques dépendant seulement d'un petit nombre de paramètres signicatifs. Pour ces deux cas (liaison montante et liaison descendante), la nouveauté consiste en l'analyse d'un système multi-cellulaire prenant en compte l'interférence en provenance de *toutes* les cellules environnantes.

Cas multi-cellulaire

Dans le cas de la liaison descendante, le cas multi-cellulaire avec codes orthogonaux est considéré. En effet, en liaison descendante, la synchronisation est particulièrement simple à effectuer, et il a été démontré que les codes orthogonaux permettent un gain important par rapport aux codes aléatoire, même en présence d'un canal sélectif en fréquence, qui détruit l'orthogonalité. Ainsi, il est usuel de considérer le cas de codes orthogonaux en liaison descendante.

En particulier, le cas d'un déploiement unidimensionnel infini de stations de base est envisagé. Nous traitons ce cas, afin de déterminer le placement optimal des stations de base dans ce cadre. Dans le cas de la liaison descendante CDMA, c'est une première étape dans l'analyse du problème complexe d'optimiser des réseaux multi-cellulaires, en utilisant une nouvelle approche basée sur la théorie des matrices unitaires aléatoires. Le but est de déterminer, pour un réseau multi-cellulaire dense et infini, la distance optimale entre les stations de base. Nous voulons ne disposer ni trop (pour diminuer les coûts de déploiement) ni pas assez (pour assurer la qualité de service aux utilisateurs) de stations de base sur la ligne. En pratique, ce déploiement peut être considéré comme le modèle d'un déploiement de stations de base le long d'une autoroute, avec les voitures comme utilisateurs qui communiquent avec les stations de base.

Un réseau CDMA émettant en liaison descendante à travers un canal sélectif en fréquence avec des codes orthogonaux où chaque utilisateur est équipé du filtre adapté linéaire est considéré. Nous supposons que les utilisateurs sont uniformément distribués le long du secteur. Seul le cas de codes orthogonaux est considéré, sous l'hypothèse que les utilisateurs sont synchronisés dans chaque cellule. Le problème est analysé dans le régime asymptotique : un réseau très dense est considéré, où le facteur d'étalement N tend vers l'infini, le nombre d'utilisateurs par mètre d tend vers l'infini mais la charge par mètre $\frac{d}{N} = \alpha$ reste constante.

Résultat Général

Dans ces conditions, le résultat général que nous obtenons est résumé dans la proposition suivante. Lorsque $N \to \infty$ avec $\frac{d}{N} = \alpha$, l'efficacité spectrale du CDMA en liaison descendante avec codes orthogonaux aléatoires et filtre adapté est donnée par

$$C(a) = \frac{\alpha}{a}\mathbb{E}_h \left[\int_{-\frac{a}{2}}^{\frac{a}{2}} \log_2 \left(1 + \frac{P(x)\left(\frac{1}{W}\int_{-\frac{W}{2}}^{\frac{W}{2}} |h(f)|^2 df\right)^2}{I(x) + \frac{\sigma^2}{W}\int_{-\frac{W}{2}}^{\frac{W}{2}} |h(f)|^2 df} \right) dx \right]$$

avec

$$\begin{aligned} I(x) = {} & \frac{\alpha a}{W} P(x) \left(\int_{-\frac{W}{2}}^{\frac{W}{2}} |h(f)|^4 df - \frac{1}{W}\left(\int_{-\frac{W}{2}}^{\frac{W}{2}} |h(f)|^2 df \right)^2 \right) \\ & + \frac{\alpha a}{W} \sum_{q \neq 0} P_q(x) \int_{-\frac{W}{2}}^{\frac{W}{2}} |h(f)|^2 |h_q(f)|^2 df. \end{aligned}$$

L'important à retenir dans cette proposition est l'effet de moyennage opéré par la théorie des matrices unitaires alétoires. Le résultat ne dépend pas des codes employés mais seulement de quelques paramètres significatifs, tels que la distribution de l'atténuation, la variance du bruit et la charge du système.

Le signal transmis subit principalement deux formes d'atténuation: la perte due au rayonnement et l'atténuation due aux trajets multiples. Dans le but de découpler les effets de ces deux formes d'atténuation, nous ôtons successivement les effets de l'une, puis de l'autre.

Influence de la Perte par Rayonnement

Dans le cas où il n'y a pas de trajets multiples, le canal n'est pas sélectif en fréquence. L'expression de l'efficacité spectrale devient:

$$C(a) = \frac{\alpha}{a} \int_{-a/2}^{a/2} \log_2 \left(1 + \frac{P(x)}{\mathbf{g}\sigma^2 + \alpha a \sum_{q \neq 0} P_q(x)} \right) dx.$$

Dans ce cas, l'orthogonalité est préservée. Il n'y a pas de terme d'interférence intra-cellulaire, seul le terme d'interférence extra-cellulaire intervient. Des simulations montrent qu'il existe une distance inter-cellulaire a optimale, qui maximise l'efficacité spectrale. Plus la perte par rayonnement est sévère, plus cette distance inter-cellulaire optimale est réduite.

Influence de l'Atténuation du Canal

Dans le cas où la perte due au rayonnement est absente et tend vers 1 (pas de perte par rayonnement), l'efficacité spectrale tend vers 0. Cependant, il est possible d'inférer sur le comportement de l'efficacité spectrale dans ce cas limite, en prenant la dérivée de l'efficacité spectrale par rapport à la distance inter-cellulaire. Après calculs, nous obtenons

$$\frac{\partial C}{\partial a} \propto \left(\frac{3}{2} - \frac{\mathbb{E}\left[|h|^4\right]}{\left(\mathbb{E}\left[|h|^2\right]\right)^2} \right).$$

Le comportement de l'efficacité spectrale dépend donc du kurtosis de l'atténation $\frac{\mathbb{E}\left[|h|^4\right]}{\left(\mathbb{E}\left[|h|^2\right]\right)^2}$. Si le kurtosis est trop grand, l'orthogonalité est sévèrement détruite, alors la seule solution est de serrer autant que possible les stations de base les unes à côté des autres. Si le kurtosis est suffisamment petit, alors il est possible pour chaque station de base de couvrir autant d'utilisateurs que les codes orthogonaux peuvent en accomoder.

Conclusions

Cette analyse fournit des indications sur les futures directions de recherche. Dans le point de vue traditionnel des systèmes cellulaires, l'avis généralement retenu est d'accroître la puissance de transmission avec la taille des cellules, pour réduire l'atténuation due au trajet. Mais les résultats obtenus dans notre analyse montre que le trajet ne représente qu'une faible partie de la perte, étant donné qu'il ne détruit pas l'orthogonalité des codes. Le premier obstacle est la selectivité en fréquence du canal, qui exerce un effet destructeur sur l'orthogonalité. Ainsi, ces considérations démontrent que l'effort doit être concentré sur la réduction des effets de la selectivité en fréquence du canal, en utilisant des techniques de diversité et d'égalisation pour restaurer l'orthogonalité.

Notons que ces résultats ne concernent que la liaison descendante, et une étude complète doit prendre en compte également la liaison montante.

Déploiement cellulaire infini, codes aléatoires, en liaison montante

Dans le cas de la liaison montante, un système semblable est étudié. Un déploiement infini de stations de base est considéré. Des mobiles sont répartis uniformément dans les cellules et utilisent une transmission CDMA pour communiquer simultanément avec la station de base qui leur est associée. Les codes considérés sont cette fois aléatoires, étant donné que les utilisateurs ne sont pas supposés être synchronisés. La transmission a lieu à travers un canal sélectif en fréquence.

En utilisant des arguments asymptotiques, des expressions explicites de mesures de performance, en termes d'efficacité spectrale, sont dérivées pour deux types de structures de récepteur à la station de base : filtre adapté et filtre optimal. En particulier, le gain potentiel obtenu avec un filtre optimum intra-cellulaire par rapport à un filtre linéaire est quantifié. L'impact de l'interférence inter-cellulaire est également quantifié pour divers types de récepteurs, pouvant éventuellement combiner les données de plusieurs cellules, et différentes distances inter-cellulaires. Ces résultats donnent une vue d'ensemble du déploiement d'un réseau cellulaire en vue d'atteindre un taux de transmission donné pour les utilisateurs.

L'analyse précédente ne prend en compte que le cas de codes aléatoires. Une façon d'obtenir un taux de transmission plus élevé peut être d'utiliser des codes orthogonaux. Dans un premier temps, la dérivation de résultats est effectuée dans un contexte unicellulaire.

Déploiement unicellulaire, codes orthogonaux, en liaison montante

Motivation

La nouveauté est l'analyse d'un système en liaison montante utilisant des codes orthogonaux. Habituellement, uniquement des codes aléatoires sont employés dans la liaison montante. Une des raisons repose sur le fait que, à cause de la sélectivité en fréquence du canal, la convolution des codes avec les différents canaux des utilisateurs peut être représentée comme un nouvel ensemble de codes avec des propriétés similaires à une séquence aléatoire.

En conséquence, même si les codes sont explicitement conçus pour assurer un accès multiple orthogonal, le canal sélectif en fréquence détruit malheureusement l'orthogonalité. Le volume de transmissions non-informatives pour synchroniser les utilisateurs dans le réseau peut alors complètement réduire à néant le gain en signal rapport à bruit dû à la réduction de l'interférence multi-utilisateurs.

Cependant, comme des études précédentes en liaison descendante [DHLdC03a] l'ont démontré, ce gain est loin d'être négligeable, surtout dans des systèmes avec beaucoup d'utilisateurs. L'idée intuitive est que, avec une bonne égalisation, un utilisateur peut restaurer l'orthogonalité en compensant l'effet de son propre canal (qui est commun à tous les utilisateurs dans la liaison descendante). Mais dans la liaison montante, un tel résultat n'est pas applicable étant donné que chaque code est affecté de manière indépendante par le canal de l'utilisateur correspondant.

En conséquence, n'importe quel mécanisme d'égalisation ne dispose que d'une efficacité limitée, dépendant principalement des caractéristiques des canaux mis en jeu.

Analyse

L'analyse est basée sur des résultats de théorie des matrices aléatoires unitaires [HP00, PR04]. En utilisant des arguments asymptotiques, ces outils permettent de dériver des expressions analytiques de l'efficacité spectrale pour le filtre adapté et le filtre adapté avec annulation successive d'interférence dans le cas général d'un canal à trajets multiples. Un cadre utile est fourni afin de déterminer si la synchronisation des utilisateurs donne une amélioration significative de performance.

Résultats

Dans le cas du filtre adapté, la proposition suivante est obtenue. Lorsque $N \to \infty$ et $\frac{K}{N} \to \alpha$, le SINR avec filtre adapté est donné par

$$\mathrm{SINR}_k^{\mathrm{orth}} = \frac{P(x_k)\left(\int_0^1 |h(f,x)|^2\, df\right)^2}{\sigma^2 \int_0^1 |h(f,x)|^2\, df + \left(\int_0^\alpha \int_0^1 P(y)\, |h(f,x)|^2\, |h(f,y)|^2\, df dy - \mu(x)\right)}$$

avec $\mu(x) = \int_0^\alpha P(y) \left| \int_0^1 h(f,x) h^*(f,y) df \right|^2 dy$ dans le cas de codes orthogonaux, et par

$$\mathrm{SINR}_k^{\mathrm{rand}} = \frac{P(x_k)\left(\int_0^1 |h(f,x)|^2\, df\right)^2}{\sigma^2 \int_0^1 |h(f,x)|^2\, df + \left(\int_0^\alpha \int_0^1 P(y)\, |h(f,x)|^2\, |h(f,y)|^2\, df dy\right)}$$

dans le cas de codes aléatoires.

L'effet de moyennage de la théorie des matrices aléatoires permet de se dispenser des codes, et de mettre en valeur les paramètres significatifs du système, tels que la charge du système, la variance du bruit et le profil de variance de la matrice des atténuations.

Seul un terme additional $\mu(x)$ distingue le SINR avec codes orthogonaux du SINR avec codes aléatoires. Comme ce terme est toujours positif, nous en déduisons immédiatement que le SINR avec codes aléatoires est toujours inférieur au SINR avec codes orthogonaux.

Cas de Délais Uniformément Répartis

Lorsque nous faisons l'hypothèse que les délais sont uniformément répartis par rapport à la bande, le rapport entre le SINR avec codes orthogonaux et le SINR avec codes aléatoires, i.e., le gain d'orthogonalité, devient

$$\frac{\mathrm{SINR}_k^{\mathrm{orth}}}{\mathrm{SINR}_k^{\mathrm{rand}}} = \frac{\sigma^2 + \alpha}{\sigma^2 + \alpha\left(1 - \frac{1}{L}\right)}.$$

Remarquablement, pour un SNR élevé ($\sigma^2 \to 0$), le gain d'orthogonalité est donné par

$$\frac{\mathrm{SINR}_k^{\mathrm{orth}}}{\mathrm{SINR}_k^{\mathrm{rand}}} = \frac{L}{L-1}.$$

Ainsi, le paramètre majeur qui détermine le gain d'orthogonalité est le nombre de trajets L. Le gain est le plus important lorsque le signal parvient à la station de base en suivant un nombre de trajets le plus réduit possible.

Autres Filtres

Les cas d'autres filtres que le filtre adapté sont encore à l'étude. Néanmoins, des simulations montrent que les conclusions restent qualitativement identiques pour d'autres filtres. Le paramètre majeur est le nombre de trajets que subit le signal transmis lors de sa propagation.

Déploiement cellulaire infini, codes orthogonaux, en liaison montante

Encore une fois, une cellule CDMA n'opère généralement pas en isolation. Afin d'achever l'étude, le cas d'un déploiement infini de cellules employant des codes orthogonaux dans la liaison montante est étudié.

Dans ce cas, nous montrons que l'interférence inter-cellule se comporte de la même manière qu'en présence de codes aléatoires. C'est uniquement l'interférence intra-cellulaire qui est affectée par l'utilisation des codes orthogonaux.

Approche Centrée sur l'Utilisateur

Dans le cas de l'approche centrée sur l'utilisateur, nous nous concentrons sur trois protocoles : ALOHA, CDMA et OFDMA.

ALOHA

Dans le cas des réseaux décentralisés, une première contribution de cette thèse est de présenter deux types de jeux dans un contexte de gestion de réseau : jeux corrélés et jeux évolutionnaires. Afin de maintenir cette introduction aussi simple que possible, le protocole d'accès multiple considéré est ALOHA. A notre connaissance, nous avons été les premiers à présenter les jeux corrélés et les jeux évolutionnaires pour étudier le comportement non-coopératif dans les réseaux sans fil.

Théorie des Jeux Evolutionnaires et ALOHA

Etant donné que les interactions entre les utilisateurs sont répétées, il était logique de présenter la notion biologique des jeux évolutionnaires dans un contexte de réseau.

Ainsi, nous considérons une grande population de terminaux communiquant en utilisant un protocole ALOHA [Abr70] avec deux niveaux possibles de puissance de transmission. Une modélisation simple est étudiée: si un unique mobile émet, sans transmissions concurrentes pendant sa période de vulnérabilité, sa transmission est correctement reçue. Si plusieurs mobiles émettent en même temps à la même puissance, toutes les transmissions sont perdues. Si un unique mobile émet à la puissance la plus haute, tandis que toutes les autres transmissions interférentes ont lieu à la puissance moins élevée, alors la transmission à puissance élevée est correctement reçue, tandis que tous les autres messages sont perdus et doivent être retransmis plus tard. Ce cadre permet d'obtenir des solutions analytiques explicites pour les mesures de performance. Nous calculons ainsi analytiquement les solutions pour plusieurs critères d'optimisation non-coopérative.

Le problème de choisir entre les deux niveaux de puissance de manière non-coopérative est posé en supposant que les mobiles sont des entités égoïstes et rationnelles. Leur stratégie consiste en choisir la probabilité de transmettre avec chaque niveau de puissance. Les gains sont fonctions des taux de transmission obtenus ainsi que du coût des niveaux de puissance. En particulier, l'impact sur le niveau de performance du système d'une stratégie de paiement est étudié.

Deux concepts d'équilibre non-coopératif sont étudiés : l'équilibre de Nash et la stratégie évolutionnairement stable (ESS). Cette dernière est dérivée de la biologie mathématique dans le contexte des jeux évolutionnaires, qui permettent de décrire et de prédire les propriétés de grandes populations dont l'évolution dépend de beaucoup d'interactions locales, chacune impliquant un nombre fini d'individus.

Théorie des Jeux Corrélés et ALOHA

Le second contexte étudié est celui des jeux corrélés.

Ajouter des mécanismes de coordination peut permettre aux mobiles d'augmenter leur taux de transmission. Cet amélioration est étudiée dans le cadre non-coopératif aussi bien que dans le cadre coopératif, dans lequel les mobiles collaborent pour la réalisation d'un but commun. Cette contribution étudie également l'optimisation multicritère, dans notre cas maximisant la sortie moyenne avec une contrainte sur la puissance d'énergie moyenne.

L'application est faite dans le contexte d'ALOHA avec fenêtres d'émission discrètes [Rob72]. Tous les mobiles sont supposés être synchronisés. Une hypothèse fréquente lors de l'étude d'ALOHA, que nous reprenons, est que si plus d'un mobile tente d'émettre un paquet pendant une fenêtre d'émission, alors tous les paquets sont perdus, et les mobiles attendent un nombre aléatoire de fenêtres d'émission avant de retenter une transmission, dans le but d'éviter des collisions répétées.

La contribution n'est pas seulement d'appliquer la notion d'équilibre corrélé dans un contexte de réseau, mais également de considérer une optimisation multicritère. Dans notre cas, chaque mobile a deux objectifs : taux de transmission moyen et consommation de puissance moyenne. Nous utilisons l'équilibre corrélé adapté au contexte d'optimisation sous contraintes de chaque joueur, maximisant son taux de transmission avec une contrainte sur sa puissance moyenne d'émission.

La coordination entre les joueurs est également utile dans le cas d'optimisation coopérative. Lorsque les joueurs ont le même objectif à maximiser, il est utile pour eux de pouvoir se coordonner. Ils peuvent bénéficier de choisir conjointement des stratégies aléatoires, ce qui peut ne pas être possible étant donné la nature distribuée du problème. Le besoin de choix conjoint des stratégies aléatoires est plus précisément dû au caractère multicritère du problème étudié.

Conclusion

Ces exemples montrent que la théorie des jeux trouve facilement des applications dans le domaine des réseaux, et donnent quelques indications sur la manière de laquelle la théorie des jeux peut être introduite dans un contexte de réseau. Une application plus substantielle en est faite dans un contexte d'allocation de ressources pour le CDMA.

CDMA

Allocation de Ressources

L'allocation de ressources est un sujet de recherche de toute première importance dans le le contexte des systèmes à utilisateurs multiples, particulièrement dans la liaison montante. Un mécanisme efficace d'allocation de puissance empèche une consommation excessive des ressources limitées des utilisateurs, tout en leur permettant d'atteindre la qualité de service qu'ils désirent.

Une allocation de puissance distribuée efficace peut être réalisée en adoptant une modélisation issue de la théorie des jeux. Cette approche a été introduite dans [Ji97] et popularisée par [GM00, MW01a]. De nombreux articles sur ce thème ont parus depuis lors.

Modèle

La nouveauté est ici de considérer le cas réaliste de transmission à travers un canal sélectif en fréquence. Le modèle est étudié dans le cadre des jeux non-atomiques. Ceci nous permet d'obtenir l'allocation de puissance comme fonction de l'énergie total du canal pour chaque utilisateur. Une autre nouvelle contribution est que, en plus des filtres linéaires, les filtres optimaux et à annulation successive d'interférence (SIC) (avec l'introduction d'un ordre des utilisateurs) sont étudiés.

La performance d'un système CDMA est analysée dans le contexte de canal sélectif en fréquence. Les utilisateurs sont supposés disposer d'information uniquement sur leur propre canal de transmission, alors que la station de base connaît parfaitement tous les canaux des utilisateurs. Ce scénario illustre le cas de mécanismes décentralisés, quand une information limitée sur le réseau est disponible au terminal.

Cette contribution représente une extension de [MPSM05] dans le cas de canaux sélectifs en fréquence. Nous ne considérons pas le cas de porteuses multiples, comme dans [MCPS06], et les résultats obtenus en diffèrent de façon importante. L'extension n'est pas triviale et requiert des résultats pointus sur les matrices aléatoires avec profil de variance dus à Girko [Gir90]. De plus, en sus des filtres linéaires étudiés dans [MPSM05], nous étudions l'amélioration fournie par les filtres optimum et SIC.

Nous dérivons des expressions simples pour l'allocation de puissance à l'équilibre de Nash lorsque le nombre de mobiles devient grand et le facteur d'étalement augmente, avec un ratio fixé. La théorie des jeux peut être utilisée pour traiter le cas d'un nombre quelconque de joueurs. Cependant, lorsque la taille du système augmente, le nombre de paramètres augmente drastiquement, et il est difficile de visualiser les expressions obtenues. Pour obtenir des expressions dépendant seulement d'un petit nombre de paramètres dans la limite d'un grand système, deux méthodologies asymptotiques sont utilisées. La première est la théorie des matrices aléatoires qui permet d'obtenir des expressions explicites de l'impact sur un mobile donné de l'interférence causée par tous les autres mobiles. La seconde est la théorie des jeux non-atomiques qui permet de calculer de bonnes approximations de l'équilibre de Nash lorsque le nombre de joueurs devient grand.

Jeu d'Allocation de Ressources

Le jeu considéré est le suivant. Les joueurs sont les mobiles présents dans la cellule. Pour chaque joueur k, sa stratégie est son choix d'allocation de puissance P_k. Son utilité est donnée par

$$u_k = \frac{\gamma(\beta_k)}{P_k},$$

où γ est une mesure de performance et β_k est le SINR de l'utilisateur k.

Usuellement, la mesure de performance γ utilisée pour définir l'utilité est une version adaptée du goodput $(1 - e^{-\beta})^M$, où M est le nombre de bits par paquet. Malheureusement, la capacité ne peut être utilisée, sous peine d'obtenir le résultat trivial qu'aucun joueur ne transmet avec une puissance strictement positive à l'équilibre.

Cette utilité est exprimée en bits/Joule. Elle prend en compte le fait que chaque utilisateur veut maximiser la quantité d'information reçue avec succès à la station de base, tout en ne consommant pas trop de puissance.

Expression du SINR

L'expression du SINR est fournie par un théorème dû à Girko [Gir90] et exploité dans [TLV05]. Par exemple, pour le filtre minimisant l'erreur quadratique moyenne (MMSE), la proposition suivante est obtenue.

Lorsque $N, K \to \infty$ avec $K/N \to \alpha$, le SINR de l'utilisateur k en sortie du filtre MMSE est donné par

$$\mathrm{SINR}_k = \beta(\frac{k}{N})$$

où $\beta : [0, \alpha] \to \mathbb{R}$ est une fonction définie par l'équation implicite

$$\beta(x) = P(x) \int_0^1 \frac{|h(f,x)|^2 \, df}{\sigma^2 + \int_0^\alpha \frac{P(y)|h(f,y)|^2 dy}{1+\beta(y)}}.$$

Ce qu'il est important de constater dans cette expression, c'est d'une part l'effet de moyennage des matrices aléatoires: les codes utilisés n'ont pas d'importance; et d'autre part, le SINR de l'utilisateur k est une fonction linéaire de son choix d'allocation de puissance P_k. Le résultat est similaire pour d'autres filtres tels que le filtre adapté ou le filtre optimum.

Analyse

A partir de l'observation de la linéarité de SINR_k en P_k, une simple différentiation nous fournit l'équilibre de Nash comme intervenant pour un SINR $\beta^\star$ solution de l'équation scalaire

$$\beta_k \gamma'(\beta_k) - \gamma(\beta_k) = 0.$$

Ce SINR cible $\beta^\star$ détermine une allocation de puissance à l'équilibre. Pour le filtre MMSE, l'allocation de puissance à l'équilibre est donnée par

$$P_k = \frac{\beta^\star}{\frac{1}{N}\sum_{n=1}^{N} \frac{|h_{nk}|^2}{\sigma^2 + \frac{1}{1+\beta^\star}\frac{1}{N}\sum_{j=1, j\neq k}^{K} P_j |h_{nj}|^2}}.$$

Cette expression semble dépendre des réalisations de l'atténuation et des allocations de puissance de tous les autres joueurs! La solution pour supprimer cette dépendance est de considérer le système dans le régime asymptotique, ce qui est déjà le cas pour utiliser les résultats de théorie des matrices aléatoires.

Jeux Non-Atomiques

En effet, dans le régime asymptotique, le jeu non-coopératif devient un jeu non-atomique, dans lequel l'impact (à travers l'interférence) de n'importe quel mobile unique sur la performance des autres mobiles est négligeable. Dans le contexte du jeu dans un réseau, le concept de solution associé est appelé équilibre de Wardrop

[War52]. Il est souvent plus facile à calculer que l'équilibre de Nash, et fournit une bonne approximation de l'équilibre de Nash [HM85]. Nous dérivons des expressions pour l'équilibre non-atomique, qui correspond généralement à une allocation de puissance non uniforme entre les utilisateurs.

Nous faisons l'hypothèse que le canal de chaque utilisateur a L trajets et que l'atténuation sur le ℓ-ème trajet de l'utilisateur k est $h_\ell(\frac{k}{N})$. Nous définissons l'énergie totale du canal de l'utilisateur k comme $E_k = \sum_{\ell=1}^{L} \left|h_\ell(\frac{k}{N})\right|^2$.

Résultats

Avec cette notation, nous obtenons les allocations de puissance à l'équilibre suivantes. Respectivement, pour le filtre adapté

$$P_k = \frac{1}{E_k}\frac{\sigma^2\beta^\star}{1-\alpha\beta^\star} \text{ for } \alpha < \frac{1}{\beta^\star},$$

et pour le filtre MMSE

$$P_k = \frac{1}{E_k}\frac{\sigma^2\beta^\star}{1-\alpha\frac{\beta^\star}{1+\beta^\star}} \text{ for } \alpha < 1 + \frac{1}{\beta^\star}.$$

Dans le cas d'un trajet unique, ces formules donnent les mêmes résultats que dans [MPSM05].

Pour le filtre optimum

$$P_k = \frac{1}{E_k}\frac{\sigma^2\beta^+}{1-\alpha\frac{\beta^+}{1+\beta^+}} \text{ for } \alpha < 1 + \frac{1}{\beta^+}$$

où β^+ est solution de l'équation (qui a toujours une solution unique)

$$\alpha \log_2\left(1+\beta^+\right) - \alpha\log_2(e)\frac{\beta^+}{1+\beta^+} + \log_2\left(1 + \frac{1}{1+\beta^+}\frac{\alpha\beta^+}{1-\alpha\frac{\beta^+}{1+\beta^+}}\right) = \alpha\log_2\left(1+\beta^\star\right).$$

Nous observons que l'allocation de puissance est une fonction linéaire de l'inverse de l'énergie totale du canal $\frac{1}{E_k}$. Etant donné que l'énergie totale du canal est une somme de variables aléatoires indépendantes et identiquement distribuées, suivant la loi des grands nombres, nous observons un effet similaire au "durcissement du canal" déjà constaté en MIMO [HMT04]. Lorsque le nombre de trajets augmente, l'allocation de puissance à l'équilibre tend à devenir uniforme parmi les utilisateurs.

Annulation Successive d'Interférence

L'équilibre non-atomique est étudié pour plusieurs récepteurs linéaires, parmi lesquels le filtre adapté et le filtre minimisant l'erreur quadratique moyenne (MMSE), ainsi que pour le filtre optimal. Néanmoins, il est possible d'améliorer encore les performances en utilisant des filtres à annulation successive d'interférences [MV01].

Cependant, pour pouvoir accomplir l'annulation successive d'interférence, les utilisateurs doivent connaître leur ordre de décodage, pour pouvoir ajuster leur taux de transmission. Deux façons d'obtenir un ordre des utilisateurs de manière distribuée sont introduites. Il est possible de donner un ordre aux utilisateurs sous des hypothèses peu exigeantes.

Dans le cas où les utilisateurs sont très nombreux, l'ordre est automatique selon la loi inverse de distribution de l'énergie totale du canal ; nous démontrons que ceci permet également un décodage des utilisateurs dans l'ordre optimal. Cet ordre est basé sur un lemme de Shamai et Verdù [SV02].

Un autre moyen est d'utiliser un arbitre, dans un cadre de jeux corrélés, qui permet d'ordonner les utilisateurs. Ceci donne lieu à une forme différente d'allocation de puissance, par rapport aux simples filtres linéaires, qui fait intervenir explicitement l'ordre de décodage des utilisateurs.

Les allocations de puissance à l'équilibre pour les filtres à annulation successive d'interférence sont les suivantes.

$$P_k^{\mathrm{MF}} = \frac{\sigma^2\beta^\star}{E_k}\left(1+\frac{1}{N}\beta^\star\right)^{K-k},$$

$$P_k^{\mathrm{MMSE}} = \frac{\sigma^2\beta^\star}{E_k}\left(1+\frac{1}{N}\frac{\beta^\star}{1+\beta^\star}\right)^{K-k}.$$

De plus, le gain obtenu à l'équilibre de Nash par rapport à l'allocation uniforme de puissance est quantifié, selon le nombre de chemins. L'originalité du travail repose sur le fait que nous montrons que, lorsque le nombre de chemins augmente, l'allocation de puissance devient de plus en plus uniforme, à cause du comportement ergodique du canal sélectif en fréquence. Ceci n'est pas sans rappeler un effet déjà observé en MIMO [HMT04], le "durcissement de canal". Le gain le plus grand, en termes d'utilité, est obtenu pour un canal non sélectif en fréquence, pour lequel les disparités d'atténuation entre utilisateurs sont les plus fortes.

OFDMA

Nous introduisons un mécanisme distribué d'allocation de porteuses pour l'accès multiple par répartition en fréquence orthogonale (OFDMA). Le choix des porteuses est fait au niveau des mobiles plutôt qu'au niveau de la station de base. Ce mécanisme de diversité multi-utilisateur fait usage de la réciprocité du canal sur chaque porteuse, pour supprimer le besoin de feedback. La nouveauté est que chaque utilisateur ne connaît que les coefficients du canal de ses porteuses, tandis que la station de base n'a aucune connaissance du canal, et la communication a lieu dans un cadre non-coopératif à un taux de transmission fixé.

L'algorithme exploite la réciprocité du canal, en supposant qu'une séquence d'entraînement est reçue par les utilisateurs avant la communication. Chaque utilisateur estime ainsi les atténuations sur ses différentes porteuses et sélectionne les porteuses garantissant le taux de transmission voulu. L'algorithme permet à chaque utilisateur d'envoyer sûrement des données à un taux prescrit, en connaissant seulement son canal, sous des hypothèses asymptotiques faibles.

Pour plusieurs modèles de canal, nous dérivons des expressions analytiques de l'efficacité spectrale des utilisateurs dans le régime asymptotique (nombre élevé de porteuses) pour deux types de filtres à la station de base : filtre adapté et filtre optimum. Le résultat est basé sur la prévisibilité de l'interférence à mesure que le nombre de porteuses augmente.

Nous montrons que dans un tel environnement non-coopératif, les utilisateurs peuvent émettre à un certain taux de transmission. De plus, dans le cas du filtre adapté, un choix judicieux du nombre de porteuses peut accroître le taux de transmission, en comparaison à l'émission sur toutes les porteuses disponibles.

Conclusion

En conclusion, nous avons fourni des éléments d'analyse aussi bien en considérant une approche centrée sur le réseau qu'une approche centrée sur l'utilisateur. Nous avons abordé le problème de l'endroit où mettre l'intelligence dans le réseau. Quand il y a beaucoup d'utilisateurs, il est légitime de mettre l'intelligence entre les mains de ceux-ci. Ainsi, différents scénarios dans lesquels l'intelligence se trouve au niveau des utilisateurs ont été étudiés. Comme le titre de cette thèse l'affirme, les deux conceptions ne sont cependant pas opposées, mais compléntaires. Il est possible de réaliser des protocoles qui, dépendant de l'état du système, pourraient jongler entre les deux paradigmes. Lorsque les conditions permettent une optimisation à faible coût, ils permettraient d'imposer le choix de l'allocation de ressources aux mobiles; tandis que, au contraire, ils laisseraient la bride sur le cou des utilisateurs pour qu'ils déterminent eux-mêmes leur meilleure option en accord avec leur connaissance limitée de l'environnement qui les entourent, lorsque la perte liée à l'opération au sein d'un milieu non-coopératif reste faible.

Chapter 1

Introduction

1.1 Multiuser Communications

Shannon's landmark 1948 publication [Sha48] singlehandedly launched the discipline of information theory. In this paper, among many other important definitions and theorems, Shannon introduces the notion of *capacity* of a channel as a relevant measure of performance. The capacity determines the achievable rate of communication between two terminals that send signals over a (noisy) channel. This foundational contribution has been the spark that ignited the publication of a multitude of works, as surveyed in [Ver98]. Nearly sixty years after its inception, the "simple" case of a single user considered in [Sha48] can be considered to have been treated in a full extent.

The more involved case of several users sharing the same communication medium was first tackled by Shannon in his 1961 single-authored publication [Sha61]. It marks the foundation of *multiuser* information theory. The work [Sha61] is devoted to the study of a two-way channel, similar to telephony, where interference occurs between signals transmitted in concurrent directions. The conventional notion of capacity is no longer relevant; nevertheless, a two-dimensional capacity region can be defined, which specifies the set of achievable rate pairs. Unfortunately, no explicit expression for this capacity region exists, even in simple particular cases. Only bounds are derived.

Shannon concludes his paper with the sentence: "In another paper we will discuss the case of a channel with two or more terminals having inputs only and one terminal with an output only, a case for which a complete and simple solution of the capacity region has been found."

This defines what is now referred to as the *multiple-access channel*. Several transmitters communicate with a single receiver over a common channel. This spawned an abundance of contributions, a selection of which can be found in the survey [Ver98], pp. 11–12. Note that, contrary to his announcement, Shannon did not publish further on the subject, therefore the actual scope of his solution remains undetermined.

Several practical protocols were proposed to implement efficient communication when several users are involved. Orthogonal protocols, like Time Division Multiple Access (TDMA) and Frequency Division Multiple Access (FDMA), already existed before the advent of Shannon's paper and were used in telegraphy. Others, like Code Division Multiple Access (CDMA), were just nascent. It was rapidly shown that orthogonal protocols, such as TDMA and FDMA, generally did not achieve the full capacity region of the multiuser channel. On the other hand, protocols with a controlled level of interference between users, such as CDMA, enable this optimization. In order to achieve the capacity region in the case of several terminals communicating simultaneously with a single base station, an equivalent of the single-user water-filling algorithm was proposed in [TH98].

The capacity region is a polygon in the plane (or a polytope in higher dimensions, when more than two users are considered). In the two-user case, the points on the border of this polygon determine pairs of rates that can be achieved simultaneously by both players, such that none of the rates can be increased without decreasing the rate of the other user (and similarly in the case of more than two users). The border of the capacity region determines the *global throughput* of the system. It is but one among several performance measures that can be optimized. For example, *delay* can be taken into account as well, as pointed in [HT98]. *Complexity* is also frequently an issue, in particular at the mobile nodes, which may dispose of only local knowledge of the system, as well as limited battery and computational power.

In addition, a multiuser system seldomly operates in isolation. On the contrary, it is usually part of a larger network. Increasingly, the interactions between several separate multiuser systems are taken into account, as the short survey [SSZ04] demonstrates.

In order to design an efficient network architecture, a lot of possibilities have been considered throughout the literature. In particular, protocol design is a vast topic. Here is a question for the reader. Think about it a few seconds. Is the best solution cellular, ad-hoc, centralized, distributed, cooperative, non cooperative, coordinated?

When asked to determine the best solution, the answer is generally "It depends". As pointed in this summary of economy [Wil05], there is rarely a single answer that is always valid to such a broad, and, more embarrassingly, seemingly subjective, question. It lacks focus, information. When presented with a choice between two (or more) alternatives, it is necessary to define objective criteria in order to circumscribe the problem; those criteria will necessarily reduce the scope of the original problem. The problem of optimizing multiuser systems can be considered from an abundance of points of view, even when only the physical layer is taken into account. To each of them can be associated a relevant performance measure that is to be optimized. However, there is necessarily a tradeoff between different performance measures such as throughput, delay, complexity.

In this thesis, we restrict ourselves to physical layer protocols such as code division multiple access and review their performances in several frameworks.

1.1.1 Definition of Terms

Cellular means that the surface considered is divided into cells. Users in each cell communicate with a base station, that is a super-node which is usually assumed to be connected (via a wired link) to other base stations, the Internet, etc. The shapes of cells are determined by different rules, either fixed surface or depending on the zones that have the maximal Signal to Noise Ratio (SNR). In 2-D, one of the models most often considered is regular hexagonal cells, in order to fill the plane. Note this is a particular case of Voronoi diagram, when base stations are regularly distributed on the plane. In 1-D, this will correspond to equal length segments on the real axis. Cellular systems are nowadays widely deployed, hence the heightened interest in their analysis, considering both theoretical and practical aspects. A recent overview of the literature dedicated to multi-cell networks is available in [SSZ04].

A system is called *ad-hoc* if it doesn't profit from such a fixed infrastructure. Nodes then communicate directly with each other and are usually assumed to be capable of auto-organization. The capacity of such networks has been thoroughly analysed, beginning with the famous article of Gupta and Kumar [GK99], and then noticing the amelioration in throughput provided by mobility, at the expense of greater delay [GT02]. The tradeoff between throughput and delay has since been extensively studied [GMPS06].

Hybrid systems, such as mesh networks, are also considered in the literature. Nodes that are far from a base station communicate with one another, in ad-hoc fashion, until the communication arrives to a node close to a base station. Such systems represent a middle-ground between centralized and decentralized systems.

A *centralized* environment is one in which there is global information about all nodes in the system. Nodes are administrated by a central controller. Cellular networks are usually assumed to benefit from centralized control provided by the base station. An equivalent term that we will use is *network centric*, since the network is considered as a global entity in this environment.

On the contrary, ad-hoc networks are generally viewed as quintessentially *decentralized* systems. There is no central controller, mobiles administrate the network by themselves. Communication algorithms in this case are called *distributed*. The respective advantages of centralization and decentralization will be illustrated by the example of resource allocation in the next section. An equivalent term that we will use is *user centric* since the main actor in such an environment is the end user.

When mobiles are considered as independent entities administrating their transmissions, two models of communications, derived from game theory, can occur. The first one is *cooperative*. In this model, mobiles work together with the purpose to achieve a common goal, for example maximize the global throughput of the system.

On the contrary, in the *non-cooperative* model, nodes are selfish. They do not care about the welfare of the system as a whole, but solely on their own gain. A solution in this framework, when no mobile can benefit from deviating singlehandedly, is called a Nash Equilibrium. Generally, the global performance at Nash Equilibrium is inferior to the optimal cooperative gain that can be attained. Additional properties of stability can be implemented, for example by introducing the mathematical biology concept of Evolutionary Stable Equilibrium.

Finally, *coordinated* refers to a subclass of game theory models. A coordinated game can occur either between cooperative or non-cooperative players. It consists in providing an additional degree of liberty to the mobiles by the presence of an arbitrator. An arbitrator is an entity that can send messages to the mobiles; it needs not have any intelligence, nor any knowledge of the system. It simply sends random signals, that players can take into account in order to jointly maximize their utility. Some useful game theory concepts will be described in more detail in a later chapter.

1.1.2 Network Centric or User Centric: an Example

In telecommunications, a centralized system is one in which most communications are administrated by one or more major central controllers. Such a system allows certain functions to be concentrated in the hubs of the system, freeing up resources in the terminals. Another benefit of centralization is the ease of maintaining accurately updated lists of data that can be easily accessed from all points. The weaknesses of centralization are centered around the heavy reliance on a few central components; if the hubs of the system are put out of operation, either accidentaly or through hostile action, the system and its peripheral components are severely affected. In addition, the complexity of computations at the central hub generally increases exponentially with the number of users, hence centralized systems pose problems of scalability when the size of the system increases.

Conversely, a decentralized environment has no source of global knowledge. Each node knows at most its own state and any information about other nodes must be gathered explicitly. The allure of decentralization lies in the promise of robustness, open-endedness and infinite scalability. Distributed procedures are especially interesting, since centralized procedures require added infrastructure, latency, and network vulnerability. The Internet itself is the largest decentralized computer system in the world. Ironically, in the 90s, many systems built on the Internet were completely centralized. Note that, in practice, extreme architectural choices in either direction are seldom the way to build a usable system.

Consider the terms described earlier in the context of resource allocation for cellular systems. Resource allocation, and especially power allocation, is of major interest in the context of multiuser systems. In the uplink multiuser systems, it is important for users to transmit with enough power to achieve their requested quality of service, but not more than necessary, in order to minimize the amount of interference caused to other users. Thus, an efficient power allocation mechanism allows to prevent an excessive consumption of the limited ressources of the users.

The most straightforward way to design a power allocation mechanism is as a centralized procedure, with the base station receiving training sequences from the users and signaling back the optimal power allocation. This is the case of global state information. The transmission power levels for all mobiles are chosen by the base station that has full information on the channel states of all mobiles. Centralized schemes in cellular systems were first introduced for TDMA/FDMA [Zan92, GVGZ93]; more recently an optimal scheme was derived for CDMA [Wu99]. In order to achieve optimal capacity, the users may also be sorted according to some rule of precedence [TH98]. However, this involves a non negligible overhead and numerous non informational transmissions. In addition, the complexity of centralized schemes increases dramatically with the number of users. Decentralized schemes may be simpler to implement in practice as the size of systems grows. As discussed in [EOA00], centralized algorithms provide useful bounds on the performance that can be attained by implementing distributed algorithms.

A way to avoid the constraints of a centralized procedure is to implement a decentralized one where each user calculates its estimation of the optimal transmission power according to its local knowledge of the system. Most of the time, a distributed algorithm means an iterative version of a centralized one. Mobiles update their power allocation according to some rule based on the limited information they retrieve from the system. This is the case of local state information. Each mobile chooses its own power level based solely on the condition of its own radio channel to the base station. Supposing that an optimal power allocation exists, a distributed iterative algorithm is derived from a differential equation in [FM93] and its convergence is proven analytically. A distributed version of the algorithm of [GVGZ93] is presented in [GVG94]. Building on these results, a general framework for power control in cellular systems is given in [Yat95]. A review of different methods of centralized and distributed power control in CDMA systems is given in [EOA00]. The survey [HT99] focuses on single-cell CDMA, and discusses the measures of performance, using some advanced tools of random matrix theory.

In this context, a natural framework is game theory, which studies competition (as well as cooperation) between independent actors. It was introduced to design efficient power allocation schemes. Following the popularization of power allocation games by [GM00, MW01a], an abundance of works can be found on the subject. Some of these works are presented in more detail in Chapter 2.

1.2 Dissertation Overview

1.2.1 Chapter 2: Game Theory and Random Matrix Theory

Game Theory provides a vast array of tools to study all kinds of interactions among selfish players that reason strategically in order to take rational decisions. With the increasing interest in deployment of self-organizing networks, it is very alluring to consider mobiles as independent actors that possess these characteristics. A few subfields of game theory are particularly promising. Evolutionary game theory, adapted from mathematical biology, is used to describe and to predict properties of large populations whose evolution depends on many local interactions, each involving a finite number of individuals. Correlated games study the impact of adding coordination mechanisms on the possible equilibria and joint optimizations that the players can perform.

Random Matrix Theory was recently introduced in Information Theory. In order to optimize a given network, intensive simulations can be performed. However, as networks grow large, simulations involve a huge number of (random) parameters, and in multiple access systems, communications interfere with one another. In addition, simulations do not readily allow to single out parameters of interest, as they depend upon so many of them. Random matrix theory gives tools to circumvent this problem. The self-averaging effect of large random matrices enables to elegantly single out parameters of interest in systems in the asymptotic regime, when number of chips, antennas or carriers and number of users both grow very large with fixed ratio. Even if the results are obtained in the asymptotic regime, they give very accurate predictions of the system's behavior in the finite size case, as shown by simulations.

The introduction of game theory and random matrix theory to the information theoretic community is rather recent. In Chapter 2, a short history of these fields and a few useful results are provided. Game theory especially is a vast mathematical domain so the introduction is necessarily very limited and partial to concepts used in wireless communications and in particular in the remainder of this work. A particular use of game theory in wireless communications is illustrated in the context of resource allocation.

1.2.2 Chapter 3: Multiuser Communication Schemes

This chapter is devoted to explaining principles of multiple-access protocols such as CDMA, ALOHA and OFDMA. In particular, models and notations used in the subsequent chapters are introduced, as well as the measures of performance: SINR, capacity, spectral efficiency, and goodput in the context of power allocation games.

1.2.3 Chapter 4: Network Centric Communications

Network centric communications are exemplified by the performance analysis of cellular CDMA systems.

Previously, single cell and networks with few interfering cells had been considered. The new contribution of this thesis lies in the analysis of infinite networks, where the contribution of all interferers is taken into account. Such networks are investigated in the downlink as well as in the uplink. The analysis makes use of results of random matrix theory and unitary random matrix theory, in order to obtain analytical expressions depending only on a small number of meaningful parameters.

Downlink Multi-Cell Orthogonal CDMA

In the downlink CDMA case, it is a first step into analyzing the complex problem of optimizing downlink CDMA multi-cell networks, using a new approach based on unitary random matrix theory. The purpose is to determine, for a dense and infinite multi-cell network, the optimal distance between base stations. A downlink frequency selective fading CDMA scheme with orthogonal codes where each user is equipped with a linear matched filter is considered. The users are assumed to be uniformly distributed along the area. Only orthogonal access codes are considered as the users are synchronized within each cell. The problem is analyzed in the asymptotic regime: very dense networks are considered where the spreading length N tends to infinity, the number of users per meter d tends to infinity but the load per meter $\frac{d}{N} = \alpha$ is constant.

Uplink Multi-Cell Random Spreading CDMA

In the uplink CDMA case, a similar setting is investigated. An infinite length base station deployment is considered and performance results, in terms of spectral efficiency, are derived for two types of receiver structures: Matched filter and Optimum filter.

Uplink Single-Cell Orthogonal CDMA

Usually, random codes are used in the uplink. The new contribution is in deriving performance results with orthogonal codes. In the single-cell context, the performance of an uplink CDMA system with orthogonal spreading is analyzed. A useful framework is provided in order to determine if synchronization of the users gives a significant performance improvement. Using asymptotic arguments, analytical expressions of the spectral efficiency for the Matched Filter and Successive Interference Cancellation Matched Filter are derived in the general case of a multipath channel.

Uplink Multi-Cell Orthogonal CDMA

In order to complete the study, the case of an infinite deployment of cells using orthogonal codes in the uplink is investigated.

1.2.4 Chapter 5: User Centric Communications

In the case of user centric communications, we focus on three protocols: ALOHA, CDMA and OFDMA.

ALOHA

In the case of decentralized networks, a first contribution of this thesis is to introduce two types of games in a networking context: correlated games and evolutionary games. In order to keep this introduction as simple as possible, the multiple-access protocol considered is ALOHA. To the best of our knowledge, we have been the first to introduce correlated games and evolutionary games to study non-cooperative behavior in wireless networks.

Since interactions between users are repeated, it was natural to introduce the biological notion of Evolutionary Games in a networking context. On the other hand, adding coordination mechanisms may enable mobiles to increase their throughput. This setting is investigated both in a non-cooperative as well as in a cooperative Correlated Games framework. This contribution also studies multi-criterion optimization, in our case maximizing the average throughput with a constraint on the average power consumption.

CDMA

Power allocation is an important topic in the context of multi-user systems, especially in the uplink, since an efficient power control mechanism allows to prevent an excessive consumption of the limited ressources of the users. An energy-efficient power allocation can be achieved by modeling power control as a game.

The contribution to power allocation games is in considering the frequency-selective model. The model is investigated in a non-atomic games framework. This enables us to obtain power allocation as function of the total channel energy for each user. In addition to linear filters, the optimal and successive interference cancellation (SIC) filters (with the introduction of an ordering of the users) are investigated. SIC filters are shown to exhibit interesting properties in relation to the accomodation of users.

OFDMA

A novel distributed carrier allocation technique for OFDMA is introduced. The choice of carriers is done at the transmitters rather than at the base station. Using the reciprocity of the channel on each carrier, the algorithm enables each user to send reliably data at a prescribed rate knowing only its channel, under mild asymptotic conditions. For several channel models, we derive analytical expressions of the cell spectral efficiency in the asymptotic regime (high number of carriers) for two filter types: matched filter and optimum filter. The result is based on the predictability of the interference as the number of carriers increases.

1.3 Published Works

During the course of this thesis, eight contributions were published: two international journal publications and six publications in refereed conferences. The dissertation is based on the first seven of these contributions, in addition to unpublished results.

[BDAC05a] "Spectral Efficiency of CDMA Uplink Cellular Networks", N. Bonneau, M. Debbah, E. Altman and G. Caire, *IEEE ICASSP 2005, Philadelphia, USA*, March 2005.

[BAD05] "An Evolutionary Game Perspective to ALOHA with Power Control", N. Bonneau, E. Altman, M. Debbah and G. Caire, *19th ITC, Beijing, China*, Aug. 29–Sep. 2, 2005.

[BDAC05b] "When to Synchronize in Uplink CDMA", N. Bonneau, M. Debbah, E. Altman and G. Caire, *IEEE ISIT 2005, Adelaide, Australia*, Sep. 4–9, 2005.

[BDHA05] "Performance of Channel Inversion Schemes for Multi-User OFDMA", N. Bonneau, M. Debbah, A. Hjørungnes and E. Altman, *2nd ISWCS, Siena, Italy*, Sep. 2005.

[BDA06] "Spectral Efficiency of CDMA Downlink Cellular Networks with Matched Filter", N. Bonneau, M. Debbah, E. Altman, *EURASIP Journal on Wireless Communications and Networking*, 2006.

[ABD06] "Correlated Equilibrium in Access Control for Wireless Communications", E. Altman, N. Bonneau and M. Debbah, *Networking 2006, Coimbra, Portugal*, May 15–19, 2006.

[BDAH07b] "Wardrop Equilibrium for CDMA Systems", N. Bonneau, M. Debbah, E. Altman and A. Hjørungnes, *RAWNET 2007, Limassol, Cyprus*, April 16, 2007.

[AAB+07] "Constrained Cost-Coupled Stochastic Games with Independent State Processes", E. Altman, K. Avrachenkov, N. Bonneau, M. Debbah, R. El-Azouzi and D. Sadoc Menasche, accepted for publication in *Operations Research Letters*, 2007.

1.4 Submitted Works

[BDAH07a] "Non-Atomic Games for Multi-User Systems", N. Bonneau, M. Debbah, E. Altman and A. Hjørungnes, submitted to *IEEE JSAC Special Issue on "Game Theory in Communication Systems"*, 2007.

Chapter 2

Game Theory and Random Matrix Theory

2.1 Game Theory for Wireless Communications

2.1.1 A Short History of Game Theory

Game theory is a field of applied mathematics that studies interactions among individuals making decisions. It relies on two assumptions. The first is the *rationality* of the individuals involved, i.e., the players choose their strategies according to their own preferences. The second is their capability of *strategic reasoning*, i.e., they also take into account the preferences of the other players.

Game theory provides a set of tools to study such interactions, which can be non-cooperative or cooperative. In the first case, players are selfish and are only concerned with maximizing their own benefit. In the second case, players cooperate in order to achieve a common goal. Therefore, it is natural that interest has been growing in recent years in studying competition aspects of networking in general, access to a common medium in particular, within the framework of game theory. The survey paper [ABA$^+$06] describing a vast array of game theoretical tools and applications, or the recent publication of a book dedicated to introducing game theory to wireless communication engineers [MD06] are but the tip of the iceberg. A survey detailing applications of non-cooperative game theory to wireless communications is also going to appear in French [AA07].

The website "A Chronology of Game Theory" [Wal95] provides a summary of some important dates of the field. The roots of game history are based on real-life situations. A mathematical analysis was originally introduced to study problems involving several players, like chess or card games. Several works touched upon the subject and lay the groundwork, during the nineteenth century and beginning of the twentieth century. It is however generally considered that the real breakthrough occurred with the publishing of *Theory of Games and Economic Behavior*, by J. von Neumann and O. Morgenstern, in 1944. This book gave a formalism for cooperative, as well as non-cooperative games. Those concepts were further extended by J. Nash[1] between 1950 and 1953. In particular, he demonstrated one of the most important

[1] J. Nash has received in 1994 the Nobel prize in economy for his contributions to game theory.

theorems of game theory, namely the existence of an *equilibrium* in non-cooperative games, henceforth denominated Nash equilibrium. Nash also studied cooperative games, which are generally considered more involved to analyze, since the players can form coalitions to cooperate for the greater good. He derived the Nash bargaining solution for such games. Nowadays, game theory is used in economics, mathematical biology, as well as wireless communications and networking.

A first contribution to networking games was, albeit in an indirect manner, [War52], which treated the case of road traffic, and found many subsequent applications in networking. Nearly fifty years ago, a game theoretic formulation of the communication process was already formulated by Blachman [Bla57], as a zero-sum noncooperative game. A saddle-point arguments shows that Gaussian noise is the worst case noise that can affect a signal in terms of mutual information between sent and received signal.

2.1.2 Useful Results and Illustrations of Game Theory

Game theory is a vast mathematical domain. This introduction is limited to giving useful insights to understand the concepts used in the remainder of the thesis. For an introductory course in game theory, refer for example to the book [OR94].

Definition 1 *A strategic game consists of*

- *A set of players S^K, consisting of K individuals;*
- *For each player $k \in S^K$, a set of strategies (or actions) $\mathbb{S}_k$;*
- *For each player $k \in S^K$, a preference relation on $\mathbb{S} = \prod_k \mathbb{S}_k$.*

In most cases of interest, the preference relation can be expressed as a utility (or payoff) function $u_k : \mathbb{S} \to \mathbb{R}$. See for example [MD06] for a detailed discussion on the conditions of existence of a utility function. Since those conditions are very broad and encompass most practical cases, we will always assume that the preference relation is expressed as a utility function in the following.

This model is very abstract; hence, it allows to cover a very wide variety of situations. There is no restriction on the set of actions available to a player. The only limit to analysis is the obligation to define a preference relation. Of course, no results can be derived directly from the model of Def. 1. The parameters have to be specified, in order to obtain results relating to the specific version of the game considered.

A strategy vector is denoted $\mathbf{p} \in \mathbb{S}$. For each player k, the strategy of this player is p_k. The vector of strategies of all players except player k is denoted $\mathbf{p}_{(-k)}$. If player k plays strategy q when all other players keep their strategies according to $\mathbf{p}$, the resulting strategy vector is denoted $\mathbf{p}_{(-k)}, q$. With the help of this notation, we state the following definition.

When each player holds an appropriate vision of the game and acts rationally, the equilibrium concept is called Nash equilibrium.

Definition 2 *A Nash equilibrium of a strategic game* $(S^K, \mathbb{S}, (u_k)_{k \in S^K})$ *is a profile* $\mathbf{p}^* \in \mathbb{S}$ *of actions such that*

$$\forall k \in S^k, \ \forall p_k \in \mathbb{S}_k, \ u_k(\mathbf{p}^*_{(-k)}, p_k) \leq u_k(\mathbf{p}^*).$$

In others words, a Nash equilibrium is a profile of actions such that no player can benefit by unilaterally deviating. It is a best response to itself. The Nash equilibrium is a concept used in non-cooperative game theory. On the other hand, when considering cooperation among players, the solution concept is called *Pareto equilibrium.* A Pareto equilibrium is a cooperative dominating solution: it is impossible to increase the payoff of a player without decreasing the payoff of another. Generally, Nash and Pareto equilibria do not coincide.

In order to give examples, the most simple games are 2-player games, with two strategies each. Those games are conveniently represented as 2×2 payoff matrices. Player 1 chooses the row, and player 2 the column. The first number in the pair is player 1's payoff, and the second number player 2's payoff. Although simple, such games provide good insight on the possible outcomes of games in general.

One of the most renowned such games is nicknamed the Prisonner's Dilemna. In this game, two inmates are given the choice between confessing and not confessing. If both confess, they get each 3 years in prison. If only one of them confesses, he is released while the other gets 4 years. If neither confesses, both get a minimal sentence of 1 year. By appropriately reevaluating the payoffs, we get a payoff matrix that has the following form.

	D	C
D	3, 3	0, 4
C	4, 0	1, 1

While the social (Pareto) optimum is obviously (D,D), it is not an equilibrium in the non-cooperative game since any player is better off by deviating. Hence, the only Nash equilibrium of the game is the suboptimal (C,C), yielding a payoff of 1 for each player.

A second example of 2×2 game is called Battle of the Sexes. In this game, a couple has to decide where to spend the evening. Their payoffs are positive only if they spend the evening together, but their actual values depend on the chosen event. The man prefers event A (theater) while the woman would rather assist to event B (dance). The payoff matrix has the following form.

	A	B
A	2, 1	0, 0
B	0, 0	1, 2

This game admits two pure Nash equilibria (A,A) and (B,B).

A third example is the game of Matching Pennies. Both players call Head or Tail. If their choices differ, player 1 pays 1 Euro to player 2. Otherwise, player 2 pays 1 Euro to player 1. Hence, both players have diametrically opposed interests: this is a zero-sum game. This game has no Nash equilibrium when players choose a single strategy.

	H	T
H	$1, -1$	$-1, 1$
T	$-1, 1$	$1, -1$

As shown by Matching Pennies, not every game has a Nash equilibrium in pure strategies, i.e., when players deterministically choose one of their strategies. It is possible to extend the possible set of strategies of the players to include nondeterministic actions, i.e., the set of probability distributions $\Delta(\mathbb{S}_k)$ over the set of strategies $\mathbb{S}_k$. The payoff associated to such a probability distribution is the average payoff over the strategies that may be employed. In this case, we talk about *mixed strategies.* Extending strategic games to mixed strategies enables to state the important theorem of Nash about the existence of a Nash equilibrium.

Theorem 1 *(Nash) Every finite game in strategic form has a mixed strategy Nash equilibrium.*

Reconsider the previous examples in terms of mixed strategies. For the Prisonner's Dilemna, nothing is changed, the only Nash equilibrium is still (C,C), in pure strategies. For the Battle of the Sexes, in addition to the two previous equilibria, (2/3 A + 1/3 B, 1/3 A + 2/3 B) is also a Nash equilibrium, yielding a payoff of $(\frac{2}{3}, \frac{2}{3})$. Finally, for Matching Pennies, the equilibria is (1/2 H + 1/2 T, 1/2 H + 1/2 T), yielding a payoff of $(0, 0)$.

What is the underlying motivation for players to randomize their strategies? Different interpretations can be given, several of which are discussed in [OR94]. One of them is the fact that we are interested in a steady state of the game. When players use mixed strategies, the steady state will be stochastic. Note that for mixed strategies, there is the underlying assumption that players are uncoordinated in their random choice. They may receive signals from "nature" enabling them to randomize, but those signals are kept strictly private and independent. If we want players to jointly randomize their strategies, the framework is that of *correlated games*, which are introduced in the next section.

2.1.3 Correlated Games

The notion of correlated equilibrium was introduced by R. Aumann[2] in [Aum74] and further studied in [Aum87, HS89, Ney97]. An algorithm for the computation of correlated equilibria is developed in [Pap05]. Correlated equilibria are generalizations of the Nash equilibrium concept; the correlated equilibria are defined in a context where there is an arbitrator who can send (private or public) signals to the players. These signals allow players to coordinate their actions, and, in particular, to perform joint randomization over strategies.

In many contexts, an arbitrator is thought of as an intelligent entity, used for helping to solve conflicts and for proposing compromises to the different sides involved. In contrast, in correlated games, an arbitrator needs not have any intelligence. It is assumed to generate signals that do not depend on the system (or on

[2]R. Aumann has received in 2005 the Nobel prize in economy for his contributions to game theory, together with Thomas Schelling.

individual) states. Moreover, it does not need to have any knowledge on the system. All the arbitrator has to do is to create some random signals (according to a randomized mechanism known by the players) that can help the synchronization (or coordination) between them.

An arbitrator may even be a virtual entity. As an example, the players can agree to use some random data (e.g., the first word they hear on the radio) as the signal or as an input to a function that allows to create a common signal (or a signal which may differ from one player to another).

In the context of non-cooperative games, each player has the possibility not to consider the signal(s) it receives. A multi-strategy obtained using the signals is a set of strategies (one strategy for each player which may depend on all the information available to the player including the signal it receives). It is said to be a correlated equilibrium if no player has an incentive to deviate unilaterally from its part of the multi-strategy. A special type of "deviation" in this definition can be of course to ignore the signals.

As an example, consider the Battle of the Sexes, already discussed above. This game has three Nash equilibria, two in pure strategies, yielding payoffs $(2,1)$ and $(1,2)$, and one in mixed strategies, yielding payoff $(\frac{2}{3}, \frac{2}{3})$. Now, if we add one degree of liberty by sending a common signal to the players randomly chosen in $(0,1)$ with equal probability, what happens? There is a new equilibrium, in which both players choose A if the received signal is 0 and B if the received signal is 1. Indeed, if the received signal is 0, player 1 knows that player 2 will choose A, so it is optimal for him to choose A as well, and reciprocally. This new equilibrium yields payoff $(\frac{3}{2}, \frac{3}{2})$.

2.1.4 Evolutionary Games

Evolutionary Game Theory can arguably be traced back to Darwin, who introduced the concept of natural selection and hence competition between genotypes and phenotypes of individuals. The way their programmed genetics enable animals to thrive determine their capability of reproduction, or *fitness*. In 1967, Bill Hamilton published a paper about a sex ratio strategy than can't be beaten, using ideas from game theory. Rationality becomes population dynamics, and utility, the Darwinian fitness. This work was extended during the 70s by John Maynard Smith. He first defined the concept of Evolutionary Stable Strategy (ESS) in 1972 in [Smi72]. Smith's seminal text *Evolution and the Theory of Games* [Smi82] appeared in 1982. In the context of mathematical biology, ESS are used to describe and to predict properties of large populations whose evolution depends on many local interactions, each involving a finite number of individuals. The payoff obtained by a strategy depends on all the strategies present in the population, which is an important game theory concept. In the biological context, the amount of reward for an individual is related to its reproduction capability. A higher reward to some behavior (which can represent more food or more chances to mate) implies a higher growth rate of individuals that adopt it.

An intuitive way of explaining what is an ESS is the following. It is a strategy that does well against itself. It expresses the fact that, if animals "playing" the ESS predominate in the population, they will tend to encounter mostly animals playing the same strategy. Therefore, the ESS must do well against other copies of itself in order to stay successful. This simplified explanation will be completed after the introduction of complementary definitions.

Competition between animals of the same species occurs frequently. Consider for example the reproduction period of stags: stags will compete in order to mate with females. A most common illustration of evolutionary games is the following. There are two attitudes: Hawk (H) and Dove (D). Animals meet randomly by pairs and compete for a resource (gain G). If both are dovish, they display (non-violently) until one of them leaves the scene. If a hawk encounters a dove, the latter flees immediately, letting the hawk get the resource all for itself. When two hawks meet, they fight for the resource, until one of them is hurt (cost C). We assume that if two animals of the same kind meet, they have an equal chance to get the resource. The associated payoff matrix is

	H	D
H	$\frac{1}{2}(G-C), \frac{1}{2}(G-C)$	$G, 0$
D	0,G	$\frac{1}{2}G, \frac{1}{2}G$

Given the payoff matrix, stags may adopt pure or mixed strategies. This is an example of random matching game: two individuals taken at random in the population compete at each turn. A strategy is called ESS if it is resistant against mutant strategies. Compared to the Nash equilibrium, it is hence characterized by a robustness property (that need not be satisfied by a Nash equilibrium): under an ESS, the populations become immune to the proliferations of mutations.

Let us define the concept of ESS in a more rigorous fashion. We restrict to the case of a symmetric game, i.e., all individuals have the same strategy set and the same utility function. The definition is simpler to state in this case, but extension to non-symmetric games is straigthforward.

A population is a set of individuals (in great numbers) who possess the same set of strategies $A = \{1, \dots, n\}$ and the same utility function $\mathbf{U}$ (in matrix form). A population state is a probability vector on A, denoted $\mathbf{p}$. $\mathbf{p}$ belongs to the simplex $\Delta^{n-1} = \{\mathbf{p} \in \mathbb{R}^n / p_i \geq 0, \sum_{i=1}^n p_i = 1\}$. p_i is the frequency of action i in the population. The fitness function $J(\cdot, \cdot)$ describing the encounter of two members of the population is $J(\mathbf{p}, \mathbf{q}) = \mathbf{p}^T \mathbf{U} \mathbf{q}$. Note that $J(\cdot, \cdot)$ is bilinear. A population can be monomorphic (all individuals randomize between the strategies) or polymorphic (each pure strategy is represented by a fraction of the population).

An ESS occurs if a small proportion of mutants cannot invade. $\mathbf{p}$ is an ESS if for all $\mathbf{q}$ distinct from $\mathbf{p}$, there exists ϵ' such that for all $\epsilon < \epsilon'$,

$$J(\mathbf{q}, \epsilon\mathbf{q} + (1-\epsilon)\mathbf{p}) < J(\mathbf{p}, \epsilon\mathbf{q} + (1-\epsilon)\mathbf{p}).$$

A simpler (and equivalent) relation is given by

$$J(\mathbf{q}, \mathbf{p}) < J(\mathbf{p}, \mathbf{p})$$

or

$$J(\mathbf{q}, \mathbf{p}) = J(\mathbf{p}, \mathbf{p}) \text{ and } J(\mathbf{q}, \mathbf{q}) < J(\mathbf{p}, \mathbf{q}).$$

In other words, a strategy $\mathbf{p}$ is ESS if it doesn't accept any best response except itself (2.1.4), or, if there exists another best response, the latter obtains a strictly inferior payoff when confronted to itself than initial strategy $\mathbf{p}$ (2.1.4). In the first case, the fraction of the mutations in the population will tend to decrease (as mutants have a lower reward, meaning a lower growth rate). $\mathbf{p}$ is then immune to mutations. In the second case, the population using $\mathbf{p}$ is "weakly" immune against a mutation $\mathbf{q}$ since as the population of mutants grows, then individuals with strategy $\mathbf{p}$ shall frequently compete with mutants; in such cases, the higher payoff for $\mathbf{p}$ ensures that the growth rate of the original population exceeds that of the mutations.

The Hawk-Dove game above can be analyzed in this light. If $C > G$, it has a unique symmetric mixed strategy equilibrium $(G/C, 1 - G/C)$, which is the only ESS. If $C < G$, the unique Nash equilibrium is when all players are Hawks. This is also the only ESS.

Remark that (2.1.4) characterizes a *strict Nash equilibrium*, meaning a Nash equilibrium that accepts no best response except itself. Replacing the strict inequality in (2.1.4) by a large inequality yields the characterization of a Nash equilibrium. Hence, there is a chain of implication regarding ESS: Strict Nash implies ESS implies Nash. Examples can be built to show that the reverse implications are generally false.

One of the most alluring aspects of evolutionary game theory and ESS is the vast array of tools that are associated to it to analyze the *dynamics* of a population. See [HS03] for a rundown of the different dynamics that can be associated to an evolutionary game and their convergence in presence of ESS.

Although the concept of ESS has been defined in the context of biological systems, it is highly relevant to engineering as well (see [VV00]). In particular, in the context of competition in the access to a common medium, we can expect that a technology that provides better performance will gain more market shares at the expense of less performant technologies.

2.1.5 Power Allocation Games

As discussed in Chapter 1, resource allocation is an important research topic in the context of multi-user wireless communications. Game theory can be naturally applied to the context of uplink resource allocation, considering that mobiles are players in the game, with their transmit powers as strategies. The utility is determined by the benefit obtained, as well as the cost incurred by the player using the strategy. Building on the framework of [Yat95], such a game theoretic approach was introduced in [Ji97] and presented in [GM00, MW01a]. Numerous works on power allocation games have followed since.

We remind that a Nash equilibrium is a stable solution, where no player has an incentive to deviate unilaterally, while a Pareto equilibrium is a cooperative dominating solution, where there is no way to improve the performance of a player without harming another one. Generally, both concepts do not coincide.

The utility generally considered takes into account both the gain from achieving a higher throughput as well as the cost of transmitting with higher power. It has been first introduced in works with an economic leaning [Ji97, SMG01, SMG02]. Customarily, it consists in throughput-to-power ratio (see Sec. 3.5.4). This particular form of the utility function is justified in [Rod03], where the author describes a widely applicable model "from first principles". Conditions under which the utility will allow to obtain non-trivial Nash equilibria (i.e., users actually transmit with nonzero powers at the equilibrium) are derived. The utility consisting of throughput-to-power ratio is shown to satisfy these conditions. In addition, it possesses a property of reliability in the sense that the transmissions occur at non-negligible rates at the equilibrium.

Unfortunately, Nash equilibria often lead to inefficient allocations, in the sense that higher rates (Pareto equilibria) could be obtained for all mobiles if they cooperated. To alleviate this problem, in addition to the non-cooperative game setting, [SMG02] introduces a pricing strategy to force users to transmit at a socially optimal rate. With this additional mechanism, communication at Pareto equilibrium is obtained.

Game theory can be used to treat the case of any number of players. However, as the size of the system increases, the number of parameters increases drastically and it is difficult to gain insight on the expressions obtained. In order to obtain simple expressions, asymptotic analysis of the system using random matrix theory is performed in [MPSM05].

Defining the utility as advised in [Rod03] as the ratio of the throughput to the transmission power, the authors obtain results of existence and unicity of a Nash equilibrium for a CDMA system. They derive explicit expressions depending only on a few parameters for the equilibrium power allocation corresponding to three linear receivers: matched filter, MMSE filter and decorrelator. This work is extended to the case of multiple carriers in [MCPS06]. In particular, it is shown that users will select and only transmit over their best carrier. As far as the attenuation is concerned, the consideration is restricted to flat fading in [MPSM05] and in [MCPS06] (each carrier experiencing flat fading in the latter). However, wireless transmissions generally suffer from the effect of multiple paths, thus becoming frequency-selective. In Sec. 5.2, we treat the case of power allocation games in frequency selective fading. In addition to the linear filters, optimal and successive interference cancellation filters are considered.

2.2 Random Matrix Theory for Wireless Communications

2.2.1 A Short History of Random Matrix Theory

Until recently, in the field of information theory, simulations were widely believed to be the only means to optimize a given network. However, as the number of users and the size of the communication systems grew, simulations had to be very intensive and did not allow to single out parameters of interest easily. This changed when simultaneously in 1999, Tse [TH99] and Verdú [VS99] introduced tools of Random Matrix Theory in order to analyse multi-user systems. Both treated the case of performance of linear receivers for CDMA systems, in the limit when the number of users as well as the spreading length tend to infinity, with a fixed ratio. In this asymptotic scenario, the use of random matrix theory leads to explicit expressions for various measures of interest such as capacity or Signal to Interference plus Noise Ratio (SINR). Interestingly, it enables to single out the main parameters of interest that determine the performance in numerous models of communication systems with more or less involved models of attenuation [TH99, VS99, ET00, SV01, TLV05]. In addition, these asymptotic results provide good approximations for the practical finite size case, as shown by simulations. A recent overview of random matrix theory, centered on the applications to information theory, is given in the book by Tulino and Verdú [TV04].

The properties of random matrices were first studied by statistical physicists. One of the first studies was done in 1928 by Wishart [Wis28]. He computed the probability density of $\mathbf{v}_1\mathbf{v}_1^H + \cdots + \mathbf{v}_n\mathbf{v}_n^H$ where $\mathbf{v}_i$ are i.i.d. Gaussian vectors. His results are among the few available concerning finite dimensional matrices. Indeed, most results of random matrix theory are asymptotic, i.e., limit properties when considering sequences of random matrices whose dimensions tend to infinity with fixed ratio.

The typical question is to characterize the distribution of (some of) the eigenvalues of random matrices. For finite matrix size this distribution itself is usually random. The real interest in random matrices surged when non-random limit distributions were derived for matrices whose dimensions tend to infinity, among others in 1955 by Wigner [Wig55] and in 1967 by Marchenko and Pastur [MP67], under simple hypotheses on the distribution of the matrix elements. The introduction of the (Cauchy-)Stieltjes transform [Pas72, SB95, Gir90] then enabled to derive distributions for more general matrix forms: correlation among the elements of the matrix, independant non-identically distributed elements.

Random matrices are also particular non-commutative random variables. The theory of non-commutative random variables is called Free Probability Theory [Bia98]. Freeness for non-commutative random variables is the analogue of independence for commutative random variables. Families of random matrices can be shown to be asymptotically free, which enables to derive results of interest on sums and products of random matrices, as was shown by Voiculescu [Voi91] and subsequent works. Free probability theory can also be used to treat the case of unitary random matrices. Results on unitary random matrices are extensively developped in [HP00].

Nowadays random matrix theory is used in numerous domains, including but not limited to Riemann hypothesis, stochastic differential equations, condensed matter physics, statistical physics, chaotic systems, numerical linear algebra, neural networks, multivariate statistics, stock exchange analysis, etc. And of course information theory [Mül03, TV04]! Information theory has even influenced work on random matrices. For example, [BS07] studies the expression of the MMSE SINR from a mathematical point of view, and provides a rigorous demonstration in a general setting of results previously used in the litterature [CM04].

2.2.2 Illustrations of Random Matrix Theory

In the following, upper case and lower case boldface symbols will be used for matrices and column vectors, respectively. $(.)^T$ will denote the transpose operator, $(.)^\star$ conjugation and $(.)^H = \left((.)^T\right)^\star$ hermitian transpose. $\mathbb{E}$ denotes the expectation operator.

Definition 3 *Let* $\mathbf{v} = [v_1, \ldots, v_N]$ *be a vector. Its* empirical distribution *is the function* $F_N^{\mathbf{v}} : \mathbb{R} \to [0,1]$ *defined by:*

$$F_N^{\mathbf{v}}(x) = \frac{1}{N}\#\{v_i \leq x \,|\, i = 1 \ldots N\}.$$

In other words, $F_N^{\mathbf{v}}(x)$ is the fraction of elements of $\mathbf{v}$ that are inferior or equal to x. In particular, if $\mathbf{v}$ is the vector of eigenvalues of a matrix $\mathbf{V}$, $F_N^{\mathbf{v}}$ is called the *empirical eigenvalue distribution* of $\mathbf{V}$.

Examples of Empirical Eigenvalue Distributions

One of the first results explicitly derived concerns a particular class of random matrices, called Wigner matrices. A Wigner matrix is an $N \times N$ symmetric matrix $\mathbf{H}$ with diagonal entries zero and upper-triangle entries i.i.d. zero mean and variance 1. As $N \to \infty$, the empirical eigenvalue distribution of $\frac{1}{\sqrt{N}}\mathbf{H}$ converges to the *semicircle law*:

$$f(\lambda) = \begin{cases} \frac{1}{2\pi}\sqrt{4-\lambda^2} & \text{if } |\lambda| \leq 2 \\ 0 & \text{if } |\lambda| \geq 2 \end{cases}$$

The semicircle law is plotted in Fig. 2.1, as well as the plot obtained by tracing the histogram of the eigenvalues of *a single* realization of a 512×512 Wigner matrix, with i.i.d. Gaussian $\mathcal{N}(0,1)$ distribution of the upper-triangle entries. The semicircle law already provides a good approximation of the eigenvalue distribution in the finite size case. Note that even though the nonzero entries are not bounded, the distribution of the eigenvalues has a bounded support.

Wigner matrices have quite a constrained form, but it also possible to obtain results for a non-symmetric matrix. If $\mathbf{H}$ is an $N \times N$ matrix with entries i.i.d. zero mean and variance 1, then the eigenvalues of $\frac{1}{\sqrt{N}}\mathbf{H}$ are uniformely distributed on the unit circle. This property is often referred to as Girko's full circle law.

The full circle law is plotted in Fig. 2.2, as well as the plot of the eigenvalues of *a single* realization of a 512×512 random matrix, with i.i.d. Gaussian $\mathcal{N}(0,1)$ distribution of the entries.

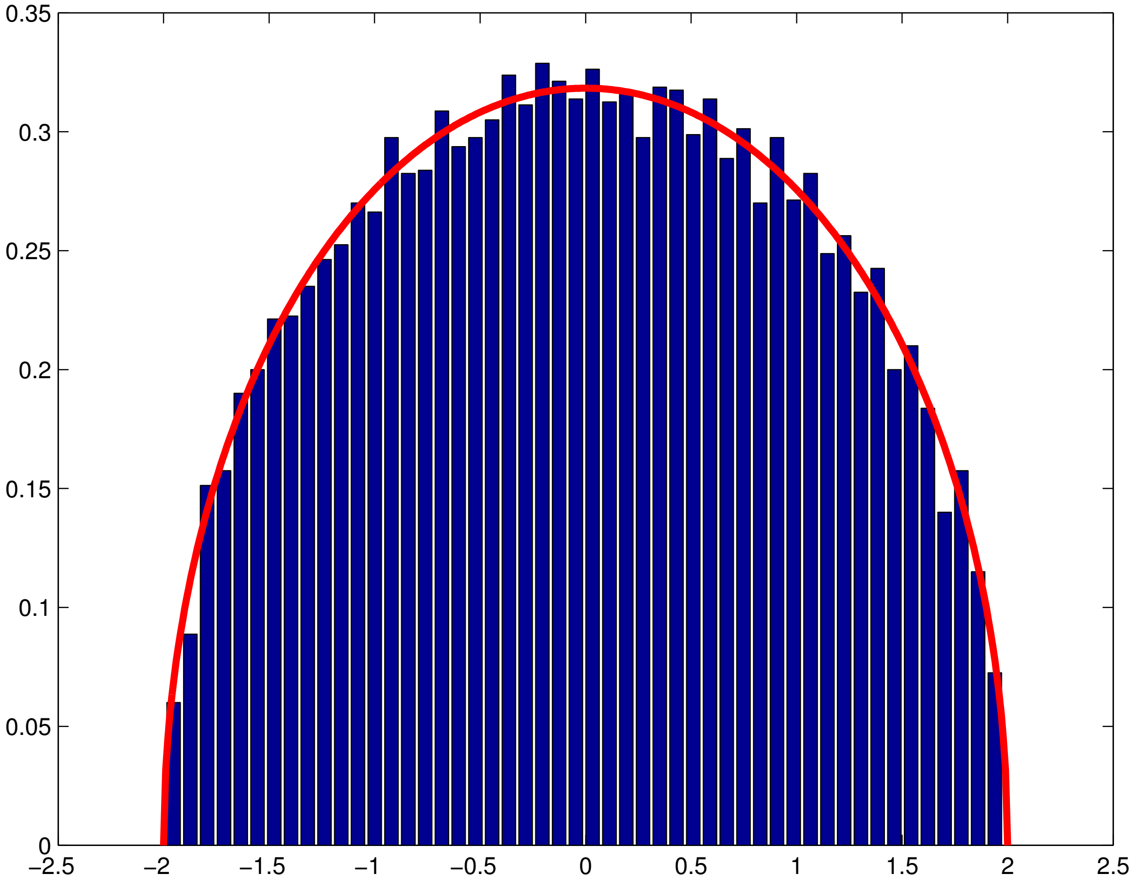

Figure 2.1: Semicircle law and simulation for a 512×512 Wigner matrix.

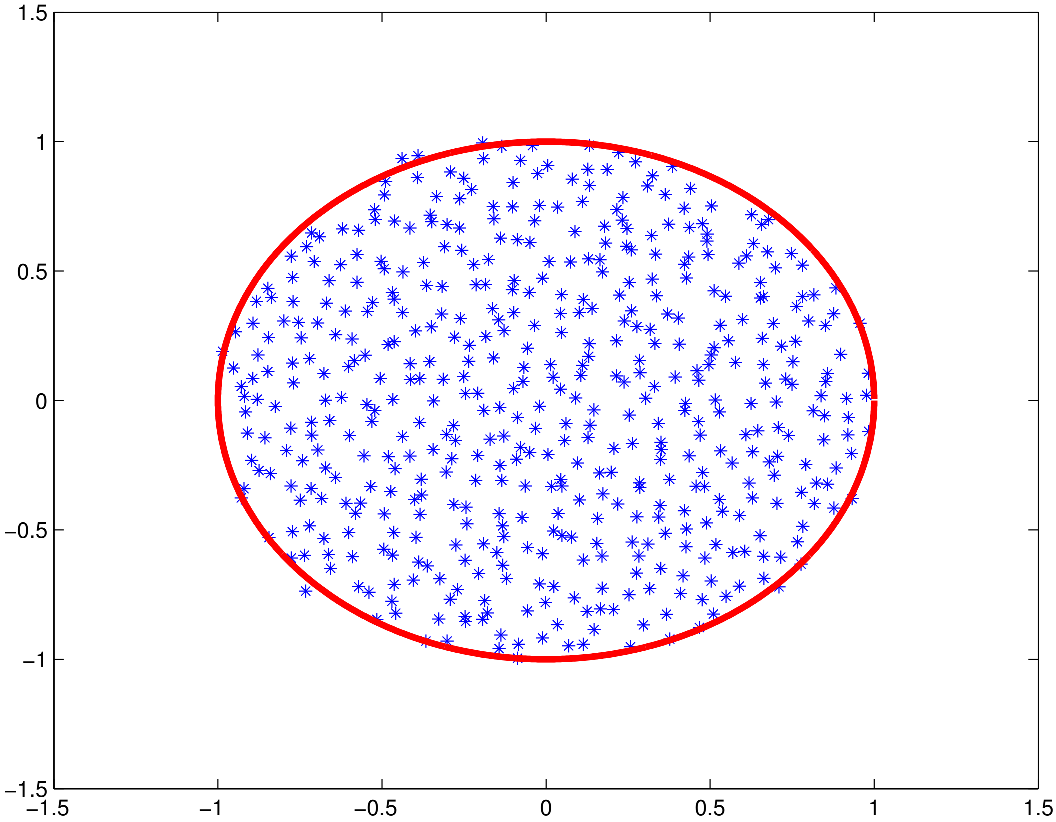

Figure 2.2: Full circle law and simulation for a 512×512 matrix.

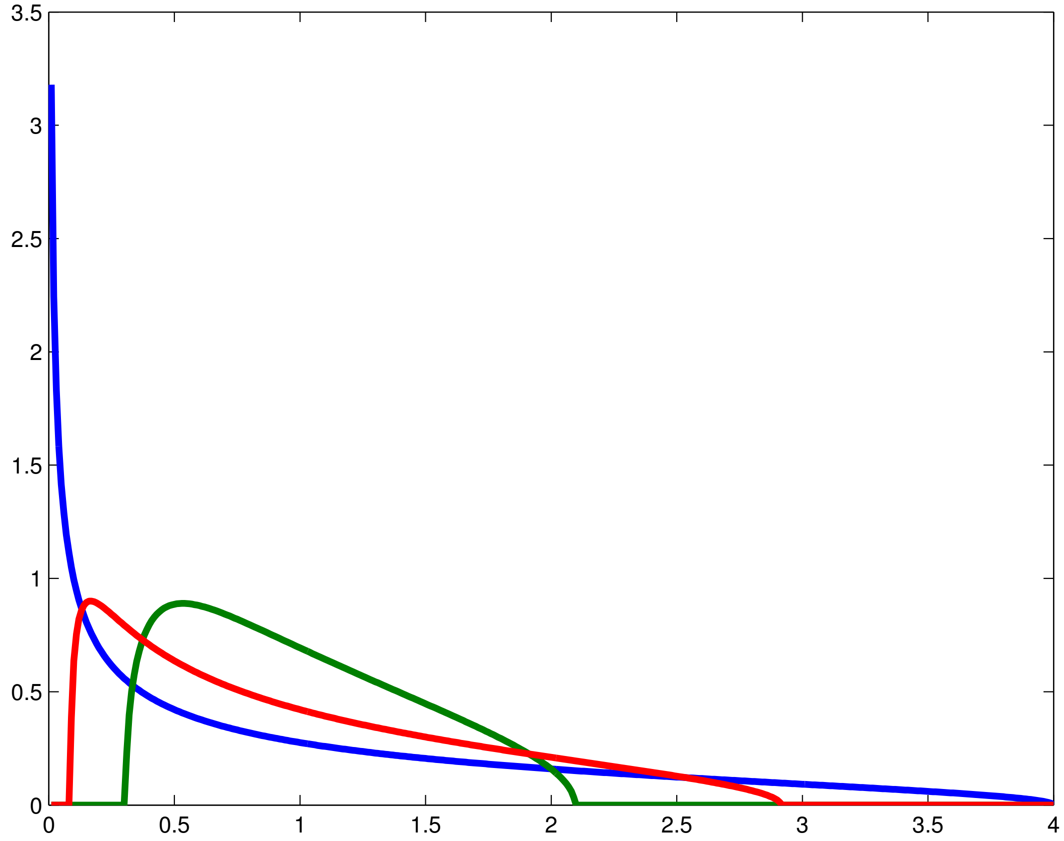

Figure 2.3: Marchenko-Pastur density function for $\alpha = 1, 0.5, 0.2$.

When nonsquare $N \times K$ matrices are under consideration, a common property to ensure asymptotic convergence of the distribution is that the ratio of the dimensions $\frac{K}{N}$ be kept constant. One of the first derivations of an explicit nonrandom limit distribution is due to Marchenko and Pastur. Let $\mathbf{H}$ be an $N \times K$ matrix, with i.i.d. zero-mean complex entries with variance $\frac{1}{N}$ and fourth moments $O\left(\frac{1}{N^2}\right)$. As $K, N \rightarrow \infty$, with $\frac{K}{N} \rightarrow \alpha$, the empirical eigenvalue distribution of $\mathbf{H}^H\mathbf{H}$ converges almost surely to a nonrandom limit distribution with density

$$f(x) = \left[1 - \frac{1}{\alpha}\right]^+ \delta(x) + \frac{\sqrt{[x-a]^+ [b-x]^+}}{2\pi\alpha x}$$

where $a = (1-\sqrt{\alpha})^2$ and $b = (1+\sqrt{\alpha})^2$.

The Marchenko-Pastur law is plotted in Fig. 2.3 for different values of α. The asymptotic analysis has an averaging effect: the limit distribution depends only on α, and not on the particular distribution of the entries of the matrices. The eigenvalues have a bounded support between $(1-\sqrt{\alpha})^2$ and $(1+\sqrt{\alpha})^2$.

Example of Application to Wireless Communications

In information theory, communication over a noisy medium between one or several transmitters and a receiver is generally considered. The model can be summarized by a single equation. Most of the information theoretic litterature focuses on vector memoryless channels of the form:

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{n}. \tag{2.1}$$

Here, $\mathbf{y}$ is the received signal vector, $\mathbf{s}$ is the transmitted signal vector, $\mathbf{n}$ is additive white Gaussian noise, and $\mathbf{H}$ is the channel matrix, representing the attenuation that affects the transmitted signal vector.

Eq. (2.1) covers the cases of a number of multiple access techniques, including but not limited to Code Division Multiple Access (CDMA), Orthogonal Frequency Division Multiple Access (OFDMA) and Multiple Input Multiple Output (MIMO). According to the technique and the channel model considered, the general form of the channel matrix $\mathbf{H}$ is determined.

One of the performance measures of a wireless communication system is called capacity (see Chapter 3). It was originally introduced by Shannon [Sha48]. Under some assumptions, the capacity for a single user of the system is given by the following expression.

$$C = \frac{1}{N} \log\det\left(\mathbf{I} + \frac{1}{\sigma^2}\mathbf{H}\mathbf{H}^H\right).$$

Given the properties of the logarithm,

$$\begin{aligned}
C &= \frac{1}{N}\sum_{i=1}^{N} \log\left(1 + \frac{1}{\sigma^2}\lambda_i\left(\mathbf{H}\mathbf{H}^H\right)\right) \\
&= \int \log\left(1 + \frac{1}{\sigma^2}\lambda\right) \frac{1}{N}\sum_{i=1}^{N} \delta\left(\lambda - \lambda_i\left(\mathbf{H}\mathbf{H}^H\right)\right) d\lambda \\
&= \int \log\left(1 + \frac{1}{\sigma^2}\lambda\right) F^{\mathbf{H}\mathbf{H}^H}(\lambda) d\lambda,
\end{aligned}$$

where $F^{\mathbf{H}\mathbf{H}^H}$ denotes the empirical eigenvalue distribution of $\mathbf{H}\mathbf{H}^H$.

Thus, as shown by the derivation above, the empirical eigenvalue distribution naturally appears in the expression of the capacity. Knowing the empirical eigenvalue distribution of a family of random matrices thus enables to get immediate insight on the performance of the corresponding communication system. In addition, even if the result is obtained in the asymptotic regime, when the dimensions of the matrix both tend to infinity with a fixed ratio, the results give very close approximations of finite-size system behavior, as shown by simulations.

There exists other performance measures that can be easily obtained as a function of the eigenvalues of the matrices involved. Thus, random matrix theory also enables to derive expressions for several other performance measures of interest, such as Signal to Interference plus Noise Ratio (SINR) or multiuser efficiency [VS99].

Unfortunately, the three laws plotted above are among the few known empirical eigenvalue distributions which have an explicit analytical expression. Generally, as in the subsequent case, the limit distributions are given by implicit equations, and can only be computed numerically.

2.2.3 Useful Results of Random Matrix Theory

Before giving an important theorem of random matrix theory, originally demonstrated by Girko [Gir90], and having since found use to derive spectral efficiency of CDMA systems under frequency selective fading [TLV05], we need some additional definitions.

Definition 4 *Let ν be a probability measure. The* Stieltjes transform *m^{ν} associated to ν is given by*

$$m^{\nu}(z) = \int \frac{1}{t-z}\nu(dt).$$

The Stieltjes transform is analytic for $\mathrm{Im}(z) > 0$. This is a one to one mapping. If the Stieltjes transform is known, the probability measure can be retrieved through the following inversion formula:

$$f(x) = \frac{d\nu}{dx}(x) = \frac{1}{\pi}\lim_{y\to 0^+} m^{\nu}(x+iy). \tag{2.2}$$

Definition 5 *Let $\mathbf{V}$ be a $N\times K$ random matrix with independent columns and entries v_{ij}. Denote by $\lfloor\cdot\rfloor$ the closest smaller integer. $\mathbf{V}$ is said to* behave ergodically *if, as $N, K\to\infty$ with $K/N\to\alpha$, for $x\in[0,1]$, the empirical distribution of*

$$\left[\left|v_{\lfloor xN\rfloor,1}\right|^2, \ldots, \left|v_{\lfloor xN\rfloor,K}\right|^2\right]$$

converges almost surely to a non-random limit distribution denoted $F_x^{\mathbf{V}}(\cdot)$ and, for $y\in[0,\alpha]$, the empirical distribution of

$$\left[\left|v_{1,\lfloor yN\rfloor}\right|^2, \ldots, \left|v_{N,\lfloor yN\rfloor}\right|^2\right]$$

converges almost surely to a non-random limit distribution denoted $F_y^{\mathbf{V}}(\cdot)$.

Definition 6 *Let $\mathbf{V}$ be a $N\times K$ random matrix that behaves ergodically as in Def. 5, such as $F_x^{\mathbf{V}}(\cdot)$ and $F_y^{\mathbf{V}}(\cdot)$ have all their moments bounded. The* two-dimensional channel profile *of $\mathbf{V}$ is the function $\rho^{\mathbf{V}}(x,y) : [0,1]\times[0,\alpha]\to\mathbb{R}$ such that, if the random variable X is uniformly distributed in $[0,1]$, then the distribution of $\rho^{\mathbf{V}}(X,y)$ equals $F_y^{\mathbf{V}}(\cdot)$ and, if the random variable Y is uniformly distributed in $[0,\alpha]$, then the distribution of $\rho^{\mathbf{V}}(x,Y)$ equals $F_x^{\mathbf{V}}(\cdot)$.*

Theorem 2 *Let $\mathbf{Y} = \mathbf{V}\odot\mathbf{W}$ be a $N\times K$ matrix, where $\odot$ is the Hadamard (element-wise) product and $\mathbf{V}$ and $\mathbf{W}$ are independent $N\times K$ random matrices. Assume that $\mathbf{V}$ behaves ergodically with channel profile $\rho^{\mathbf{V}}(x,y)$ as in Def. 6 and that $\mathbf{W}$ has i.i.d. entries with zero mean and variance $\frac{1}{N}$. Then, as $N, K\to\infty$ with $K/N\to\alpha$, the empirical eigenvalue distribution of $\mathbf{YY}^H$ converges almost surely to a non-random limit distribution function whose Stieltjes transform $m^{\mathbf{YY}^H}$ is given by:*

$$\begin{aligned} m^{\mathbf{YY}^H}(z) &= \lim_{N\to\infty}\frac{1}{N}\,\mathrm{Trace}\left(\left(\mathbf{YY}^H - z\mathbf{I}\right)^{-1}\right) \\ &= \int_0^1 u(x,z)dx \end{aligned}$$

and $u(x,z)$ satisfies the fixed point equation:

$$u(x,z) = \frac{1}{\int_0^\alpha \frac{\rho^{\mathbf{V}}(x,y)dy}{1+\int_0^1 \rho^{\mathbf{V}}(x',y)u(x',z)dx'} - z}. \tag{2.3}$$

The solution to equation (2.3) *exists and is unique in the class of functions $u(x,z) \geq 0$, analytic for $Im(z) > 0$, and continuous on $x \in [0,1]$.*

When considering a random matrix product such as in Th. 2, it looks tiresome to first get the Stieltjes transform from (2.3), and then retrieve the empirical eigenvalue distribution using the inversion formula (2.2). However, it is not necessary to do both steps. If we go back to wireless communications, taking the equation previously derived for the capacity

$$C = \int \log\left(1 + \frac{1}{\sigma^2}\lambda\right) F^{\mathbf{HH}^H}(\lambda)d\lambda,$$

and differentiating according to σ^2, we obtain

$$\begin{aligned}
\frac{\partial C}{\partial \sigma^2} &= \int \frac{-\frac{1}{\sigma^4}\lambda}{1+\frac{1}{\sigma^2}\lambda} F^{\mathbf{HH}^H}(\lambda)d\lambda \\
&= -\frac{1}{\sigma^2}\int \frac{\frac{1}{\sigma^2}\lambda + 1 - 1}{\frac{1}{\sigma^2}\lambda + 1} F^{\mathbf{HH}^H}(\lambda)d\lambda \\
&= -\frac{1}{\sigma^2} + \int \frac{1}{\lambda+\sigma^2} F^{\mathbf{HH}^H}(\lambda)d\lambda \\
&= -\frac{1}{\sigma^2} + m^{\mathbf{HH}^H}\left(-\sigma^2\right),
\end{aligned}$$

where $m^{\mathbf{HH}^H}$ is the Stieltjes transform of the empirical eigenvalue distribution of $\mathbf{HH}^H$.

Therefore, the capacity is directly expressed as a function of the Stieltjes transform of the empirical eigenvalue distribution of $\mathbf{HH}^H$. Finding the Stieltjes transform is often enough! Th. 2 is thus used in [TLV05] to derive formulas for the SINR and spectral efficiency of CDMA under frequency-selective fading.

2.2.4 Useful Results of Unitary Random Matrix Theory

Random matrix theory also provides results on unitary matrices [HP00]. A unitary matrix $\mathbf{V} = [v_{ik}]$ is a $N \times N$ matrix with complex entries such as $\mathbf{VV}^H = \mathbf{V}^H\mathbf{V} = \mathbf{I}$. Note that the entries are therefore dependent.

$$\begin{aligned}
&\sum_{k=1}^N |v_{ik}|^2 = \sum_{i=1}^N |v_{ik}|^2 = 1, \quad \text{for all } 1 \leq i,k \leq N, \\
&\sum_{l=1}^N v_{il}v_{lk}^* = 0, \quad \text{for all } i \neq k.
\end{aligned}$$

The following definition is given in [PR04]. Since the set $\mathcal{U}(N)$ of $N \times N$ unitary matrices forms a compact topological group with respect to the matrix multiplication and the usual topology, there exists a unique nonzero left and right invariant measure. It is known as the Haar measure. A unitary random matrix $\mathbf{V}$ is Haar distributed if it takes its values uniformly in $\mathcal{U}(N)$, i.e., if for any subset H of $\mathcal{U}(N)$, the probability that $\mathbf{V} \in H$ is equal to the normalized Haar measure μ of H:

$$\mathbb{P}\left(\mathbf{V} \in H\right) = \mu(H).$$

Given that the left invariance characterizes the Haar measure, to show that a unitary random matrix $\mathbf{V}$ is Haar distributed, it is sufficient to show that for any $\mathbf{U} \in \mathcal{U}(N)$, $\mathbf{UV}$ has the same distribution as $\mathbf{V}$. The Gram-Schmidt orthonormalization procedure can be used on the column vectors of a $N \times N$ Gaussian matrix with independent entries to obtain a Haar unitary matrix. If $\mathbf{X}$ is a Gaussian i.i.d. matrix, then $\mathbf{V} = \mathbf{X}\left(\mathbf{X}^H\mathbf{X}\right)^{-1/2}$ is Haar unitary [DHLdC03a]:

- $\mathbf{VV}^H = \mathbf{X}(\mathbf{X}^H\mathbf{X})^{-1/2}(\mathbf{X}^H\mathbf{X})^{-1/2}\mathbf{X}^H = \mathbf{I}$,
- $\mathbf{UV} = \mathbf{UX}(\mathbf{X}^H\mathbf{U}^H\mathbf{UX})^{-1/2}$ has the same distribution as $\mathbf{V}$.

Results are known on the moments of entries of Haar distributed random matrices. The entries of such a matrix $\mathbf{V}$ satisfy [HP00]:

$$\mathbb{E}\left[|v_{ik}|^2\right] = \frac{1}{N}, \quad \text{for all } 1 \leq i,k \leq N, \tag{2.4}$$

$$\mathbb{E}\left[|v_{ik}|^4\right] = \frac{2}{N(N+1)}, \quad \text{for all } 1 \leq i,k \leq N, \tag{2.5}$$

$$\mathbb{E}\left[|v_{ik}|^2 |v_{il}|^2\right] = \frac{1}{N(N+1)}, \quad \text{for all } l \neq k,\ 1 \leq i,k \leq N, \tag{2.6}$$

$$\mathbb{E}\left[{v_{ik}}^* v_{il} v_{jk} {v_{jl}}^*\right] = -\frac{1}{N(N^2-1)}, \quad \text{for all } k \neq l,\ i \neq j. \tag{2.7}$$

All other combinations of degree inferior or equal to 4 of elements of $\mathbf{V}$ have expectation equal to zero.

In actual CDMA systems, as far as orthogonal codes are concerned, Walsh-Hadamard codes are generally used. However, in order to get interpretable expressions of the SINR, isometric $N \times K$ matrices $[\mathbf{v}_1, \ldots, \mathbf{v}_K]$ obtained by extracting $K < N$ columns from a Haar unitary matrix $\mathbf{X}\left(\mathbf{X}^H\mathbf{X}\right)^{-\frac{1}{2}}$ will be considered. Although of limited practical use, these random matrices represent a very useful analytical tool as simulations [DHLdC03a] show that their use provides similar performances as Walsh-Hadamard codes. Those results are used in Chapter 4 in the downlink CDMA case, when orthogonal, rather than i.i.d. spreading codes are used.

Chapter 3

Wireless Networks at Large

In this part, the principles of Code Division Multiple Access (CDMA), ALOHA, and Orthogonal Frequency Division Multiple Access (OFDMA) are explained. Since CDMA is the most investigated multiuser scheme in this thesis, the model is most detailed for CDMA. References are included for OFDMA, which has been extensively treated in the litterature.

The models and notations used are introduced. Comments are given on the measures of performance like Signal to Interference plus Noise Ratio (SINR), capacity, spectral efficiency and goodput .

3.1 Channel Modeling

Suppose we transmit a signal over a wireless channel. Different forms of attenuation will affect the signal. In this section, I will concentrate on the Single-Input Single-Output (SISO) case, when there is a single antenna both at the transmitter and the receiver.

The wireless channel is usually highly volatile and changes over time. The signal is affected by several forms of attenuation. *Fading* is the effect due to the presence of scatterers between the transmit and the receive antenna. Scatterers are assumed to induce some attenuation, usually modeled as a random variable, and some delay in the signal propagation. One of the most frequent assumption is that it is possible to group scatterers with same propagation delay in L distinct clusters. In this case, the signal is said to propagate through a (discrete) multipath channel, described by the impulse response:

$$c(t,\tau) = \sum_{\ell=0}^{L-1} \eta_\ell(t)\delta(\tau - \tau_\ell(t)). \tag{3.1}$$

L is the number of paths. The attenuation $\eta_\ell(t)$ comes from the aggregate coefficients of the scatterer cluster ℓ and is a random variable. The delay associated with the ℓ-th multipath is $\tau_\ell(t)$. In the models, we will consider that transmission takes place over a short time interval, or with low mobility, and drop the dependency on the time index t for simplicity. If there is a single path, then the channel is *flat fading*: its effect on the signal does not vary with frequency. On the contrary, if

$L > 1$, the channel is *frequency selective*: signal transmitted at different frequencies will experience different attenuations.

Another non determistic effect witnessed during the propagation in a wireless channel is called *shadowing*, which is caused by trees, hills or buildings, i.e., objects that do not scatter the signal, but only attenuate its power. Usually, Rayleigh fading varies much more rapidly than shadowing. In the remainder of this thesis, the effect of shadowing will not be taken into account.

Finally, the propagation of any waveform through the air is affected by *path loss*, which is the deterministic attenuation of the signal with the distance. It is generally modeled in a polynomial form as proportional to $d^{-\beta}$, where d is the distance between the receive antenna and the transmit antenna, and β is the path loss factor, generally between 2 and 4.

3.2 Code Division Multiple Access

Code Division Multiple Acces (CDMA) is a multi-access technique enabling high rate wireless communications between the different nodes. A review of the state of CDMA was made fifteen years ago in [Lee91]. For a thorough introduction to CDMA, consult the book [Vit95].

Contrary to Time Division Multiple Access (TDMA) or Frequency Division Multiple Access (FDMA), CDMA is generally a non-orthogonal protocol. Users and base stations transmit at the same time and on the same frequency band. They make use of spreading codes to simultaneously transmit their information, so that the receiving end can discriminate between the interfering users. However, even when orthogonal codes are used, since frequency-selective fading destroys orthogonality, communications will suffer from multiple-access interference. Using channel coding at the transmitter and/or signal processing at the receiver, the signal of interest is then reconstructed.

CDMA is implemented in the UMTS standard. UMTS (Universal Mobile Telecommunications System) is one of the 3G mobile phone technologies, currently offered by operators in many continents.

3.2.1 CDMA Communication Model

Different models will be used in the subsequent parts of this thesis. They are uplink multi-cell and downlink multi-cell, uplink single-cell and downlink single-cell CDMA. Downlink may not be considered separately as it turns out to be a particular instance of uplink, where the signals intended for different users experience the same value of the fading when they reach the user under study [TLV05, Cot06]. The crucial similarities in the models are pointed out.

An important problem that arises in the design of CDMA systems concerns the deployment of an efficient architecture to cover the users. In a single-cell setting (without outside interference), the performances of CDMA have been thoroughly investigated by the information theory community, with no fading [TH99, VS99] or frequency-flat fading [ET00, SV01]. Extensions with a more involved attenuation model, frequency-selective fading, are derived in [TLV05].

However, when several cells are involved, the communications in a cell are perturbated by intra-cell (users within the cell) as well as inter-cell interference (users outside the cell). In order to get tractable expressions, previous studies of CDMA multi-access schemes were restricted to the study of a few interfering nodes, with Wyner's model [ZSV01] or with simple interference models [Lee91, GJP+91, TW94, SV97, CMV98, KM99, KA01, ZA01, LLA03, SZS04].

However, none has taken explicitly into account the cumulative effect from all interfering nodes with realistic path loss and fading models. Increasing the number of nodes in a given area yields indeed a better coverage but increases at the same time inter-node interference. The gain provided by a cellular network is not at all straightforward and depends on many parameters: path loss, type of codes used, receiving filter, channel characteristics.

Concerning the multi-cell scenario, without loss of generality and in order to ease the understanding, we focus our analysis on a one dimensional (1-D) network. This scenario represents for example the case of the deployment of base stations along a motorway (users, i.e., cars are supposed to move along the motorway). An infinite length base station deployment is considered (see Fig. 3.1). The base stations are supposed equidistant with inter-base station distance a. The spreading length N is fixed and is independent of the number of users. The number of users per cell is $K = da$ (d is the density of the network, with $d/N = \alpha$) and the load of each cell is $\frac{K}{N} = \alpha a$. Note that as the size of the cell increases, each cell accommodates more users (with the constraint $da \leq N$ if orthogonal codes are used).

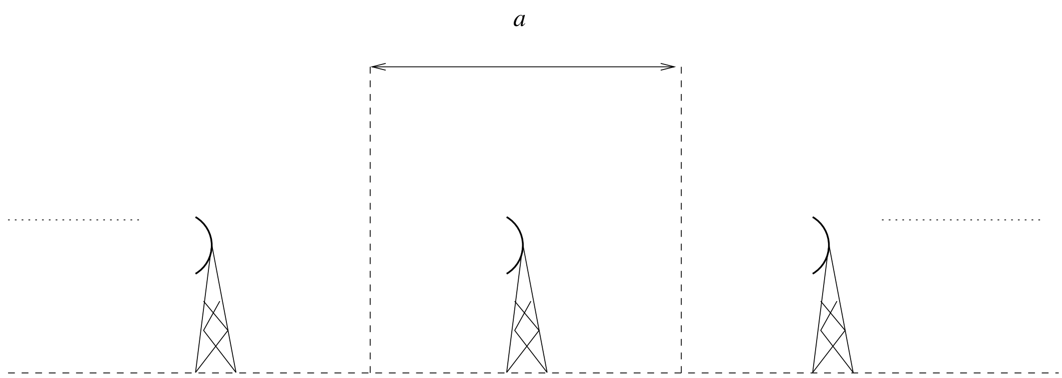

Figure 3.1: Representation of a CDMA Cellular Network

3.2.2 Uplink Multi-Cell

User k wants to transmit the signal s_{kn}, $n \in \mathbb{Z}$. The general case of wide-band CDMA is considered where the signal transmitted by user k has complex envelope

$$r_k(t) = \sum_n s_{kn} v_k(t - nT). \tag{3.2}$$

In (3.2), $v_k(t)$ is an weighted sum of elementary modulation pulses $\psi(t)$ which satisfy the Nyquist criterion with respect to the chip interval T_c ($T = NT_c$):

$$v_k(t) = \sum_{\ell=1}^{N} v_{k\ell}\psi(t - (\ell - 1)T_c).$$

In CDMA, all users transmit simultaneously, thus creating interference. The vector $\mathbf{v}_k$ with elements $v_{k\ell}$ is the spreading code of user k. The spreading code enables to mitigate the interference in the sum of transmitted signals $\sum_k r_k(t)$.

However, the signals are transmitted over the wireless channel, which attenuates the signal. We consider a frequency selective channel with impulse response $c_k(\tau)$. The base station will receive a convolution of the channel impulse reponse $c_k(\tau)$ with the sum of transmitted signals $\sum_k r_k(t)$. Without loss of generality, we consider the signal received at the base station located at the origin. Under the assumption of slowly-varying fading, the continuous time received signal $y(t)$ at the base station has the form:

$$\begin{aligned} y(t) = &\sum_n \sum_{k=1}^{K} s_{kn} \int \sqrt{P_k}\sqrt{P(x_k)}c_k(\tau)v_k(t - nT - \tau)d\tau \\ &+ \sum_n \sum_{k=K+1}^{\infty} s_{kn} \int \sqrt{P_k}\sqrt{P(x_k)}c_k(\tau)v_k(t - nT - \tau)d\tau + n(t). \end{aligned} \quad (3.3)$$

where $n(t)$ is the complex white Gaussian noise.

Any user k is determined by his position x_k, with respect to the considered base station, located at the origin.

In (3.3), $\sqrt{P_k}$ represents the power allocation of user k. $\sqrt{P(x_k)}$ represents the attenuation of the signal sent by user k due to potential path loss, i.e., deterministic attenuation, as opposed to (random) fading. We will suppose that the attenuation $P(x_k)$ can be represented as an even integrable function $P(x)$.

The signal (after pulse matched filtering by $\psi^*(-t)$) is sampled at the chip rate to get a discrete-time signal that has the form:

$$\mathbf{y} = \sum_{k=1}^{K} \mathbf{C}_k\mathbf{v}_k\sqrt{P_k}\sqrt{P(x_k)}s_k + \sum_{k=K+1}^{\infty} \mathbf{C}_k\mathbf{v}_k\sqrt{P_k}\sqrt{P(x_k)}s_k + \mathbf{n}. \quad (3.4)$$

$\sum_{k=1}^{K} \mathbf{C}_k\mathbf{v}_k\sqrt{P_k}\sqrt{P(x_k)}s_k$ is the useful signal, transmitted by users actually communicating with the receiving antenna. $\sum_{k=K+1}^{\infty} \mathbf{C}_k\mathbf{v}_k\sqrt{P_k}\sqrt{P(x_k)}s_k$ is the inter-cell interference.

$\mathbf{C}_k$ is a $N \times N$ Toeplitz matrix representing the frequency selective fading for the k-th user, $\mathbf{v}_k$ is a $N \times 1$ vector representing the spreading code of the k-th user, and $\mathbf{n}$ is an $N \times 1$ Additive White Gaussian Noise (AWGN) vector with covariance matrix $\sigma^2\mathbf{I}_N$.

We consider the case of a multipath channel similar to (3.1). Under the assumption that the number of paths from user k to the base station is given by L_k, the model of the channel is given by

$$c_k(\tau) = \sum_{\ell=0}^{L_k-1} \eta_{k\ell}\psi(\tau - \tau_{k\ell}), \quad (3.5)$$

where we assume that the channel is invariant during the time considered. In order to compare channels at the same signal to noise ratio, we constrain the distribution of the i.i.d. fading coefficients $\eta_{k\ell}$ such as:

$$\mathbb{E}\left[\eta_{k\ell}\right] = 0 \text{ and } \mathbb{E}\left[|\eta_{k\ell}|^2\right] = \frac{\varrho}{L_k}. \tag{3.6}$$

Usually, fading coefficients $\eta_{k\ell}$ are supposed to be independent with decreasing variance as the delay increases. In all cases, ϱ is the average power of the channel, such as $\mathbb{E}\left[|c_k(\tau)|^2\right] = \sum_{\ell=0}^{L_k-1} \mathbb{E}\left[|\eta_{k\ell}|^2\right] = \varrho$, for all channels considered. For the fading process $c_k(\tau)$, the frequency response of the channel at the receiver is given by:

$$h_k(f) = \sum_{\ell=0}^{L_k-1} \eta_{k\ell} e^{-j2\pi f \tau_{k\ell}} \left|\Psi(f)\right|^2. \tag{3.7}$$

where we assume that the transmit filter $\Psi(f)$ and the receive filter $\Psi^*(-f)$ are such that, given the bandwidth W,

$$\Psi(f) = \begin{cases} 1 & \text{if } -\frac{W}{2} \leq f \leq \frac{W}{2} \\ 0 & \text{otherwise.} \end{cases} \tag{3.8}$$

Sampling at the various frequencies $f_1 = -\frac{W}{2}$, $f_2 = -\frac{W}{2} + \frac{1}{N}W$, $\ldots$, $f_N = -\frac{W}{2} + \frac{N-1}{N}W$, we obtain the coefficients h_{ik}, $1 \leq i \leq N$, as

$$h_{ik} = h_k(f_i) = \sum_{\ell=0}^{L_k-1} \eta_{k\ell} e^{-j2\pi \frac{i}{N} W \tau_{k\ell}} e^{j\pi W \tau_{k\ell}}. \tag{3.9}$$

Note that $\mathbb{E}\left[|h_{ik}|^2\right] = \varrho$.

Since the users are supposed to be synchronized with the base stations and for sake of simplicity, we will consider in all the following that users add a cyclic prefix of length equal to the channel impulse response length to their code sequence.[1] This case is similar to uplink MC-CDMA [FP93, Lin96]. As a consequence, matrices $\{\mathbf{C}_k\}$ are circulant [Bin90] and can all be diagonalized in the Fourier basis $\mathbf{F}$ [Gra06]. Model (3.4) simplifies therefore to:

$$\mathbf{y} = \sum_{k=1}^{K} \mathbf{F}\mathbf{H}_k\mathbf{F}^H\mathbf{v}_k\sqrt{P_k}\sqrt{P(x_k)}s_k + \sum_{k=K+1}^{\infty} \mathbf{F}\mathbf{H}_k\mathbf{F}^H\mathbf{v}_k\sqrt{P_k}\sqrt{P(x_k)}s_k + \mathbf{n}. \tag{3.10}$$

where $\mathbf{H}_k$ is a diagonal matrix with diagonal elements $\{h_{ik}\}_{i=1\ldots N}$. For each user k, the coefficients h_{ik} are the discrete Fourier transform of the channel impulse response.

In order to simplify further (3.10), we make assumptions on the codes that the users employ. The codes can be i.i.d. random or orthogonal. In the i.i.d. case, we assume that users employ Gaussian i.i.d. codes with zero mean and variance $1/N$

[1] Note that in the asymptotic case (when $N \to \infty$), the result holds without the need of a cyclic prefix as long as the channel is absolutely summable [Gra06].

[ET00]. In the orthogonal case, we assume that, for each cell, the codes are columns extracted from a Haar distributed unitary matrix (see Sec. 2.2.4). Note that each cell uses a different isometric code matrix.

These assumptions enable us to state simply our results, however almost all of the results are valid for any distribution of the codes as long as it has mean zero and variance $1/N$. Namely, we make use of the fact that every unitary transformation of a Gaussian i.i.d. vector is a Gaussian i.i.d. vector, and every unitary transformation of a Haar distributed unitary matrix has the same distribution as the initial matrix. In particular, since $\mathbf{F}$ is unitary, $\mathbf{F}^H\mathbf{n}$ and $\mathbf{w}_k = \mathbf{F}\mathbf{v}_k$ have respectively the same distribution as $\mathbf{n}$ and $\mathbf{v}_k$ for all k. We multiply $\mathbf{y}$ in (3.10) with $\mathbf{F}^H$ and obtain without any change in the statistics:

$$\mathbf{y} = \sum_{k=1}^{K} \mathbf{H}_k \mathbf{w}_k \sqrt{P_k}\sqrt{P(x_k)} s_k + \sum_{k=K+1}^{\infty} \mathbf{H}_k \mathbf{w}_k \sqrt{P_k}\sqrt{P(x_k)} s_k + \mathbf{n}. \tag{3.11}$$

Writing (3.11) in a more compact form, we finally obtain

$$\mathbf{y} = \left(\mathbf{H}\sqrt{\mathbf{P}} \odot \mathbf{W}\right)\mathbf{s} + \sum_{l=1}^{\infty}\left(\mathbf{H}_l\sqrt{\mathbf{P}_l} \odot \mathbf{W}_l\right)\mathbf{s}_l + \mathbf{n} \tag{3.12}$$

where $\odot$ is the Hadamard (element-wise) product.

In (3.12), $\mathbf{H}$ (for the considered cell) and $\mathbf{H}_l$, $l \geq 1$, (corresponding to each interfering cell) are the frequency selective fading matrices, of size $N \times K$:

$$\mathbf{H} = \begin{bmatrix} h_{11} & h_{12} & \dots & h_{1K} \\ \vdots & \vdots & & \vdots \\ h_{N1} & h_{N2} & \dots & h_{NK} \end{bmatrix}$$

$$\mathbf{H}_l = \begin{bmatrix} h_{1(lK+1)} & h_{1(lK+2)} & \dots & h_{1((l+1)K)} \\ \vdots & \vdots & & \vdots \\ h_{N(lK+1)} & h_{N(lK+2)} & \dots & h_{N((l+1)K)} \end{bmatrix}.$$

$\sqrt{\mathbf{P}}$ (for the considered cell) and $\sqrt{\mathbf{P}_l}$, $l \geq 1$, (corresponding to each interfering cell) summarize the power allocation and power attenuation in diagonal matrix form, of size $K \times K$:

$$\sqrt{\mathbf{P}} = \begin{bmatrix} \sqrt{P_1}\sqrt{P(x_1)} & & \\ & \ddots & \\ & & \sqrt{P_K}\sqrt{P(x_K)} \end{bmatrix}$$

$$\sqrt{\mathbf{P}_l} = \begin{bmatrix} \sqrt{P_{lK+1}}\sqrt{P(x_{lK+1})} & & \\ & \ddots & \\ & & \sqrt{P_{(l+1)K}}\sqrt{P(x_{(l+1)K})} \end{bmatrix}.$$

$\mathbf{W}$ and $\mathbf{W}_l$ are $N \times K$ random spreading matrices, with either columns of Gaussian i.i.d. entries $\sim \mathcal{N}\left(0, \frac{1}{N}\right)$, or orthogonal columns extracted from a Haar distributed unitary matrix:

$$\mathbf{W} = \left[\mathbf{w}_1 | \mathbf{w}_2 | \cdots | \mathbf{w}_K\right]$$

$$\mathbf{W}_l = \left[\mathbf{w}_{lK+1} | \mathbf{w}_{lK+2} | \cdots | \mathbf{w}_{(l+1)K}\right].$$

Note that asymptotically (as $N \to \infty$), for a given multipath channel of length L, model (3.12) is also valid for the case of uplink DS-CDMA since all Toeplitz matrices can be asymptotically diagonalized in a Fourier Basis [Gra06, Hac04].

In the following, we will assume that the frequency selective fading matrices $\mathbf{H}_l$ behave ergodically, as in Def. 5. The two-dimensional channel profile of $\mathbf{H}_l\sqrt{\mathbf{P}_l}$ is denoted $\rho(f,x) = P(x)\left|h(f,x)\right|^2$, $f \in [0,1]$, $x \in [\alpha(la-a/2), \alpha(la+a/2)]$. f is the frequency index and x is the user index. In the notations of (3.7), if $\frac{k}{N} \leq x < \frac{k+1}{N}$ is the index of user k, this gives

$$h(f,x) = \sum_{\ell=0}^{L_k-1} \eta_{k\ell} e^{-j2\pi W \tau_{k\ell} f} e^{j\pi W \tau_{k\ell}}. \tag{3.13}$$

This enables us to use Th. 2 in order to obtain expressions for the SINR.

In the case of flat fading, $h_{ik} = h_k$ for all i. Denoting $\mathbf{D}$ the diagonal matrix with diagonal elements $h_1, \ldots, h_K$, we obtain the following equality:

$$\mathbf{H}\sqrt{\mathbf{P}} \odot \mathbf{W} = \begin{bmatrix} 1 & \ldots & 1 \\ \vdots & \ddots & \vdots \\ 1 & \ldots & 1 \end{bmatrix} \mathbf{D}\sqrt{\mathbf{P}} \odot \mathbf{W} = \mathbf{W}\mathbf{D}\sqrt{\mathbf{P}}.$$

Injecting this in (3.11), the received signal expression reduces to:

$$\mathbf{y} = \sum_{k=1}^{K} \mathbf{w}_k h_k \sqrt{P(x_k)} s_k + \sum_{k=K+1}^{\infty} \mathbf{w}_k h_k \sqrt{P(x_k)} s_k + \mathbf{n}. \tag{3.14}$$

This case is treated in [BDAC05a].

3.2.3 Downlink Multi-Cell

For the downlink case, the multi-cell model is a particular instance of the uplink case, due to the fact that each base station signal will experience the same fading realization. Since I devote a part of this thesis to a particular instance of downlink optimization, I detail completely this model as well, in order to introduce the notations, that differentiate between cells rather than between users, and point out the similarities.

The general case of downlink wide-band CDMA is considered where the signal intended to be transmitted by the base station in cell p to user j has complex envelope

$$x_{pj}(t) = \sum_n s_{pj}(n) v_{pj}(t - nT). \tag{3.15}$$

In (3.15), $v_{pj}(t)$ is a weighted sum of elementary modulation pulses $\psi(t)$ which satisfy the Nyquist criterion with respect to the chip interval T_c ($T = NT_c$):

$$v_{pj}(t) = \sum_{\ell=1}^{N} v_{pj\ell}\psi(t - (\ell-1)T_c).$$

The vector $\mathbf{v}_{pj}$ with elements $v_{pj\ell}$ is the spreading code that the base station in cell p uses to communicate with user j. The signals are transmitted over frequency selective channels with impulse response $c_{pj}(\tau)$. Under the assumption of slowly-varying fading, the continuous time signal $y_{pj}(t)$ received by user j in cell p has the form:

$$y_{pj}(t) = \sum_q \sum_n \sum_{k=1}^{K} s_{qk}(n) \int \sqrt{P_{qk}} \sqrt{P_q(x_j)} c_{qj}(\tau) v_{qj}(t - nT - \tau) d\tau + n(t). \quad (3.16)$$

where $n(t)$ is the complex white Gaussian noise.

In (3.16), the index q stands for the cell, the index n for the transmitted symbol and the index k for the user (in each cell, there are K users). User j is determined by his position x_j. The signal (after pulse matched filtering by $\psi^*(-t)$) is sampled at the chip rate to get a discrete-time received signal of user j in cell p of a downlink CDMA system that has the form:

$$\mathbf{y}_p(x_j) = \sum_q \sqrt{P_q(x_j)} \mathbf{C}_{qj} \mathbf{V}_q \mathbf{s}_q + \mathbf{n}. \quad (3.17)$$

We can obtain directly (3.17) from (3.4) by denoting the $K \times 1$ transmit vector of cell q for its K users as $\mathbf{s}_q = [\sqrt{P_{q1}} s_q(1), \ldots, \sqrt{P_{qK}} s_q(K)]^T$ and assuming the fading and path loss realizations from base station q to user j are identical for all those signals. $\sqrt{P_{qk}}$ represents the power control attributed to the signal destined to user k by base station q. $P_q(x_j)$ represents the path loss between base station q and user j whereas $\mathbf{C}_{qj}$ represents the $N \times N$ Toeplitz structured frequency selective channel matrix between base station q and user j. $\mathbf{n} = [n(1), \ldots, n(N)]^T$ is an $N \times 1$ noise vector with i.i.d. Gaussian entries with zero mean and variance σ^2. Each base station has an isometric $N \times K$ code matrix $\mathbf{V}_q = [\mathbf{v}_{q1}, \ldots, \mathbf{v}_{qK}]$. $\mathbf{V}_q$ is assumed to be obtained by extracting K columns from a Haar distributed matrix, as discussed in Sec. 2.2.4. Note that each cell uses a different isometric code matrix. User j is subject to intra-cell interference from other users of cell p as well as inter-cell interference from all the other cells.

Notice that the assumption on the code structure model enables us to simplify model (3.17) as previously. Matrices $\{\mathbf{C}_{qj}\}$ can all be diagonalized in the Fourier basis $\mathbf{F}$ [Gra06]. As a consequence, model (3.17) is equivalent to:

$$\mathbf{y}_p(x_j) = \sum_q \sqrt{P_q(x_j)} \mathbf{H}_{qj} \mathbf{W}_q \mathbf{s}_q + \mathbf{n}. \quad (3.18)$$

where $\mathbf{H}_{qj}$ is a diagonal matrix with the Discrete Fourier Transform of the channel impulse response $h_{qj}(i)$ given by (3.9) as diagonal elements [Gra06].

The general path loss $P_q(x_j)$ depends on a path loss factor which characterizes the type of attenuation. The greater the factor, the more severe the attenuation. In the downlink case, we will derive expressions for an exponential path loss $P_q(x_j) = Pe^{-\gamma|x_j - m_q|}$ [FBS04], where m_q are the coordinates of base station q. Note that in the usual model, the attenuation is generally of the polynomial form: $P_q(x_j) = \frac{P}{(|x_j - m_q|)^\beta}$. We use the exponential form for the sake of calculation simplicity and therefore put the framework in the most severe path loss scenario in favor of the multi-cell approach.

3.2.4 Uplink Single-Cell

We consider a single uplink multi-user system cell, i.e., inter-cell interference free case. The model for single-cell uplink is essentially the same as for multi-cell uplink without the inter-cell interference term. The spreading length is denoted N. The number of users in the cell is K. When considering a single cell, we do not take the size of the cell into account. For example, as far as orthogonal codes are concerned, path loss does not affect orthogonality and can be neglected. Therefore, the size of the cell is not an relevant parameter. The load is simply $\alpha = K/N$.

Under the same assumptions, and with the similar notations, we obtain equations similar to (3.11) and (3.12).

$$\mathbf{y} = \sum_{k=1}^{K} \mathbf{H}_k \mathbf{w}_k \sqrt{P_k} s_k + \mathbf{n} \tag{3.19}$$

$$\mathbf{y} = \big(\mathbf{H}\sqrt{\mathbf{P}} \odot \mathbf{W}\big)\mathbf{s} + \mathbf{n}. \tag{3.20}$$

Model (3.20) is essentially similar as the one studied in [TLV05].

3.2.5 Downlink Single-Cell

A single downlink multi-user system cell is inter-cell interference free. The model for single-cell uplink is essentially the same as for multi-cell uplink without the inter-cell interference term. As pointed out in [TLV05], it is a particular case of uplink single-cell. It is treated as such extensively in [DHLdC03a, TLV05].

3.3 ALOHA

ALOHA is one of the simplest access schemes for users sharing a common medium. Unslotted ALOHA was introduced by [Abr70]. In the context of a discrete time system, the slotted ALOHA protocol [Rob72] enables to enhance throughput, under the constraint that users are synchronized.

3.3.1 ALOHA Communication Model

Users simply transmit their packets and wait for an acknowledgement.

The time required to transmit a packet is one unit. In unslotted ALOHA [Abr70], if a packet is transmitted at time t, then any other transmission during the so-called vulnerable period $[t-1, t+1]$ will cause a collision. In slotted ALOHA [Rob72], if two (or more) packets are transmitted during the same slot, this will create a collision. If there is a collision between packets, all the packets are assumed to be lost. Users then wait a random amount of time (or a random amount of time slots in slotted ALOHA) before retransmitting. The waiting time is random in order to prevent repeated collisions.

When users transmit at different powers, the capture phenomenon may enable a single higher power transmission to be correctly decoded [Rob72, LKZ98].

Note that propagation is not taken into account, only collisions between concurrent transmissions are, in a control access protocol setting. This model is used in Secs. 5.1.1 and 5.1.2.

3.3.2 ALOHA Performance

ALOHA is typically a user centric protocol. There is no central controller involved and in particular no collision control mechanism. Improvements of ALOHA involving collision control have been introduced such as CSMA, IEEE 802.11, etc.

Several previous papers have already studied ALOHA or slotted ALOHA in a non-cooperative context. The papers [MW01b, JK02, AAJ03, MW03, ABAJ04] have studied ALOHA for a non-cooperative choice of transmission probabilities. Several papers study slotted ALOHA with power diversities in the context of the cooperative formulation [Met76, LKZ98, SHH02]. In [ABBA05] the authors have studied the performance of slotted ALOHA in a non-cooperative context, modeled as a game, in which both retransmission probabilities as well as power levels are chosen by the players. A Markov chain formulation has been obtained, whose numerical solutions enable to study the system performance.

3.4 Orthogonal Frequency Division Multiple Access

A N carrier Orthogonal Frequency Division Multiplexing (OFDM) system [Bin90] using a cyclic prefix or zero-padding [Muq01] for preventing inter-block interference is known to be equivalent in the Frequency Domain to N flat fading parallel transmission channels.

Orthogonal Frequency Division Multiple Access (OFDMA) is the extension to multiple users of the OFDM digital modulation scheme. Multiple access is achieved in OFDMA by assigning subsets of subcarriers to individual users. This allows simultaneous low data rate transmission from several users. This emerging multiple access technology is already in use in WiMAX.

OFDMA is an alternative to combining OFDM with time division multiple access (TDMA). Users requesting low rates can send continuously with low transmission power instead of punctually using a single high-power carrier. Constant delay, and shorter delay, can be achieved.

OFDMA can also be described as a combination of frequency division and time division multiple access, where the resources are partitioned in the time-frequency space, and slots are assigned along the OFDM symbol index as well as OFDM subcarrier index.

3.4.1 OFDMA Communication Model

We consider the uplink of a single cell network. K users are simultaneously communicating with a base station using OFDM modulation over N carriers.

On each carrier i, user k sends the information $s_k(i)$. $s_k(i)$ is the transmitted data such as $\mathbb{E}\left[|s_k(i)|^2\right] = 1$. A set $\mathbb{M}_i \subseteq \{1, \ldots, K\}$ of users can select the same frequency carrier i, which introduces interference. As a consequence, the received signal on carrier i at the base station is given by:

$$y(i) = \sum_{k \in \mathbb{M}_i} h_k(i)\sqrt{P_k(i)}s_k(i) + n(i) \tag{3.21}$$

where $P_k(i)$ is the power control and $n(i)$ is additive white Gaussian noise.

3.4.2 OFDMA Performance

In order to optimize the users' rate, scheduling of the users is required. An efficient scheduling algorithm is based on multi-user diversity schemes [KH95]. The algorithm requires an estimation by the scheduler (usually the base station) of the N carriers of the K users. Based on this feedback information about the channel conditions, adaptive user-to-subcarrier assignment can be achieved. Only users with the highest carrier-to-noise ratio (CNR) are allowed to transmit. An interesting property of OFDMA is that such an assignment maintains fairness among users [WL04]. If the assignment is done sufficiently fast, this scheme further improves the OFDM robustness to fast fading and narrow-band cochannel interference, and enables to increase the system spectral efficiency. In order to control the data rate and error probability individually for each user, a different number of subcarriers can be assigned to each user.

One drawback of OFDMA is that the scheduler needs complete channel state information, which creates many non-informational transmissions. To reduce the feedback load, selective multiuser diversity algorithms have been introduced: only the users that have a CNR above a threshold send feedback to the scheduler [GA04]. Multiple feedback thresholds can be used [HAGØ05] and are generally found numerically. Another drawback is that, for high mobility, channel conditions vary in a fast manner and the algorithm becomes inaccurate.

As far as power control is concerned, OFDMA presents challenges as the optimal policies perform adaptive joint allocation of subcarriers and powers. Several algorithms have been designed in the single-cell case [WCLM99, JL03, HJL05]. In the context of fixed wireless networks, when inter-cell interference is present, [KAS05] introduces a subcarrier and bit loading algorithm that satisfies users requested rates. A centralized version of the algorithm is introduced as a constrained optimization problem. A distributed version of the algorithm is also presented, when users iteratively adjust their choice of subcarriers and power allocation. However, even in the case when the set of requested rates can be achieved, the convergence of the distributed scheme is not guaranteed.

Since subcarriers are narrow in bandwidth, the fading coefficients on adjacent subcarriers are correlated. A way to alleviate this correlation is to group adjacent carriers in clusters, that can be assumed to experience independant fading values (see [BZ06] and references therein). In [BZ06], a distributed scheme for resource allocation is described, for which each user needs to know only local information about the system to transmit at its requested rate, whenever it is possible. The success of transmissions simply has to be acknowledged by the base station. It is shown that this scheme performs almost as well as an equivalent centralized scheme.

3.5 Performance Measures

3.5.1 Signal to Interference plus Noise Ratio

At the receiver, the received signal $\mathbf{y}$ will be processed in order to retrieve the original symbols. The optimal detectors usually demand a prohibitive calculation complexity with the increasing number of users. However, simpler, almost optimal, detectors can be implemented by linear filters. A linear filter for user k is a vector $\mathbf{g}_k$ that enables to obtain an estimate of the original symbol. For example, starting from uplink single-cell model (3.20), after filtering,

$$\mathbf{g}_k^H \mathbf{y} = \mathbf{g}_k^H (\mathbf{H}\sqrt{\mathbf{P}} \odot \mathbf{W})\mathbf{s} + \mathbf{g}_k^H \mathbf{n}. \tag{3.22}$$

Denote $\mathbf{h}_k$ the k-th column of $\mathbf{H}$, and $\mathbf{H}_{(-k)}$ the matrix $\mathbf{H}$ with $\mathbf{h}_k$ removed, and similarly for the code matrix $\mathbf{W}$. From (3.22), the *Signal to Interference plus Noise Ratio* (SINR) for user k is immediately deduced.

$$\mathrm{SINR}_k = \frac{P_k \mathbf{g}_k^H (\mathbf{h}_k \odot \mathbf{w}_k)(\mathbf{h}_k \odot \mathbf{w}_k)^H \mathbf{g}_k}{\sigma^2 \mathbf{g}_k^H \mathbf{g}_k + \mathbf{g}_k^H (\mathbf{H}_{(-k)}\sqrt{\mathbf{P}_{(-k)}} \odot \mathbf{W}_{(-k)})(\mathbf{H}_{(-k)}\sqrt{\mathbf{P}_{(-k)}} \odot \mathbf{W}_{(-k)})^H \mathbf{g}_k}.$$

The simplest linear filter is the Matched Filter $\mathbf{g}_k = \sqrt{P_k}(\mathbf{h}_k \odot \mathbf{w}_k)$, which simply takes into account only the code of the user of interest. The minimum mean square error (MMSE) Filter is the filter that minimizes the mean square error $\mathbb{E}\left[\left|s_k - \mathbf{g}_k^H \mathbf{y}\right|\right]$. It can be proven that it is given by the following expression.

$$\mathbf{g}_k = \left(\sigma^2 \mathbf{I}_N + (\mathbf{H}\sqrt{\mathbf{P}} \odot \mathbf{W})(\mathbf{H}\sqrt{\mathbf{P}} \odot \mathbf{W})^H\right)^{-1} (\mathbf{h}_k \odot \mathbf{w}_k).$$

Remark that since $\mathbf{H}$ and $\mathbf{W}$ are random matrices, the SINR itself is generally a random variable.

3.5.2 Capacity

In his seminal paper [Sha48], Shannon introduces a quantity called *capacity*. This performance measure enables to quantify the number of bits of information per time and frequency unit the system is able to deliver. The capacity of a channel is the maximal quantity of information that can be transmitted through this channel.

In a multi-user communication system, it has been shown [GVR02] that the interference plus noise for randomly spread systems can be considered as Gaussian when K and N are large enough. When the SINR of user k is defined, the capacity of this user is given by:

$$C_k = \log_2(1 + \mathrm{SINR}_k).$$

The capacity of the channel is the sum of individual capacities

$$C = \sum_{k=1}^{K} \log_2(1 + \mathrm{SINR}_k).$$

Hence, the capacity of the channel is also generally a random variable.

Two cases can occur:

- If the channel is ergodic, the capacity can be averaged over the channel realizations to obtain the *ergodic capacity*

$$\bar{C} = \mathbb{E}\left[\sum_{k=1}^{K} \log_2(1 + \mathrm{SINR}_k)\right].$$

- If the channel is static, what counts is the minimal rate at which users will be able to transmit at least $(1-q)\%$ of the time, called *outage capacity*

$$C_0 = \arg\max_R \left\{R \geq 0 \mid \mathbb{P}\left(C < R\right) \leq q\right\}.$$

  For example, if $q = 0.01$, users will be able to transmit at least 99% of the time at rate C_0.

3.5.3 Spectral Efficiency

In the context of a multi-cell network, capacity is specialized in a quantity called *spectral efficiency*, which quantifies the number of bits/s/Hz the system is able to deliver to all the users given a certain inter-cell distance. In the infinite multi-cell setting detailed above, due to invariance by translation, the spectral efficiency per cell is the same for all cells. Since the network capacity is infinite, the measure of performance in this case is the number of bits per second per hertz per meter (bits/s/Hz/m) the system is able to deliver defined by:

$$C = \frac{1}{FNTa} I(\mathbf{s}, \mathbf{y}). \tag{3.23}$$

where F is the frequency reuse (meaning F adjacent cells use F different frequencies), T is the chip time (set to 1 in the rest of the report) and $I(\mathbf{s}, \mathbf{y})$ is the mutual information between the received signal and the transmitted signal for a given receiver structure. The network capacity is a linear scaling factor of C.

For linear detectors such as Matched Filter or MMSE Filter, a SINR can be defined, the spectral efficiency is simply written:

$$C = \frac{1}{FNa} \sum_{k=1}^{K} \log_2(1 + \mathrm{SINR}_k). \tag{3.24}$$

In general, the SINR is a random variable, and we will consider the mean spectral efficiency, averaged over the fading distribution.

3.5.4 Goodput

Another performance measure that is frequently used, notably in power allocation games, is the *goodput*, or rather an adapted version of it [GM00].

The wireless system transmits packets with L informational bits. Those bits are coded, so that $M > L$ bits are effectively transmitted. The transmission rate is R bits/s. If the SINR of user k is β_k, the probability of correct reception is $q(\beta_k)$, where $q(\cdot)$ is a function that depends on the details of the modulation, coding, interleaving, wireless channel, etc. Hence, the number of transmissions required to receive a packet correctly is a random variable. The goodput can be expressed as

$$\gamma(\beta_k) = R\frac{L}{M}q(\beta_k). \tag{3.25}$$

The probability of correct reception is generally expressed as $q(\beta_k) = (1 - \mathrm{BER}_k)^M$, where BER is the bit error rate. For example, in the case of transmission with binary phase-shift-keying modulation with no channel coding, $q(\beta_k) = \left(1 - Q\left(\sqrt{2\beta_k}\right)\right)^M$, where $Q(\cdot)$ is the normal distribution function.

The goodput is a customary performance in power allocation games. In the game theoretical framework, the users are considered as players in a game, their choice of transmit powers represent their strategies. The utility measures the gain of a user as a result of the strategy this user plays. In [Rod03], the author derives what he calls Throughput to Power Ratio (TPR) under minimal requirements. The utility of user k is expressed as

$$u_k = \frac{\gamma(\beta_k)}{P_k}. \tag{3.26}$$

The utility is expressed in bit/Joule, so it represents the amount of successful transmissions obtained from every Joule of energy spent by the user. This is a relevant performance measure, as each mobile wants to use its (limited) battery power to transmit the maximum possible amount of information.

In (3.26), the function $\gamma(\cdot)$ is at least C^2 and should satisfy conditions detailed in [Rod03] in order to obtain an "interesting" equilibrium. However, the goodput $\gamma(\beta_k)$, which is proportional to $q(\beta_k) = (1 - \mathrm{BER}_k)^M$, does not satisfy the conditions. Namely, this quantity is not zero when the transmitted power is zero. Using this function in the utility would lead to the unsatisfying conclusion that mobiles should not transmit at all, since the (improbable) event of a correct guess gives them infinite utility.

Therefore, it is customary to consider an adapted version of the goodput. An *efficiency function* $f(\beta_k)$ is introduced [GM00], which should mimic closely the behavior of $q(\beta_k)$ while satisfying the desirable property $f(0) = 0$. A usual way to achieve this is simply to add a factor 2 in front of the BER, $f(\beta_k) = (1 - 2\mathrm{BER}_k)^M$. More specifically, the function $f(\beta_k) = (1 - \exp(-\beta_k))$ is widely used as such an efficiency function [GM00, MPSM05].

Summing up, from (3.25) and (3.26), the following expression is used for the utility in the setting of power allocation games:

$$u_k = R\frac{L}{M}\frac{f(\beta_k)}{P_k}.$$

Chapter 4

Network Centric Communications

4.1 Downlink Multi-Cell Orthogonal CDMA

In this section, the downlink multi-cell model detailed in Chapter 3 is used. It is a first step into analyzing the complex problem of downlink CDMA multi-cell networks, using a new approach based on unitary random matrix theory. Asymptotic results for the performance of a downlink CDMA system with orthogonal spreading and multi-cell interference have been derived when considering a finite dense network [Deb04]. Our contribution is in extending this analysis to an infinite network [BDA06].

The purpose is to determine, for a dense and infinite multi-cell network, the optimal distance between base stations. A downlink frequency selective fading CDMA scheme where each user is equipped with a linear matched filter is considered. The users are assumed to be uniformly distributed along the area. Only orthogonal access codes are considered as the users are synchronized within each cell. The problem is analyzed in the asymptotic regime: very dense networks are considered where the spreading length N tends to infinity, the number of users per meter d tends to infinity but the load per meter $\frac{d}{N} = \alpha$ is constant. The analysis is mainly based on asymptotic results of unitary random matrices [HP00, PR04].

4.1.1 Model

In all the following, without loss of generality, we will focus on user j of cell p. The received signal of this user is given by (3.18):

$$\mathbf{y}_p(x_j) = \sum_q \sqrt{P_q(x_j)}\mathbf{H}_{qj}\mathbf{W}_q\mathbf{s}_q + \mathbf{n}.$$

4.1.2 Performance Analysis

General SINR formula

We assume that the users do not know the codes of the other cells as well as the codes of other users within the same cell. As a consequence, user j of cell p is equipped with the matched filter receiver $\mathbf{g}_{pj}$ defined by $\mathbf{g}_{pj} = \mathbf{H}_{pj}\mathbf{w}_{pj}$.

The output of the matched filter is given by:

$$\begin{aligned}\mathbf{g}_{pj}^H \mathbf{y}_p(x_j) &= \sqrt{P_p(x_j)}\mathbf{g}_{pj}^H \mathbf{H}_{pj}\mathbf{w}_{pj}s_p(j) \\ &\quad + \sqrt{P_p(x_j)}\mathbf{g}_{pj}^H \mathbf{H}_{pj}\mathbf{W}_p^{(-j)} \begin{bmatrix} s_p(1) \\ \vdots \\ s_p(K) \end{bmatrix}_{(K-1)\times 1} \\ &\quad + \sum_{q\neq p}\sqrt{P_q(x_j)}\mathbf{g}_{pj}^H \mathbf{H}_{qj}\mathbf{W}_q\mathbf{s}_q + \mathbf{g}_{pj}^H\mathbf{n}. \end{aligned} \tag{4.1}$$

where $\mathbf{W}_p^{(-j)} = [\mathbf{w}_{p1}, \ldots, \mathbf{w}_{p(j-1)}, \mathbf{w}_{p(j+1)}, \ldots, \mathbf{w}_{pK}]$. From (4.1), we obtain the expression for the output SINR of user j in cell p with coordinates x_j and code $\mathbf{w}_{pj}$:

$$\mathrm{SINR}(x_j, \mathbf{w}_{pj}) = \frac{S^*(x_j)}{I_1(x_j) + I_2(x_j) + \sigma^2 \mathbf{w}_{pj}^H \mathbf{H}_{pj}^H \mathbf{H}_{pj}\mathbf{w}_{pj}}. \tag{4.2}$$

where

$$S^*(x_j) = P_p(x_j)\left|\mathbf{w}_{pj}^H \mathbf{H}_{pj}^H \mathbf{H}_{pj}\mathbf{w}_{pj}\right|^2. \tag{4.3}$$

$$I_1(x_j) = \sum_{q\neq p} P_q(x_j)\mathbf{w}_{pj}^H \mathbf{H}_{pj}^H \mathbf{H}_{qj}\mathbf{W}_q \mathbf{W}_q^H \mathbf{H}_{qj}^H \mathbf{H}_{pj}\mathbf{w}_{pj}. \tag{4.4}$$

$$I_2(x_j) = P_p(x_j)\mathbf{w}_{pj}^H \mathbf{H}_{pj}^H \mathbf{H}_{pj}\mathbf{W}_p^{(-j)} {\mathbf{W}_p^{(-j)}}^H \mathbf{H}_{pj}^H \mathbf{H}_{pj}\mathbf{w}_{pj}. \tag{4.5}$$

Note that the SINR is a random variable with respect to the channel model. For a fixed d (or $K = da$) and N, it is extremely difficult to get some insight on expression (4.2). In order to provide a tractable expression, we will analyze (4.2) in the asymptotic regime ($N \to \infty$, $d \to \infty$ but $\frac{d}{N} \to \alpha$) and show in particular that $\mathrm{SINR}(x_j, \mathbf{w}_{pj})$ converges almost surely to a random value $\mathrm{SINR}_{\lim}(x_j, p)$ independent of the code $\mathbf{w}_{pj}$. Usual analysis based on random matrices use the ratio $\frac{K}{N}$ [VS99] also known as the load of the system. In our case, the ratio $\frac{K}{N}$ is equal to αa.

Proposition 1 *When N grows towards infinity and $\frac{d}{N} \to \alpha$, the SINR of user j in cell p in downlink CDMA with orthogonal spreading codes and matched filter is given by:*

$$\mathrm{SINR}_{\lim}(x_j, p) = \frac{P_p(x_j)\left(\frac{1}{W}\int_{-\frac{W}{2}}^{\frac{W}{2}} |h_{pj}(f)|^2 df\right)^2}{I_1(x_j) + I_2(x_j) + \frac{\sigma^2}{W}\int_{-\frac{W}{2}}^{\frac{W}{2}} |h_{pj}(f)|^2 df}. \tag{4.6}$$

$$I_1(x_j) = \frac{\alpha a}{W}\sum_{q\neq p} P_q(x_j)\int_{-\frac{W}{2}}^{\frac{W}{2}} |h_{pj}(f)|^2 |h_{qj}(f)|^2 df.$$

$$I_2(x_j) = \frac{\alpha a}{W} P_p(x_j)\left(\int_{-\frac{W}{2}}^{\frac{W}{2}} |h_{pj}(f)|^4 df - \frac{1}{W}\left(\int_{-\frac{W}{2}}^{\frac{W}{2}} |h_{pj}(f)|^2 df\right)^2\right).$$

Proof *See Appendix 7.1.*

Spectral Efficiency

We would like to quantify the number of bits/s/Hz the system is able to provide to all the users. The mean (with respect to the position of the users and the fading) spectral efficiency of cell p is given by:

$$C_p = \frac{1}{N}\mathbb{E}_{x,h}\left[\sum_{j=1}^{K}\log_2\left(1+\mathrm{SINR}(x_j,\mathbf{w}_{pj})\right)\right]. \tag{4.7}$$

In the asymptotic case and due to invariance by translation, the spectral efficiency per cell is the same for all cells. As a consequence, the network spectral efficiency is infinite. Without loss of generality, we will consider a user in cell 0 ($x_j \in [-\frac{a}{2}, \frac{a}{2}]$) and the corresponding asymptotic SINR is denoted $\mathrm{SINR}_{\mathrm{lim}}(x_j)$. Assuming the same distribution for all the users in cell 0, we drop the index j. The measure of performance in this case is the number of bits per second per hertz per meter (b/s/Hz/meter) the system is able to deliver:

$$C = \frac{1}{a}\frac{K}{N}\mathbb{E}_{x,h}\left[\log_2\left(1+\mathrm{SINR}_{\mathrm{lim}}(x)\right)\right] = \alpha\mathbb{E}_{x,h}\left[\log_2\left(1+\mathrm{SINR}_{\mathrm{lim}}(x)\right)\right]. \tag{4.8}$$

According to the size of the network, the total spectral efficiency scales linearly with the factor C. If we suppose that in each cell, the statistics of the channels are the same, then denoting by $P_0(x) = P(x)$, $h_0(f) = h(f)$ and $L_0 = L$, we obtain the following proposition from Prop. 1.

Proposition 2 *When the spreading length N grows towards infinity and $\frac{d}{N} \to \alpha$, the asymptotic spectral efficiency per meter of downlink CDMA with random orthogonal spreading codes, general path loss, and matched filter is given by:*

$$C(a) = \frac{\alpha}{a}\mathbb{E}_h\left[\int_{-\frac{a}{2}}^{\frac{a}{2}}\log_2\left(1+\frac{P(x)\left(\frac{1}{W}\int_{-\frac{W}{2}}^{\frac{W}{2}}|h(f)|^2df\right)^2}{I(x)+\frac{\sigma^2}{W}\int_{-\frac{W}{2}}^{\frac{W}{2}}|h(f)|^2df}\right)dx\right]. \tag{4.9}$$

$$\begin{aligned}I(x) &= \frac{\alpha a}{W}\sum_{q\neq 0}P_q(x)\int_{-\frac{W}{2}}^{\frac{W}{2}}|h(f)|^2|h_q(f)|^2df\\ &\quad+\frac{\alpha a}{W}P(x)\left(\int_{-\frac{W}{2}}^{\frac{W}{2}}|h(f)|^4df-\frac{1}{W}\left(\int_{-\frac{W}{2}}^{\frac{W}{2}}|h(f)|^2df\right)^2\right)\end{aligned}$$

with $a \in [0, \frac{1}{\alpha}]$.

In the next two paragraphs, two simplifying assumptions are introduced, in order to get expressions that are more easy to manipulate.

Equispaced Delays

For ease of understanding of the impact of the number of paths on the orthogonality gain, we suppose that in each cell q, all the users have the same number of paths L_q and the delays are uniformly distributed according to the bandwidth:

$$\tau_{q\ell} = \frac{\ell}{W},\ 1 \leq \ell \leq L_q. \tag{4.10}$$

Hence, replacing $h(f)$ and $h_q(f)$ with their expression with respect to the temporal coefficients (3.7) and using (4.10), (4.9) from Prop. 2 reduces to

$$C(a) = \frac{\alpha}{ar}\mathbb{E}_{\eta}\left[\int_{-\frac{a}{2}}^{\frac{a}{2}} \log_2\left(1 + \frac{P(x)\left(\sum_{\ell=0}^{L-1}|\eta_\ell|^2\right)^2}{I(x) + \sigma^2\sum_{\ell=0}^{L-1}|\eta_\ell|^2}\right)dx\right]. \tag{4.11}$$

$$I(x) = \alpha a\sum_{q\neq 0} P_q(x)\left(\left(\sum_{\ell=0}^{L-1}|\eta_\ell|^2\right)\left(\sum_{\ell=0}^{L_q-1}|\eta_{q\ell}|^2\right) + \sum_{\ell=0}^{\min(L,L_q)-1}\sum_{\ell'\neq\ell}\eta_\ell{\eta_{\ell'}}^*{\eta_{q\ell}}^*\eta_{q\ell'}\right)$$
$$+ \alpha aP(x)\left(\sum_{\ell=0}^{L-1}\sum_{\ell'\neq\ell}|\eta_\ell|^2|\eta_{\ell'}|^2\right).$$

In the case of a single path, i.e., $L_q = 1$ for all q, the signal is only affected by flat-fading, therefore orthogonality is preserved and intra-cell interference vanishes.

$$C(a) = \frac{\alpha}{ar}\mathbb{E}_{\eta}\left[\int_{-\frac{a}{2}}^{\frac{a}{2}} \log_2\left(1 + \frac{P(x)|\eta|^2}{\alpha a\sum_{q\neq 0}P_q(x)|\eta_q|^2 + \sigma^2}\right)dx\right]. \tag{4.12}$$

Exponential path loss and ergodic case

In the case of an exponential path loss, explicit expressions of the spectral efficiency can be derived when $L_q \to \infty$ for all q, referred in the following as the ergodic case. Although L_q grows large, we suppose L_q to be negligible with respect to N (see Appendix 7.1). The impact of frequency reuse is also considered. In other words, r adjacent cells may use different frequencies to reduce the amount of interference. This point is a critical issue to determine the impact of frequency reuse on the spectral efficiency of downlink CDMA networks.

Proposition 3 *When the spreading length N and the number of paths L_q (for all q) grow towards infinity with $\frac{d}{N} \to \alpha$ and $\frac{L_q}{N} \to 0$, the asymptotic spectral efficiency per meter of downlink CDMA with random orthogonal spreading codes, exponential path loss, frequency reuse r and matched filter is given by:*

$$C(a) = \frac{\alpha}{ar}\int_{-\frac{a}{2}}^{\frac{a}{2}} \log_2\left(1 + \frac{Pe^{-\gamma|x|}\left(\mathbb{E}\left[|h|^2\right]\right)^2}{I(x) + \sigma^2\mathbb{E}\left[|h|^2\right]}\right)dx. \tag{4.13}$$

$$I(x) = \alpha aP\left(\mathbb{E}\left[|h|^2\right]\right)^2\frac{2e^{-\gamma ar}}{1-e^{-\gamma ar}}\cosh(\gamma x)$$
$$+ \alpha aPe^{-\gamma|x|}\left(\mathbb{E}\left[|h|^4\right] - \left(\mathbb{E}\left[|h|^2\right]\right)^2\right)$$

with $a \in [0, \frac{1}{\alpha}]$.

Proof *See Appendix 7.1.*

In the case where $a \to 0$, the number of b/s/Hz/meter depends only on the fading statistics, the path loss and the factor $\alpha = \frac{d}{N}$ through:

$$C(0) = \alpha \log_2 \left(1 + \frac{P\mathbb{E}\left[|h|^2\right]}{\sigma^2 + \frac{2\alpha P\mathbb{E}\left[|h|^2\right]}{\gamma}} \right). \tag{4.14}$$

Proof *Let* $a \to 0$ *in* (4.13).

4.1.3 Discussion

In all the following discussion, $P = 1$, $\sigma^2 = 10^{-7}$, $\alpha = 10^{-2}$ and $r = 1$ (unless specified otherwise).

Path Loss versus Orthogonality

We would like to quantify the impact of path loss on the overall performance of the system when considering downlink unfaded CDMA. In this case,

$$C(a) = \frac{\alpha}{a} \int_{-\frac{a}{2}}^{\frac{a}{2}} \log_2 \left(1 + \frac{P(x)}{I(x) + \sigma^2} \right) dx. \tag{4.15}$$

$$I(x) = \alpha a \sum_{q \neq 0} P_q(x)$$

with $a \in [0, \frac{1}{\alpha}]$.

In Figs. 4.1 and 4.2, we have plotted the spectral efficiency per meter with respect to the inter-cell distance for an exponential ($\gamma = 1, 2, 3$) and polynomial ($\beta = 4$) path loss, without frequency-selective fading. Remarkably, for each path loss factor, there is an optimum inter-cell distance which maximizes the users' spectral efficiency. This surprising result shows that there is no need into packing base stations without bound if one can remove completely the effect of frequency-selective fading. It can be shown that optimal spacing depends mainly on the path loss factor γ and increases with a decreasing path loss factor.

Ergodic fading versus Orthogonality

We would like to quantify the impact of the channel statistics on the inter-cell distance. In other words, in the case of limited path loss, should one increase or reduce the cell size? A neat framework can be formulated in the case of exponential path loss with vanishing values of the path loss factor γ and ergodic fading. Although the spectral efficiency tends to zero, one can infer on the behavior of the derivative of the spectral efficiency which is given by:

$$\frac{\partial C}{\partial a} \propto \left(\frac{3}{2} - \frac{\mathbb{E}\left[|h|^4\right]}{\left(\mathbb{E}\left[|h|^2\right]\right)^2} \right) \tag{4.16}$$

with $a \in [0, \frac{1}{\alpha}]$.

Proof *See Appendix 7.1.*

This simplified case (exponential path loss with ergodic fading) is quite instructive on the impact of frequency-selective fading on orthogonal downlink CDMA. In the ergodic case and with limited path loss, the optimum number of cells depends only on how "peaky" the channel is through the kurtosis $T = \frac{\mathbb{E}\left[|h|^4\right]}{\left(\mathbb{E}\left[|h|^2\right]\right)^2}$. If $T > \frac{3}{2}$, orthogonality is severely destroyed by the channel and one has to decrease the cell size whereas if $T \leq \frac{3}{2}$, one can increase the cell size[1].

Number of paths versus Orthogonality

We would like to quantify the impact of the number of multi-paths on the overall performance of the system. In Fig. 4.3 and 4.4, we have plotted the spectral efficiency per meter with respect to the inter-cell distance for an exponential ($\gamma = 1$) and polynomial ($\beta = 4$) path loss, in each case for numbers of multi-paths $L = 1$, $L = 2$, $L = 10$ (supposing an equal number of paths is generated by each cell) and Rayleigh fading. For $L = 1$, fading does not destroy orthogonality and as a consequence, an optimum inter-cell distance is obtained as in the non-fading case. However, for any value of $L > 1$, the optimum inter-cell distance is equal to 0.

Impact of reuse factor

In Fig. 4.5, we consider a realistic case with ergodic Rayleigh frequency-selective fading and $\gamma = 2$ and reuse factor $r = 1, 2, 3$. The spectral efficiency has been plotted for various values of the inter-cell distance. The curve shows that the users' rate decreases with increasing inter-cell distance, which is mainly due to frequency-selective fading. Note that the best spectral efficiency is achieved for a reuse factor of 1, meaning that all base stations should use all the available bandwidth.

General discussion

We would like to show that, in a cellular system, multipath fading is in fact more dramatic than path loss and restoring orthogonality through diversity (multiple antennas at the base station) and equalization techniques (MMSE,...) pays off. To visually confirm this fact, Fig. 4.6 plots for a path loss factor $\gamma = 2$ the spectral efficiency per meter with respect to the inter-cell distance in the ergodic Rayleigh frequency-selective fading, non-fading and inter-cell interference free case (i.e. $\frac{1}{a}\int_{-\frac{a}{2}}^{\frac{a}{2}} \log_2(1 + \frac{P(x)}{\sigma^2})dx$). The figure shows that one can more than triple the spectral efficiency per meter by restoring orthogonal multiple access for any inter-cell distance. Moreover, for small values of the inter-cell distance, greater gains can be achieved if one removes inter-cell interference (by exploiting the statistics of the inter-cell interference for example). Note also that even with fading and inter-cell interference, the capacity gain with respect to the number of base stations is not linear and therefore, based on economic constraints, the optimal inter-base station distance can be determined.

[1] The value $\frac{3}{2}$ is mainly dependent on the type of path loss (exponential, polynomial...)

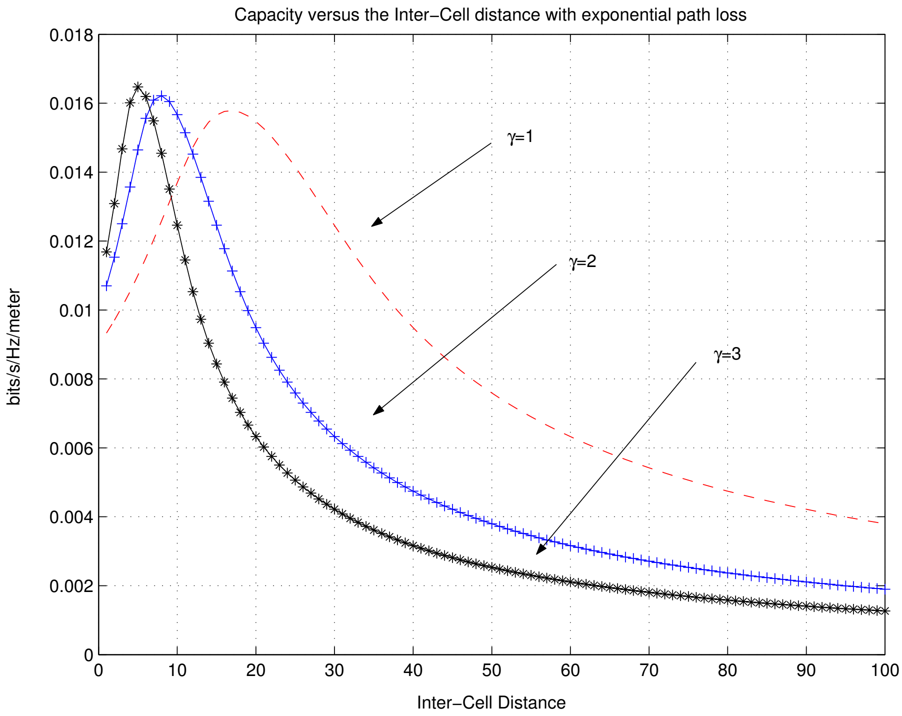

Figure 4.1: Spectral efficiency versus inter-cell distance (in meters) in the case of exponential path loss and no fading: $\sigma^2 = 10^{-7}$, $P = 1$, $\gamma = 1, 2, 3$.

Hence, based on the quality of service targets for each user, the optimum inter-cell distance can be straightforwardly derived.

4.1.4 Conclusion

Using asymptotic arguments, an explicit expression of the spectral efficiency was derived and was shown to depend only on a few meaningful parameters. This gives some insight in terms of future research directions. In the "traditional point of view" of cellular systems, the general guidance to increase the cell size has always been related to an increase in the transmitted power to reduce path loss. However, these results show that path loss is only the second part of the story and the first obstacle is on the contrary frequency-selective fading since path loss does not destroy orthogonal multiple-access whereas frequency-selective fading does. These considerations show therefore that all the effort must be focused on combating frequency-selective fading through diversity and equalization techniques in order to restore orthogonality. Finally, note that these results deal only with the downlink and any deployment strategy should take also into account the uplink traffic, as in the next section.

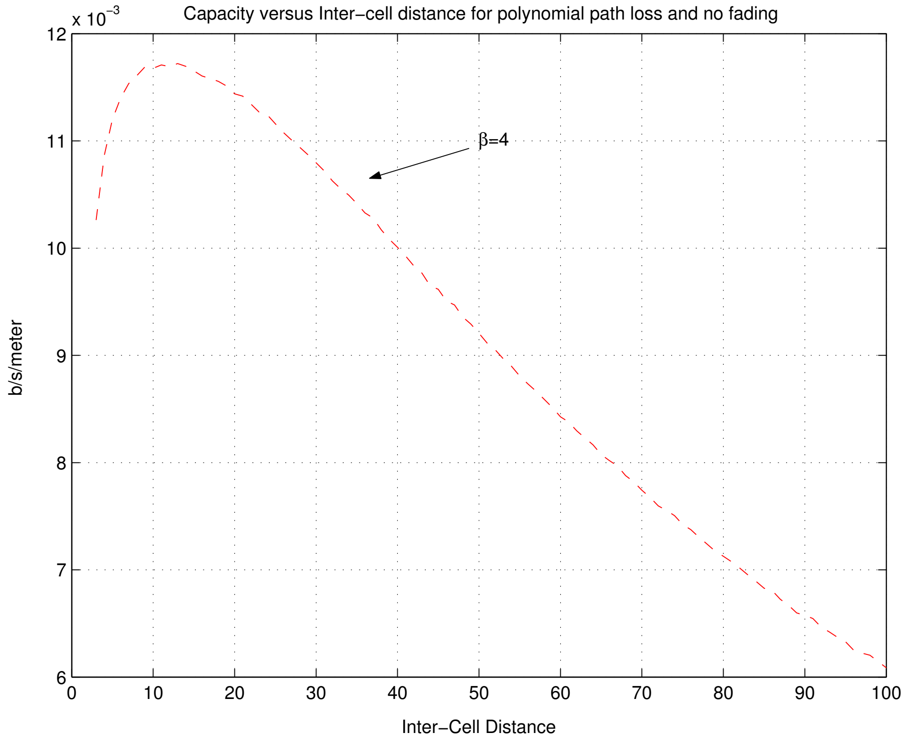

Figure 4.2: Spectral efficiency versus inter-cell distance (in meters) in the case of polynomial path loss ($\beta = 4$) and no fading: $\sigma^2 = 10^{-7}$, $P = 1$.

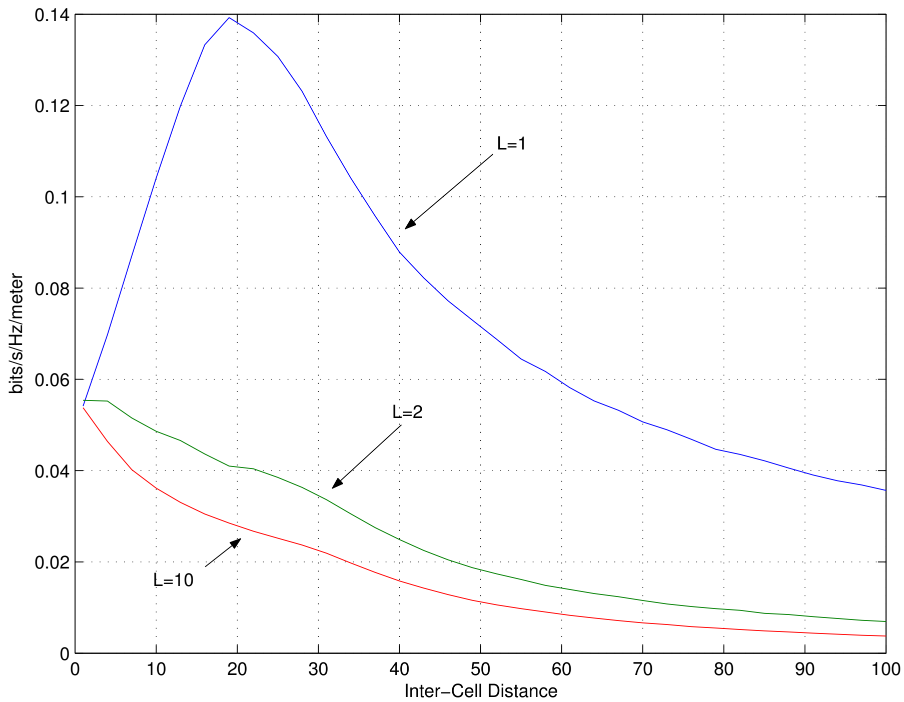

Figure 4.3: Spectral efficiency versus inter-cell distance (in meters) in the case of exponential path loss and multipath fading: $\sigma^2 = 10^{-7}$, $P = 1$, $\gamma = 1$, $L = 1, 2, 10$.

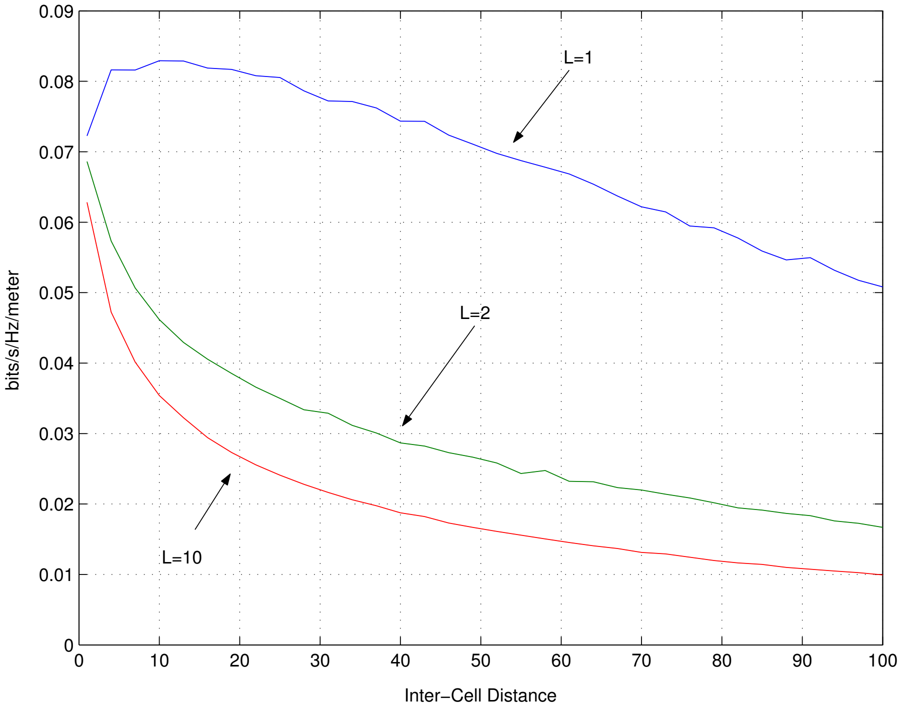

Figure 4.4: Spectral efficiency versus inter-cell distance (in meters) in the case of polynomial path loss ($\beta = 4$) and multipath fading: $\sigma^2 = 10^{-7}$, $P = 1$, $L = 1, 2, 10$.

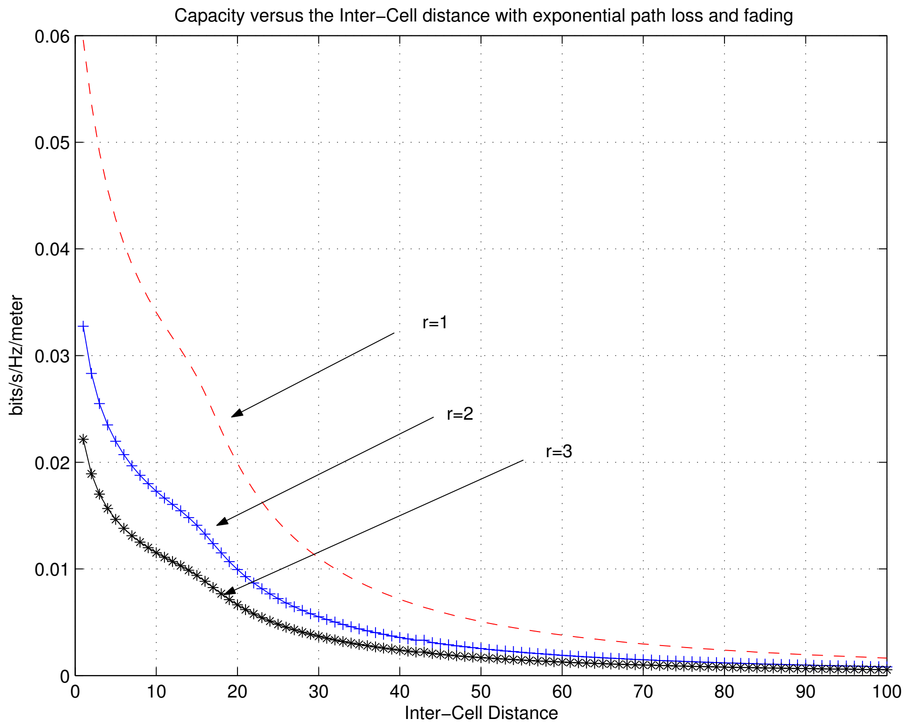

Figure 4.5: Effect of the reuse factor: spectral efficiency versus inter-cell distance (in meters) in the case of exponential path loss and fading: $\sigma^2 = 10^{-7}$, $\gamma = 2$, $P = 1$, $r = 1, 2, 3$.

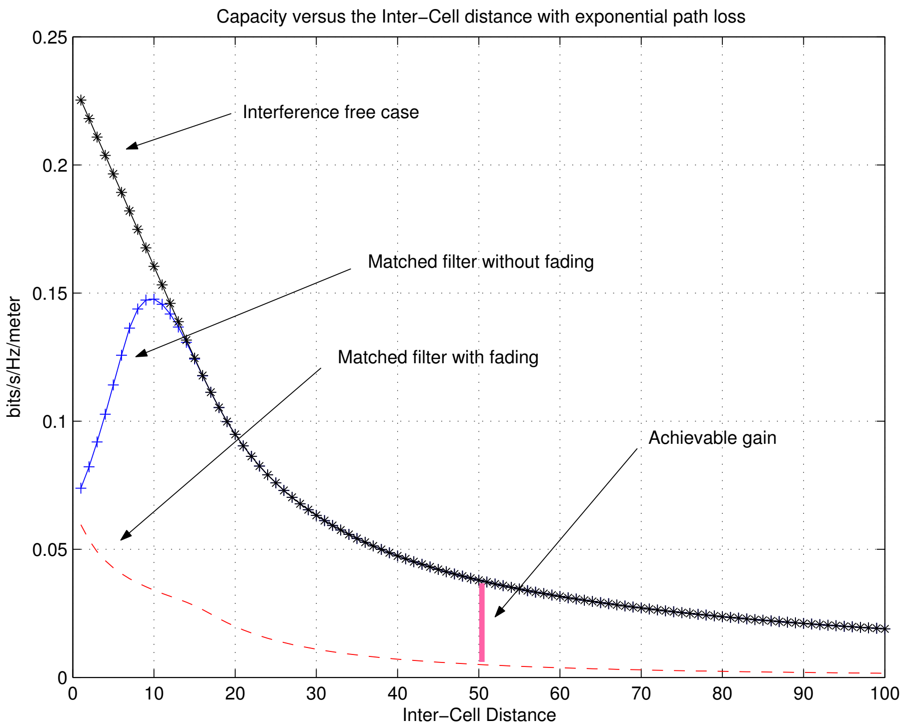

Figure 4.6: Spectral efficiency versus inter-cell distance (in meters) in the case of exponential path loss with fading, without fading and interference free case (one cell): $\sigma^2 = 10^{-7}$, $P = 1$, $\gamma = 2$.

4.2 Uplink Multi-Cell Random Spreading CDMA

In this section, the uplink counterpart of the model studied in Sec. 4.1 is studied. In the context of flat fading, this is the topic of our contribution [BDAC05a]. It is extended here to the more realistic case of frequency selective fading.

4.2.1 Model

We consider an infinite linear deployement of cells. The cell size is a and the density of users is d, so that there are $K = da$ users per cell. The spreading length is N, with $d/N = \alpha$, so that the load is $K/N = \alpha a$. The model of communication is exactly (3.12) from Sec. 3.2.2.

$$\mathbf{y} = \left(\mathbf{H}\sqrt{\mathbf{P}} \odot \mathbf{W}\right)\mathbf{s} + \sum_{l=1}^{\infty}\left(\mathbf{H}_l\sqrt{\mathbf{P}_l} \odot \mathbf{W}_l\right)\mathbf{s}_l + \mathbf{n}. \tag{4.17}$$

In the following, we assume that the frequency selective fading matrices $\mathbf{H}_l$ behave ergodically, as in Def. 5. The two-dimensional channel profile of $\mathbf{H}_l\sqrt{\mathbf{P}_l}$ is denoted $\rho(f,x) = P(x)\left|h(f,x)\right|^2$, $f \in [0,1]$, $x \in [\alpha(la - a/2), \alpha(la + a/2)]$. f is the frequency index and x is the user index. This enables us to use Th. 2 in order to obtain expressions for the SINR.

In the case of flat fading, $h_{ik} = h_k$ for all i. Eq. (4.17) reduces to:

$$\mathbf{y} = \sum_{k=1}^{K}\mathbf{w}_k h_k \sqrt{P(x_k)} s_k + \sum_{k=K+1}^{\infty}\mathbf{w}_k h_k \sqrt{P(x_k)} s_k + \mathbf{n}.$$

This case is treated in [BDAC05a].

4.2.2 Matched Filter

Single Cell

In this section, we assume that $\mathbf{s}_l = 0$ for all $l \geq 1$. Let $\mathbf{h}_k$ be the k-th column of $\mathbf{H}$, and $\mathbf{H}_{(-k)}$ be $\mathbf{H}$ with $\mathbf{h}_k$ removed. Similarly, let $\mathbf{w}_k$ be the k-th column of $\mathbf{W}$, and $\mathbf{W}_{(-k)}$ be $\mathbf{W}$ with $\mathbf{w}_k$ removed. Let $\sqrt{\mathbf{P}}_{(-k)}$ be $\sqrt{\mathbf{P}}$ with the k-th column and line removed. Let $\mathbf{G}_0 = \mathbf{H}\sqrt{\mathbf{P}} \odot \mathbf{W}$. Finally, let $\mathbf{G}_{(-k)} = \mathbf{H}_{(-k)}\sqrt{\mathbf{P}}_{(-k)} \odot \mathbf{W}_{(-k)}$.

Supposing perfect CSI at the receiver, the matched filter for the k-th user is given by $\mathbf{g}_k = \left(\mathbf{h}_k\sqrt{P(x_k)} \odot \mathbf{w}_k\right)$. This leads to the following expression for the SINR of user k

$$\mathrm{SINR}_k = \frac{\left|\mathbf{g}_k^H \mathbf{g}_k\right|^2}{\sigma^2 \mathbf{g}_k^H \mathbf{g}_k + \mathbf{g}_k^H\left(\mathbf{G}_{(-k)}\mathbf{G}_{(-k)}^H\right)\mathbf{g}_k}.$$

Proposition 4 *[TLV05] As $N, K \to \infty$ with $K/N \to \alpha a$, the SINR of user k at the output of the matched filter is given by*

$$\mathrm{SINR}_k = \beta^{MF}(x_k)$$

where $\beta^{MF} : [-\alpha a/2, \alpha a/2] \to \mathbb{R}$ *is given by*

$$\beta^{MF}(x) = P(x) \frac{\left(\int_0^1 |h(f,x)|^2 \, df\right)^2}{\sigma^2 \int_0^1 |h(f,x)|^2 \, df + \int_{-\alpha a/2}^{\alpha a/2} \int_0^1 P(y) \, |h(f,y)|^2 \, |h(f,x)|^2 \, df dy}. \tag{4.18}$$

Denoting $\mathrm{SINR}_k = \beta_k^{\mathrm{MF}}$, Prop. 4 enables us to extract an approximation of the value of the SINR of user k in the finite size case

$$\beta_k^{\mathrm{MF}} = \frac{P_k \left(\frac{1}{N} \sum_{n=1}^N |h_{nk}|^2\right)^2}{\frac{\sigma^2}{N} \sum_{n=1}^N |h_{nk}|^2 + \frac{1}{N^2} \sum_{j \neq k} \sum_{n=1}^N P_j \, |h_{nj}|^2 \, |h_{nk}|^2}. \tag{4.19}$$

Single Cell Processing

In the case of the cellular network, there is also interference from outside the cell. In addition to the notations of Sec. 4.2.2, let $\mathbf{G}_l = \mathbf{H}_l \sqrt{\mathbf{P}_l} \odot \mathbf{W}_l$. Supposing perfect CSI at the receiver, the matched filter for the k-th user is given by $\mathbf{g}_k = \left(\mathbf{h}_k \sqrt{P(x_k)} \odot \mathbf{w}_k\right)$. This leads to the following expression for the SINR of user k

$$\mathrm{SINR}_k = \frac{\left|\mathbf{g}_k^H \mathbf{g}_k\right|^2}{\sigma^2 \mathbf{g}_k^H \mathbf{g}_k + \mathbf{g}_k^H \left(\mathbf{G}_{(-k)} \mathbf{G}_{(-k)}^H + \sum_{l=1}^{\infty} \mathbf{G}_l \mathbf{G}_l^H\right) \mathbf{g}_k}.$$

Proposition 5 *As* $N, K \to \infty$ *with* $K/N \to \alpha a$*, the SINR of user* k *at the output of the matched filter is given by*

$$\mathrm{SINR}_k = \beta^{MF}(x_k)$$

where $\beta^{MF} : [-\alpha a/2, \alpha a/2] \to \mathbb{R}$ *is given by*

$$\beta^{MF}(x) = P(x) \frac{\left(\int_0^1 |h(f,x)|^2 \, df\right)^2}{\sigma^2 \int_0^1 |h(f,x)|^2 \, df + \int_{-\alpha a/2}^{+\infty} \int_0^1 P(y) \, |h(f,y)|^2 \, |h(f,x)|^2 \, df dy}. \tag{4.20}$$

Proof *For all* $1 \leq k \leq K$ *and* $l \geq 1$*,* $\mathbf{g}_k$ *and* $\mathbf{G}_l$ *have independent entries, using standard arguments [TLV05],*

$$\lim_{N \to \infty} \mathbf{g}_k^H \mathbf{G}_l \mathbf{G}_l^H \mathbf{g}_k = \int_{\alpha(la - a/2)}^{\alpha(la + a/2)} \int_0^1 P(y) \, |h(f,y)|^2 \, |h(f,x)|^2 \, df dy. \tag{4.21}$$

Given the fact that $P(y)$ *is assumed to be an integrable function and that the fading is ergodic, integrals from* (4.21) *can be summed for* $l \geq 1$*.*

Compared to the single cell case of Sec. 4.2.2, there is an additional interference term $\int_{\alpha a/2}^{+\infty} \int_0^1 P(y) \, |h(f,y)|^2 \, |h(f,x)|^2 \, df dy$ in (4.20). In the finite size case,

$$\beta_k^{\mathrm{MF}} = \frac{P_k \left(\frac{1}{N} \sum_{n=1}^N |h_{nk}|^2\right)^2}{\frac{\sigma^2}{N} \sum_{n=1}^N |h_{nk}|^2 + \frac{1}{N^2} \sum_{j \neq k} \sum_{n=1}^N P_j \, |h_{nj}|^2 \, |h_{nk}|^2}. \tag{4.22}$$

Compared to the flat fading case in [BDAC05a], i.i.d. frequency selective fading has an averaging effect, and only the distribution of the sum of the square norms of the fading coefficients appears in the spectral efficiency.

Joint Multi-Cell Processing

In the case of joint multi-cell processing, the matched filter gives the same SINR results for each user, but is applied to a much greater (infinite) number of users.

4.2.3 Optimum Receiver

Single Cell

We recall that for a complex Gaussian process Y with zero-mean and auto-covariance Q, the differential entropy is [CT91]

$$H(Y) = \log_2 \det(2\pi e Q) \tag{4.23}$$

In this section, we assume that $\mathbf{s}_l = 0$ for all $l \geq 1$. In the case of a single cell, without inter-cell interference, the mutual information between $\mathbf{Y}$ and $\mathbf{s}$ at the output of the optimum receiver is given by:

$$\begin{aligned} I(\mathbf{s}, \mathbf{y}) &= H(\mathbf{y}) - H(\mathbf{y}/\mathbf{s}) \\ &= \log_2 \det(\mathbf{R} + \sigma^2 I) - \log_2 \det(\sigma^2 I) \end{aligned}$$

where $\mathbf{R} = \left(\mathbf{H}\sqrt{\mathbf{P}} \odot \mathbf{W}\right)\left(\mathbf{H}\sqrt{\mathbf{P}} \odot \mathbf{W}\right)^H$. By invariance, the spectral efficiency of any cell (3.23) is given by

$$C = \frac{1}{Na} I(\mathbf{s}, \mathbf{y}) \tag{4.24}$$

Proposition 6 *When $N \to \infty$ and $\frac{d}{N} \to \alpha$, the spectral efficiency with i.i.d. random spreading and optimum filter is:*

$$C = \frac{1}{a \ln(2)} \int_{+\infty}^{\sigma^2} \left(m^{\mathbf{R}}(-z) - \frac{1}{z} \right) dz \tag{4.25}$$

where $m^{\mathbf{R}}(\cdot)$ is the Stieltjes transform of the empirical distribution function of the eigenvalues of $\mathbf{R}$ given by Th. 2:

$$m^{\mathbf{R}}(z) = \int_0^1 u(f, z) df$$

and $u(f, z)$ satisfies the fixed point equation:

$$u(f, z) = \frac{1}{\int_{-\alpha a/2}^{\alpha a/2} \frac{P(x)|h(f,x)|^2 dx}{1 + \int_0^1 P(x)|h(f',x)|^2 u(f',z) df'} - z}.$$

Proof *Note that*

$$\begin{aligned} C &= \frac{1}{Na} \sum_{i=1}^{N} \log_2 \left(\lambda_i^{\mathbf{R}} + \sigma^2 \right) - \log_2 \left(\sigma^2 \right) \\ &\xrightarrow[N \to \infty]{} \frac{1}{a} \int \left(\log_2 \left(\lambda + \sigma^2 \right) - \log_2 \left(\sigma^2 \right) \right) dF^{\mathbf{R}}(\lambda) \end{aligned}$$

where $\{\lambda_i^{\mathbf{R}}\}_{i=1\ldots N}$ *is the set of eigenvalues of* $\mathbf{R}$*, and* $F^{\mathbf{R}}$ *is the empirical distribution function of the eigenvalues. Differentiating this expression with respect to* σ^2 *we obtain:*

$$\frac{\partial C}{\partial \sigma^2} = \frac{1}{a \ln 2}\left(m^{\mathbf{R}}\left(-\sigma^2\right) - \frac{1}{\sigma^2}\right)$$

where $m^{\mathbf{R}}$ *is the Stieltjes transform of* $F^{\mathbf{R}}$*.*

Another way to obtain the optimum capacity in the single-cell case is the following. Assuming perfect cancellation of decoded users, successive interference cancellation with MMSE filter achieves the optimum capacity [Mö01]. The following proposition ensues from this fact.

Proposition 7 *[TLV05] As* $N, K \to \infty$ *with* $K/N \to \alpha$*, the optimal capacity is given by:*

$$C = \frac{\alpha}{a}\int_{-\alpha a/2}^{\alpha a/2} \log_2\left(1 + \beta^{SIC}(x)\right) dx$$

where $\beta^{SIC} : [-\alpha a/2, \alpha a/2] \to \mathbb{R}$ *is a function defined by the implicit equation*

$$\beta^{SIC}(x) = P(x)\int_0^1 \frac{|h(f,x)|^2\, df}{\sigma^2 + \int_{-\alpha a/2}^{x} \frac{P(y)|h(f,y)|^2 dy}{1+\beta^{SIC}(y)}}. \tag{4.26}$$

Single Cell Processing

The mutual information between $\mathbf{y}$ and $\mathbf{s}$ at the output of the optimum receiver (based only on the knowledge of the intra-cell signatures) is given by:

$$\begin{aligned} I(\mathbf{s},\mathbf{y}) &= H(\mathbf{y}) - H(\mathbf{y}/\mathbf{s}) \\ &= \log_2 \det(\mathbf{R}_0^+ + \sigma^2 I) - \log_2 \det(\mathbf{R}^+ + \sigma^2 I) \end{aligned}$$

where $\mathbf{R}^+ = \sum_{l=1}^{\infty}\left(\mathbf{H}_l\sqrt{\mathbf{P}_l}\odot \mathbf{W}_l\right)\left(\mathbf{H}_l\sqrt{\mathbf{P}_l}\odot \mathbf{W}_l\right)^H$ and $\mathbf{R}_0^+ = \mathbf{R} + \mathbf{R}^+$.

Proposition 8 *When* $N \to \infty$ *and* $\frac{d}{N} \to \alpha$*, the spectral efficiency with i.i.d. random spreading and optimum filter is:*

$$C = \frac{1}{a\ln(2)}\int_{+\infty}^{\sigma^2}\left(m^{\mathbf{R}_0^+}(-z) - m^{\mathbf{R}^+}(-z)\right) dz \tag{4.27}$$

where $m^{\mathbf{R}_0^+}(z)$ *and* $m^{\mathbf{R}^+}(z)$ *are the Stieltjes transforms of the empirical distribution functions of the eigenvalues of* $\mathbf{R}_0^+$ *and* $\mathbf{R}^+$ *given by Theorem 2:*

$$\begin{aligned} m^{\mathbf{R}_0^+}(z) &= \int_0^1 t(f,z) df, \\ m^{\mathbf{R}^+}(z) &= \int_0^1 v(f,z) df \end{aligned}$$

and $t(f,z)$, $v(f,z)$ satisfy the fixed point equations:

$$t(f,z) = \frac{1}{\int_{-\alpha a/2}^{+\infty} \frac{P(x)|h(f,x)|^2 dx}{1+\int_0^1 P(x)|h(f',x)|^2 t(f',z) df'} - z},$$

$$v(f,z) = \frac{1}{\int_{\alpha a/2}^{+\infty} \frac{P(x)|h(f,x)|^2 dx}{1+\int_0^1 P(x)|h(f',x)|^2 v(f',z) df'} - z}.$$

Proof *Note that*

$$\begin{aligned} C &= \frac{1}{Na} \sum_{i=1}^{N} \log_2\left(\lambda_i^{\mathbf{R}_0^+} + \sigma^2\right) - \log_2\left(\lambda_i^{\mathbf{R}^+} + \sigma^2\right) \\ &\xrightarrow[\substack{N,K\to\infty \\ K/N\to Cste}]{} \frac{1}{a} \int \log_2\left(\lambda + \sigma^2\right)\left(dF^{\mathbf{R}_0^+}(\lambda) - dF^{\mathbf{R}^+}(\lambda)\right) \end{aligned}$$

where $\{\lambda_i^{\mathbf{R}_0^+}\}_{i=1\ldots N}$ and $\{\lambda_i^{\mathbf{R}^+}\}_{i=1\ldots N}$ are the sets of eigenvalues of $\mathbf{R}_0^+$ and $\mathbf{R}^+$, and $F^{\mathbf{R}_0^+}$ and $F^{\mathbf{R}^+}$ are the empirical distribution functions of the eigenvalues. If we derive this expression with respect to σ^2 we obtain:

$$\begin{aligned} \frac{\partial C}{\partial \sigma^2} &= \frac{1}{a\ln 2}\left(\int \frac{1}{\sigma^2+\lambda} dF^{\mathbf{R}_0^+}(\lambda) - \int \frac{1}{\sigma^2+\lambda} dF^{\mathbf{R}^+}(\lambda)\right) \\ &= \frac{1}{a\ln 2}\left(m^{\mathbf{R}_0^+}\left(-\sigma^2\right) - m^{\mathbf{R}^+}\left(-\sigma^2\right)\right). \end{aligned}$$

Joint Multi-Cell Processing

Proposition 9 *When $N \to \infty$ and $\frac{d}{N} \to \alpha$, the spectral efficiency with i.i.d. random spreading, optimum filter and joint multi-cell processing is:*

$$C = \frac{1}{a\ln(2)} \int_{+\infty}^{\sigma^2} \left(m^{\mathbf{R}_0^+}(-z) - \frac{1}{z}\right) dz. \tag{4.28}$$

4.2.4 Simulations

In the following, we consider two special flat-fading cases.

- The unfaded case, i.e., the distribution function of the fading is $\delta(t-1)$ (δ is the Dirac function)
- The Rayleigh fading case, i.e., the distribution function of the fading is $\exp(-t)$.

The path loss is of the polynomial type $P(x) = 1/(|x|+1)^\beta$. In Fig. 4.7, we have plotted the spectral efficiency for $\beta = 2$, $\alpha = 0.01$ and $\sigma^2 = 10^{-7}$. In addition to the matched filter and optimum filter, we have drawn the curves for the MMSE filter (see Sec. 3.5.1) in both cases of single cell processing (partial knowledge Wiener filter) and joint multi-cell processing (full knowledge Wiener filter). Contrary to the case for downlink CDMA, spectral efficiency always decreases when the inter-cell distance increases. One can see that the fading does not have a great impact as faded and

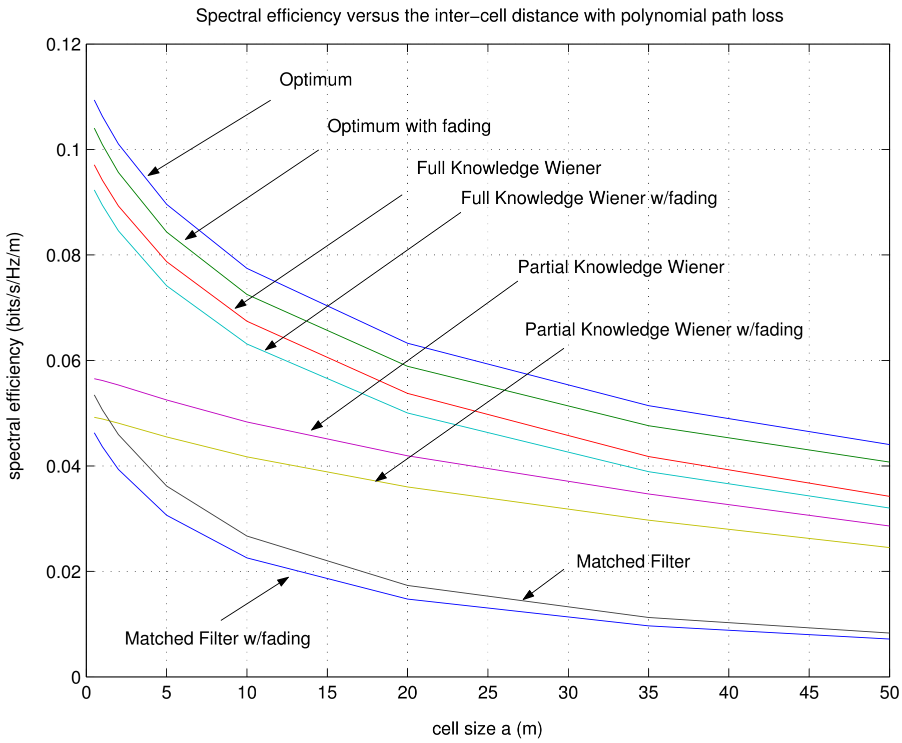

Figure 4.7: Results for $\alpha = 0.01$, $P(x) = 1/(|x|+1)^{\beta}$, $\beta = 2$.

unfaded curves are very close. Additionally, the curves show that optimum intra-cell processing can more than double the spectral efficiency with respect to the use of the matched filter or the Wiener filter. The relative gap is even higher for increasing inter-cell distance (in which case the cell system is overloaded). In fact, the curve given by (4.28) (not plotted due to scaling factors) shows that inter-cell interference reduces spectral efficiency by a factor of 3 for the range of values of a considered in Fig. 4.7.

4.2.5 Conclusion

Using asymptotic arguments, an explicit expression of the spectral efficiency of multi-cell networks has been derived considering realistic path loss and fading models. We have shown in particular the potential gain in cellular environments of optimum intra-cell processing with respect to various receivers. The impact of inter-cell interference has also been quantified for various inter-cell distances. The results are especially useful for the deployment of cellular networks for a given target user rate. Note that although the model under consideration applies to 1-D networks, it is straightforward to extend the analysis to 2-D networks (in the case of a regular pattern).

4.3 Uplink Single-Cell Orthogonal CDMA

4.3.1 Motivation

Usual studies of uplink CDMA schemes suppose a multiple access communication scheme where each user modulates his signal with a pseudo-random i.i.d. sequence [TH99, SV01, TLV05]. One of the reasons relies on the fact that, due to the multi-path channel, the convolution of the codes with the different channels of the users can be represented as a new set of codes with properties similar to a pseudo-random sequence due to the randomness of the channel. As a consequence, even if codes are designed for an orthogonal multiple access scheme, the multi-path channel unfortunately destroys orthogonality. Therefore, the signaling overhead for synchronizing the users in the network (by estimation/reshifting of each users channel delay in a GSM mode) may overcome the SINR improvement due to the reduction of the multi-user interference.

However, recently, Debbah et al. [DHLdC03a] showed that, as far as downlink is concerned, a non-negligible gain can be achieved if one uses orthogonal codes to serve the users, especially for highly loaded systems. The intuitive idea is that by appropriate equalization, a user can restore orthogonal access in the network by compensating the effect of his own channel (which is also common to all the users in the downlink). The gain is mainly a function of the load (see [DLdC04]) and the type of equalizer. However, in the uplink, such a result can not be applied as each user code is distorted independently by a different channel. As a consequence, the ability of any equalization scheme to restore orthogonality is very limited and is mostly dependent on the channel (orthogonality destroying) fading characteristics.

The goal is to assess more precisely how multi-path fading affects the performance of uplink-CDMA. In particular, for a given statistical environment, the gain of using orthogonal codes in the uplink is theoretically quantified. The setting is analyzed for the simple receiver structure of the Matched Filter as well as the Successive Interference Cancellation (SIC) Matched Filter. In order to obtain interpretable expressions, the problem is analyzed in the asymptotic regime: a high number of users is considered where the spreading length N tends to infinity, the number of users K tends to infinity but the ratio $\frac{K}{N} \rightarrow \alpha$ is constant.

The results are based on random unitary matrix theory [HP00, PR04]. This tool enables us to express the SINR in a very simple form in the large system limit. Moreover, the theoretical results are shown to be very accurate predictions of the system's behavior in the finite size case (spreading length N of 256). This section is based upon [BDAC05b].

4.3.2 Model

We consider a single uplink multi-user system cell, i.e., inter-cell interference free case. The spreading length is denoted N. The number of users in the cell is K. The load is $\alpha = K/N$. The general case of wide-band CDMA is considered, as in (3.20):

$$\mathbf{y} = \left(\mathbf{H}\sqrt{\mathbf{P}} \odot \mathbf{W}\right)\mathbf{s} + \mathbf{n}. \tag{4.29}$$

Users are assumed to employ orthogonal codes. Although not restrictive and in order to derive tractable expressions of the SINR, vectors $\mathbf{w}_k = [w_{1k}, \ldots, w_{Nk}]^T$ are supposed to be columns extracted from a Haar distributed unitary matrix, as described in Sec. 2.2.4. Hence, $\mathbf{W}$ is an $N \times K$ unitary spreading matrix.

$$\mathbf{W} = \left[\mathbf{w}_1 | \mathbf{w}_2 | \cdots | \mathbf{w}_K\right] \text{ where } \mathbf{w}_k = \begin{bmatrix} w_{1k} \\ \vdots \\ w_{Nk} \end{bmatrix}.$$

In the following, we assume that the frequency selective fading matrice $\mathbf{H}$ behaves ergodically, as in Def. 5. The two-dimensional channel profile of $\mathbf{H}$ is denoted $\rho(f,x) = P(x)\left|h(f,x)\right|^2,\ f \in [0,1],\ x \in [0,\alpha]$. f is the frequency index and x is the user index. This enables us to use results on unitary random matrices in order to obtain expressions for the SINR.

4.3.3 Matched Filter

Let $\mathbf{h}_k$ be the k-th column of $\mathbf{H}$, and $\mathbf{H}_{(-k)}$ be $\mathbf{H}$ with $\mathbf{h}_k$ removed. Similarly, let $\mathbf{w}_k$ be the k-th column of $\mathbf{W}$, and $\mathbf{W}_{(-k)}$ be $\mathbf{W}$ with $\mathbf{w}_k$ removed. Let $\sqrt{\mathbf{P}}_{(-k)}$ be $\sqrt{\mathbf{P}}$ with the k-th column and line removed. Let $\mathbf{G}_0 = \mathbf{H}\sqrt{\mathbf{P}} \odot \mathbf{W}$. Finally, let $\mathbf{G}_{(-k)} = \mathbf{H}_{(-k)}\sqrt{\mathbf{P}}_{(-k)} \odot \mathbf{W}_{(-k)}$.

Supposing perfect CSI at the receiver, the matched filter for the k-th user is given by $\mathbf{g}_k = \left(\mathbf{h}_k\sqrt{P(x_k)} \odot \mathbf{w}_k\right)$. This leads to the following expression for the SINR of user k

$$\mathrm{SINR}_k = \frac{\left|\mathbf{g}_k^H \mathbf{g}_k\right|^2}{\sigma^2 \mathbf{g}_k^H \mathbf{g}_k + \mathbf{g}_k^H\left(\mathbf{G}_{(-k)}\mathbf{G}_{(-k)}^H\right)\mathbf{g}_k}. \tag{4.30}$$

Proposition 10 *For given channel coefficients, the* SINR*, with orthogonal codes, converges almost surely to a deterministic value as* $N \to \infty$ *and* $\frac{K}{N} \to \alpha$*, namely:*

$$\mathrm{SINR}_k^{orth} = P(x_k)\frac{\xi(x_k)^2}{\sigma^2\xi(x_k) + (\nu(x_k) - \mu(x_k))}$$

where

$$\xi(x) = \int_0^1 \left|h(f,x)\right|^2 df,$$
$$\nu(x) = \int_0^\alpha \int_0^1 P(y)\left|h(f,x)\right|^2 \left|h(f,y)\right|^2 df dy,$$
$$\mu(x) = \int_0^\alpha P(y)\left|\int_0^1 h(f,x)h^*(f,y)df\right|^2 dy.$$

Proof *See Appendix 7.1.*

Note that in the case of random codes (i.i.d. elements with zero mean and variance $\frac{1}{N}$), the SINR has the following expression (see [TLV05]):

$$\mathrm{SINR}_k^{\mathrm{rand}} = \frac{\xi(x_k)^2}{\sigma^2 \xi(x_k) + \nu(x_k)}.$$

The additional term $\mu(x_k)$ characterizes the orthogonality gain of the channel and ranges from 0 to $\nu(x_k)$. The orthogonality gain is function of the selectivity of the channel as well as the correlation between the channels. For example, if the channels are all the same, $\mathrm{SINR}_k^{\mathrm{orth}} = \frac{\xi(x_k)}{\sigma^2}$ and the orthogonality gain is maximal.

4.3.4 Discussion

Since path loss does not affect orthogonality, from now on, we set $P(x) = 1$ for all users. We consider the case of a multipath channel. User k transmits through a channel with impulse response as in (3.5)

$$c_k(\tau) = \sum_{\ell=0}^{L-1} \eta_{k\ell} \delta(\tau - \tau_{k\ell}).$$

where we assume that the channel is invariant during the time considered. As in (3.9), the coefficients h_{ik} are given by the Discrete Fourier Transform of the fading process.

$$h_{ik} = \sum_{\ell=0}^{L-1} \eta_{k\ell} e^{-j2\pi \frac{i}{N} W \tau_{k\ell}} e^{j\pi W \tau_{k\ell}}.$$

We will use two simplifying hypotheses. The first one is that, in order to compare channels at the same signal to noise ratio, we constrain the fading coefficients to be complex Gaussian i.i.d. random variables with zero mean and variance $\frac{\varrho}{L}$.

$$\mathbb{E}\left[\eta_{k\ell}\right] = 0 \text{ and } \mathbb{E}\left[|\eta_{k\ell}|^2\right] = \frac{\varrho}{L}. \tag{4.31}$$

The second one is that, for ease of understanding of the impact of the number of paths on the orthogonality gain, the delays are supposed to be uniformly distributed according to the bandwidth

$$\tau_{k\ell} = \frac{\ell}{W}. \tag{4.32}$$

From (4.32), the expression of $h(f, x)$ from (3.13) is simply written

$$h(f, x) = \sum_{\ell=0}^{L_k - 1} \eta_{k\ell} e^{-j2\pi\ell f} e^{j\pi\ell}.$$

As a consequence, we obtain immediately:

$$\xi(x_k) = \sum_{\ell=0}^{L-1} |\eta_{k\ell}|^2.$$

As far as the terms $\nu(x_k)$ and $\mu(x_k)$ are concerned, we obtain (see Appendix 7.1 for derivations):

$$\nu(x_k) = \alpha\varrho \sum_{\ell=0}^{L-1} |\eta_{k\ell}|^2 , \tag{4.33}$$

$$\mu(x_k) = \alpha\frac{\varrho}{L} \sum_{\ell=0}^{L-1} |\eta_{k\ell}|^2 . \tag{4.34}$$

This gives us the following expressions for the SINR:

$$\mathrm{SINR}_k^{\mathrm{orth}} = \frac{\sum_{\ell=0}^{L-1} |\eta_{k\ell}|^2}{\sigma^2 + \alpha\varrho\left(1-\frac{1}{L}\right)},$$

$$\mathrm{SINR}_k^{\mathrm{rand}} = \frac{\sum_{\ell=0}^{L-1} |\eta_{k\ell}|^2}{\sigma^2 + \alpha\varrho}.$$

We observe that the orthogonal gain depends only on the four parameters σ^2, α, ϱ and L:

$$\frac{\mathrm{SINR}_k^{\mathrm{orth}}}{\mathrm{SINR}_k^{\mathrm{rand}}} = \frac{\sigma^2 + \alpha\varrho}{\sigma^2 + \alpha\varrho\left(1-\frac{1}{L}\right)}.$$

Remarkably, at high SNR ($\sigma^2 \to 0$), the SINR gain is given by:

$$\frac{\mathrm{SINR}_k^{\mathrm{orth}}}{\mathrm{SINR}_k^{\mathrm{rand}}} = \frac{L}{L-1}.$$

Hence, in the case of a two-path channel, one can increase by 3 dB the SINR by synchronizing the users whereas for a 5-path channel, the synchronization gain is less than 1 dB.

4.3.5 Simulations

In Fig. 4.8, the asymptotic SINR gain has been plotted versus the number of paths L for an SNR of 10 dB (SNR$=\frac{\varrho}{\sigma^2}$). The simulated SINR gain for $N = 256$ has also been plotted. The simulation curve has been obtained by generating at random a single fading matrix and a single spreading code matrix. On Fig. 4.8, we can observe that the simulations are very close to the theoretical formulas for a realistic spreading length.

In Fig. 4.9, the mean spectral efficiency of the system with Matched filter has been plotted. For the case of orthogonal spreading, the mean spectral efficiency is given by:

$$\begin{aligned} C &= \alpha \mathbb{E}_{\{\eta_k\}} \left[\log_2 \left(1 + \frac{\sum_{\ell=0}^{L-1} |\eta_{k\ell}|^2}{\alpha \left(\frac{1}{C\frac{E_{\mathrm{b}}}{N_0}} + \varrho\left(1-\frac{1}{L}\right) \right)} \right) \right] \\ &= \alpha \int_0^{+\infty} \log_2 \left(1 + \frac{x}{\alpha \left(\frac{1}{C\frac{E_{\mathrm{b}}}{N_0}} + \varrho\left(1-\frac{1}{L}\right) \right)} \right) p(x)dx \end{aligned}$$

where x represents the random variable $\sum_{\ell=0}^{L-1} |\eta_{k\ell}|^2$, and $p(x)$ is its distribution, given by a Chi-Squared distribution with $2L$ degrees of freedom

$$p(x) = \frac{x^{L-1}e^{-Lx/\varrho}}{(L-1)!\,(\varrho/L)^L}.$$

In order to assess the gap with more complex receivers, the performance of the Successive Interference Cancellation Matched Filter [MV01] has been plotted in Fig. 4.9 in addition to the simple Matched Filter (MF). The principle of SIC receivers is quite simple: assuming ergodic channels, users are ordered and are decoded successively. At each step, supposing that the user has been encoded at the appropriate decoding rate, the signal is decoded and its contribution to the interference is then perfectly substracted. This removes some of the inter-user interference and therefore increases the SINR of the following decoded users. The SINR of the k-th decoded user is then:

$$\mathrm{SINR}_k^{\mathrm{orth}} = \frac{\sum_{\ell=0}^{L-1} |\eta_{k\ell}|^2}{\sigma^2 + \frac{K-k+1}{N}\varrho\left(1-\frac{1}{L}\right)},$$

since the contributions of the $k-1$ first decoded users have already been substracted.

In the limit when $N \to \infty$ and $\frac{K}{N} \to \alpha$, C is then given by the implicit equation:

$$C = \int_0^{\alpha}\int_0^{+\infty} \log_2\left(1 + \frac{x}{y\left(\frac{1}{C\frac{E_b}{N_0}} + \varrho\left(1-\frac{1}{L}\right)\right)}\right) p(x)dxdy.$$

Figure 4.9 shows C for various values of L with orthogonal codes, with or without successive interference cancellation (SIC), as well as comparative plots of C obtained with random i.i.d. spreading codes. The following results are obtained:

- i.i.d. spreading always provides a lower spectral efficiency than orthogonal spreading, with respect to the same filter.
- In the case of orthogonal spreading, the spectral efficiency is higher for low values of L for any receiver (to ensure orthogonality between users). This is in contrast with the case of i.i.d. spreading where L must be high in order to decrease the randomness of the fading.
- As L increases, the gap between orthogonal and i.i.d. spreading reduces for any kind of receiver. This result has already been shown previously through the orthogonality gain expression.
- For $L > 2$, the gain of using a SIC scheme with respect to the Matched filter is equivalent for i.i.d. and orthogonal spreading.

Other receivers can be considered, like the MMSE and optimum filters. However, the study involves more sophisticated tools for the orthogonal case, and the theoretical analysis of these receivers is still under study. In the case of i.i.d. codes, the results rely on a theorem due to Girko [Gir90, TLV05].

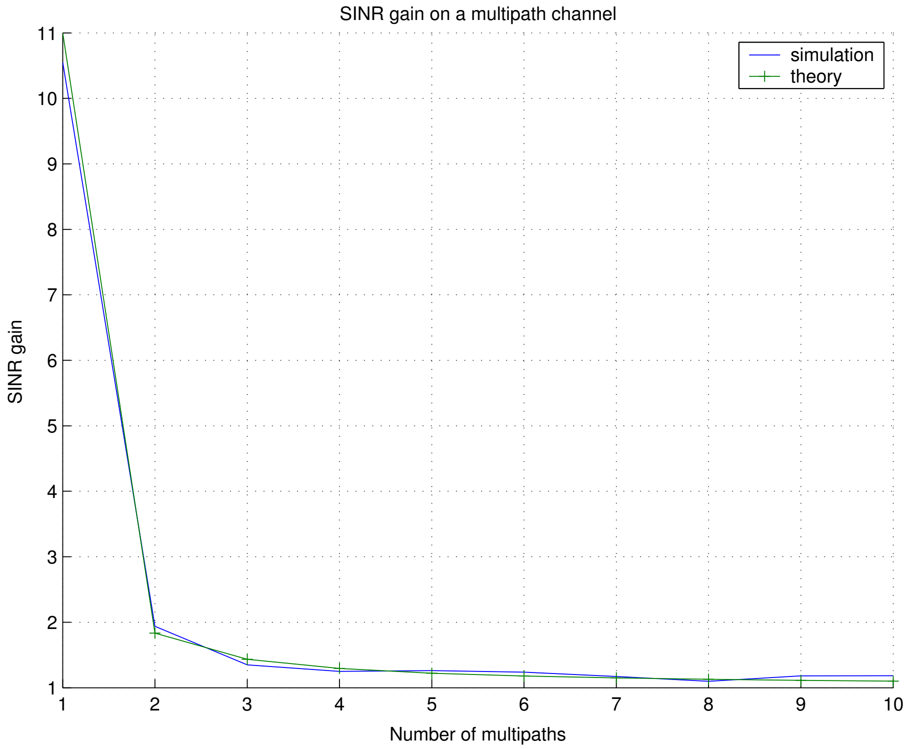

Figure 4.8: SINR gain on a multipath channel, $\alpha = 1$, SNR = 10dB.

In order to evaluate the potential gains, simulations are shown for both orthogonal and i.i.d. random codes, with 3 different filters: matched filter (MF), MMSE filter and optimum filter, on channels with respectively L=1 path (Fig. 4.10) and L=5 paths (Fig. 4.11). The curves prompt the same comments as Fig. 4.9: though there is always a gain in spectral efficiency with orthogonal codes, this gain decreases as L increases for any receiver. However, note that in the particular case of the optimum receiver, i.i.d. codes achieve the single user Gaussian bound when the load tends to infinity. Therefore, i.i.d. codes can outperform the performance of orthogonal codes (if the system is working at high loads, see Fig. 4.11).

4.3.6 Conclusion

Using asymptotic arguments, an explicit expression of the SINR of an uplink CDMA cell using orthogonal spreading codes and Matched Filter has been derived considering a realistic frequency selective fading model. The orthogonality gain has been shown to depend mainly on the number of paths and the load of the system through very simple insightful expressions. As a consequence, the need to synchronize the users is mainly a function of the environment at hand and one could think of adaptive synchronization protocols for future multiple access CDMA schemes to increase the rate.

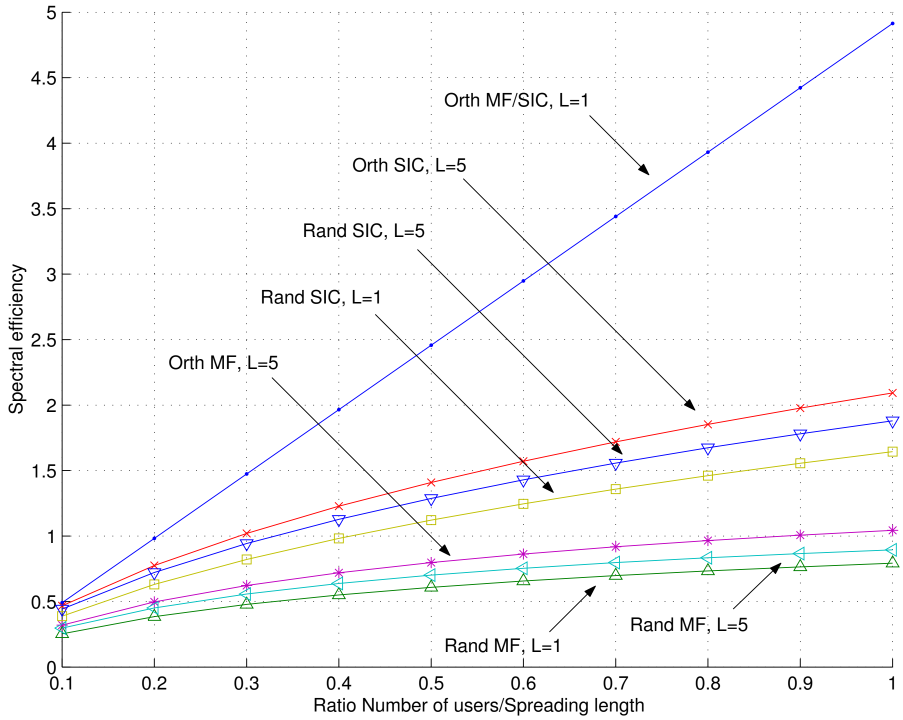

Figure 4.9: Spectral efficiency of the multipath channel, SNR = 10dB

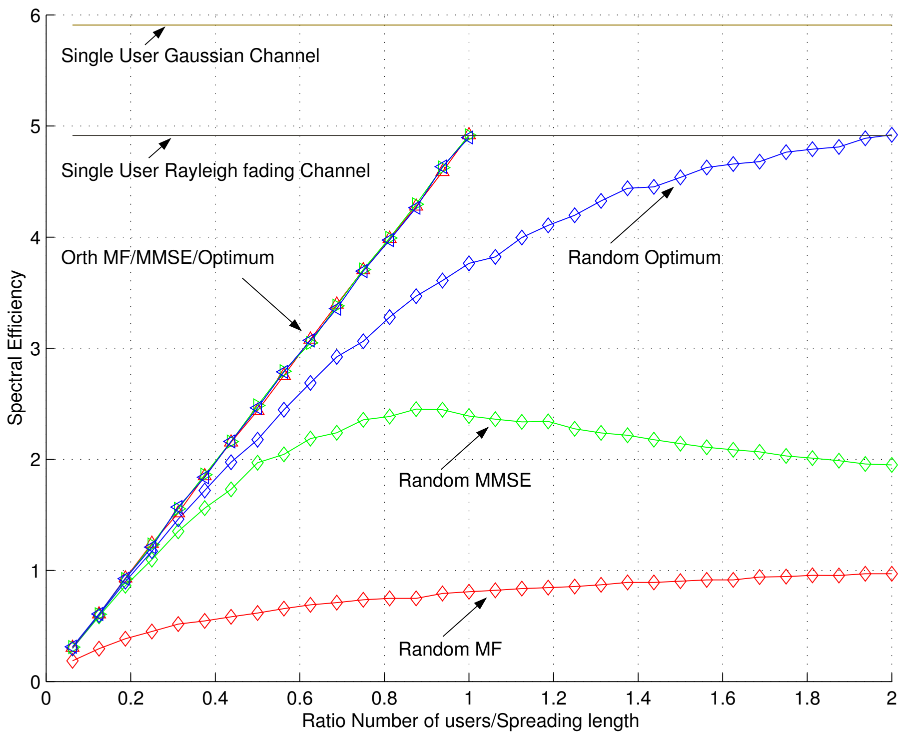

Figure 4.10: Simulations on the multipath channel, L=1, SNR = 10dB

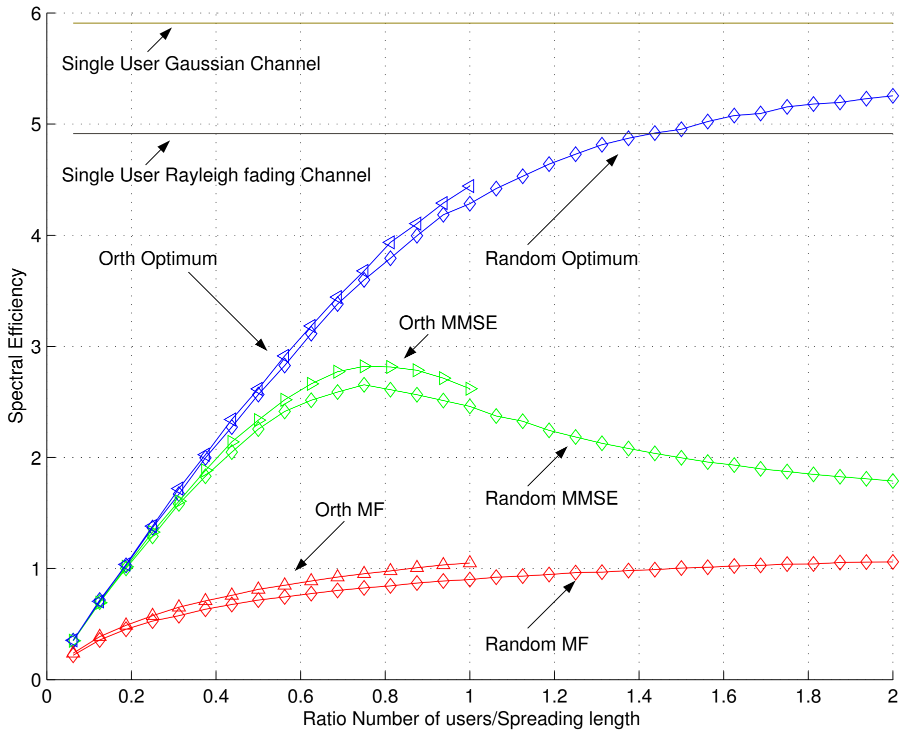

Figure 4.11: Simulations on the multipath channel, L=5, SNR = 10dB

4.4 Uplink Multi-Cell Orthogonal CDMA

In this section, the infinite deployment of cells using orthogonal uplink CDMA as in Sec. 4.3 is studied.

4.4.1 Model

We consider an infinite linear deployement of cells. The cell size is a and the density of users is d, so that there are $K = da$ users per cell. The spreading length is N, with $d/N = \alpha$, so that the load is $K/N = \alpha a$. The model of communication is similar to (3.12) from Sec. 3.2.2.

$$\mathbf{y} = \big(\mathbf{H}\sqrt{\mathbf{P}} \odot \mathbf{W}\big)\mathbf{s} + \sum_{l=1}^{\infty}\big(\mathbf{H}_l\sqrt{\mathbf{P}_l} \odot \mathbf{W}_l\big)\mathbf{s}_l + \mathbf{n}. \tag{4.35}$$

Users are assumed to employ orthogonal codes. Although not restrictive and in order to derive tractable expressions of the SINR, vectors $\mathbf{w}_k = [w_{1k}, \dots, w_{Nk}]^T$ are supposed to be columns extracted from a Haar distributed unitary matrix, as described in Sec. 2.2.4. Hence, $\mathbf{W}$ and $\mathbf{W}_l,\ 1 \le l \le \infty$, are $N \times K$ independent unitary spreading matrices.

In the following, we assume that the frequency selective fading matrices $\mathbf{H}_l$ behave ergodically, as in Def. 5. The two-dimensional channel profile of $\mathbf{H}_l\sqrt{\mathbf{P}_l}$ is denoted $\rho(f,x) = P(x)\left|h(f,x)\right|^2,\ f \in [0,1],\ x \in [\alpha(la - a/2), \alpha(la + a/2)]$. f is the frequency index and x is the user index. This enables us to use results on unitary random matrices in order to obtain expressions for the SINR.

4.4.2 Matched Filter

Let $\mathbf{h}_k$ be the k-th column of $\mathbf{H}$, and $\mathbf{H}_{(-k)}$ be $\mathbf{H}$ with $\mathbf{h}_k$ removed. Similarly, let $\mathbf{w}_k$ be the k-th column of $\mathbf{W}$, and $\mathbf{W}_{(-k)}$ be $\mathbf{W}$ with $\mathbf{w}_k$ removed. Let $\sqrt{\mathbf{P}}_{(-k)}$ be $\sqrt{\mathbf{P}}$ with the k-th column and line removed. Let $\mathbf{G}_0 = \mathbf{H}\sqrt{\mathbf{P}} \odot \mathbf{W}$. Finally, let $\mathbf{G}_{(-k)} = \mathbf{H}_{(-k)}\sqrt{\mathbf{P}}_{(-k)} \odot \mathbf{W}_{(-k)}$ and $\mathbf{G}_l = \mathbf{H}_l\sqrt{\mathbf{P}_l} \odot \mathbf{W}_l$. .

Supposing perfect CSI at the receiver, the matched filter for the k-th user is given by $\mathbf{g}_k = \left(\mathbf{h}_k\sqrt{P(x_k)} \odot \mathbf{w}_k\right)$. This leads to the following expression for the SINR of user k

$$\mathrm{SINR}_k = \frac{\left|\mathbf{g}_k^H\mathbf{g}_k\right|^2}{\sigma^2\mathbf{g}_k^H\mathbf{g}_k + \mathbf{g}_k^H\left(\mathbf{G}_{(-k)}\mathbf{G}_{(-k)}^H + \sum_{l=1}^{\infty}\mathbf{G}_l\mathbf{G}_l^H\right)\mathbf{g}_k}. \tag{4.36}$$

Proposition 11 *As $N, K \to \infty$ with $K/N \to \alpha a$, the SINR of user k at the output of the matched filter is given by*

$$\mathrm{SINR}_k^{orth} = P(x_k)\frac{\xi(x_k)^2}{\sigma^2\xi(x_k) + (\nu(x_k) - \mu(x_k)) + \nu^+(x_k)}$$

where

$$\xi(x) = \int_0^1 |h(f,x)|^2 \, df,$$

$$\nu(x) = \int_{-\alpha a/2}^{\alpha a/2} \int_0^1 P(y) \, |h(f,x)|^2 \, |h(f,y)|^2 \, df dy,$$

$$\mu(x) = \int_{-\alpha a/2}^{\alpha a/2} P(y) \left| \int_0^1 h(f,x) h^*(f,y) df \right|^2 dy,$$

$$\nu^+(x) = \int_{\alpha a/2}^{+\infty} \int_0^1 P(y) \, |h(f,x)|^2 \, |h(f,y)|^2 \, df dy.$$

Proof *See Appendix 7.1.*

Note that in the case of random codes (i.i.d. elements with zero mean and variance $\frac{1}{N}$), the SINR has the following expression (see Sec. 4.2):

$$\mathrm{SINR}_k^{\mathrm{rand}} = P(x_k) \frac{\xi(x_k)^2}{\sigma^2 \xi(x_k) + \nu^+(x_k)}$$

The additional term $\mu(x_k)$ characterizes the orthogonality gain of the channel and ranges from 0 to $\nu(x_k)$. The orthogonality gain is function of the selectivity of the channel as well as the correlation between the channels. For example, if the channels are all the same,

$$\mathrm{SINR}_k^{\mathrm{orth}} = \frac{\varrho}{\sigma^2 + \nu^+(x_k)},$$

and the orthogonality gain is maximal. Note that using orthogonal codes does not affect the inter-cell interference term, and reduces only the intra-cell interference. The intensity of the inter-cell interference depends mainly on the type of path loss, the more severe the path loss, the less severe the inter-cell interference.

4.4.3 Conclusion

Using asymptotic arguments, an explicit expression of the SINR of a multi-cell uplink CDMA network using orthogonal spreading codes has been derived considering a realistic frequency selective fading model. The orthogonality gain affects only intra-cell interference, while the inter-cell interference is not reduced compared to the random spreading case.

This concludes the chapter on network centric communications. We considered only the case of CDMA in this performance analysis. However, numerous results also exist in the case of other protocols, in particular, see Sec. 3.4.2 for examples in the case of OFDMA.

Chapter 5

User Centric Communications

5.1 ALOHA

In this chapter, we introduce in a networking context the two types of games presented earlier, namely Evolutionary Games and Correlated Games. A population of mobile terminals competes over the access to a common channel. In order to keep things simple, the model considered here is intentionally as simple as possible, i.e., ALOHA [Abr70, Rob72]. The results presented here come from the two publications [BAD05] and [ABD06].

In Sec. 5.1.1, a large population of communicating terminals using an unslotted ALOHA protocol [Abr70] with two possible levels of transmission power is studied. An alternative simpler modeling approach is investigated, which enables to obtain explicit analytical expressions for the performance measures. This enables to compute analytically the solutions for various non-cooperative optimization criteria.

The problem of how to choose between both power levels is posed in non-cooperative fashion, when mobiles are selfish and rational players. Their strategy consists in choosing the probability to transmit with either power level. The considered payoffs are functions of the obtained throughputs and of the cost for the power levels. In particular, the impact on the system performance of pricing for the power levels is studied.

Two non-cooperative optimization concepts are studied: the Nash equilibrium and the Evolutionary Stable Strategy (ESS). The latter was introduced in mathematical biology in the context of Evolutionary Games, which allows to describe and to predict properties of large populations whose evolution depends on many local interactions, each involving a finite number of individuals (see Chapter 2).

To the best of our knowledge, this contribution is the first to apply evolutionary games to study non-cooperative behavior in wireless networks.

In Sec. 5.1.2, the framework of correlated games is applied to slotted ALOHA [Rob72]. All mobiles are thus supposed to be synchronized. As is frequently assumed when studying slotted ALOHA, we assume that if more than one mobile attempts to send a packet at time slot t then all transmitted packets are lost and mobiles wait a random amount of slots before retransmitting their packets, in order to avoid repeated collisions.

We consider both the cooperative as well as the non-cooperative approaches. For each case we study the impact of adding coordination mechanisms on the throughput.

The contribution is not only in applying the notion of correlated equilibrium in the context of networking, but also in extending it to the multi-criterion case; in our case, each mobile (player) has two objectives: expected throughput and expected power consumption. We use the correlated equilibrium setting adapted to the context of constrained optimization by each player (maximizing the average throughput with a constraint on the average power consumption).

Coordination between players turns out to be useful also in the case of cooperative optimization. Indeed, the coordination may be needed also in the so called *team problem* [BC82, ABA$^+$06], i.e., when various players have the same common objective that they maximize (e.g., the global throughput). Users may benefit from performing joint randomizations, which may not be possible without coordination due to the distributed nature of the problem. The need for joint randomization in the team setting is due to the multi-objective nature of the problem (more precisely to the constraints on the expected power consumption).

5.1.1 Evolutionary Game Perspective to ALOHA

Unslotted ALOHA Model

We consider an infinite population of mobile terminals. We use a model similar to [BG87] for unslotted ALOHA where the global arrival of new packets from all mobiles follows a Poisson process with intensity λ.

We assume that for each packet, its source can choose the transmitted power among two levels. All packets of the lower power level involved in a collision are assumed to be lost and will have to be retransmitted later. In addition, if more than one packet of the higher power level is involved in a collision then all packets are lost. The power differentiation thus allows one packet of the higher power level to be successfully transmitted in collisions that do not involve other packets of the higher power level. This is the capture phenomenon [Rob72, LKZ98].

In this section, we study the choice of power levels. A strategy for a mobile corresponds to the choice of a power level. This can be a deterministic choice or a randomized one. We assume that the power level choice for a retransmitted packet is the same as the power level at which it was transmitted the first time. Thus, if the whole population uses the same strategy q for transmissions (meaning that the higher power level is chosen with probability q, and the lower with probability $\bar{q} = 1 - q$) then the rate of arrival of packets that will be transmitted with higher power level is given by λq.

We consider a non-cooperative approach in which each mobile determines its power level so as to maximize its payoff, which has two components:

- $P_{succ}(p, q)$, which is the success probability when it chooses the higher power level (level 1) with probability p and the lower one (level 2) with probability $\bar{p} = 1 - p$, given that all other mobiles choose the higher power level with probability q and the lower one with probability $\bar{q} = 1 - q$ (we shall keep using q below as the strategy of other mobiles).

- $\pi(p)$, which represents the cost of a packet transmission when choosing the higher power level with probability p. $\pi(p)$ can be linear: if a is the cost for the higher power level and b ($b < a$) for the lower one then we have $\pi(p) = ap + b\bar{p}$. In this case $\pi(p)$ can represent in particular the *expected transmission power*. $\pi(p)$ can also be chosen as an arbitrary function $\pi_2(p)$ representing the *pricing* paid by the users. In this case, $\pi_2(p)$ is generally assumed to be strictly convex and increasing.

We shall consider the following payoff function, given as the ratio between the packet success probability and expected consumed power: $J_r(p,q) = \frac{P_{succ}(p,q)}{\pi(p)}$. This type of payoff can represent in particular the *power efficiency*, i.e., the expected number of packets that can be transmitted per a unit power transmitted [SMG01].

Using the approach of [BG87], we assume that the point process describing packets that are either transmitted with power level i or retransmitted at a power level i ($i = 1, 2$) is a Poisson process with intensity $g_i = g_i(\lambda, q)$ (it depends on the arrival process of packets as well as on the fraction of packets sent with each power level).

Computing the Performance Measures

Retransmission rates The function $\mathrm{LambertW}(\cdot)$ is defined in the following way. If $z \geq -\exp(-1)$, $x = \mathrm{LambertW}(z)$ is the root greater than or equal to -1 of the equation $z = x\exp(x)$.

Remark 1 *The function* $\mathrm{LambertW}(\cdot)$ *satisfies the following property.*

$$\exp(\mathrm{LambertW}(z)) = \frac{z}{\mathrm{LambertW}(z)}. \tag{5.1}$$

Theorem 3

- *Assuming that* $\lambda q \leq \frac{1}{2}\exp(-1)$, *we have*

$$g_1(\lambda, q) = -\frac{1}{2}\mathrm{LambertW}(-2\lambda q). \tag{5.2}$$

- *Assume*

$$\begin{cases} \lambda q \leq \frac{1}{2}\exp(-1), \\ \text{if } q \leq \frac{1}{1+\exp(-1)} \text{ then } \lambda\bar{q}\exp\left(\exp(-1)\frac{q}{\bar{q}}\right) \leq \frac{1}{2}\exp(-1). \end{cases}$$

Then,

$$g_2(\lambda, q) = -\frac{1}{2}\mathrm{LambertW}\left(\frac{-2\bar{q}g_1}{q}\right) = -\frac{1}{2}\mathrm{LambertW}\left(\frac{\bar{q}}{q}\mathrm{LambertW}(-2\lambda q)\right). \tag{5.3}$$

- *Under the same conditions, we have*

$$P_{succ}(p,q) = p\exp(-2g_1) + \bar{p}\exp(-2(g_1+g_2)) = \lambda\left(\frac{pq}{g_1} + \frac{\bar{p}\bar{q}}{g_2}\right). \tag{5.4}$$

- *If the conditions on* (λ, q) *are not met, then there is no possible steady state.*

Proof *The success probability of a higher power level (re)transmission is given by* $\exp(-2g_1)$. *Thus the rate of departure of type 1 packets (i.e., the rate of successful packet transmissions and retransmissions of higher power level) is given by* $g_1 \exp(-2g_1)$.

Since at steady state, λq, *the rate of arrival of type 1 packets equals to the rate of departure of type 1 packets, we have*

$$\lambda q = g_1 \exp(-2g_1). \tag{5.5}$$

Eq. (5.5) *has a solution if* $2\lambda q \leq \exp(-1)$, *thus we obtain* (5.2). *It follows from* (5.2) *that* $g_1(\lambda, 0) = 0$ *and* $g_1(\lambda, 1) = -\frac{1}{2}\,\mathrm{LambertW}(-2\lambda)$.

The success probability for type 2 packets is given by $\exp\left(-2[g_1 + g_2]\right)$. *Thus the rate of departure of type 2 packets is given by* $g_2 \exp\left(-2[g_1 + g_2]\right)$. *Hence at steady state:*

$$\lambda \bar{q} = g_2 \exp\left(-2[g_1 + g_2]\right) \tag{5.6}$$

Using (5.5), *we can write* $\exp(-2g_1) = \dfrac{\lambda q}{g_1}$, *and substituting in equation* (5.6), *we obtain*

$$\frac{-2\bar{q} g_1}{q} = -2g_2 \exp(-2g_2). \tag{5.7}$$

Eq. (5.7) *has a solution if* $\frac{2\bar{q} g_1}{q} \leq \exp(-1)$. *Since* $g_1 \leq \frac{1}{2}$, *this is always verified if* $q \geq \frac{1}{1+\exp(-1)}$. *If* $q \leq \frac{1}{1+\exp(-1)}$, *this condition becomes* $\mathrm{LambertW}(-2\lambda q) \geq \exp(-1)\frac{q}{\bar{q}}$, *and since the function* $x \mapsto x \exp(x)$ *is increasing for* $x \geq -1$, *we can apply it to both sides of the inequality and get* (5.3). *It follows from* (5.6) *that* $g_2(\lambda, 1) = 0$ *and* $g_2(\lambda, 0) = -\frac{1}{2}\,\mathrm{LambertW}(-2\lambda)$.

As (5.6) *implies* $\exp(-2[g_1 + g_2]) = \dfrac{\lambda \bar{q}}{g_2}$, *we obtain the global success probability expression* (5.4).

We observe that $P_{succ}(p, q)$ is a linear function in p:

$$P_{succ}(p, q) = \lambda p \left(\frac{q}{g_1} - \frac{\bar{q}}{g_2} \right) + \frac{\lambda \bar{q}}{g_2}.$$

The coefficient multiplicating p is zero for $q = 1$ and strictly positive otherwise. More specifically, when $q = 1$, we have $\exp(-2g_1(\lambda, 1)) = -\frac{2\lambda}{\mathrm{LambertW}(-2\lambda)}$ and $g_2(\lambda, 1) = 0$, so that

$$P_{succ}(p, 1) = -\frac{2\lambda}{\mathrm{LambertW}(-2\lambda)}. \tag{5.8}$$

When $q = 0$, we have $g_1(\lambda, 0) = 0$ and $\exp(-2g_2(\lambda, 0)) = -\frac{2\lambda}{\mathrm{LambertW}(-2\lambda)}$, so that

$$P_{succ}(p, 0) = p - \bar{p}\frac{2\lambda}{\mathrm{LambertW}(-2\lambda)}. \tag{5.9}$$

Optimization issues We first seek to find the maximum throughput that can be achieved (through the choice of λ and q). (5.5) together with (5.6) imply that the global throughput of the system is

$$\Theta = g_1 \exp(-2g_1) + g_2 \exp(-2[g_1 + g_2]). \tag{5.10}$$

To obtain g_2 that gives the maximum throughput for a fixed g_1, we differentiate (5.10) with respect to g_2 and equate to zero. This gives $g_2^* = \frac{1}{2}$. g_2^* does not depend on g_1 and the optimization of Θ corresponds to the optimization of the single-variable function $g_1 \exp(-2g_1) + \frac{1}{2}\exp(-2g_1 - 1)$. Therefore, $g_1^* = \frac{1}{2}(1 - \exp(-1))$, and

$$\Theta^* = \frac{1}{2}\exp\left(\exp(-1) - 1\right). \tag{5.11}$$

These values are obtained for $\lambda = \Theta^* = \frac{1}{2}\exp\left(\exp(-1) - 1\right)$ and $q = 1 - \exp(-1)$, which satisfy the conditions of Th. 3. We observe that this throughput is higher (by a factor $\exp(\exp(-1))$) than the maximum stable throughput with a single power level, which is equal to $\frac{1}{2}\exp(-1)$ for unslotted ALOHA. Such optimal performance can usually be obtained only in a cooperative setting, for example when a regulator enforces a common policy for all mobiles.

Nash Equilibrium

Power Efficiency case The payoff function for a mobile using strategy p while the population uses strategy q is given by

$$J_r(p,q) = \frac{P_{succ}(p,q)}{\pi(p)} = \frac{\lambda\left[\left(\frac{q}{g_1} - \frac{\bar{q}}{g_2}\right)p + \frac{\bar{q}}{g_2}\right]}{(a-b)p + b} = \frac{\lambda\left(\frac{q}{g_1} - \frac{\bar{q}}{g_2}\right)}{a-b}\left[1 + \frac{\frac{\bar{q}g_1}{qg_2 - \bar{q}g_1} - \frac{b}{a-b}}{p + \frac{b}{a-b}}\right]. \tag{5.12}$$

We begin by checking whether the boundary cases $q = 1$ and $q = 0$ are Nash equilibria.

When $q = 1$, we obtain from (5.8)

$$J_r(p,1) = \frac{-\frac{2\lambda}{\mathrm{LambertW}(-2\lambda)}}{(a-b)p + b}. \tag{5.13}$$

$J_r(p,1)$ is strictly decreasing over $p \in [0,1]$. Thus $p = 0$ optimizes $J_r(p,1)$, so that $q = 1$ is not a Nash equilibrium.

When $q = 0$, we obtain from (5.9)

$$J_r(p,0) = \frac{p - \bar{p}\frac{2\lambda}{\mathrm{LambertW}(-2\lambda)}}{(a-b)p + b}. \tag{5.14}$$

Differentiating (5.14),

$$\frac{\partial J_r}{\partial p}(p,0) = \frac{b + a\frac{2\lambda}{\mathrm{LambertW}(-2\lambda)}}{((a-b)p + b)^2}, \tag{5.15}$$

we conclude the following:

- If $-\frac{2\lambda}{\text{LambertW}(-2\lambda)} \geq \frac{b}{a}$ (i.e., $\lambda \leq -\frac{b}{2a}\ln\frac{b}{a}$) then $J_r(p,0)$ is non-increasing over $p \in [0,1]$. Therefore $q = 0$ is a Nash equilibrium.
- If $-\frac{2\lambda}{\text{LambertW}(-2\lambda)} < \frac{b}{a}$ (i.e., $\lambda > -\frac{b}{2a}\ln\frac{b}{a}$) then $J_r(p,0)$ is strictly increasing over $p \in [0,1]$. Thus $q = 0$ is not a Nash equilibrium.

To obtain other equilibria, consider now $q \in (0,1)$. The following theorem is deduced from (5.12).

Theorem 4 *$J_r(p,q)$ does not depend on p if $\bar{q}g_1(a-b) = (qg_2 - \bar{q}g_1)b$. If $\frac{b}{a} \geq \exp(-1)$, there are three solutions to this equation, $q = 0$, $q = 1$, and*

$$q^* = 1 - \frac{\frac{b}{a}\ln\left(\frac{b}{a}\right)}{\text{LambertW}\left(-2\lambda\left(\frac{b}{a}\right)^{b/a}\right)}.$$

When $q^* \in (0,1)$ (which is not necessarily the case), q^* is a Nash equilibrium.

Note that if we allow the parameters $\frac{b}{a} \in [\exp(-1),1)$ and $\lambda \in (0, \frac{1}{2}\exp(-1))$ to fluctuate independently, then it is rather straightforward that the expression of q^* can take any value in $(-\infty, 1)$. However, we observe that for $\lambda = -\frac{b}{2a}\ln\frac{b}{a}$, $q^* = 0$, which implies that if $\lambda > -\frac{b}{2a}\ln\frac{b}{a}$ then $q^* \in (0,1)$, and if $\lambda \leq -\frac{b}{2a}\ln\frac{b}{a}$ then $q^* \in (-\infty, 0]$.

To find other possible equilibria, compute

$$\frac{\partial J_r}{\partial p}(p,q) = \lambda \frac{qg_2b - \bar{q}g_1a}{g_1g_2\left((a-b)p+b\right)^2}. \tag{5.16}$$

The solutions to $\frac{\partial J_r}{\partial p}(p,q) = 0$ reduce to the case studied in Th. 4.

Pricing case When the cost function $\pi_2(p)$ is strictly convex, and we consider the payoff function

$$J_r(p,q) = \frac{P_{succ}(p,q)}{\pi_2(p)} = \frac{\lambda\left(\frac{pq}{g_1} + \frac{\bar{p}\bar{q}}{g_2}\right)}{\pi_2(p)},$$

non-trivial potential Nash equilibria are determined by the following implicit equation:

$$q^* = 1 - \frac{r(q^*)\ln(r(q^*))}{\text{LambertW}\left(-2\lambda(r(q^*))^{r(q^*)}\right)} \text{ where } r(p) = \frac{\frac{\pi_2(p)}{\pi_2{}'(p)} - p}{\frac{\pi_2(p)}{\pi_2{}'(p)} - p + 1}.$$

Evolutionary Stable Strategy

In the biological context, the payoff, or fitness, for an individual is related to its reproduction capability. A higher reward to some behavior (which can represent more food or more chances to mate) implies a higher growth rate of individuals that adopt it.

Assume a population uses a strategy q^*. This could be obtained either by a fraction q^* of the population playing one strategy and the remainder $\bar{q}^*$ playing the other, or by each individual randomizing between the strategies. A small fraction (identified as mutations) adopts another distribution p over the two strategies.

The definition of an ESS q^* is given by the following relation, for all $p \neq q^*$.

$$J(p, q^*) < J(q^*, q^*), \tag{5.17}$$

or

$$J(p, q^*) = J(q^*, q^*) \text{ and } J(p, p) < J(q^*, p). \tag{5.18}$$

If (5.17) is verified, then the fraction of the mutations in the population will tend to decrease (as it has a lower fitness, meaning a lower growth rate). q^* is then immune to mutations.

In the special case of (5.18), a population using q^* is "weakly" immune against a mutation using p. If the mutant's population grows, then we shall frequently have individuals with strategy q^* competing with mutants; in such cases, the condition $J(q^*, p) > J(p, p)$ ensures that the growth rate of the original population exceeds that of the mutations.

Computing ESS: Power Efficiency Evolutionary Stable Strategies constitute a subset of the set of Nash equilibria, therefore we only have to check whether the Nash equilibria satisfy either condition (5.17) or (5.18).

Assume that $\lambda < -\frac{b}{2a} \ln \frac{b}{a}$ then the Nash equilibrium $q^* = 0$ (see (5.15)) is also an ESS, since (5.15) implies that for all $p \neq 0$, condition (5.17) holds.

Assume that $\frac{b}{a} \geq \exp(-1)$ and $\lambda > -\frac{b}{2a} \ln \frac{b}{a}$ then the Nash equilibrium in Th. 4 exists, and is $q^* \in (0, 1)$. We note that the utility of a player does not depend on his choice of p: $J_r(q^*, q^*) = J_r(p, q^*)$, $\forall p$. Thus condition (5.17) does not hold. However, from (5.16), if $p < q^*$, then $\frac{\partial J_r}{\partial q}(q, p) > 0$ and if $p > q^*$, then $\frac{\partial J_r}{\partial q}(q, p) < 0$, which means that for all $p \neq q^*$, condition (5.18) holds. Therefore q^* is an ESS. Finally, for $\lambda = -\frac{b}{2a} \ln \frac{b}{a}$, (5.17) does not hold, but (5.18) does and $q^* = 0$ is an ESS.

Therefore, all Nash equilibria previously exhibited are ESS as well. As stated in the previous subsection, this fact has the following interpretation. All these equilibria are robust against small perturbations either in the "immune" or the "weakly immune" sense.

Numerical Results

Figs. 5.1 and 5.2 show curves delimiting the atteignable regions of (λ, q) in steady state. Remember that the system is in steady state if the throughput is equal to the arrival rate λ. The different atteignable regions are located on the left of the curves pictured. If the arrival rate is larger than certain values (possibly dependent on the transmission probability q), then there is no possible steady state.

As a means of comparison, the bound on the throughput of an ALOHA scheme with a single power level has been plotted. This bound is equal to $\frac{1}{2}\exp(-1)$. The atteignable region with one power level lies on the left of the dashdotted straight line. For two power levels, (λ, q) must satisfy the two conditions of Theorem 3 and the bound on the obtainable throughput is always larger than $\frac{1}{2}\exp(-1)$. In particular, the point defined by (5.11) appears as the rightmost point on the dashed curve in both figures. The atteignable region using two power levels lies on the left of the dashed curve. It is to be noted that these theoretical results can be applied to any ratio $b/a < 1$ of the two powers; however, if b/a is too close to 1, then the hypothesis of capture phenomenon becomes questionable.

In the case of a linear cost function, an interesting result occurs when $b/a = \exp(-1)$. For this particular value, the Nash equilibria curve will merge with the bound curve up to the point defined by (5.11), as seen in Fig. 5.1. It can be shown using the theoretical formulas, that this is the only case in which the optimum throughput represents a Nash equilibrium. For $b/a > \exp(-1)$, the Nash equilibria (and ESS) points are always on the left of the bound. However, there still exists Nash equilibria points up to a certain throughput greater than $\frac{1}{2}\exp(-1)$. In addition, for all b/a, the highest λ such that there exists a corresponding Nash equilibrium which lies on the bound curve, as can be seen on Fig. 5.2.

The case of an exponential pricing is shown in Fig. 5.3. The curve obtained for the Nash equilibria shows a similar behaviour as the case of a linear payoff.

In Fig. 5.4, we show the curves of the payoff function at the Nash equilibrium for some values of the parameter b/a, as well as for an exponential pricing, with regard to the rate of arrival λ (which is also the throughput at steady state). For the linear cost function, two distinct decreasing regions can be distinguished in each case, the first one corresponding to the trivial Nash equilibria and the second one to the equilibria of Th. 4. For the exponential pricing, the curve is plotted only for the non-trivial Nash equilibria we have found. These curves show that even though the throughput with a lower value of b/a is higher, the payoff for a mobile is lower. These results provide an insight on the potential gains when using several power levels in a non-cooperative framework in wireless communications.

5.1.2 Correlated Equilibrium in ALOHA

Slotted ALOHA Model

We consider a finite population of K mobile terminals. Each mobile has a unique i.d. number ranging between 1 and K. Time is slotted.

Let $\mathcal{N} = \{0,1\}^K$ represent the set of all 2^K subsets of $\{1,\dots,K\}$. At each time slot, a subset of mobiles $\mathbf{Z}(t) \in \mathcal{N}$ is assumed to be active. The number of active terminals at time t is equal to the Hamming weight $|\mathbf{Z}(t)| = \sum_{i=1}^{K} Z_i(t)$ of $\mathbf{Z}(t)$ and denoted by $N(t)$. $\mathbf{Z}(t)$ (and thus $N(t) = |\mathbf{Z}(t)|$) are assumed to be stationary ergodic processes.

Each active mobile is assumed to be saturated, i.e., it always has packets to send. At each time slot, a random subset of mobiles is active. If at a time slot, more than one active mobile attempts to transmit then there is a collision and all packets transmitted in the time slot are lost.

Let q_i denote the probability that mobile i transmits a packet when active (we call q_i the *strategy*). If $\mathbf{z} \in \mathcal{N}$, let $\zeta(\mathbf{z})$ be the probability that the subset $\mathbf{z}$ of mobiles is active at a slot and let $\pi_n = \sum_{|\mathbf{z}|=n} \zeta(\mathbf{z})$ be the probability that there are n active mobiles at a slot. In particular, the probability that mobile i is the only

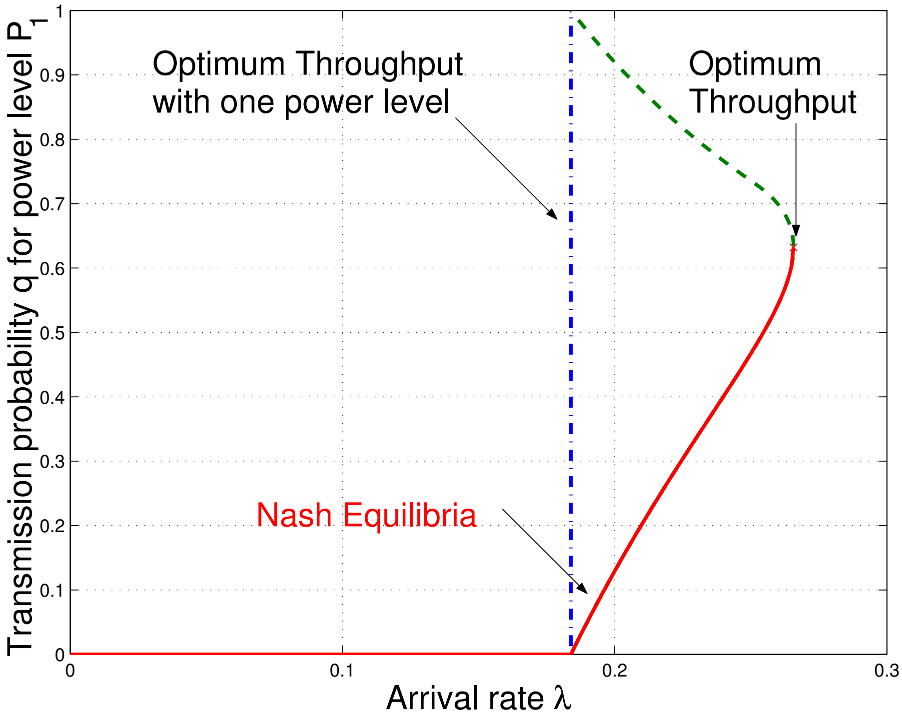

Figure 5.1: Atteignable throughput, Nash equilibria points with power efficiency payoff function, $b/a = \exp(-1)$.

active mobile in a slot is $\zeta(\mathbf{e}_i)$ where $\mathbf{e}_i$ is the vector whose elements are all zero except for the ith entry which equals one.

The probability of a successful transmission at a time slot is

$$\begin{aligned} \Theta_{\text{all}}(q_1, \dots, q_K) &= \mathbb{E}_{\mathbf{Z}}\left[\sum_{i \in \mathbf{Z}} q_i \prod_{j \in \mathbf{Z} \setminus \{i\}} (1 - q_j)\right] \\ &= \sum_{\mathbf{z} \in \mathcal{N}} \zeta(\mathbf{z}) \sum_{i \in \mathbf{z}} q_i \prod_{j \in \mathbf{z} \setminus \{i\}} (1 - q_j). \end{aligned} \tag{5.19}$$

which is also the system throughput. The expected average throughput per mobile is Θ_{all}/K. The throughput of mobile i conditioned on being active is given by

$$\begin{aligned} \Theta_i^{\text{act}}(q_1, \dots, q_K) &= \mathbb{E}_{\mathbf{Z}}\left[q_i \prod_{j \in \mathbf{Z} \setminus \{i\}} (1 - q_j) \middle| i \in \mathbf{Z}\right] \\ &= q_i \sum_{\substack{\mathbf{z} \in \mathcal{N} \\ i \in \mathbf{z}}} \zeta(\mathbf{z}) \prod_{j \in \mathbf{z} \setminus \{i\}} (1 - q_j). \end{aligned} \tag{5.20}$$

In the following, the purpose of cooperative optimization will be to maximize the system throughput Θ_{all}, whereas in a non-cooperative setting, each mobile will attempt to maximize selfishly its conditional throughput Θ_i^{act}, which we call its *utility*.

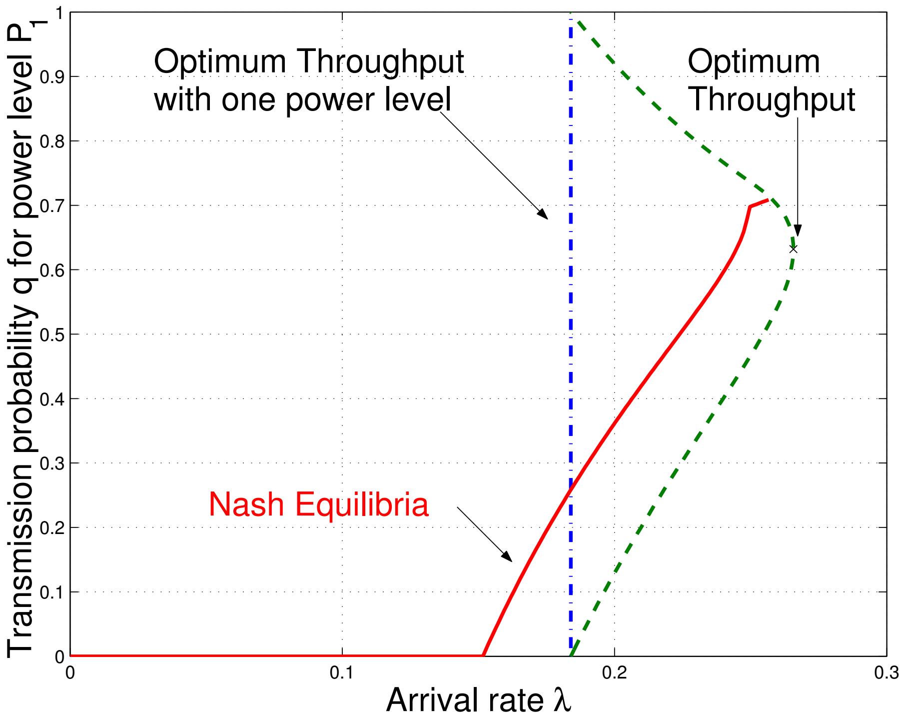

Figure 5.2: Atteignable throughput, Nash equilibria points with power efficiency payoff function, $b/a = \exp(-0.5)$.

No Coordination Mechanism

General Case The maximal throughput that can be attained is obtained by maximizing the system throughtput $\Theta_{\text{all}}(q_1, \ldots, q_K)$ given by (5.19) over $(q_1, \ldots, q_K) \in [0,1]^K$. Since $\Theta_{\text{all}}(q_1, \ldots, q_K)$ is a multivariate polynomial, hence continuous in $(q_1, \ldots, q_K)$, and $[0,1]^K$ is a compact set, the existence of a maximum is immediate. For given $\{\zeta(\mathbf{z})/\mathbf{z} \in \mathcal{N}\}$, computing this maximum is a constrained optimization problem [Lue84].

If the mobiles are non-cooperative and care only for their own throughput then it is immediate from (5.20) that the only Nash equilibrium is wheN all mobiles transmit with $q_i = 1$. The global throughput is then π_1 and the expected average throughput per mobile is π_1/K.

In the non-cooperative case, we are also interested by the *conditional* throughput, i.e., the throughput of a mobile averaged over the activity periods of the mobile. The conditional throughput of mobile i when $q_i = 1$ for all mobiles is given by $\zeta(\mathbf{e}_i)$.

Power Considerations In reality mobile users are sensitive to power consumption. Their objective is to maximize the system throughput (in the cooperative case) or the individual throughput (in the non-cooperative case) under the constraints $q_i \leq q_i^{\max}$ for some constant $q_i^{\max}$, for all users i. In the cooperative case, we can model the choice of transmission probability q_i as a constrained optimization problem. In the non-cooperative case, it is easy to see that the Nash equilibrium is obtained with $q_i = q_i^{\max}$ for all mobiles. From (5.19), this gives at the Nash

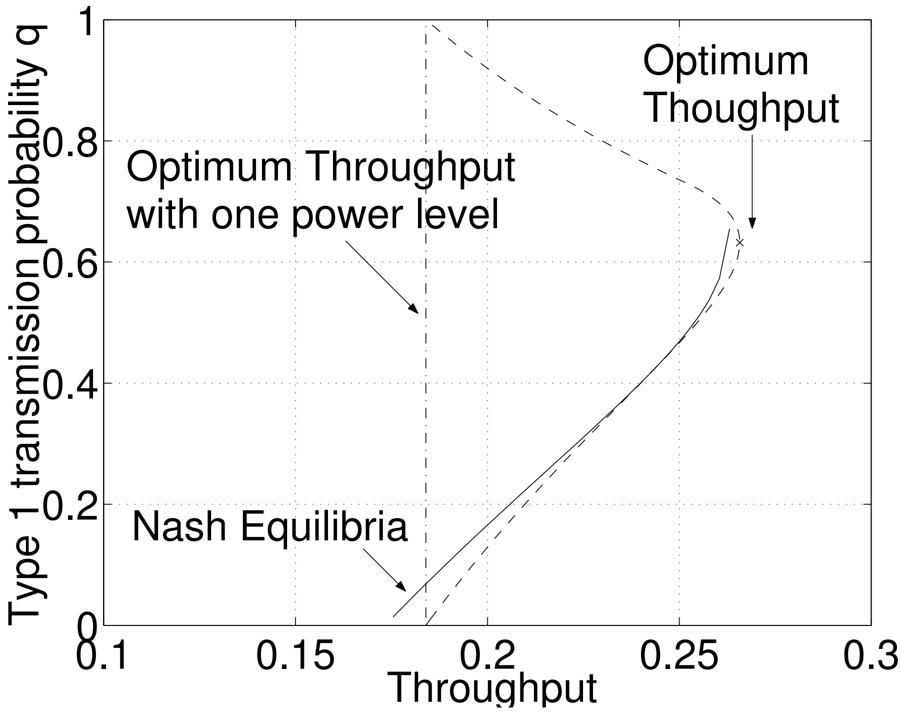

Figure 5.3: Atteignable throughput, Nash equilibria points with exponential pricing $\pi_2(p) = \exp(p)$.

equilibrium the throughput of

$$\Theta_{\mathrm{all}}(q_1^{\max}, \ldots, q_K^{\max}) = \sum_{\mathbf{z} \in \mathcal{N}} \zeta(\mathbf{z}) \sum_{i \in \mathbf{z}} q_i^{\max} \prod_{j \in \mathbf{z} \setminus \{i\}} (1 - q_j^{\max}),$$

and from (5.20), the conditional throughput as

$$\Theta_i^{\mathrm{act}}(q_1^{\max}, \ldots, q_K^{\max}) = q_i^{\max} \sum_{\substack{\mathbf{z} \in \mathcal{N} \\ i \in \mathbf{z}}} \zeta(\mathbf{z}) \prod_{j \in \mathbf{z} \setminus \{i\}} (1 - q_j^{\max}).$$

Coordination, Correlated Equilibrium and Optimization

Coordination Mechanism If the base station had full information and could schedule transmissions of the mobiles then full utilisation (i.e., a throughput of $1 - \pi_0$) could be achieved by a TDMA type approach. We consider however the case where the base station has no control over the mobiles and has no information on their power constraints nor on their number. It can only serve as an arbitrator, in the sense that was discussed in the introduction.

We therefore consider the following coordination mechanism. We assume that at each time slot t, the base station can send a signal to all mobiles in the form of a random variable $X(t)$, uniformly distributed over the integers $\{0, \ldots, \kappa - 1\}$ for some integer $\kappa \geq 2$. We assume for simplicity that K is a multiple of κ. The process $X(t)$ is assumed to be independent of $\mathbf{Z}(t)$. Our goal is to look at the incentive given to the selfish mobiles so that possibly, for a given coordination signal, only a

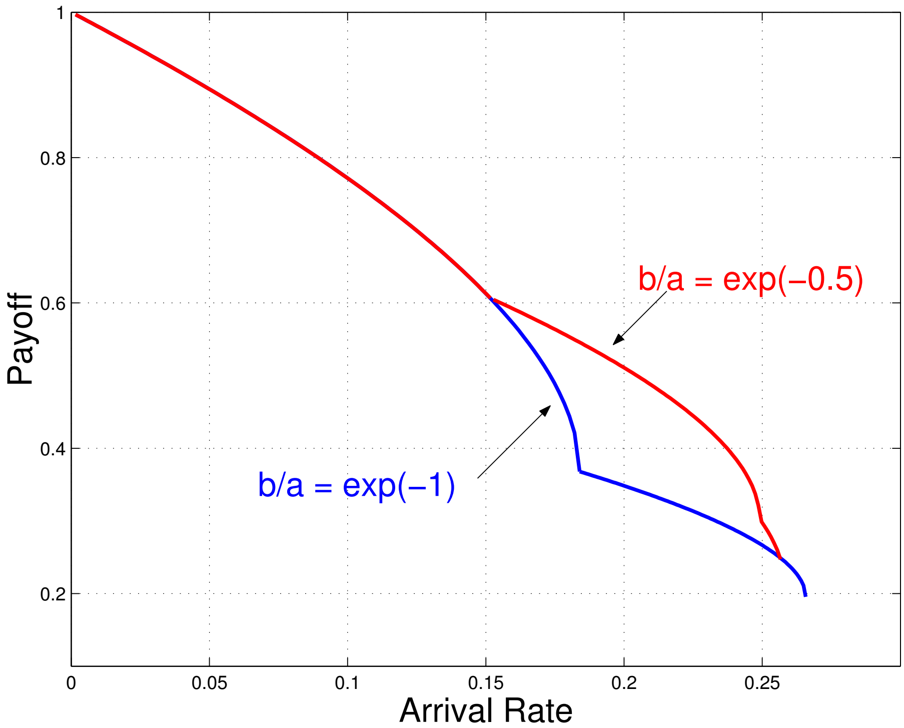

Figure 5.4: Values of the payoff function, power efficiency $b/a = \exp(-1)$ and $b/a = \exp(-0.5)$, and exponential pricing.

fraction of the mobiles has a nonzero transmission probability. Thus we introduce the coordination mechanism detailed in the following.

Transmission Strategy for Mobiles In absence of any coordination mechanism, a strategy of a mobile would be the probability of transmitting a packet. In the presence of the coordination mechanism, a mobile has the possibility to use a larger notion of strategies.

Definition 7 *We define the set of correlated policies as follows.*

- *We partition the set of all mobiles into κ subgroups S_j, $j = 1, \ldots, \kappa$ where S_j contains a mobile i if and only if $i = j - 1 \pmod{\kappa}$ (denoted $i \equiv j - 1$).*
- *A correlated strategy of a mobile is described using two real numbers in the unit interval: p_i and q_i.*
- *At time t, an active mobile i transmits a packet with probability p_i if and only if $i \in S_{X(t)}$. Otherwise it transmits with probability q_i.*

The class of correlated strategies includes in particular the non-correlated strategies. Thus, in the non-cooperative setting, a mobile has always the possibility of ignoring the signals $X(t)$ by using $p_i = q_i$. The latter can be viewed as a non-correlated strategy.

We call (p_i, q_i) the strategy of mobile i. For two K-dimensional vectors $\mathbf{p}$ and $\mathbf{q}$ we define $(\mathbf{p}, \mathbf{q})$ to be a multi-strategy for all mobiles, where mobile i uses the ith entry (p_i, q_i) of the vectors $(\mathbf{p}, \mathbf{q})$. Let

$$\mathcal{U} = \{(\mathbf{p}, \mathbf{q})/\forall i \in \{1, \ldots, K\},\ p_i \in [0,1],\ q_i \in [0,1]\}$$

denote the class of all multi-strategies.

Define $(\mathbf{p}, \mathbf{q})^{-i}$ to be the set of $K-1$ strategies of all mobiles except for mobile i, and set $\Big((\mathbf{p}, \mathbf{q})^{-i}, (p', q')_i\Big)$ to be the policy where all mobiles other than the ith one use the policies described by $(\mathbf{p}, \mathbf{q})^{-i}$ whereas the ith mobile uses policy (p', q').

Power Considerations We assume that mobile i has a constraint on the average power it can use while active. In our model, this power constraint is directly linked to the probabilities of transmission of the mobile. More precisely, the average power consumption during activity periods of a mobile with parameters (p, q) is

$$\mathrm{Pow}(p, q) = \frac{p}{\kappa} + \frac{(\kappa - 1)q}{\kappa}. \tag{5.21}$$

We then assume that mobile i has the power constraint

$$\mathrm{Pow}(p_i, q_i) \leq q_i^{\max} \text{ where } q_i^{\max} \leq 1. \tag{5.22}$$

Let U_i^{cons} denote the class of strategies of mobile i satisfying (5.22). Let

$$\mathcal{U}^{\mathrm{cons}} = \{\mathbf{u} \in \mathcal{U}/\forall i \in \{1, \ldots, K\},\ u_i \in U_i^{\mathrm{cons}}\}$$

denote the class of multi-strategies $\mathbf{u}$ for which for each i, $u_i = (p_i, q_i)$ satisfies (5.22).

Definition 8 *A multi-strategy $\mathbf{u} \in \mathcal{U}^{cons}$ is said to be a correlated equilibrium if for all i and $(p', q') \in U_i^{cons}$*

$$\Theta_i^{act}(\mathbf{u}) \geq \Theta_i^{act}\Big(\mathbf{u}^{-i}, (p', q')_i\Big). \tag{5.23}$$

Definition 9 *A multi-strategy $\mathbf{u}^* \in \mathcal{U}^{cons}$ is said to be correlated optimal if for all feasible multi-strategies $\mathbf{u} \in \mathcal{U}^{cons}$,*

$$\Theta_{all}(\mathbf{u}^*) \geq \Theta_{all}(\mathbf{u}). \tag{5.24}$$

The expressions for $\Theta_{\mathrm{all}}(\mathbf{u})$ and $\Theta_i^{\mathrm{act}}(\mathbf{u})$ can be written as

$$\begin{aligned}\Theta_{\mathrm{all}}(\mathbf{u}) = \sum_{\mathbf{z} \in \mathcal{N}} \zeta(\mathbf{z}) \sum_{i \in \mathbf{z}} \Bigg(& \frac{p_i}{\kappa} \prod_{\substack{j \in \mathbf{z} \setminus \{i\} \\ j \equiv i}} (1 - p_j) \prod_{\substack{j \in \mathbf{z} \setminus \{i\} \\ j \not\equiv i}} (1 - q_j) \\ & + \frac{q_i}{\kappa} \sum_{\substack{k=1 \\ k \not\equiv i}}^{\kappa} \prod_{\substack{j \in \mathbf{z} \setminus \{i\} \\ j \equiv k}} (1 - p_j) \prod_{\substack{j \in \mathbf{z} \setminus \{i\} \\ j \not\equiv k}} (1 - q_j) \Bigg)\end{aligned} \tag{5.25}$$

and

$$\Theta_i^{\text{act}}(\mathbf{u}) = \frac{p_i}{\kappa} \sum_{\substack{\mathbf{z}\in\mathcal{N}\\ i\in\mathbf{z}}} \zeta(\mathbf{z}) \prod_{\substack{j\in\mathbf{z}\setminus\{i\}\\ j\equiv i}} (1-p_j) \prod_{\substack{j\in\mathbf{z}\setminus\{i\}\\ j\not\equiv i}} (1-q_j)$$
$$+ \frac{q_i}{\kappa} \sum_{\substack{\mathbf{z}\in\mathcal{N}\\ i\in\mathbf{z}}} \zeta(\mathbf{z}) \sum_{\substack{k=1\\ k\not\equiv i}}^{\kappa} \prod_{\substack{j\in\mathbf{z}\setminus\{i\}\\ j\equiv k}} (1-p_j) \prod_{\substack{j\in\mathbf{z}\setminus\{i\}\\ j\not\equiv k}} (1-q_j)\,. \tag{5.26}$$

$\Theta_i^{\text{act}}(\mathbf{u})$ is an affine function of p_i and q_i. Therefore, in order to maximize $\Theta_i^{\text{act}}(\mathbf{u})$, the inequality in (5.22) will be an equality: each mobile will transmit at the maximum of its possibilities. In the next section, we investigate how the power is split between p_i and q_i for each mobile in a particular case.

Symmetric case Solving the constrained optimization problems of (5.19) or (5.25), as well as finding Nash or correlated equilibria, becomes rapidly intractable in the general case when the number of mobiles K (and hence the number of variables in the multivariate polynomials involved) increases. To simplify the analysis, we consider a symmetric case when the coefficients $\zeta(\mathbf{z})$ depend only on $|\mathbf{z}|$, and the power constraints $q_i^{\max} = q^{\max}$ are the same for all users.

We consider a simple model when mobiles are independently active with a probability π. This corresponds to the model used in [KL75] for users with a single packet buffer, when the probability of arrival of a new packet is equal to the probability of retransmission of a backlogged packet. In this case, the coefficients in (5.19) and (5.25) become symmetric, since for all $\mathbf{z}$ such that $|\mathbf{z}| = n$, $\zeta(\mathbf{z})$ are equal:

$$\zeta(\mathbf{z}) = \pi^{|\mathbf{z}|}(1-\pi)^{K-|\mathbf{z}|} \tag{5.27}$$

In the non-cooperative case, we can restrict to the same strategy (p, q) being used by all users, and investigate if a single user deviating from this strategy benefits by using a different strategy $(\hat{p}, \hat{q})$. Recall that $\pi_n = \sum_{|\mathbf{z}|=n} \zeta(\mathbf{z})$. Let

$$\ell = \frac{K}{\kappa}, \qquad \lambda = K - \frac{K}{\kappa}.$$

After some manipulations, (5.26) can be rewritten as:

$$\Theta^{\text{act}}(\mathbf{u}) = \frac{\hat{p}}{\kappa} \sum_{n=1}^{K} \frac{1}{\binom{K}{n}} \pi_n$$
$$\times \sum_{k=\max(0,n-1-\lambda)}^{\min(\ell-1,n-1)} \binom{\ell-1}{k} \binom{\lambda}{n-1-k} (1-p)^k (1-q)^{n-1-k}$$
$$+ \frac{(\kappa-1)\hat{q}}{\kappa} \sum_{n=1}^{K} \frac{1}{\binom{K}{n}} \pi_n$$
$$\times \sum_{k=\max(0,n-\lambda)}^{\min(\ell,n-1)} \binom{\ell}{k} \binom{\lambda-1}{n-1-k} (1-p)^k (1-q)^{n-1-k}. \tag{5.28}$$

The power constraints (5.22) give us

$$\hat{p} = \kappa q^{\max} - (\kappa - 1)\hat{q}. \tag{5.29}$$

Replacing $\hat{p}$ by this expression in (5.28), we obtain $\Theta^{\text{act}}(\mathbf{u})$ as an affine function in $\hat{q}$. Hence, the optimal $\hat{q}$ will be either

$$\max(0, \frac{\kappa q^{\max} - 1}{\kappa - 1}) \text{ or } \min(1, \frac{\kappa q^{\max}}{\kappa - 1})$$

depending on the sign of the coefficient

$$\begin{aligned} &\sum_{n=1}^{K} \frac{1}{\binom{K}{n}} \pi_n \sum_{k=\max(0,n-\lambda)}^{\min(\ell,n-1)} \binom{\ell}{k} \binom{\lambda - 1}{n - 1 - k} (1-p)^k (1-q)^{n-1-k} \\ &- \sum_{n=1}^{K} \frac{1}{\binom{K}{n}} \pi_n \sum_{k=\max(0,n-1-\lambda)}^{\min(\ell-1,n-1)} \binom{\ell - 1}{k} \binom{\lambda}{n - 1 - k} (1-p)^k (1-q)^{n-1-k}. \end{aligned} \tag{5.30}$$

This gives us a simple formula to investigate whether or not a given value of (p, q) that saturates (5.22) is a correlated equilibrium: replace (p, q) by their values in (5.30) and estimate the sign of the expression. If the chosen q satisfies

$$q = \max(0, \frac{\kappa q^{\max} - 1}{\kappa - 1})$$

and the sign of (5.30) is negative or if the chosen q satisfies

$$q = \min(1, \frac{\kappa q^{\max}}{\kappa - 1})$$

and the sign of (5.30) is positive, then (p, q) is indeed a correlated equilibrium.

Numerical Results

We use the terms *cooperative* and *non-cooperative* to describe the behavior of mobiles, whereas the term *coordination* refers to the presence of a common signal. Without coordination, the equilibrium concept in the non-cooperative case is the *Nash equilibrium*, whereas it is the *correlated equilibrium* with coordination.

In this section, we consider the setting of Sec. 5.1.2. However, an interesting result is that, even in this symmetric case, the optimal throughput is neither reached by saturating the power constraints $q_i^{\max}$ for all users nor for a symmetric attribution of the channel (i.e., the same strategy for all users).

In Fig. 5.5, we have plotted the system throughput Θ_{all} versus the probability of being active π with and without coordination, according to (5.19) and (5.25), for 6 users, without power constraints ($q_i^{\max} = 1$ for all users). We observe that the optimal throughput with the same strategy for all mobiles reaches a plateau and stays constant, no matter how active the mobiles are. With coordination, the value of this plateau is increased.

With a non-symmetric attribution of the strategies, a higher system throughput can be achieved. The linear portion of the curve, for $\pi > 0.5$, is actually obtained by letting only one user transmit; for $\pi \leq 0.5$, it is optimal to let several users transmit. Without power constraints, the optimal throughput with coordination is the same as without coordination.

The system throughput reached at Nash equilibrium (i.e., $q = 1$ for all mobiles) is close to the optimum for low values of π (when few mobiles are active), but rapidly decreases and approaches 0 as the probability of being active increases. Note that without power constraints, Nash and correlated equilibrium coincide, therefore the coordination mechanism does not increase the throughput in the non-cooperative case. We remark that the curve for Nash equilibrium corresponds to the throughput calculated in [KL75], which is simply $K\pi(1-\pi)^{K-1}$.

The optimal curves in Fig. 5.5 are obtained without power constraints. With power constraints the optimal curves will always be lower.

In Fig. 5.6, we have plotted the system throughput Θ_{all} obtained in the correlated equilibrium with power constraint $q^{\max} = 0.25$. In the case $\kappa = 3$, two correlated equilibria are possible: $p = 0.75$, $q = 0$ or $p = 0$, $q = 0.375$ (denoted respectively as 1 and 2 in the figure). In the case $\kappa = 2$, there are two correlated equilibria as well: $p = 0.5$, $q = 0$ and $p = 0$, $q = 0.5$ (both give the same system throughput). As a comparison, we have plotted the optimal throughput that can be obtained under the power constraint $q^{\max} = 0.25$ in the cooperative case, as well as the throughput obtained in the Nash equilibrium without coordination.

Non-cooperative throughput is improved compared to the case without power constraints. With strong power constraints, we observe that the coordination mechanism allows to obtain higher values of the throughput in the non-cooperative case. For some probabilities π, non-cooperative global throughput almost reaches the values obtained in the cooperative case.

5.1.3 Conclusion

This chapter was devoted to introducing two game theoretical concepts in a networking context. It shows that game theory is a relevant concept in the engineering of communication systems and gives insight on how to apply game theoretical tools to networking and wireless communications.

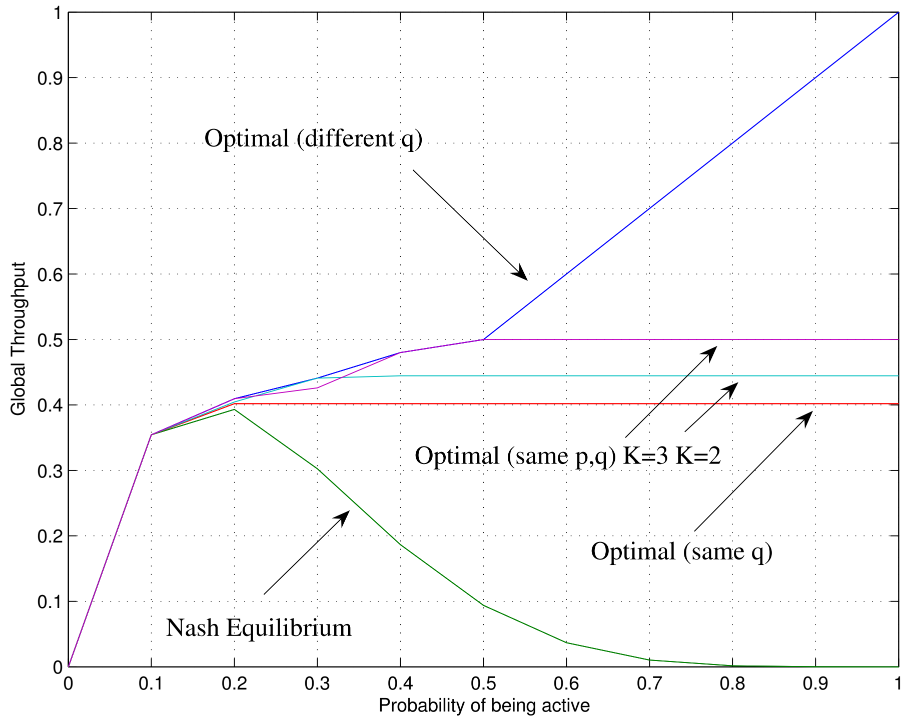

Figure 5.5: System throughput versus the probability of being active for a mobile with and without coordination, for 6 users, without power constraints.

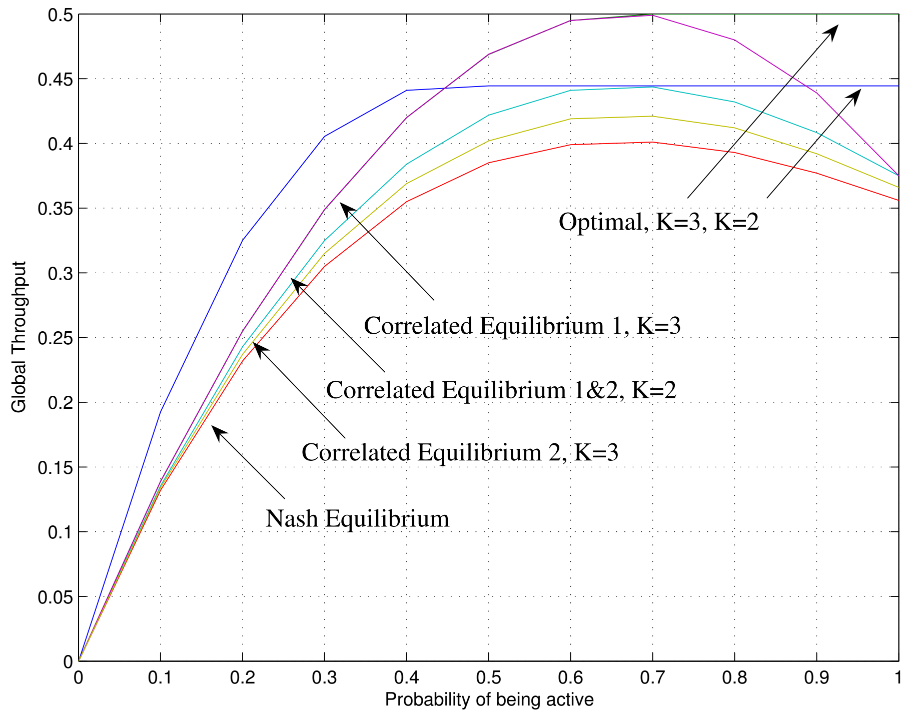

Figure 5.6: System throughput versus the probability of being active for a mobile with power constraint $q^{\max} = 0.25$, for 6 users.

5.2 Non-Atomic Games for CDMA

The performance of a CDMA system is analyzed in the context of frequency selective fading channels. Using game theoretic tools, a useful framework is provided in order to determine the optimal power allocation when users know only their own channel (while perfect channel state information is assumed at the base station). The realistic case of frequency selective channels for uplink CDMA is considered. This scenario illustrates the case of decentralized schemes, where limited information on the network is available at the terminal.

This represents an extension of [MPSM05] in the case of frequency-selective fading. We do not consider the case of multiple carriers, as in [MCPS06], and the results are very different to those obtained in that work. The extension is not trivial and involves advanced results on random matrices with non-equal variances due to Girko [Gir90] whereas classical results rely on the work of Silverstein [SB95]. A part of this work was published as a conference paper [BDAH07b]. Moreover, in addition to the linear filters studied in [MPSM05], we study the enhancements provided by the optimum and successive interference cancellation filters.

The goal is to derive simple expressions for the non-cooperative Nash equilibrium power allocation as the number of mobiles becomes large and the spreading length increases. Game theory can be used to treat the case of any number of players. However, as the size of the system increases, the number of parameters increases drastically and it is difficult to gain insight on the expressions obtained. In order to obtain expressions depending only on few parameters in the large system limit, two asymptotic methodologies are combined. The first is asymptotic random matrix theory which allows us to obtain explicit expressions of the impact of all other mobiles on any given tagged mobile. The second is the theory of non-atomic games which computes good approximations of the Nash equilibrium as the number of mobiles grows.

In the asymptotic regime, the non-cooperative game becomes a non-atomic one, in which the impact (through interference) of any single mobile on the performance of other mobiles is negligible. In the networking game context, the related solution concept is often called Wardrop equilibrium [War52]; it is often much easier to compute than the original Nash equilibrium [ABA$^+$06], and yet, the former equilibrium is a good approximation for the latter, see details in [HM85]. The non-atomic equilibrium is derived, and is shown to correspond generally to a non-uniform power allocation for the users.

The non-atomic Nash equilibrium is studied in this section for several linear receivers, namely the matched filter and the MMSE filter, as well as non-linear filters, such as the successive interference cancellation (SIC) [MV01] version of those filters. However, in order to perform SIC, the users need to know their decoding order, in order to adjust their rates. Two ways of obtaining an ordering of the users in a distributed manner are introduced. The ordering can be determined simply in a distributed manner under weak hypotheses. This gives rise to a different kind of power allocation, that depends explicitly on the order in which the users are decoded.

Moreover, the gain of the non-uniform power allocation with respect to uniform power allocation is quantified, according to the number of paths. The originality of the work lies in the fact that we show that as the number of paths increases, the optimal power allocation becomes more and more uniform due to the ergodic behavior of all the CDMA channels. This is reminiscent of an effect ("channel hardening") already revealed in MIMO [HMT04]. The highest gain (in terms of utility) is obtained in the case of flat fading (which also favors dis-uniform power allocation between the users).

5.2.1 Model

We consider a single uplink multi-user system cell, i.e., inter-cell interference free case. The spreading length is denoted N. The number of users in the cell is K. The load is $\alpha = K/N$. The general case of wide-band CDMA is considered, as in (3.20):

$$\begin{aligned} \mathbf{y} &= \sum_{k=1}^{K} \mathbf{D}_k \mathbf{w}_k \sqrt{P_k} s_k + \mathbf{n} \\ &= \left(\mathbf{H}\sqrt{\mathbf{P}} \odot \mathbf{W}\right)\mathbf{s} + \mathbf{n} \end{aligned} \tag{5.31}$$

where $\odot$ is the Hadamard (element-wise) product.

Notations are the same as in Chapter 3 and are summarized here for clarity. In (5.31), $\mathbf{H}$ is the frequency selective fading matrix, of size $N \times K$. $\sqrt{\mathbf{P}}$ is the root square of the diagonal power control matrix, of size $K \times K$. $\mathbf{W}$ is an $N \times K$ random spreading matrix.

In the following, we will assume that the frequency selective fading matrix $\mathbf{H}$ behaves ergodically, as in Def. 5. The two-dimensional channel profile of $\mathbf{H}\sqrt{\mathbf{P}}$ is denoted $\rho(f,x) = P(x)\,|h(f,x)|^2$, $f \in [0,1]$, $x \in [0,\alpha]$. f is the frequency index and x is the user index. This enables us to use Th. 2 in order to obtain expressions for the SINR.

It is also assumed that the power of all users is upper bounded by $P_{\max}$ and the square norm of the fading, on all paths, for all users, is upper bounded by $h_{\max}$.

5.2.2 Asymptotic SINR Expressions

Let $\mathbf{h}_k$ be the k-th column of $\mathbf{H}$, and $\mathbf{H}_{(-k)}$ be $\mathbf{H}$ with $\mathbf{h}_k$ removed. Similarly, let $\mathbf{w}_k$ be the k-th column of $\mathbf{W}$, and $\mathbf{W}_{(-k)}$ be $\mathbf{W}$ with $\mathbf{w}_k$ removed. Let $\sqrt{\mathbf{P}}_{(-k)}$ be $\sqrt{\mathbf{P}}$ with the k-th column and line removed. Finally, let $\mathbf{G}_{(-k)} = \mathbf{H}_{(-k)}\sqrt{\mathbf{P}}_{(-k)} \odot \mathbf{W}_{(-k)}$.

Matched Filter

Supposing perfect CSI at the receiver, the matched filter for the k-th user is given by $\mathbf{g}_k = \sqrt{P_k}\,(\mathbf{h}_k \odot \mathbf{w}_k)$. This leads to the following expression for the SINR of user k

$$\mathrm{SINR}_k = \frac{\left|\mathbf{g}_k^H \mathbf{g}_k\right|^2}{\sigma^2 \mathbf{g}_k^H \mathbf{g}_k + \mathbf{g}_k^H \left(\mathbf{G}_{(-k)}\mathbf{G}_{(-k)}^H\right)\mathbf{g}_k}.$$

Proposition 12 *[TLV05] As $N, K \to \infty$ with $K/N \to \alpha$, the SINR of user k at the output of the matched filter is given by*

$$\mathrm{SINR}_k = \beta^{MF}\left(\frac{k}{N}\right)$$

where $\beta^{MF} : [0, \alpha] \to \mathbb{R}$ is given by

$$\beta^{MF}(x) = P(x)\frac{(H(x))^2}{\sigma^2 H(x) + \int_0^\alpha \int_0^1 P(y)\left|h(f,y)\right|^2 \left|h(f,x)\right|^2 df dy} \tag{5.32}$$

and $H(x) = \int_0^1 \left|h(f,x)\right|^2 df$.

Denoting $\mathrm{SINR}_k = \beta_k^{\mathrm{MF}}$, Prop. 12 enables us to extract an approximation of the value of the SINR of user k in the finite size case

$$\beta_k^{\mathrm{MF}} = \frac{P_k\left(\frac{1}{N}\sum_{n=1}^N \left|h_{nk}\right|^2\right)^2}{\frac{\sigma^2}{N}\sum_{n=1}^N \left|h_{nk}\right|^2 + \frac{1}{N^2}\sum_{j\neq k}\sum_{n=1}^N P_j\left|h_{nj}\right|^2\left|h_{nk}\right|^2}. \tag{5.33}$$

We observe that $P_k\frac{\partial \beta_k^{\mathrm{MF}}}{\partial P_k} = \beta_k^{\mathrm{MF}}$.

MMSE Filter

Supposing perfect CSI at the receiver, the MMSE filter for the k-th user is given by $\mathbf{g}_k^{\mathrm{MMSE}} = \mathbf{R}^{-1}\mathbf{g}_k$, where $\mathbf{R} = \left(\left(\mathbf{H}\sqrt{\mathbf{P}} \odot \mathbf{W}\right)\left(\mathbf{H}\sqrt{\mathbf{P}} \odot \mathbf{W}\right)^H + \sigma^2 \mathbf{I}_N\right)$. This leads to the following expression for the SINR of user k [TH99]

$$\mathrm{SINR}_k = \mathbf{g}_k^H\left(\mathbf{G}_{(-k)}\mathbf{G}_{(-k)}^H + \sigma^2\mathbf{I}_N\right)^{-1}\mathbf{g}_k. \tag{5.34}$$

Proposition 13 *[TLV05] As $N, K \to \infty$ with $K/N \to \alpha$, the SINR of user k at the output of the MMSE receiver is given by:*

$$\mathrm{SINR}_k = \beta^{MMSE}\left(\frac{k}{N}\right)$$

where $\beta^{MMSE} : [0, \alpha] \to \mathbb{R}$ is a function defined by the implicit equation

$$\beta^{MMSE}(x) = P(x)\int_0^1 \frac{\left|h(f,x)\right|^2 df}{\sigma^2 + \int_0^\alpha \frac{P(y)\left|h(f,y)\right|^2 dy}{1+\beta^{MMSE}(y)}}. \tag{5.35}$$

Denoting $\mathrm{SINR}_k = \beta_k^{\mathrm{MMSE}}$, Prop. 13 enables us to extract an approximation of the value of the SINR of user k in the finite size case

$$\beta_k^{\mathrm{MMSE}} = P_k\frac{1}{N}\sum_{n=1}^N \left|h_{nk}\right|^2 \frac{1}{\sigma^2 + \frac{1}{N}\sum_{j\neq k}\frac{P_j\left|h_{nj}\right|^2}{1+\beta_j^{\mathrm{MMSE}}}}. \tag{5.36}$$

From (5.34), we observe that $P_k \frac{\partial \beta_k^{\text{MMSE}}}{\partial P_k} = \beta_k^{\text{MMSE}}$.
From Prop. 13, we have the capacity of user k

$$C_k^{\text{MMSE}} = \frac{1}{N} \log_2 \left(1 + \beta_k^{\text{MMSE}}\right).$$

The global capacity of the system is

$$C^{\text{MMSE}} = \int_0^{\alpha} \log_2 \left(1 + \beta^{\text{MMSE}}(x)\right) dx. \tag{5.37}$$

Optimal Filter

The term optimal filter designates a filter capable of decoding the received signal at the bound given by Shannon's capacity. Hence it is difficult to define an SINR associated to it. However, results of random matrix theory can still be applied. Let $\mathbf{Y} = \left(\mathbf{H}\sqrt{\mathbf{P}} \odot \mathbf{W}\right)$. The definition of Shannon's capacity per dimension for our system is

$$C_{(N)}^{\text{OPT}} = \frac{1}{N} \log_2 \det \left(\mathbf{I}_N + \frac{1}{\sigma^2}\mathbf{Y}\mathbf{Y}^H\right). \tag{5.38}$$

As $N, K \to \infty$ with $K/N \to \alpha$,

$$C_{(N)}^{\text{OPT}} \to \int \log_2 \left(1 + \frac{1}{\sigma^2} t\right) \nu(dt) \tag{5.39}$$

where ν is the empirical eigenvalue distribution of $\mathbf{Y}\mathbf{Y}^H$, as in Def. 3. If we differentiate the asymptotic value C^{OPT} of (5.39) with respect to σ^2, we obtain

$$\begin{aligned}
\frac{\partial C^{\text{OPT}}}{\partial \sigma^2} &= \log_2(e) \int \frac{-\frac{1}{\sigma^4} t}{1 + \frac{1}{\sigma^2} t} \nu(dt) \\
&= \log_2(e) \int \frac{\sigma^2 \left(-\frac{1}{\sigma^4} t - \frac{1}{\sigma^2} + \frac{1}{\sigma^2}\right)}{\sigma^2 \left(1 + \frac{1}{\sigma^2} t\right)} \nu(dt) \\
&= \log_2(e) \left(\int \frac{1}{t + \sigma^2} \nu(dt) - \frac{1}{\sigma^2} \int \nu(dt)\right) \\
&= \log_2(e) \left(m^{\nu}(-\sigma^2) - \frac{1}{\sigma^2}\right)
\end{aligned} \tag{5.40}$$

where $m^{\nu}(\cdot)$ is the Stieltjes transform of the empirical eigenvalue distribution of $\mathbf{Y}\mathbf{Y}^H$. From Th. 2, $m^{\nu}(\cdot)$ is given by

$$m^{\nu}(z) = \int_0^1 u(f, z) df$$

where $u(f, z)$ is given by (2.3) with $\rho^{\mathbf{H}\sqrt{\mathbf{P}}}(f, x) = \rho(f, x) = P(x) \left|h(f, x)\right|^2$. Given that if $\sigma^2 = +\infty$, $C^{\text{OPT}} = 0$, it is immediate to obtain C^{OPT} from (5.40) as

$$C^{\text{OPT}} = \log_2(e) \int_{\sigma^2}^{+\infty} m^{\nu}(-z) - \frac{1}{z} dz. \tag{5.41}$$

Proposition 14 *C^{OPT} and C^{MMSE} are related through the following equality*

$$C^{OPT} = C^{MMSE} - \log_2(e) \int_0^{\alpha} \frac{\beta^{MMSE}(x)}{1+\beta^{MMSE}(x)} dx$$
$$+ \int_0^1 \log_2 \left(1 + \frac{1}{\sigma^2} \int_0^{\alpha} \frac{\rho(f,x)}{1+\beta^{MMSE}(x)} dx \right) df. \tag{5.42}$$

Proof *See Appendix 7.2.*

The additional term in the right-hand side of (5.42) corresponds to the non-linear processing gain. It quantifies the gain in terms of capacity that can be achieved between pure linear MMSE and non-linear filtering.

Assuming perfect cancellation of decoded users, successive interference cancellation with MMSE filter achieves the optimum capacity [Mö01]. The following proposition ensues from this fact.

Proposition 15 *[TLV05] As $N, K \to \infty$ with $K/N \to \alpha$, the optimal capacity is given by:*

$$C^{OPT} = \int_0^{\alpha} \log_2 \left(1 + \beta^{SIC}(x)\right) dx$$

where $\beta^{SIC} : [0, \alpha] \to \mathbb{R}$ is a function defined by the implicit equation

$$\beta^{SIC}(x) = P(x) \int_0^1 \frac{|h(f,x)|^2 \, df}{\sigma^2 + \int_0^x \frac{P(y)|h(f,y)|^2 dy}{1+\beta^{SIC}(y)}}. \tag{5.43}$$

Prop. 15 enables us to extract an expression that is analog to the SINR for the optimal filter. Similarly to the case of β^{MMSE} in Sec. 5.2.2, the derivative of this expression obeys the property $P_k \frac{\partial \beta_k^{\mathrm{SIC}}}{\partial P_k} = \beta_k^{\mathrm{SIC}}$.

5.2.3 Games and Equilibria

From now on, we denote $\mathrm{SINR}_k = \beta_k$, whichever filter is actually used.

Power Allocation Game

A game with a unique strategy set for all users is defined by a triple $\{S, \mathbb{S}, (u_k)_{k \in S}\}$ where S is the set of *players*, $\mathbb{S}$ is the set of *strategies*, and $(u_k)_{k \in S}$ is the set of *utility functions*, $u_k : \mathbb{S}^{|S|} \to \mathbb{R}$.

In our setting, the players are simply the users, indexed by the set $S^K = \{1, \ldots, K\}$. The strategy for a mobile is its power allocation P_k, which we will assume belongs to a compact interval $\mathbb{S} = [0, P_{\max}] \subseteq \mathbb{R}$. The utility measures the gain of a user as a result of the strategy this user plays. In [Rod03], the author derives what he calls Throughput to Power Ratio (TPR) under minimal requirements. The utility of user k is expressed

$$u_k = \frac{\gamma_k}{P_k}. \tag{5.44}$$

Denote $\gamma_k = \gamma(\beta_k)$, where $\gamma(\cdot)$ is the same function for all users. In (5.44), γ is at least C^2 and should satisfy conditions detailed in [Rod03] in order to obtain an "interesting" equilibrium.

For example, in the simulations, we consider the goodput $\gamma(\beta_k)$, which is proportional to $\left(1 - e^{-\beta_k}\right)^M$ where M is the number of bits transmitted in a CDMA packet, as detailed in Sec. 3.5.4.

This utility is expressed in *bits per joule*. In the non-cooperative game setting, each user wants to selfishly maximize its utility. A Nash equilibrium is obtained when no user can benefit by unilaterally deviating from its strategy.

To obtain the maximum utility achievable by user k, we differentiate u_k with respect to the power P_k and equate to 0. We obtain

$$P_k \frac{\partial \beta_k}{\partial P_k} \gamma'(\beta_k) - \gamma(\beta_k) = 0. \tag{5.45}$$

For all filters under consideration, (5.32), (5.35) and (5.43) imply $P_k \frac{\partial \beta_k}{\partial P_k} = \beta_k$, thus (5.45) reduces an equation on β_k

$$\beta_k \gamma'(\beta_k) - \gamma(\beta_k) = 0. \tag{5.46}$$

Eq. (5.46) is particularly interesting in the case when there exists a unique solution $\beta^\star$.

The existence of a solution to (5.46) is guaranteed as long as the function $\gamma(\cdot)$ is a quasiconcave function of the SINR, i.e., there exists a point below which the function is non-decreasing, and above which the function is non-increasing [SMG02, Rod03]. In addition, we assume that the function $\gamma(\cdot)$ takes value $\gamma(0) = 0$, so that users cannot achieve an infinite utility by not transmitting. This occurs for several functions $\gamma(\cdot)$ of interest, in particular the goodput [MPSM05], which we will use for simulations. Unfortunately, the capacity can not be used as a function $\gamma(\cdot)$, since it leads to the trivial result $\beta^\star = 0$ for this utility function. The uniqueness of the solution $\beta^\star$ to (5.46) is due to fact that the SINR of each user is a strictly increasing function of its transmit power. Given the target SINR $\beta^\star$, we obtain the strategy of users in the next section.

5.2.4 Power Allocation in the Nash Equilibrium

Flat Fading

In this subsection, we show that the results of [MPSM05] for Matched and MMSE filters are a special case of our setting when $L = 1$ (flat fading case). In addition, we derive the power allocation for the Optimum filter. When there is only one path, for each user k, denoted by its index $\frac{k}{N} = x \in [0, \alpha]$, $h(f, x)$ does not depend on f. Given the target SINR $\beta^\star$, we have explicit expressions of the power with which user k transmits for the various receivers.

In Appendix 7.2, we show that the influence of the strategy of a player on the payoffs of other players is (asymptotically) "small". It justifies the fact that we can obtain an equilibrium in the asymptotic setting, without the need for players to possess all the information on the system. Their local information is sufficient. In the asymptotic limit, we obtain results similar to Wardrop equilibrium: the strategy used by each user does not influence the strategy of other users.

Matched filter

From Prop. 12, the continuous formulation is

$$P(x) = \frac{\beta^\star \left(\sigma^2 + \int_0^\alpha P(y) \left|h(y)\right|^2 dy\right)}{\left|h(x)\right|^2}$$

or equivalently in a discrete form

$$P_k = \frac{\beta^\star \left(\sigma^2 + \frac{1}{N} \sum_{j=1, j\neq k}^{K} P_j \left|h_j\right|^2\right)}{\left|h_k\right|^2}. \tag{5.47}$$

Summing (5.47) over $k = 1, \ldots, K$, we obtain a closed form expression for the minimum power with which user k transmits when using the matched filter

$$P_k = \frac{1}{\left|h_k\right|^2} \frac{\sigma^2 \beta^\star}{1 - \alpha\beta^\star} \text{ for } \alpha < \frac{1}{\beta^\star}. \tag{5.48}$$

MMSE filter

From Prop. 13, the continuous formulation is

$$P(x) = \frac{\beta^\star \left(\sigma^2 + \frac{1}{1+\beta^\star} \int_0^\alpha P(y) \left|h(y)\right|^2 dy\right)}{\left|h(x)\right|^2}.$$

or equivalently in a discrete form

$$P_k = \frac{\beta^\star \left(\sigma^2 + \frac{1}{1+\beta^\star} \frac{1}{N} \sum_{j=1, j\neq k}^{K} P_j \left|h_j\right|^2\right)}{\left|h_k\right|^2}. \tag{5.49}$$

Summing (5.49) over $k = 1, \ldots, K$, we obtain a closed form expression for the minimum power with which user k transmits when using the MMSE filter

$$P_k = \frac{1}{\left|h_k\right|^2} \frac{\sigma^2 \beta^\star}{1 - \alpha \frac{\beta^\star}{1+\beta^\star}} \text{ for } \alpha < 1 + \frac{1}{\beta^\star}. \tag{5.50}$$

Both (5.48) and (5.50) are the same results as in [MPSM05].

Optimum filter

Each user maximizes its utility for a SINR equal to $\beta^\star$. However, in the case of the optimum filter, the SINR is not defined directly. It is nevertheless possible to extract an equivalent quantity from the expression of the capacity, since the value of the capacity of user k at the equilibrium is given by $C^\star = \frac{1}{N} \log_2 (1 + \beta^\star)$.

Proposition 16 *The power allocation is given by*

$$P_k = \frac{1}{|h_k|^2} \frac{\sigma^2 \beta^+}{1 - \alpha \frac{\beta^+}{1+\beta^+}} \textit{ for } \alpha < 1 + \frac{1}{\beta^+} \tag{5.51}$$

where β^+ *is the solution to*

$$\alpha \log_2\left(1+\beta^+\right) - \alpha \log_2(e) \frac{\beta^+}{1+\beta^+} + \log_2\left(1 + \frac{1}{1+\beta^+} \frac{\alpha\beta^+}{1 - \alpha\frac{\beta^+}{1+\beta^+}}\right) = \alpha \log_2\left(1+\beta^\star\right). \tag{5.52}$$

Proof *See Appendix 7.2.*

Frequency Selective Fading

In the context of frequency selective fading, for each user k, denoted by its index $\frac{k}{N} = x \in [0, \alpha]$, there are $L > 1$ paths with respective attenuations $h_\ell(x)$, $\ell = 1, \ldots, L$, which are i.i.d. random variables with some known distribution. We suppose that $h_\ell(x)$ has mean zero, and the distributions of the real part and imaginary part of $h_\ell(x)$ are even functions, as for example the Gaussian distribution, which we consider in the simulations. $h(f, x)$ depends on f through $h(f, x) = \sum_{\ell=1}^{L} h_\ell(x) e^{-2\pi i f(\ell-1)}$. Given the target SINR $\beta^\star$, the Nash equilibrium power allocation is determined by implicit equations for the various receivers.

Matched filter

The continuous formulation is

$$P(x) = \beta^\star \frac{\sigma^2 H(x) + \int_0^1 \int_0^\alpha P(y) \left|h(f,y)\right|^2 \left|h(f,x)\right|^2 df dy}{\left(H(x)\right)^2}$$

or equivalently in a discrete form

$$P_k = \beta^\star \frac{\frac{\sigma^2}{N} \sum_{n=1}^{N} |h_{nk}|^2 + \frac{1}{N} \sum_{n=1}^{N} |h_{nk}|^2 \frac{1}{N} \sum_{j \neq k}^{K} P_j \left|h_{nj}\right|^2}{\left(\frac{1}{N} \sum_{n=1}^{N} |h_{nk}|^2\right)^2}. \tag{5.53}$$

In (5.53), $h_{nk} = h\left(\frac{n-1}{N}, \frac{k}{N}\right)$.

In this expression, the power allocation of user k seems to depend on the power allocation and fading realization of all the other users. However, when the number of users tends to infinity, the strategy of any single user does not have any influence on the payoff of user k, as shown in Appendix 7.2. Hence, the appropriate framework is non-atomic games. The expression $\frac{1}{N} \sum_{j=1}^{K} P_j \left|h_{nj}\right|^2$ is asymptotically a constant (not depending on n), denoted Ω.

$$\Omega = \frac{\alpha \beta^\star \sigma^2 \frac{1}{K} \sum_{j=1}^{K} \frac{|h_{nj}|^2}{E_j}}{1 - \alpha\beta^\star \frac{1}{K} \sum_{j=1}^{K} \frac{|h_{nj}|^2}{E_j}} \tag{5.54}$$

where $E_j = \frac{1}{N}\sum_{m=1}^{N}|h_{mj}|^2$.

As $K \to \infty$, we can apply the Central Limit Theorem to the sum of random variables

$$\frac{1}{K}\sum_{j=1}^{K}\frac{|h_{nj}|^2}{E_j}. \tag{5.55}$$

It tends to its expectation, which is equal to 1 (see Appendix 7.2).

It follows that asymptotically $\Omega = \frac{\alpha\beta^\star\sigma^2}{1-\alpha\beta^\star}$ (and simulations in Sec. 5.2.6 prove that this approximation is valid for moderate finite values of N). From (5.53), we obtain a formula similar to (5.48)

$$P_k = \frac{1}{E_k}\frac{\sigma^2\beta^\star}{1-\alpha\beta^\star} \text{ for } \alpha < \frac{1}{\beta^\star}. \tag{5.56}$$

MMSE filter

The continuous formulation is

$$P(x) = \frac{\beta^\star}{\int_0^1 \frac{|h(f,x)|^2 df}{\sigma^2 + \frac{1}{1+\beta^\star}\int_0^\alpha P(y)|h(f,y)|^2 dy}} \tag{5.57}$$

or equivalently in a discrete form

$$P_k = \frac{\beta^\star}{\frac{1}{N}\sum_{n=1}^{N}\frac{|h_{nk}|^2}{\sigma^2 + \frac{1}{1+\beta^\star}\frac{1}{N}\sum_{j=1,j\neq k}^{K}P_j|h_{nj}|^2}}. \tag{5.58}$$

In (5.58), $h_{nk} = h\left(\frac{n-1}{N}, \frac{k}{N}\right)$.

As previously, when the number of users tends to infinity, $\frac{1}{N}\sum_{j=1}^{K}P_j|h_{nj}|^2$ is asymptotically a constant (not depending on n), denoted Ω.

$$\Omega = \frac{\alpha\beta^\star\sigma^2\frac{1}{K}\sum_{j=1}^{K}\frac{|h_{nj}|^2}{E_j}}{1 - \frac{\alpha\beta^\star}{1+\beta^\star}\frac{1}{K}\sum_{j=1}^{K}\frac{|h_{nj}|^2}{E_j}} \tag{5.59}$$

where $E_j = \frac{1}{N}\sum_{m=1}^{N}|h_{mj}|^2$.

It follows that asymptotically $\Omega = \frac{\alpha\beta^\star\sigma^2}{1-\alpha\frac{\beta^\star}{1+\beta^\star}}$, we obtain a formula similar to (5.50)

$$P_k = \frac{1}{E_k}\frac{\sigma^2\beta^\star}{1-\alpha\frac{\beta^\star}{1+\beta^\star}} \text{ for } \alpha < 1 + \frac{1}{\beta^\star}. \tag{5.60}$$

Optimum filter

Each user maximizes its utility for a SINR equal to $\beta^\star$. However, in the case of the optimum filter, the SINR is not defined directly. It is nevertheless possible to extract an equivalent quantity from the expression of the capacity, since the value of the capacity of user k at the equilibrium is given by $C^\star = \frac{1}{N}\log_2(1+\beta^\star)$.

Proposition 17 *Asymptotically, as $N, K \to \infty$, the power allocation is given by*

$$P_k = \frac{1}{E_k} \frac{\sigma^2 \beta^+}{1 - \alpha \frac{\beta^+}{1+\beta^+}} \text{ for } \alpha < 1 + \frac{1}{\beta^+} \tag{5.61}$$

where β^+ is the solution to

$$\alpha \log_2 \left(1 + \beta^+\right) - \alpha \log_2(e) \frac{\beta^+}{1 + \beta^+} + \log_2 \left(1 + \frac{1}{1+\beta^+} \frac{\alpha \beta^+}{1 - \alpha \frac{\beta^+}{1+\beta^+}}\right) = \alpha \log_2 \left(1 + \beta^\star\right). \tag{5.62}$$

Proof *The proof is similar to the proof of Prop. 16.*

We observe that for all filters considered, the optimal power allocation is a constant times the inverse of the *total* energy of the channel E_j. Via Parseval's Theorem, $E_j = \sum_{\ell=1}^{L} \left|h_\ell\left(\frac{j}{N}\right)\right|^2$. It is a sum of i.i.d. random variables. As the number of paths increases, the optimal power allocation tends to a uniform power allocation. This is an effect similar to "channel hardening" [HMT04]: as the number of paths increases, the variance of the distribution of the channel energy decreases and the Nash equilibrium power allocation becomes more and more uniform for all users.

5.2.5 Successive Interference Cancellation

The optimal filter gives a bound on the performance that can be achieved through (non-linear) filtering at the base station. In order to improve the performance of the system, we introduce Successive Interference Cancellation (SIC) [MV01] at the base station. Under the assumption of perfect decoding, SIC improves immensely the performance of linear filters (Matched Filter or MMSE Filter). The MMSE SIC filter actually achieves the optimum filter bound, under the assumption of perfect decoding. The principle of SIC receivers is quite simple: users are ordered and are decoded successively. At each step, supposing that the user has been encoded at the appropriate decoding rate, the signal is decoded and its contribution to the interference is then perfectly subtracted. This removes some of the inter-user interference and therefore increases the SINR of the following decoded users.

The challenge is that the users must transmit at the appropriate rate to avoid the catastrophic occurrence of imperfect decoding. Usually, the ordering of users is done in a centralized way, at the base station which then advertises it to the users. However, for the protocol to remain distributed, users should be able to decide, based on their local information, at which rate to transmit.

At equilibrium, the rate is determined by the SINR $\beta^\star$, and it is the transmission power of the user that is determined according to its rank of decoding. The equilibrium power allocation can be determined in a simple manner when the number of multipaths is finite ($L < \infty$) and the number of users is very high ($K \to \infty$). In Sec. 5.2.5, we make use of the fact that the whole law of E_j is realized in this case, so that users automatically know their rank of decoding. Another manner to give a (random) ordering of decoding is to introduce an additional degree of liberty in the system. In Sec. 5.2.5, we develop a correlated game framework that enables users to learn their rank of decoding in a simple way. In the following, we assume that each user has a unique has a unique i.d. number j ranging between 1 to K.

Ordering when $K \to \infty$

If the number of users $K \to \infty$, with L fixed, the whole law of the total channel energy will be realized. More specifically, we make use of the following lemma from [SV02].

Lemma 1 *Denote by $D(\cdot)$ the cumulative distribution function of the total channel energy coefficients E_j, and by $(E_{\pi(1)}, \dots, E_{\pi(K)})$ the vector of coefficients ordered by decreasing order. Then, $E_{\pi(j)}$ converges in probability, as $K \to \infty$, to $D^{-1}\left(\frac{K-j}{K}\right)$ for $j = 1, \dots, K$.*

Assume the base station advertises to the users that they will be decoded by decreasing total channel energy. Each user knows, according to the realization of its fading, its rank in the decoding order given by K times 1 minus the cumulative distribution function $D(\cdot)$ of the total channel energy E_j.

$$\mathrm{rank}_j = K(1 - D(E_j)).$$

In case that the base station advertises to the users that they will be decoded by increasing total channel energy, user j will have rank $\mathrm{rank}_j = KD(E_j)$.

Correlated Equilibrium

We wish to introduce a simple mechanism that enables players to coordinate and to know in which order they will be decoded. We place ourselves in the context of correlated games, as introduced in Sec. 2.1.3

The simplest and most intuitive coordination mechanism is given by a common signal which users as well as the base station overhear before each transmission. There are $K!$ possible permutations of K users. Hence, the arbitrator broadcasts a signal to the users belonging to the set $\{0, \dots, K! - 1\}$. Each of these numbers corresponds to a permutation π of $\{1, \dots, K\}$ that gives the (random) ordering of decoding as $\mathrm{rank}_j = \pi(j)$. The users can then adjust their transmit power according to this ordering. In terms of size of the message, this is equivalent to the case when the base station decides the decoding order and broadcasts it to the users, or sends K individual messages of $\ln(K)$ bits containing the rank, since $\ln(K!) = K\ln(K) + o(K\ln(K))$. However, there is no need of either any knowledge of the system or computations at the base station in the case of the correlated mechanism.

SIC Power Allocations

In both cases, once the users know their order, they can calculate their transmit power according to the filter that is used. The equilibrium still occurs when all users reach the SINR $\beta^\star$. A single user will not benefit by deviating, since it would decrease its utility. From now on, index k denotes the rank of decoding.

In the case of the matched filter with SIC, the SINR of the user decoded at rank k is

$$\beta_k^{\mathrm{MF}} = \frac{P_k \left(\frac{1}{N}\sum_{n=1}^{N} |h_{nk}|^2\right)^2}{\frac{\sigma^2}{N}\sum_{n=1}^{N} |h_{nk}|^2 + \frac{1}{N^2}\sum_{j>k}\sum_{n=1}^{N} P_j |h_{nj}|^2 |h_{nk}|^2}. \tag{5.63}$$

From (5.63), we get the equilibrium power allocation of user k as

$$P_k = \beta^\star \frac{\frac{\sigma^2}{N}\sum_{n=1}^{N} |h_{nk}|^2 + \frac{1}{N^2}\sum_{j>k}\sum_{n=1}^{N} P_j |h_{nj}|^2 |h_{nk}|^2}{\left(\frac{1}{N}\sum_{n=1}^{N} |h_{nk}|^2\right)^2}. \tag{5.64}$$

In the case of the MMSE filter with SIC, the SINR of the user decoded at rank k is

$$\beta_k^{\mathrm{MMSE}} = P_k \frac{1}{N}\sum_{n=1}^{N} |h_{nk}|^2 \frac{1}{\sigma^2 + \frac{1}{N}\sum_{j>k}\frac{P_j |h_{nj}|^2}{1+\beta_j^{\mathrm{MMSE}}}}. \tag{5.65}$$

From (5.65), we get the equilibrium power allocation of user k as

$$P_k = \frac{\beta^\star}{\frac{1}{N}\sum_{n=1}^{N}\frac{|h_{nk}|^2}{\sigma^2 + \frac{1}{1+\beta^\star}\frac{1}{N}\sum_{j>k}^{K} P_j |h_{nj}|^2}}. \tag{5.66}$$

For flat fading, a simple recursion gives the equilibrium power allocation (see Appendix 7.2). We obtain respectively

$$P_k^{\mathrm{MF}} = \frac{\sigma^2\beta^\star}{|h_k|^2}\left(1 + \frac{1}{N}\beta^\star\right)^{K-k}, \tag{5.67}$$

$$P_k^{\mathrm{MMSE}} = \frac{\sigma^2\beta^\star}{|h_k|^2}\left(1 + \frac{1}{N}\frac{\beta^\star}{1+\beta^\star}\right)^{K-k}. \tag{5.68}$$

As far as frequency-selective fading is concerned, this gives us the form of the asymptotic expressions. Asymptotically, the power allocation of one user will not depend on the power allocation of the other users, as shown in Appendix 7.2. With a similar reasoning as in Sec. 5.2.4, the expressions mimic (5.67) and (5.68) with the total channel energy E_k replacing $|h_k|^2$, i.e.,

$$P_k^{\mathrm{MF}} = \frac{\sigma^2\beta^\star}{E_k}\left(1 + \frac{1}{N}\beta^\star\right)^{K-k}, \tag{5.69}$$

$$P_k^{\mathrm{MMSE}} = \frac{\sigma^2\beta^\star}{E_k}\left(1 + \frac{1}{N}\frac{\beta^\star}{1+\beta^\star}\right)^{K-k}. \tag{5.70}$$

These expressions are also validated by simulations.

Since MMSE SIC with perfect decoding is equivalent to the optimum filter, we thus obtain a second possible equilibrium power allocation for the optimum filter. In Sec. 5.2.6, we investigate which is the power allocation which minimizes total amount of power needed to transmit at equilibrium SINR. In the case of automatic ordering of the users, one question is whether it is best to order the users by increasing or decreasing total fading energy. The answer is the following: it is always best to decode the users by decreasing total channel energy $E_1 < \cdots < E_k$ (see Appendix 7.2).

An interesting feature of equilibrium power allocation (5.69) and (5.70) is that there is no limitation on the number of users than can be accomodated by the system, contrary to the previous case of (5.56), (5.60) and (5.61). The limitation is only imposed by the increasing power needed for each new user decoded last, which grows without bound as an exponential.

5.2.6 Numerical Results

In all the following, we consider that $P_{\max}$ is chosen sufficiently high so that users can actually transmit at the equilibrium power allocation values. For the simulations, we consider the usual case of Rayleigh fading. Although Rayleigh distribution is not bounded from above, simulations show that the results still hold.

We consider a CDMA system with $K = 32$ users and a spreading factor $N = 256$. The noise variance is $\sigma^2 = 10^{-10}$. For a number of bits in a CDMA packet $M = 100$, the goodput is $\gamma(\beta) = \left(1 - e^{-\beta}\right)^{100}$, and $\beta^\star = 6.48$. The capacity achieved at the Nash Equilibrium is $C = \alpha \log_2 (1 + \beta^\star) = 0.39$ bits/s. Unfortunately, the capacity itself cannot be used as a relevant performance measure in the definition of the utility, because in this case the maximal utility is obtained when not sending.

We have performed simulations over 10000 realizations. Fig. 5.7 shows the good fit of theoretic values calculated directly from (5.56), (5.60) and (5.61) (thick straight lines) with the simulation points for various numbers of multipaths (losanges). The values of the utility do not depend on the number of multipaths. We see that optimum filter requires the minimal power, and matched filter the maximal power to achieve the required goodput.

In Fig. 5.8 we have plotted the average utility versus the number of multipaths L. Multipaths are supposed to be i.i.d. Rayleigh distributed with variance $1/L$, in order for the channels to have the same energy. Two cases are considered: the utility obtained in the Nash equilibrium, according to the power allocation given by (5.53) and (5.58), and the utility in the case where all nodes transmit at the same power. For comparison purposes, the sum of the uniform powers is equal to the sum of the powers used in the Nash equilibrium. In addition, simulations (not reproduced here) show that this value gives the higher average utility for a uniform power allocation.

The utility does not vary with L in the Nash equilibrium: the Central Limit Theorem applies to the utility, which is a constant times the random variable E_k in the Nash equilibrium. The utility with uniform powers is always inferior to the utility in the Nash equilibrium. However, as L increases, the gap decreases, as the variance of E_k decreases, and the equilibrium power allocation becomes uniform.

In Fig. 5.9 we have plotted the average of the inverse power of the users in the Nash equilibrium for each of the investigated schemes. We plot the average inverse power because of the direct relation to the utility for the users. The higher this average, the higher the utility for the user. The SIC filters are always more efficient

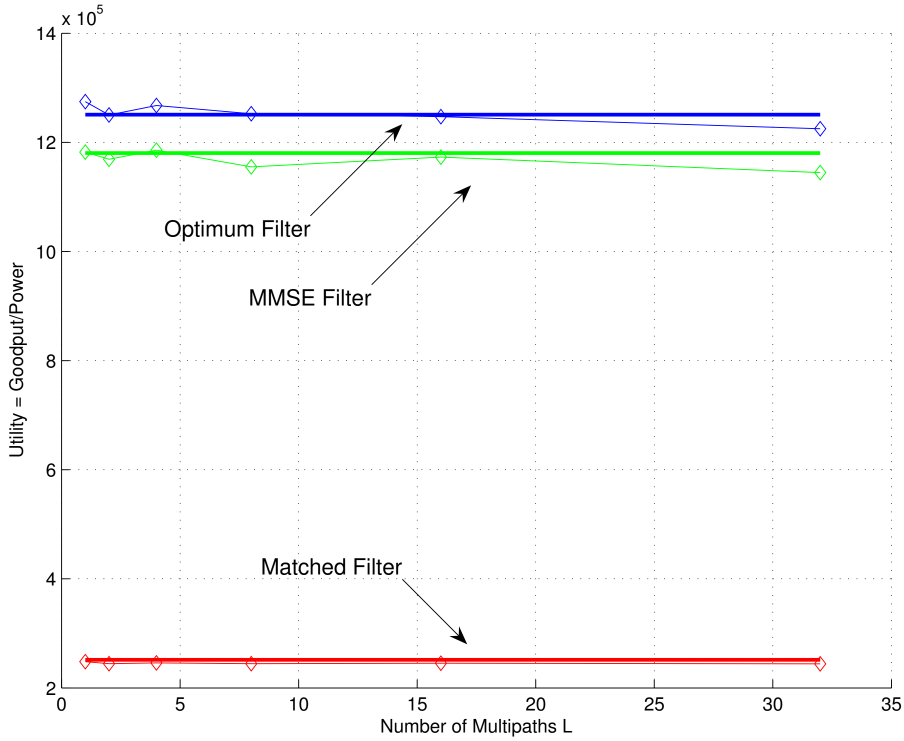

Figure 5.7: Comparison of theoretic values and simulations for utilities in the Nash equilibrium.

than their linear counterparts. However, for a load $\alpha < 0.12$ and optimum filter[1], it is better to use the first variation of power allocation (5.61) than use MMSE SIC (5.70). This relation is reversed when $\alpha > 0.12$. In addition to the theoretical curves, Monte-Carlo simulations were performed both with random ordering (circles) and ordering by decreasing total channel energy (crosses), for $L = 8$ multipaths. Simulations show that the optimal ordering improves the power efficiency of the successive interference cancellation filters.

In Fig. 5.10, we investigate the amelioration provided by optimal ordering as a function of the number of multipaths. The simulations are done for $K = 128$ users, in order to be in the "interesting" zone $\alpha > 0.12$. As expected, as the number of paths increases, the total channel energy is more and more the same for each channel and the gain provided by ordering the users decreases. However, when the number of users is very large and they benefit from automatic ordering, we see that the utility with the MMSE SIC equilibrium power allocation is the maximal utility that can be obtained in the non-cooperative setting.

5.2.7 Conclusion

Using tools of random matrices, we have derived the equilibrium power allocation in a game-theoretic framework applied to asymptotic CDMA with cyclic prefix, under

[1]The value of α is obtained as solution of the equation $\alpha\beta^\star \frac{\beta^\star}{1+\beta^\star}(1 - \alpha\frac{\beta^+}{1+\beta^+}) = \beta^+(1 - \exp(-\alpha\frac{\beta^\star}{1+\beta^\star}))$.

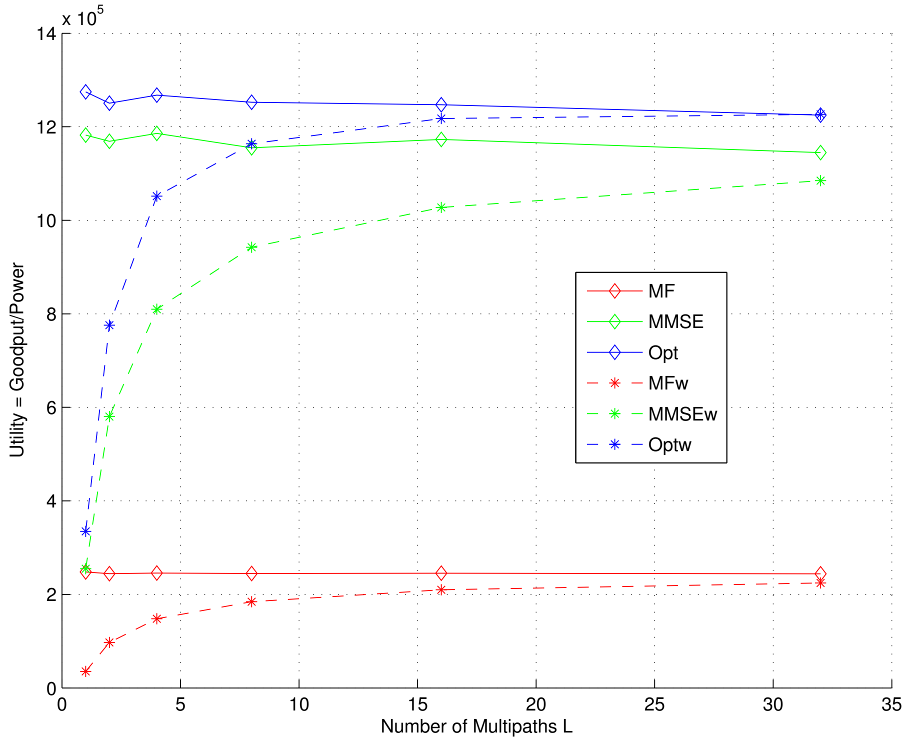

Figure 5.8: Simulation of utilities in the Nash equilibrium and constant power allocations versus L.

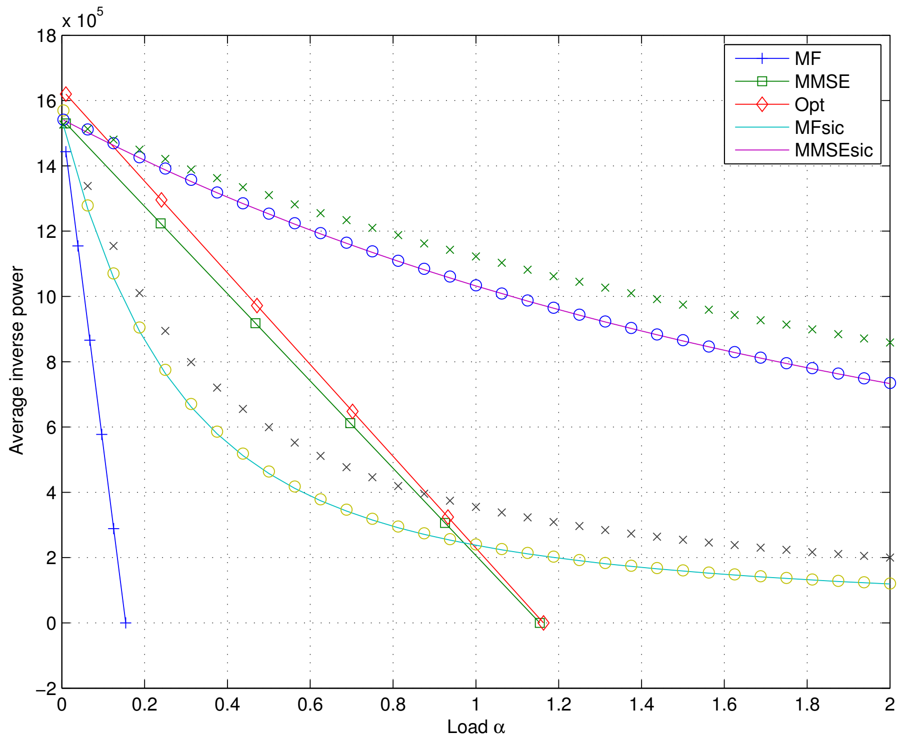

Figure 5.9: Average inverse power used by the different filters.

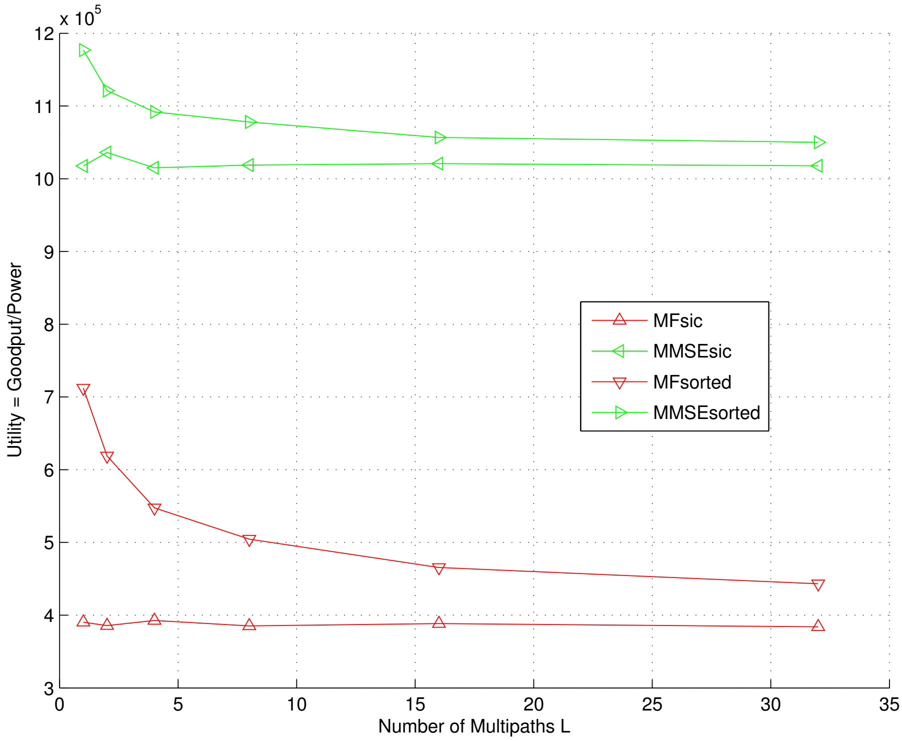

Figure 5.10: Simulation of utilities in the Nash equilibrium with SIC filter with and without optimal ordering, versus L.

frequency-selective fading. Three receivers are considered: matched filter, MMSE and optimum filter (given by Shannon's capacity). In addition, distributed ordering mechanisms are introduced and the successive interference cancellation variants of the linear filters are studied. For each user, this power allocation depends only on the total energy of the channel of the user under consideration. For a frequency-flat channel, the power allocation among users is highly dis-uniform, whereas when the number of multipaths increases, the power allocation tends more and more to a uniform one.

5.3 Channel Inversion Schemes for OFDMA

A novel multi-user diversity scheme for OFDMA is described which alleviates the need of feedback. Namely, each user knows only the channel coefficients of its N carriers whereas the scheduler has no channel knowledge.

The algorithm exploits the reciprocity of the channel. A broadcast training sequence is sent by the base station to all the users at the beginning of the communication. Each user estimates its channel and based on an algorithm detailed afterwards selects the carriers ensuring the required data rate.

Under mild asymptotic conditions, the algorithm enables each user to send reliably data at a prescribed rate knowing only its channel. For several channel models, we derive analytical expressions of the cell spectral efficiency in the asymptotic regime (high number of carriers) for two filter types: matched filter and optimum filter. This section is based on [BDHA05].

5.3.1 Model

The scheme considers Time Division Duplex (TDD) mode slotted transmissions. The channel is assumed to have coherence time $T_C = DNT$, where T is the time to transmit one information symbol, and NT is the time to transmit one OFDM symbol with N symbols. At the beginning of a slot, the scheduler sends a broadcast training sequence to all the users. It is a known sequence of $G < D$ OFDM symbols. Each user k estimates its N carriers $h_k(i)$, $i \in \{1, \ldots, N\}$ and can transmit during the remaining time $(D - G)NT$. The spectral efficiency should therefore be reduced by a factor $\frac{D-G}{D}$. In all the following, we will however suppose that the channel is perfectly estimated and $\frac{D-G}{D} \to 1$.

Each user has a total power budget PW, where P is the power spectral density and W is the available bandwidth[2]. Constrained on its power budget, user k selects S_k carriers, where S_k depends on the particular realization of the fading. On each selected carrier i, user k sends the information $x_k(i) = \frac{s_k(i)}{h_k(i)}$, where $s_k(i)$ is the transmitted data such as $\mathbb{E}\left[|s_k(i)|^2\right] = \frac{PW}{N}$. Therefore, the scheduler does not need to know the channel state information in this "channel inversion scheme" which alleviates the need of a feedback mechanism.

Each user chooses a set $\mathbb{S}_k \subseteq \{1, \ldots, N\}$ of S_k ($1 \leq S_k \leq N$) carriers such as $\sum_{i \in \mathbb{S}_k} \frac{PW}{N|h_k(i)|^2} \leq PW$. Thus $\mathbb{S}_k$ is the subset of cardinal S_k of $\{1, \ldots, N\}$ such that $\mathbb{S}_k$ contains the S_k best carriers that satisfy

$$\frac{1}{N} \sum_{i \in \mathbb{S}_k} \frac{1}{|h_k(i)|^2} \leq 1. \tag{5.71}$$

Note that in this non-cooperative scenario, for each carrier i, a set $\mathbb{M}_i \subseteq \{1, \ldots, K\}$ of users can select the same frequency carrier i, which introduces interference. As a consequence, the received signal on carrier i at the base station is given by an

[2]Note that in this setting the inter-carrier spacing is $\frac{W}{N}$.

equation similar to (3.21).

$$y(i) = \sum_{k \in \mathbb{M}_i} s_k(i) + n(i)$$

where $n(i)$ is a zero mean gaussian noise with variance $\frac{N_0 W}{N}$.

5.3.2 Spectral Efficiency

Let $\mathrm{card}(\mathbb{M}_i) = M_i$. Note that $\sum_{i=1}^{N} M_i = \sum_{k=1}^{K} S_k$. Assuming that each transmitter simultaneously transmits Gaussian-like signals using a different random code book (known by the base station), the spectral efficiency of the cell is given by:

- For the optimum filter:

$$\gamma_{\text{optimum}}(K) = \frac{1}{N} \sum_{k=1}^{K} \sum_{i \in \mathbb{S}_k} \frac{1}{M_i} \log_2 \left(1 + \frac{M_i P}{N_0}\right). \tag{5.72}$$

- For the matched filter:

$$\gamma_{\text{matched}}(K) = \frac{1}{N} \sum_{k=1}^{K} \sum_{i \in \mathbb{S}_k} \log_2 \left(1 + \frac{P}{(M_i - 1)P + N_0}\right). \tag{5.73}$$

By optimum, we refer to a joint decoding of all the users with separate code books [CT91] (or successive stripping of the users where the equivalence is shown in [VG97]). The matched filter corresponds to the case where all the users (except the user of interest) are considered as background noise. The spectral efficiencies in (5.72) and (5.73) are expressed in bits/seconds/Hz and are a priori random variables that depend on the realization of the channels. The rates are achievable if the transmitters know exactly these rates.

In the following, we will consider the channel in the asymptotic regime (high number of carriers N) and show that under some assumptions on the channel statistics, the transmitters can send reliably data at a predictable rate irrespective of a particular realization of the interfering channels. Let $\frac{P}{N_0} = \text{snr}$; for $M \geq 1$ we define the single-user capacities

$$\gamma^{\text{su}}_{\text{optimum}}(M) = \frac{1}{M} \log_2 (1 + M \,\text{snr}), \tag{5.74}$$

$$\gamma^{\text{su}}_{\text{matched}}(M) = \log_2 \left(1 + \frac{\text{snr}}{(M-1)\,\text{snr} + 1}\right). \tag{5.75}$$

Using the fact that $\text{snr} = \gamma^{\text{su}}_{\text{optimum}}(M) \frac{E_{\text{b}}}{N_0}$ (respectively $\text{snr} = \gamma^{\text{su}}_{\text{matched}}(M) \frac{E_{\text{b}}}{N_0}$) in these expressions, the single-user capacities are solutions of implicit equations. Concerning the single-user optimum capacity (5.74), we remark that if $M \geq 1$, $x = M \gamma^{\text{su}}_{\text{optimum}}(M)$ is the solution of an implicit equation that does not depend on M.

$$x = \log_2 \left(1 + x \frac{E_{\text{b}}}{N_0}\right). \tag{5.76}$$

x is known as the Gaussian single user bound in an AWGN single user transmission. The total spectral efficiencies from (5.72) and (5.73) can then be calculated as

$$\gamma_{\text{optimum}}(K) = \frac{1}{N}\sum_{k=1}^{K}\sum_{i\in S^k}\gamma_{\text{optimum}}^{\text{su}}(M_i), \tag{5.77}$$

$$\gamma_{\text{matched}}(K) = \frac{1}{N}\sum_{k=1}^{K}\sum_{i\in S^k}\gamma_{\text{matched}}^{\text{su}}(M_i). \tag{5.78}$$

Eqs. (5.77) and (5.78) can be equivalently written

$$\gamma_{\text{optimum}}(K) = \frac{1}{N}\sum_{i=1}^{N} M_i\gamma_{\text{optimum}}^{\text{su}}(M_i), \tag{5.79}$$

$$\gamma_{\text{matched}}(K) = \frac{1}{N}\sum_{i=1}^{N} M_i\gamma_{\text{matched}}^{\text{su}}(M_i). \tag{5.80}$$

5.3.3 Gaussian Case

Let us first analyze the case of a Gaussian (no fading) uplink multiple access network. The results are useful for comparison purposes with the fading case of Sec. 5.3.4. For each user k and each carrier i, $h_k(i) = 1$. Therefore, $\mathbb{S}_k = \{1,\ldots,N\}$ in the setting of the model. Eqs. (5.77) and (5.78) reduce to:

$$\gamma_{\text{optimum}}(K) = K\gamma_{\text{optimum}}^{\text{su}}(K) = x,$$
$$\gamma_{\text{matched}}(K) = K\gamma_{\text{matched}}^{\text{su}}(K).$$

In the case of the optimum receiver (for which the complexity increases with the number of users as joint processing is performed), the cell spectral efficiency is independent of the number of users and is equal to the gaussian single user bound. However, for the matched filter, the spectral efficiency is a decreasing function of K, as

$$\lim_{K\to\infty}\gamma_{\text{matched}}(K) = \lim_{K\to\infty} K\log_2\left(1+\frac{\text{snr}}{(K-1)\,\text{snr}+1}\right) = \frac{1}{\ln(2)}.$$

To increase the matched filter spectral efficiency of the system, suppose that only a certain proportion β of the carriers is to be used. In this setting, $\mathbb{S}_k \subseteq \{1,\ldots,N\}$ is a set of $S_k = \beta N$ carriers chosen at random for each user. Since the carriers are chosen at random, the distribution of M_i (number of users transmitting on carrier i with $1 \le M_i \le K$) is binomial with parameter β.

$$\mathbb{P}(M_i = M) = \binom{K}{M}\beta^M(1-\beta)^{K-M}. \tag{5.81}$$

Using (5.81), (5.79) and (5.80) can be written as

$$\begin{aligned}\gamma_{\text{optimum}}(K) &= \sum_{M=1}^{K}\binom{K}{M}\beta^M(1-\beta)^{K-M} M\gamma_{\text{optimum}}^{\text{su}}(M)\\ &= x\left(1-(1-\beta)^K\right)\end{aligned} \tag{5.82}$$

and

$$\gamma_{\text{matched}}(K) = \sum_{M=1}^{K} \binom{K}{M} \beta^M (1-\beta)^{K-M} M \gamma_{\text{matched}}^{\text{su}}(M). \tag{5.83}$$

Relation (5.82) shows that $\beta = 1$ is optimum for the optimum receiver. Moreover, the cell spectral efficiency is an increasing function of the number of users K that tends to the single user bound when $K \to \infty$, whatever the value of β, at the expense of a decoding complexity.

For the matched filter, for a given value of $\frac{Eb}{N_0}$ and number of users K, there is an optimum value of β as shown in Fig. 5.11. The reduction in interference achieved by optimizing β can more than double the spectral efficiency, as shown in Fig. 5.12.

5.3.4 Fading Case

Independent Fading

In this section, we consider that the fading coefficients $h_k(i)$ are i.i.d. complex random variables, such as the probability distribution function of $|h_k(i)|^2$ is $f(u)$. This can be the case, for example if subcarriers are grouped in clusters [BZ06]. In the asymptotic regime, under the assumption that the users know the channel statistics and the total number of users K, we have the following proposition.

Proposition 18 *As $N \to \infty$, the mean spectral efficiency with optimum filter has the following asymptotic expression:*

$$\gamma_{optimum}(K) = \sum_{M=1}^{K} \binom{K}{M} p^M (1-p)^{K-M} M \gamma_{optimum}^{su}(M) = x\left(1-(1-p)^K\right). \tag{5.84}$$

As $N \to \infty$, the mean spectral efficiency with matched filter has the following asymptotic expression:

$$\gamma_{matched}(K) = \sum_{M=1}^{K} \binom{K}{M} p^M (1-p)^{K-M} M \gamma_{matched}^{su}(M). \tag{5.85}$$

The parameter p is given by:

$$p = \int_{u^*}^{+\infty} f(u)du \tag{5.86}$$

where u^* is solution of

$$\int_{u^*}^{+\infty} \frac{f(u)}{u} du. \tag{5.87}$$

The Gaussian channel of Sec. 5.3.3 is actually a particular type of uncorrelated fading channel with probability distribution function $\delta(u-1)$, which gives $p = 1$. We observe that (5.84) and (5.85) are similar to (5.82) and (5.83) with parameter $\beta = p$: in the asymptotic regime, the uncorrelated fading channel is equivalent to the Gaussian channel with a proportion $\beta = p$ of carriers used.

Proof *We use the ergodicity of $h_k(i)$, $i = 1 \dots N$, to show that sums involving $h_k(i)$ tend asymptotically to fixed values. As $N \to \infty$, the sum of inverse of square norms of the complex random variables $h_k(i)$ tends to an integral with respect to the distribution of $|h_k(i)|^2$. Namely, for some u^*,*

$$\frac{1}{N} \sum_{i \in \mathbb{S}_k} \frac{1}{|h_k(i)|^2} \to \int_{u^*}^{+\infty} \frac{f(u)}{u} du. \tag{5.88}$$

Injecting (5.88) *in* (5.71), *we obtain* (5.87). *Using the fact that*

$$\frac{1}{N} \operatorname{card}(\mathbb{S}_k) = \frac{1}{N} \sum_{i=1}^{N} \mathbf{1}_{\{|h_k(i)|^2 > u^*\}} \to \int_{u^*}^{+\infty} e^{-u} du$$

and

$$p = \lim_{N \to \infty} \frac{1}{N} \operatorname{card}(\mathbb{S}_k),$$

we obtain p as in (5.86). *According to this analysis, asymptotically it is as if each user chose a set of pN carriers at random. The distribution of M is therefore given by a binomial distribution (5.81) with parameter p and we obtain (5.84) and (5.85) from (5.79) and (5.80) as in Sec. 5.3.3.*

For example, in the particular case when $h_k(i)$ have a Gaussian distribution $\mathcal{N}(0,1)$, then the distribution of $|h_k(i)|^2$ is Chi-squared with 2 degrees of freedom and (5.86) reduces to

$$p = \int_{\mathrm{Ei}_1^{-1}(1)}^{+\infty} e^{-u} du = \exp\left(- \mathrm{Ei}_1^{-1}(1)\right),$$

where Ei_1 is the exponential integral defined by: $\mathrm{Ei}_1(t) = \int_t^{\infty} \frac{e^{-u}}{u} du$. Given the channel model, p is therefore a known parameter and is approximately equal to 0.7674.

Fig. 5.13 shows the mean spectral efficiency of the matched filter and the optimum filter for various number of users K. Realistic Monte-Carlo simulations have also been performed for $N = 256$. The theoretical curves and the simulated curves for both filters match. In other words, in a finite system, a user is able to send data without knowing the interference generated by other users. It also appears clearly that for the matched filter, the optimum number of users in the cell to be considered is one, result already proved in the downlink case in [JL03].

As far as the optimum filter is concerned, the spectral efficiency increase is substantial and reaches the Gaussian single user bound. Note that one can increase the spectral efficiency of the matched filter by choosing an optimized subset of the carriers as in the Gaussian case of Sec. 5.3.3.

Totally Correlated Fading

If the fading is totally correlated on all carriers of each user, then for all i, $h_k(i) = h_k$, and S_k is a random variable distributed as a rounded version of $N|h_k|^2$, which has known distribution $\frac{1}{N} f\left(\frac{u}{N}\right)$. More specifically,

$$0 \leq \ell \leq N-1, \mathbb{P}\left(S_k = \ell\right) = \mathbb{P}\left(\frac{\ell}{N} \leq |h_k|^2 < \frac{\ell+1}{N}\right) = \int_{\frac{\ell}{N}}^{\frac{\ell+1}{N}} f(u)du,$$

$$\mathbb{P}\left(S_k = N\right) = \mathbb{P}\left(|h_k|^2 \geq 1\right) = \int_{1}^{+\infty} f(u)du.$$

From the distribution of the S_k, it is possible to determine the distribution of M via the relations

$$\mathbb{P}\left(M_i = M\right) = \sum_{\ell_1=1}^{N} \cdots \sum_{\ell_K=1}^{N} \mathbb{P}\left(M_i = M | S_1 = \ell_1, \ldots, S_K = \ell_K\right) \mathbb{P}\left(S_1 = \ell_1\right) \cdots \mathbb{P}\left(S_K = \ell_K\right)$$

and

$$\mathbb{P}\left(M_i = M | S_1 = \ell_1, \ldots, S_K = \ell_K\right) = \sum_{\substack{\mathbb{M}_i \subseteq \{1,\ldots,K\} \\ \mathrm{card}(\mathbb{M}_i) = M}} \prod_{k \in \mathbb{M}_i} \frac{\ell_k}{N} \prod_{k \notin \mathbb{M}_i} \left(1 - \frac{\ell_k}{N}\right).$$

The expectation of the spectral efficiency can then be obtained from (5.79) or (5.80).

Effect of the Number of Paths

In order to assess the effect of channel correlation on the cell spectral efficiency, we consider the case of a multipath channel, as in (3.5). Under the assumption that the number of paths from user k to the base station is given by L, the model of the channel is given by

$$c_k(\tau) = \sum_{\ell=0}^{L-1} \eta_{k\ell} \psi(\tau - \tau_{k\ell}).$$

where we assume that the channel is invariant during the time considered. In order to compare channels at the same signal to noise ratio, we constrain the distribution of the i.i.d. fading coefficients $\eta_{k\ell}$ such as:

$$\mathbb{E}\left[\eta_{k\ell}\right] = 0 \text{ and } \mathbb{E}\left[|\eta_{k\ell}|^2\right] = \frac{\varrho}{L}.$$

Usually, fading coefficients $\eta_{k\ell}$ are supposed to be independent with decreasing variance as the delay increases. In all cases, ϱ is the average power of the channel, such as $\mathbb{E}\left[|c_k(\tau)|^2\right] = \sum_{\ell=0}^{L_k-1} \mathbb{E}\left[|\eta_{k\ell}|^2\right] = \varrho$, for all channels considered. For each user k, let $h_k(i)$ be the Discrete Fourier Transform of the fading process $c_k(\tau)$. The frequency response of the channel at the receiver is given by:

$$h_k(f) = \sum_{\ell=0}^{L_k-1} \eta_{k\ell} e^{-j2\pi f \tau_{k\ell}} \left|\Psi(f)\right|^2.$$

where we assume that the transmit filter $\Psi(f)$ and the receive filter $\Psi^*(-f)$ are such that, given the bandwidth W,

$$\Psi(f) = \begin{cases} 1 & \text{if } -\frac{W}{2} \leq f \leq \frac{W}{2} \\ 0 & \text{otherwise.} \end{cases}$$

Sampling at the various frequencies $f_1 = -\frac{W}{2}$, $f_2 = -\frac{W}{2} + \frac{1}{N}W$, $\ldots$, $f_N = -\frac{W}{2} + \frac{N-1}{N}W$, we obtain the coefficients $h_k(i)$, $1 \leq i \leq N$, as

$$h_k(i) = \sum_{\ell=0}^{L_k-1} \eta_{k\ell} e^{-j2\pi \frac{i}{N} W \tau_{k\ell}} e^{j\pi W \tau_{k\ell}}.$$

Note that $\mathbb{E}\left[|h_{ik}|^2\right] = \varrho$.

For simplicity sake, the delays are supposed to be uniformly distributed according to the bandwidth

$$\tau_{kp} = \frac{p}{W}.$$

In figure 5.14, the spectral efficiency has been plotted versus the number of users at 10 dB for 2 and 16 paths, for fading coefficients having a Gaussian distribution $\mathcal{N}\left(0, \frac{1}{L}\right)$. Interestingly, for the optimum filter, the spectral efficiency decreases with correlation whereas for the matched filter, the results are completely opposite. As the number of users increases, the difference tends however to disappear.

5.3.5 Conclusion

An OFDMA scheme making use of the reciprocity of the channel to alleviate the need for feedback has been proposed and its performances analyzed. Surprisingly, we show that in a non-cooperative environment with channel fading, a user can send reliable data at a prescribed rate knowing only his channel. The result is based on the predictability of the interference as the number of carriers increases. Moreover, in the case of the matched filter, we show that a judicious choice of the number of carriers can dramatically increase the rate (in comparison with the full use of all the carriers). These results put forward the gain achieved by non-cooperative reciprocal transmissions. In order to assess the performance with respect to fully centralized transmissions, the effect of channel estimation and time-variations should be taken into account.

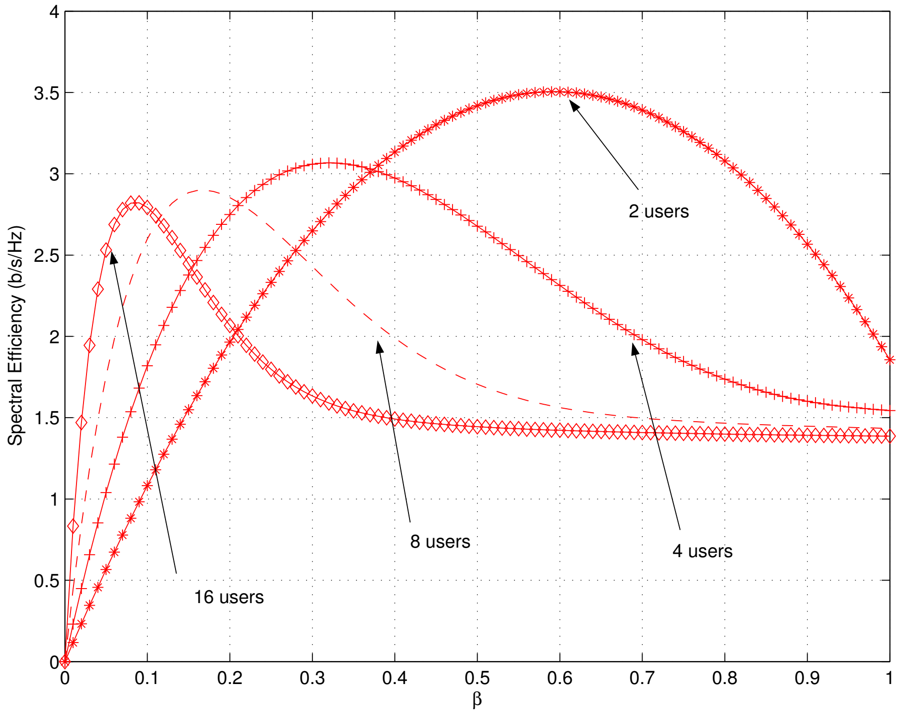

Figure 5.11: Spectral efficiency versus β for the matched filter in the Gaussian case at 10dB

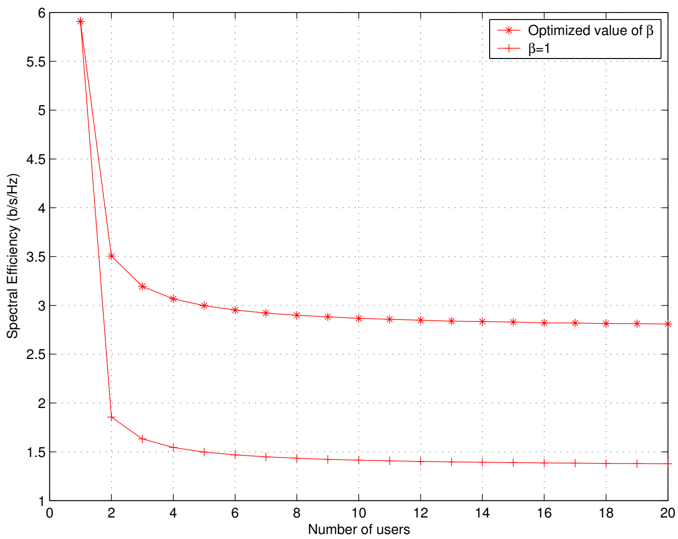

Figure 5.12: Spectral efficiency versus the number of users for the matched filter in the Gaussian case at 10dB

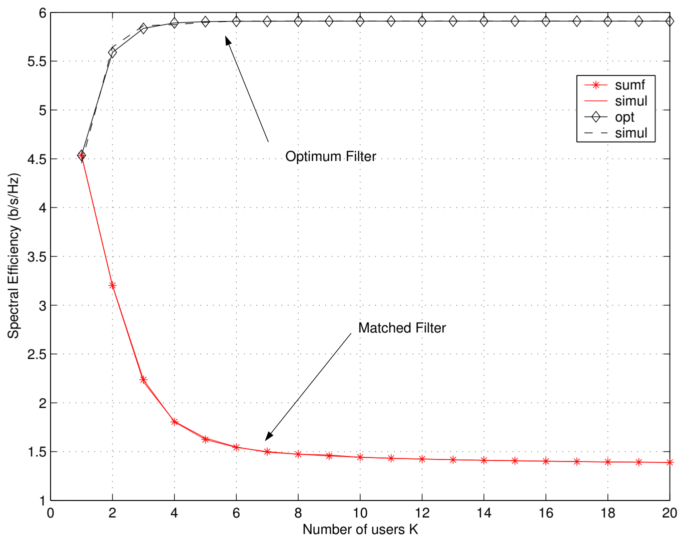

Figure 5.13: Spectral efficiency versus the number of users for Rayleigh channel at 10dB

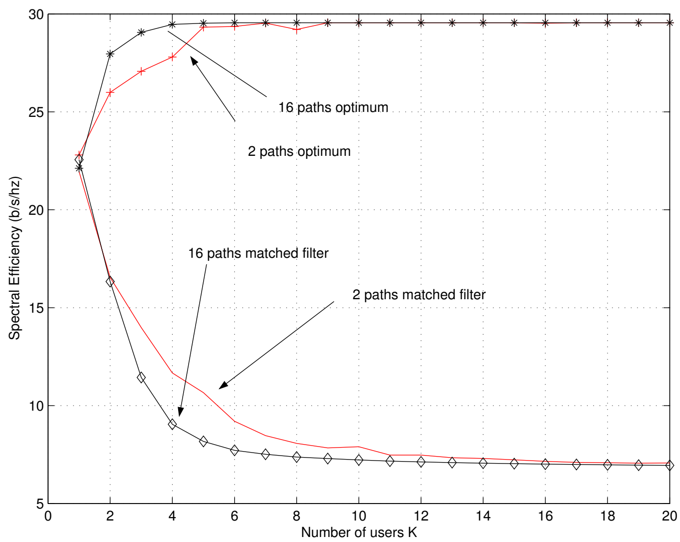

Figure 5.14: Spectral efficiency versus the number of users for 2 and 16 paths at 10dB

Chapter 6

Conclusions and Further Work

6.1 Summary and Conclusions

We have investigated network centric communications, restricted to the case of CDMA cellular networks.

In the case of downlink CDMA, results are provided in the asymptotic regime for an infinite cellular deployment. We show that the increase in throughput is not linear with the cell size, so that packing base stations is not necessarily optimal. In real life situations, the cost relative to adding additional base stations must also be taken into account to determine the optimal distance. Depending on the severity of the path loss (exponential or polynomial), communications are more or less affected by inter-cell interference. However, the path loss represents only a minor part of the problem, since it does not destroy orthogonality; the main effort should be put on alleviating the orthogonality-destroying effect of frequency-selective fading.

In the case of uplink CDMA, several settings are investigated. In contrast to previous works that consider only random codes for the uplink, the case of orthogonal codes is considered in the single-cell setting. It is shown that the synchronization of the users provides a non-negligible enhancement in the throughput when considering flat-fading. However, as the number of multipaths increases, the advantage becomes limited. As in the downlink case, reducing the orthogonality-destroying effect of frequency-selective fading is a priority.

In the infinite cellular deployment, both random and orthogonal codes are considered. It is shown that orthogonal codes are useful only to combat intra-cell interference. Hence, the amelioration provided by synchronizing the users is dependent on the severity of the path loss, which determines the importance of inter-cell interference with respect to intra-cell interference.

When using random codes, we also show the potential gain in cellular environments of optimum intra-cell processing with respect to linear receivers, as well as the gain of joint multi-cell processing.

These works represent a testimony of the amazing powers of random matrix theory. The averaging effect enables to determine explicit formulas for relevant measures of performance and single out elegantly the parameters of interest. What's more, even if these results are derived in the asymptotic regime, they provide accurate predictions for the finite size case. We derived results in the case of an infinite cellular deployment. They can also be useful in the case of finite size networks, with a fixed number of cells.

The objective of the network centric approach was to obtain bounds on the performance of cellular multiuser systems.

We have investigated user centric communications, restricted to the case of three protocols: ALOHA/CDMA/OFDMA.

In the context of ALOHA, we studied non-cooperative interactions between mobiles in two game theoretical frameworks: evolutionary games and correlated games. As far as we know, we have been the first to introduce these subfields of game theory in the context of networking. Both are especially interesting, since they provide additional insight on possible amelioration of future user centric schemes of transmission. This represents a clear demonstration of the usefulness of game theory in a network context.

We have investigated power allocation games for CDMA in frequency-selective fading. Introducting non-atomic games has enabled to obtain explicit expressions for the resource allocation of the users, needing only local information. Distributed means of ordering the users are introduced, in order to use more efficient successive interference cancellation filters. In addition to linear filters, the resource allocation for optimal and SIC filtering is derived, and the gain provided by those filters is analyzed.

As far as OFDMA is concerned, a novel power allocation scheme is investigated, where the base station needs not have any knowledge of the system. The scheme is based on the reciprocity of the channel. We show that users can communicate reliably at a prescribed rate, knowing only their own channel.

In conclusion, we provided elements of analysis of both network centric and user centric communications. We have tackled the problem of where to put the intelligence in the network. When there are lots of users, it is legitimate to put the intelligence at the hands of the users. Therefore, scenarios where intelligence is given to users were investigated. As the title of the thesis states, both conceptions are not opposed, but complementary. One could think about protocols that, depending on the state of the system, impose the choice of resource allocation to the mobiles or on the contrary let the users determine themselves their best option according to their (limited) knowledge of the system.

6.2 Perspectives

Other filters for the orthogonal uplink CDMA case. At the moment, we have no explicit expressions for the MMSE filter in the CDMA uplink case. An analogue of Girko's Theorem [Gir90] (see Th. 2) needs to be found in the case of unitary random matrices.

Comparison between CDMA and OFDMA. In OFDMA, carriers are flat fading. The interesting feature of OFDMA is that carriers can be chosen and the transmission can occur independently at different power for each carrier. However, fading is correlated over adjacent carriers. A way to alleviate this correlation is to group carriers together.

Nash equilibrium and Pareto equilibrium. Pareto equilibrium, when it is not possible to increase the gain of one user without decreasing the gain of another one, are often difficult to characterize. Answers are starting to emerge. A recent paper, which the authors were kind enough to provide us a preliminar version of, investigates the efficiency of the Nash equilibrium relative to the Pareto equilibrium in the context of resource allocation games [BP97]. In particular, it is shown that under some conditions, the Nash equilibrium coincides with the Pareto equilibrium for underloaded systems, while the difference is barely noticeable for overloaded systems. However, this question is still unanswered in general.

To an analogue of the price of anarchy? The price of anarchy is a customary concept in routing [Rou05]. The price of anarchy is equal to the ratio of the utility obtained by selfish users to the utility they would obtain by (cooperative) optimal routing. As such, it measures the loss suffered by the users when they are left to fend off for themselves, compared to the case where there is a central controller. How can an analogue be designed at the protocol level, involving the ratio between utility achieved in Nash and Pareto equilibrium? The study of its behavior can be used to measure the impact of selfish users on the efficiency of communications.

Protocols adapted to high mobility. While most of the time, static networks and static attenuations are considered, actually communications occur in dynamic conditions. How to take into account a fast varying network topology? Centralized attribution of resources may need a lot of feedback and computation, hence time to adapt to changing conditions. Similarly, most distributed protocols are iterative versions of centralized ones and need time to converge. Power allocation games are a first step towards an answer to this question.

Fair comparison between the protocols in function of mobility. Intuitively, there is clearly a tradeoff between centralization and mobility. From which moment is there a gain in letting the users adopt a selfish behavior and how can this be determined in a fast and efficient way?

To an information theory for protocols? Information theory for protocols represents a very general and broad goal. It would enable to measure the impact of acquisition of information and redundancy on the performance of protocols, and compare different communication protocols in a fair manner.

Chapter 7

Appendix

7.1 Proofs for Chapter 4

Proof of Prop. 1

Term S^*: Let us focus on the term S^* of (4.3). As $N \to \infty$, (4.3) becomes:

$$
\begin{aligned}
S^* &= P_p(x_j) \left| \mathbf{w}_{pj}^H \mathbf{H}_{pj}^H \mathbf{H}_{pj} \mathbf{w}_{pj} \right|^2 = P_p(x_j) \left| \sum_{i=1}^{N} |h_{pj}(i)|^2 \, |w_{pj}(i)|^2 \right|^2 \\
&= \sum_{i=1}^{N} |h_{pj}(i)|^4 \, |w_{pj}(i)|^4 + \sum_{i=1}^{N} \sum_{\substack{l=1 \\ l \neq i}}^{N} |h_{pj}(i)|^2 \, |w_{pj}(i)|^2 \, |h_{pj}(l)|^2 \, |w_{pj}(l)|^2 .
\end{aligned}
$$

Using (2.5) and (2.6), it is rather straightforward to show that

$$
S^* \to P_p(x_j) \lim_{N \to \infty} \frac{2}{N(N+1)} \sum_{i=1}^{N} |h_{pj}(i)|^4 + \frac{1}{N(N+1)} \sum_{i=1}^{N} \sum_{\substack{l=1 \\ l \neq i}}^{N} |h_{pj}(i)|^2 \, |h_{pj}(l)|^2 \tag{7.1}
$$

in the mean square sense. The first term in the limit in (7.1) tends to 0. Therefore,

$$
S^* \xrightarrow[N \to \infty]{} P_p(x_j) \left(\frac{1}{W} \int_{-\frac{W}{2}}^{\frac{W}{2}} |h_{pj}(f)|^2 df \right)^2 . \tag{7.2}
$$

(7.2) stems from the fact that as $N \to \infty$, the eigenvalues $|h_{pj}(i)|^2$ of $\mathbf{H}_{pj}^H \mathbf{H}_{pj}$ correspond to the squared frequency response of the channel in the case of a Toeplitz structure of $\mathbf{H}_{pj}$ (see [Gra06]).

Term I_1: Let us now derive the term I_1 of (4.4). It can be shown that (since $\mathbf{w}_{pj}$ is independent of $\mathbf{W}_q$, see proof in [DHLdC03b])

$$
\mathbf{w}_{pj}^H \mathbf{H}_{pj}^H \mathbf{H}_{qj} \mathbf{W}_q \mathbf{W}_q^H \mathbf{H}_{qj}^H \mathbf{H}_{pj} \mathbf{w}_{pj} - \frac{1}{N} \mathrm{trace}(\mathbf{W}_q \mathbf{W}_q^H \mathbf{H}_{qj}^H \mathbf{H}_{pj} \mathbf{H}_{pj}^H \mathbf{H}_{qj}) \to 0. \tag{7.3}
$$

Therefore, each term I_q^* in the sum in (4.4) can be calculated as:

$$\begin{aligned} I_q^* &= \mathbf{w}_{pj}^H \mathbf{H}_{pj}^H \mathbf{H}_{qj} \mathbf{W}_q \mathbf{W}_q^H \mathbf{H}_{qj}^H \mathbf{H}_{pj} \mathbf{w}_{pj} \\ &\to \frac{1}{N} \mathrm{trace}(\mathbf{W}_q \mathbf{W}_q^H \mathbf{H}_{qj}^H \mathbf{H}_{pj} \mathbf{H}_{pj}^H \mathbf{H}_{qj}) \\ &\to \frac{1}{N} \sum_{k=1}^{K} \sum_{i=1}^{N} |h_{pj}(i)|^2 \, |h_{qj}(i)|^2 \, |w_{qk}(i)|^2. \end{aligned}$$

Using (2.4), it is rather straightforward to show that

$$I_q^* \to \lim_{N\to\infty} \frac{1}{N^2} \sum_{k=1}^{K} \sum_{i=1}^{N} |h_{pj}(i)|^2 \, |h_{qj}(i)|^2$$

in the mean square sense. Therefore,

$$I_q^* \xrightarrow[d/N\to\alpha]{N\to\infty} \frac{\alpha a}{W} \int_{-\frac{W}{2}}^{\frac{W}{2}} |h_{pj}(f)|^2 |h_{qj}(f)|^2 df.$$

Term I_2: Finally, let us derive the asymptotic expression of I_2 in (4.5). The proof follows here a different procedure as $\mathbf{w}_{pj}$ is not independent of $\mathbf{W}_p^{(-j)}$. However, one can show that

$$\begin{aligned} I_2 &= P_p(x_j) \mathbf{w}_{pj}^H \mathbf{H}_{pj}^H \mathbf{H}_{pj} \mathbf{W}_p^{(-j)} \mathbf{W}_p^{(-j)^H} \mathbf{H}_{pj}^H \mathbf{H}_{pj} \mathbf{w}_{pj} \\ &= P_p(x_j) \sum_{\substack{k=1 \\ k\neq j}}^{K} \left(\mathbf{w}_{pj}^H \mathbf{H}_{pj}^H \mathbf{H}_{pj} \mathbf{w}_{pk} \right)^2 \\ &= P_p(x_j) \sum_{\substack{k=1 \\ k\neq j}}^{K} \left(\sum_{i=1}^{N} |h_{pj}(i)|^2 \, w_{pj}(i)^* w_{pk}(i) \right)^2 \\ &= P_p(x_j) \sum_{\substack{k=1 \\ k\neq j}}^{K} \sum_{i=1}^{N} \sum_{l=1}^{N} |h_{pj}(i)|^2 \, |h_{pj}(l)|^2 \, w_{pj}(i)^* w_{pj}(l) w_{pk}(i) w_{pk}(l)^*. \end{aligned}$$

and using (2.6) and (2.7), it can be shown that I_2 converges in the mean square sense to

$$P_p(x_j) \lim_{N\to\infty} \frac{1}{N(N+1)} \sum_{\substack{k=1 \\ k\neq j}}^{K} \sum_{i=1}^{N} |h_{pj}(i)|^4 - \frac{1}{N(N^2-1)} \sum_{\substack{k=1 \\ k\neq j}}^{K} \sum_{i=1}^{N} \sum_{\substack{l=1 \\ l\neq i}}^{N} |h_{pj}(i)|^2 \, |h_{pj}(l)|^2 .$$

Therefore,

$$I_2 \xrightarrow[d/N\to\alpha]{N\to\infty} P_p(x_j) \frac{\alpha a}{W} \left(\int_{-\frac{W}{2}}^{\frac{W}{2}} |h_{pj}(f)|^4 - \frac{1}{W} \left(\int_{-\frac{W}{2}}^{\frac{W}{2}} |h_{pj}(f)|^2 df \right)^2 \right).$$

Proof of Prop. 3

Note that as $L_q \to \infty$,

$$\sum_{\ell=0}^{L_q-1} |\eta_{q\ell}|^2 \to \mathbb{E}\left[\sum_{\ell=0}^{L_q-1} |\eta_{q\ell}|^2\right] = \mathbb{E}\left[|h|^2\right],$$

$$\begin{aligned}\sum_{\ell=0}^{L-1}\sum_{\ell'\neq\ell} |\eta_\ell|^2 |\eta_{\ell'}|^2 &= \left(\sum_{\ell=0}^{L-1} |\eta_\ell|^2\right)^2 + \sum_{\ell=0}^{L-1}\sum_{\ell'\neq\ell} |\eta_\ell|^2 |\eta_{\ell'}|^2 - \left(\sum_{\ell=0}^{L-1} |\eta_\ell|^2\right)^2 \\ &\to \mathbb{E}\left[|h|^4\right] - \left(\mathbb{E}\left[|h|^2\right]\right)^2,\end{aligned}$$

and $\sum_{\ell'\neq\ell} \eta_\ell \eta_{\ell'}{}^* \eta_{q\ell}{}^* \eta_{q\ell'} \to 0$.

The rest of the proof is mainly an application of Prop. 1 where we consider a path loss of the form $Pe^{-\gamma(|x-qar|)}$ (γ is a decaying factor) between the user x ($x \in \left[-\frac{a}{2}, \frac{a}{2}\right]$) and base station q ($q \in \mathbb{Z}$) with coordinates $m_q = qa$. In this case, the inter-cell interference has an explicit form:

$$\sum_{q\neq 0} P_q(x) = P\sum_{q=-\infty}^{+\infty} e^{-\gamma|x-qar|} = Pe^{-\gamma x}\sum_{q=1}^{+\infty} e^{-\gamma qar} + Pe^{\gamma x}\sum_{q=1}^{+\infty} e^{-\gamma qar} = \frac{2Pe^{-\gamma ar}}{1-e^{-\gamma ar}}\cosh(\gamma x).$$

Proof of Equation (4.16)

Differentiating (4.13) with respect to a, we obtain

$$\begin{aligned}\frac{\partial C(a)}{\partial a} &= -\frac{1}{a}C(a) + \frac{\alpha}{a}\log_2\left(1 + \frac{Pe^{-\gamma\frac{a}{2}}\left(\mathbb{E}\left[|h|^2\right]\right)^2}{I(\frac{a}{2}) + \sigma^2\mathbb{E}\left[|h|^2\right]}\right) \\ &- \frac{2\alpha}{a\ln 2}\int_0^{\frac{a}{2}} \frac{Pe^{-\gamma x}\left(\mathbb{E}\left[|h|^2\right]\right)^2 \frac{\partial I(x)}{\partial a}}{\left(I(x)+\sigma^2\mathbb{E}\left[|h|^2\right]\right)\left(I(x)+\sigma^2\mathbb{E}\left[|h|^2\right] + Pe^{-\gamma x}\left(\mathbb{E}\left[|h|^2\right]\right)^2\right)}dx. \quad (7.4)\end{aligned}$$

Let $\gamma \to 0$ in (4.13). In this case,

$$C(a) = \frac{2\alpha}{a}\int_0^{\frac{a}{2}} \log_2\left(1 + \frac{P(1-\gamma x + \frac{\gamma^2}{2}x^2)\left(\mathbb{E}\left[|h|^2\right]\right)^2}{I(x) + \sigma^2\mathbb{E}\left[|h|^2\right]}\right)dx + O(\gamma^3)$$

where

$$\begin{aligned}I(x) &= \alpha a P\left(\mathbb{E}\left[|h|^2\right]\right)^2 \frac{2e^{-\gamma a}}{1-e^{-\gamma a}} + \alpha a P\left(\mathbb{E}\left[|h|^4\right] - \left(\mathbb{E}\left[|h|^2\right]\right)^2\right) + O(\gamma) \\ &= \frac{2\alpha P}{\gamma}\left(\mathbb{E}\left[|h|^2\right]\right)^2 + \alpha a P\left(\mathbb{E}\left[|h|^4\right] - 2\left(\mathbb{E}\left[|h|^2\right]\right)^2\right) + O(\gamma)\end{aligned}$$

since $\frac{2}{e^{\gamma a}-1} = \frac{2}{\gamma a} - 1 + O(\gamma)$. We have therefore:

$$\frac{1}{I(x)+\sigma^2\mathbb{E}\left[|h|^2\right]} = \frac{1}{\frac{2\alpha P}{\gamma}\left(\mathbb{E}\left[|h|^2\right]\right)^2\left(1+\gamma\left(\frac{a}{2}\left(\frac{\mathbb{E}\left[|h|^4\right]}{\left(\mathbb{E}\left[|h|^2\right]\right)^2}-2\right)+\frac{\sigma^2}{2\alpha P\mathbb{E}\left[|h|^2\right]}\right)+O(\gamma^2)\right)}$$

$$= \frac{\gamma}{2\alpha P\left(\mathbb{E}\left[|h|^2\right]\right)^2}\left(1-\gamma\left(\frac{a}{2}\left(\frac{\mathbb{E}\left[|h|^4\right]}{\left(\mathbb{E}\left[|h|^2\right]\right)^2}-2\right)+\frac{\sigma^2}{2\alpha P\mathbb{E}\left[|h|^2\right]}\right)+O(\gamma^2)\right)$$

$$= \frac{\gamma}{2\alpha P\left(\mathbb{E}\left[|h|^2\right]\right)^2}(1-\gamma B)+O(\gamma^3).$$

Hence,

$$\log_2\left(1+\frac{Pe^{-\gamma x}\left(\mathbb{E}\left[|h|^2\right]\right)^2}{I(x)+\sigma^2\mathbb{E}\left[|h|^2\right]}\right) = \log_2\left(1+\left(1-\gamma x+\frac{\gamma^2}{2}x^2\right)\frac{\gamma(1-\gamma B)}{2\alpha}\right)+O(\gamma^3)$$

$$= \frac{1}{\ln 2}\left(\gamma\frac{1-\gamma x}{2\alpha}(1-\gamma B)+\frac{\gamma^2}{8\alpha^2}\right)+O(\gamma^3)$$

$$= \frac{\gamma}{2\alpha\ln 2}\left(1-\gamma\left(x+B-\frac{1}{4\alpha}\right)\right)+O(\gamma^3).$$

Integrating, we obtain

$$C(a) = \frac{\gamma}{2\ln 2}\left(1-\gamma\left(\frac{a}{2}+B-\frac{1}{4\alpha}\right)\right)+O(\gamma^3)$$

which gives

$$\frac{1}{a}C(a) = \frac{\gamma}{2a\ln 2}\left(1-\gamma\left(\frac{a}{2}\left(\frac{\mathbb{E}\left[|h|^4\right]}{\left(\mathbb{E}\left[|h|^2\right]\right)^2}-\frac{3}{2}\right)+\frac{\sigma^2}{2\alpha P\mathbb{E}\left[|h|^2\right]}-\frac{1}{4\alpha}\right)\right)+O(\gamma^3). \tag{7.5}$$

The second term in (7.4) is treated in the same way to obtain a similar expression as (7.5)

$$\frac{\alpha}{a}\log_2\left(1+\frac{Pe^{-\gamma\frac{a}{2}}\left(\mathbb{E}\left[|h|^2\right]\right)^2}{I(\frac{a}{2})+\sigma^2\mathbb{E}\left[|h|^2\right]}\right)$$
$$= \frac{\gamma}{2a\ln 2}\left(1-\gamma\left(\frac{a}{2}\left(\frac{\mathbb{E}\left[|h|^4\right]}{\left(\mathbb{E}\left[|h|^2\right]\right)^2}-1\right)+\frac{\sigma^2}{2\alpha P\mathbb{E}\left[|h|^2\right]}-\frac{1}{4\alpha}\right)\right)+O(\gamma^3). \tag{7.6}$$

It means that the difference of (7.5) and (7.6) reduces to a term $-\frac{\gamma^2}{8\ln 2}+O(\gamma^3)$. Concerning the third term in (7.4),

$$\frac{\partial I(x)}{\partial a} = \frac{1}{a}I(x) - \alpha a P\left(\mathbb{E}\left[|h|^2\right]\right)^2\frac{2\gamma e^{-\gamma a}}{(1-e^{-\gamma a})^2}\cosh(\gamma x)$$
$$= \alpha P\left(\mathbb{E}\left[|h|^4\right]-2\left(\mathbb{E}\left[|h|^2\right]\right)^2\right)+O(\gamma).$$

and

$$\frac{1}{I(x)+\sigma^2\mathbb{E}\left[|h|^2\right]+Pe^{-\gamma x}\left(\mathbb{E}\left[|h|^2\right]\right)^2}$$
$$= \frac{1}{\frac{2\alpha P}{\gamma}\left(\mathbb{E}\left[|h|^2\right]\right)^2\left(1+\gamma\left(\frac{a}{2}\left(\frac{\mathbb{E}\left[|h|^4\right]}{\left(\mathbb{E}\left[|h|^2\right]\right)^2}-2\right)+\frac{\sigma^2}{2\alpha P\mathbb{E}\left[|h|^2\right]}+\frac{1}{2\alpha}\right)+O(\gamma^2)\right)}$$
$$= \frac{\gamma}{2\alpha P\left(\mathbb{E}\left[|h|^2\right]\right)^2}\left(1-\gamma\left(\frac{a}{2}\left(\frac{\mathbb{E}\left[|h|^4\right]}{\left(\mathbb{E}\left[|h|^2\right]\right)^2}-2\right)+\frac{\sigma^2}{2\alpha P\mathbb{E}\left[|h|^2\right]}+\frac{1}{2\alpha}\right)+O(\gamma^2)\right)$$
$$= \frac{\gamma}{2\alpha P\left(\mathbb{E}\left[|h|^2\right]\right)^2}(1-\gamma D)+O(\gamma^3).$$

Hence,

$$\frac{2\alpha}{a\ln 2}\int_0^{\frac{a}{2}}\frac{Pe^{-\gamma x}\left(\mathbb{E}\left[|h|^2\right]\right)^2\frac{\partial I(x)}{\partial a}}{\left(I(x)+\sigma^2\mathbb{E}\left[|h|^2\right]\right)\left(I(x)+\sigma^2\mathbb{E}\left[|h|^2\right]+Pe^{-\gamma x}\left(\mathbb{E}\left[|h|^2\right]\right)^2\right)}dx$$
$$= \frac{2\alpha}{a\ln 2}\int_0^{\frac{a}{2}}\frac{P\left(\mathbb{E}\left[|h|^2\right]\right)^2\alpha P\left(\mathbb{E}\left[|h|^4\right]-2\left(\mathbb{E}\left[|h|^2\right]\right)^2\right)\gamma^2}{\left(2\alpha P\left(\mathbb{E}\left[|h|^2\right]\right)^2\right)^2}dx+O(\gamma^3)$$
$$= \frac{\gamma^2}{4\ln 2}\left(\frac{\mathbb{E}\left[|h|^4\right]}{\left(\mathbb{E}\left[|h|^2\right]\right)^2}-2\right)+O(\gamma^3).$$

We conclude

$$\frac{\partial C}{\partial a} = -\frac{\gamma^2}{4\ln 2}\left(\frac{\mathbb{E}\left[|h|^4\right]}{\left(\mathbb{E}\left[|h|^2\right]\right)^2}-\frac{3}{2}\right)+O(\gamma^3).$$

Remark: 2-D Network

In the case of a 2-D network, the expression for the general SINR (4.6) from Prop. 1 is still valid, if we admit that $x_j = (x_j^1, x_j^2)$ represents the coordinates of the user considered, d is the density of users *per square meter* and $a = |\mathcal{C}|$ is the *surface* of the cell $\mathcal{C}$. We assume a regular partitionning of the plane, for example in hexagonal cells. The expression (4.9) for the spectral efficiency from Prop. 2 can be immediately rewritten with a double integration over the surface of the cell:

$$C(a) = \frac{\alpha}{a}\mathbb{E}_h\left[\iint_{\mathcal{C}}\log_2\left(1+\frac{P(x^1,x^2)\left(\frac{1}{W}\int_{-\frac{W}{2}}^{\frac{W}{2}}|h(f)|^2df\right)^2}{I(x^1,x^2)+\frac{\sigma^2}{W}\int_{-\frac{W}{2}}^{\frac{W}{2}}|h(f)|^2df}\right)dx^1dx^2\right]. \quad (7.7)$$

$$I(x^1,x^2) = \frac{\alpha a}{W}\sum_{q\neq 0}P_q(x^1,x^2)\int_{-\frac{W}{2}}^{\frac{W}{2}}|h(f)|^2|h_q(f)|^2df$$
$$+\frac{\alpha a}{W}P(x^1,x^2)\left(\int_{-\frac{W}{2}}^{\frac{W}{2}}|h(f)|^4df-\frac{1}{W}\left(\int_{-\frac{W}{2}}^{\frac{W}{2}}|h(f)|^2df\right)^2\right)$$

with $a\in[0,\frac{1}{\alpha}]$.

Appendix for uplink single-cell orthogonal CDMA

We give here an intuitive proof that the two terms $\left|\mathbf{g}_k^H\mathbf{g}_k\right|$ and $\mathbf{g}_k^H\left(\mathbf{G}_{(-k)}\mathbf{G}_{(-k)}^H\right)\mathbf{g}_k$ in the expression of the SINR (4.30) for the Matched Filter converge in expectation to obtain Prop. 10. Namely,

$$\left|\mathbf{g}_k^H\mathbf{g}_k\right| = P(x_k)\sum_{i=1}^{N}|h_{ik}|^2|w_{ik}|^2 \underset{N\to\infty}{\sim} P(x_k)\xi(x_k), \tag{7.8}$$

$$\mathbf{g}_k^H\left(\mathbf{G}_{(-k)}\mathbf{G}_{(-k)}^H\right)\mathbf{g}_k = P(x_k)\sum_{\ell\neq k}^{K}P(x_\ell)\left|\sum_{i=1}^{N}h_{ik}^*h_{i\ell}w_{ik}^*w_{i\ell}\right|^2 \underset{N\to\infty}{\sim} P(x_k)\left(\nu(x_k)-\mu(x_k)\right). \tag{7.9}$$

Note that (7.8) is immediate:

$$\mathbb{E}_w\left[\sum_{i=1}^{N}|h_{ik}|^2|w_{ik}|^2\right] = \frac{1}{N}\sum_{i=1}^{N}|h_{ik}|^2 \xrightarrow[N\to\infty]{} \xi(x_k).$$

Using the fact that (see Sec. 2.2.4)

$$\begin{aligned}
\mathbb{E}\left[|w_{ik}|^2|w_{i\ell}|^2\right] &= \frac{1}{N(N+1)},\ \ell>1,\\
\mathbb{E}\left[w_{ik}^*w_{i\ell}w_{lk}w_{l\ell}^*\right] &= -\frac{1}{N(N^2-1)},\ \ell>1,\ i\neq l,
\end{aligned}$$

the expression $\mathbb{E}_w\left[\left|\sum_{i=1}^N h_{ik}^*h_{i\ell}w_{ik}^*w_{i\ell}\right|^2\right]$ is equal to:

$$\sum_{i=1}^{N}\sum_{l=1}^{N}h_{ik}^*h_{i\ell}h_{lk}h_{l\ell}^*\mathbb{E}_w\left[w_{ik}^*w_{i\ell}w_{lk}w_{l\ell}^*\right] = \frac{1}{N(N+1)}\sum_{i=1}^{N}|h_{ik}|^2|h_{i\ell}|^2 \tag{7.10}$$

$$-\frac{1}{N(N^2-1)}\sum_{i=1}^{N}\sum_{l\neq i}h_{ik}^*h_{lk}h_{i\ell}h_{l\ell}^*. \tag{7.11}$$

When N becomes large, the terms (7.10) and (7.11) tend respectively to

$$\frac{1}{N+1}\int_0^1|h(f,x_k)|^2|h(f,x_\ell)|^2df$$

and

$$\frac{1}{N+1}\int_0^1 h^*(u,x_k)h(u,x_\ell)du\int_0^1 h(v,x_k)h^*(v,x_\ell)dv.$$

Note that absolute convergence can be proven using Lemma 2.25 from [Pea05].

Derivation of (4.33) **and** (4.34). As far as the term $\nu(x_k)$ is concerned,

$$\begin{aligned}
\nu(x_k) &= \int_0^\alpha \int_0^1 |h(f,x_k)|^2 |h^*(f,y)|^2 \, df dy \\
&= \frac{1}{N} \sum_{\ell \neq k}^{K} \sum_{m,n,p,q=0}^{L-1} \eta_{km}\eta_{kn}^*\eta_{\ell p}\eta_{\ell q}^* \int_0^1 e^{-j2\pi(m-n+p-q)f} e^{j\pi(m-n+p-q)} df \\
&= \frac{1}{N} \sum_{\ell \neq k}^{K} \left(\sum_{m=0}^{L-1} |\eta_{km}|^2 \sum_{p=0}^{L-1} |\eta_{\ell p}|^2 + \underline{\sum_{m-n+p-q=0} \eta_{km}\eta_{kn}^*\eta_{\ell p}\eta_{\ell q}^*} \right)
\end{aligned}$$

where the underlined sum is taken over all 4-tuples $(m,n,p,q) \in \{0,\dots,L-1\}^4$ such as $m-n+p-q=0$, $m \neq n$, $p \neq q$. The expectation of the underlined sum is obviously 0, hence we obtain (4.33).

As for $\mu(x_k)$,

$$\begin{aligned}
\mu(x_k) &= \int_0^\alpha \left| \int_0^1 h(f,x_k)h^*(f,y)df \right|^2 dy \\
&= \frac{1}{N} \sum_{\ell \neq k}^{K} \left| \sum_{p=0}^{L-1} \sum_{q=0}^{L-1} \eta_{kp}\eta_{\ell q}^* \int_0^1 e^{-j2\pi(p-q)f} e^{j\pi(p-q)} df \right|^2 \\
&= \frac{1}{N} \sum_{\ell \neq k}^{K} \left(\sum_{p=0}^{L-1} |\eta_{kp}|^2 |\eta_{\ell p}|^2 + \sum_{p=0}^{L-1} \sum_{q \neq p} \eta_{kp}\eta_{kq}^*\eta_{\ell q}\eta_{\ell p}^* \right)
\end{aligned}$$

The expectation of the second sum is obviously 0, hence we obtain (4.34).

Appendix for uplink multi-cell orthogonal CDMA

We give here an intuitive proof that the terms $\mathbf{g}_k^H \left(\mathbf{G}_l \mathbf{G}_l^H\right) \mathbf{g}_k$ in the expression of the SINR (4.36) for the Matched Filter converge in expectation to obtain Prop. 11. Namely,

$$\begin{aligned}
\mathbf{g}_k^H \left(\mathbf{G}_l \mathbf{G}_l^H\right) \mathbf{g}_k &= P(x_k) \sum_{\ell=lK+1}^{(l+1)K} P(x_\ell) \left| \sum_{i=1}^{N} h_{ik}^* h_{i\ell} w_{ik}^* w_{i\ell} \right|^2 \\
&\underset{N\to\infty}{\sim} P(x_k) \int_{\alpha(la-a/2)}^{\alpha(la+a/2)} \int_0^1 P(y) |h(f,y)|^2 |h(f,x)|^2 df dy. \qquad (7.12)
\end{aligned}$$

Using the fact that w_{ik} and $w_{i\ell}$ are elements from two independant Haar distributed unitary matrices, so that

$$\begin{aligned}
\mathbb{E}\left[|w_{ik}|^2 |w_{i\ell}|^2\right] &= \frac{1}{N^2}, \ \ell > 1, \\
\mathbb{E}\left[w_{ik}^* w_{i\ell} w_{lk} w_{l\ell}^*\right] &= 0, \ \ell > 1, \ i \neq l,
\end{aligned}$$

the expression $\mathbb{E}_w \left[\left| \sum_{i=1}^N h_{ik}^* h_{i\ell} w_{ik}^* w_{i\ell} \right|^2 \right]$ is equal to:

$$\sum_{i=1}^{N} \sum_{l=1}^{N} h_{ik}^* h_{i\ell} h_{lk} h_{l\ell}^* \mathbb{E}_w \left[w_{ik}^* w_{i\ell} w_{lk} w_{l\ell}^*\right] = \frac{1}{N^2} \sum_{i=1}^{N} |h_{ik}|^2 |h_{i\ell}|^2 .$$

Note that absolute convergence can be proven using Lemma 2.25 from [Pea05].

7.2 Proofs for Chapter 5

Proof of Prop. 14

Notice that when $\sigma^2 \to \infty$, $C^{\mathrm{OPT}} = 0$, $C^{\mathrm{MMSE}} = 0$ and $\beta^{\mathrm{MMSE}}(x) = \beta(x) = 0$. Thus we only have to prove that the derivatives of either side of (5.42) are equal.

Using $\rho(f,x) = P(x)\left|h(f,x)\right|^2$, (5.35) can be rewritten

$$\beta(x) = \int_0^1 \frac{\rho(f,x)df}{\sigma^2 + \int_0^\alpha \frac{\rho(f,y)2dy}{1+\beta(y)}}. \tag{7.13}$$

From (2.3), $\int_0^1 \rho(f,x)u(f,-\sigma^2)df$ satisfies the same implicit equation (7.13) as $\beta(x)$ and thus

$$u(f,-\sigma^2) = \frac{1}{\int_0^\alpha \frac{\rho(f,y)dy}{1+\beta(y)} + \sigma^2}. \tag{7.14}$$

Using (7.13) and (7.14), we can rewrite

$$\begin{aligned}
\int_0^1 u(f,-\sigma^2)df - \frac{1}{\sigma^2} &= \int_0^1 \frac{1}{\int_0^\alpha \frac{\rho(f,y)dy}{1+\beta(y)} + \sigma^2}df - \int_0^1 \frac{1}{\sigma^2}df \\
&= \int_0^1 \frac{-\int_0^\alpha \frac{\rho(f,x)}{1+\beta(x)}dx}{\sigma^2\left(\int_0^\alpha \frac{\rho(f,y)dy}{1+\beta(y)} + \sigma^2\right)}df \\
&= \int_0^\alpha \frac{\frac{-1}{(1+\beta(x))}}{\sigma^2} \int_0^1 \frac{\rho(f,x)df}{\int_0^\alpha \frac{\rho(f,y)dy}{1+\beta(y)} + \sigma^2}dx \\
&= -\int_0^\alpha \frac{\beta(x)}{\sigma^2\left(1+\beta(x)\right)}dx.
\end{aligned}$$

Thus from (5.40)

$$\frac{\partial C^{\mathrm{OPT}}}{\partial \sigma^2} = -\log_2(e)\int_0^\alpha \frac{\beta(x)}{\sigma^2\left(1+\beta(x)\right)}dx. \tag{7.15}$$

Differentiating (5.37) with respect to σ^2, we obtain

$$\frac{\partial C^{\mathrm{MMSE}}}{\partial \sigma^2} = \log_2(e)\int_0^\alpha \frac{1}{1+\beta(x)}\frac{\partial \beta}{\partial \sigma^2}(x)dx. \tag{7.16}$$

Let $\pi(x) = \frac{1}{\sigma^2(1+\beta(x))}$. From (7.15) and (7.16), we obtain

$$\frac{\partial C^{\mathrm{OPT}}}{\partial \sigma^2} - \frac{\partial C^{\mathrm{MMSE}}}{\partial \sigma^2} = -\log_2(e)\int_0^\alpha \left(\beta(x) + \sigma^2\frac{\partial \beta}{\partial \sigma^2}(x)\right)\pi(x)dx. \tag{7.17}$$

From (5.35), we have

$$\begin{aligned}
\int_0^\alpha \sigma^2 \beta(x) \frac{\partial \pi}{\partial \sigma^2}(x) dx &= \int_0^\alpha \int_0^1 \frac{\sigma^2 \rho(f,x) df}{\sigma^2 + \int_0^\alpha \sigma^2 \rho(f,y)\pi(y) dy} \frac{\partial \pi}{\partial \sigma^2}(x) dx \\
&= \int_0^1 \frac{\int_0^\alpha \rho(f,x) \frac{\partial \pi}{\partial \sigma^2}(x) dx}{1 + \int_0^\alpha \rho(f,y)\pi(y) dy} df \\
&= \frac{1}{\log_2(e)} \frac{\partial}{\partial \sigma^2} \int_0^1 \log_2 \left(1 + \int_0^\alpha \rho(f,y)\pi(y) dy \right) df.
\end{aligned}$$

Observing that

$$\int_0^\alpha \left(\beta(x) + \sigma^2 \frac{\partial \beta}{\partial \sigma^2}(x) \right) \pi(x) + \sigma^2 \beta(x) \frac{\partial \pi}{\partial \sigma^2}(x) dx = \frac{\partial}{\partial \sigma^2} \int_0^\alpha \sigma^2 \beta(x) \pi(x) dx,$$

we obtain (5.42) from Prop. 14.

Influence of Other Players' Strategies

We want to prove that asymptotically, in the game $\{S^K, \mathbb{S}, (u_k)_{k \in S^K}\}$, the strategy of a single player does not have any influence on the payoff of the other players. In other words, for all $k \neq i \in S^K$, for all $\mathbf{p} = (P_1, \ldots, P_K) \in \mathbb{S}^K$, for all $P'_i \in \mathbb{S}$,

$$\left| u_k(\mathbf{p}) - u_k(P'_i, \mathbf{p}_{(-i)}) \right| \to 0, \text{ as } N \to \infty.$$

Remember that $u_k = \frac{\gamma(\beta_k)}{P_k}$, and γ is at least C^2. Let $(\beta_1, \ldots, \beta_K)$ be the SINRs associated with the power allocation $\mathbf{p}$ and $(\beta'_1, \ldots, \beta'_K)$ the SINRs associated with the power allocation $(P'_i, \mathbf{p}_{(-i)})$. Then a simple Taylor expansion of γ in β'_k gives

$$\gamma(\beta'_k) = \gamma(\beta_k) + (\beta'_k - \beta_k) \frac{\partial \gamma}{\partial \beta}(\beta_k) + o(\beta'_k - \beta_k). \tag{7.18}$$

According to (7.18), it is sufficient to show that

$$\left| \frac{\beta'_k - \beta_k}{P_k} \right| \to 0, \text{ as } N \to \infty. \tag{7.19}$$

Matched Filter For the matched filter, the inequality is obtained directly from (5.33). The denominator of (5.33) is always greater than $\frac{\sigma^2}{N} \sum_{n=1}^N |h_{nk}|^2$. Hence,

$$\begin{aligned}
\left| \frac{\beta'_k - \beta_k}{P_k} \right| &\leq \left| \frac{P_k \frac{1}{N}(P'_i - P_i) \frac{1}{N} \sum_{n=1}^N |h_{ni}|^2 |h_{nk}|^2}{P_k \sigma^4} \right| \\
&\leq \frac{P_{\max} h_{\max}^2}{\sigma^4 N}.
\end{aligned}$$

MMSE Filter For the MMSE filter, the inequality is obtained from (5.34), Lemma 1 from [ET00] and Lemma 2.1 from [BS07], which we both reproduce below for convenience.

Lemma 2 *[ET00] Let $\mathbf{C}$ be a $N \times N$ complex matrix with uniformely bounded spectral radius for all N: $\sup_N(|\mathbf{C}|) < \infty$. Let $\mathbf{w} = \frac{1}{\sqrt{N}}[w_1, \dots, w_N]^T$ where $\{w_i\}_{i=1\dots N}$ are i.i.d. complex random variables with zero mean, unit variance and finite eighth moment. Then:*

$$\mathbb{E}\left[\left|\mathbf{w}^H\mathbf{C}\mathbf{w} - \frac{1}{N}\operatorname{tr}\mathbf{C}\right|^4\right] \leq \frac{C}{N^2}$$

where C is a constant that does not depend on N or $\mathbf{C}$.

Lemma 3 *[BS07] Let $\sigma^2 > 0$, $\mathbf{A}$ and $\mathbf{B}$ $N \times N$ with $\mathbf{B}$ Hermitian nonnegative definite, and $\mathbf{q} \in \mathbb{C}^N$. Then*

$$\operatorname{tr}\left(\left((\mathbf{B} + \sigma^2\mathbf{I})^{-1} - (\mathbf{B} + \mathbf{q}\mathbf{q}^H + \sigma^2\mathbf{I})^{-1}\right)\mathbf{A}\right) \leq \frac{\|\mathbf{A}\|}{\sigma^2}.$$

In Lemma 3, $\|\mathbf{A}\|$ is the spectral norm of $\mathbf{A}$, i.e., the square root of the largest singular value of $\mathbf{A}$.

From (7), we can write

$$\beta_k = P_k{\mathbf{w}_k}^H\mathbf{H}_k^H\left(\mathbf{G}_{(-k)}\mathbf{G}_{(-k)}^H + \sigma^2\mathbf{I}_N\right)^{-1}\mathbf{H}_k\mathbf{w}_k,$$
$$\beta_k' = P_k{\mathbf{w}_k}^H\mathbf{H}_k^H\left(\mathbf{G}_{(-k)}'\mathbf{G}_{(-k)}^H{}' + \sigma^2\mathbf{I}_N\right)^{-1}\mathbf{H}_k\mathbf{w}_k$$

where

$$\mathbf{G}_{(-k)}'\mathbf{G}_{(-k)}^H{}' = \mathbf{G}_{(-k)}\mathbf{G}_{(-k)}^H + (P_i' - P_i)(\mathbf{h}_i \odot \mathbf{w}_i)(\mathbf{h}_i \odot \mathbf{w}_i)^H.$$

A corollary of Lemma 2 is that for either matrix $\mathbf{C} = \mathbf{H}_k^H\left(\mathbf{G}_{(-k)}\mathbf{G}_{(-k)}^H + \sigma^2\mathbf{I}_N\right)^{-1}\mathbf{H}_k$ or matrix $\mathbf{C} = \mathbf{H}_k^H\left(\mathbf{G}_{(-k)}'\mathbf{G}_{(-k)}^H{}' + \sigma^2\mathbf{I}_N\right)^{-1}\mathbf{H}_k$, we obtain [ET00]

$$\left|{\mathbf{w}_k}^H\mathbf{C}\mathbf{w}_k - \frac{1}{N}\operatorname{tr}\mathbf{C}\right| \to 0, \text{ as } N \to \infty.$$

Matrix $\mathbf{B} = \mathbf{G}_{(-k)}\mathbf{G}_{(-k)}^H$ is Hermitian nonnegative definite, as for all $\mathbf{w} \in \mathbb{C}^N$, $\mathbf{w}^H\mathbf{G}_{(-k)}\mathbf{G}_{(-k)}^H\mathbf{w} = \left\|\mathbf{G}_{(-k)}\mathbf{w}\right\|^2 \geq 0$. Diagonal matrix $\mathbf{A} = \mathbf{H}_k\mathbf{H}_k^H$ has spectral norm $\left\|\mathbf{H}_k\mathbf{H}_k^H\right\| \leq h_{\max}^2$. Using Lemmas 2 and 3, as $N \to \infty$, we obtain

$$\left|\frac{\beta_k' - \beta_k}{P_k}\right| \to 0, \text{ as } N \to \infty.$$

Optimum and Successive Interference Cancellation Filters The analog of the SINR derived for the optimum filter stems from the MMSE filter with SIC. The SINR for SIC filters have similar expressions with less interfering users appearing in the denominator. Hence the result is immediate.

Proof of Prop. 16

Given $C^\star$, we can use (5.42) to obtain a Nash equilibrium power allocation in the following way. We rewrite (5.42) assuming that the target SINR for the MMSE filter is β^+.

$$\alpha \log_2\left(1+\beta^+\right) - \alpha \log_2(e)\frac{\beta^+}{1+\beta^+} + \log_2\left(1+\frac{1}{\sigma^2\left(1+\beta^+\right)}\int_0^\alpha P(y)\left|h(y)\right|^2 dy\right) = \alpha\log_2\left(1+\beta^\star\right). \quad (7.20)$$

In the left-hand side of (7.20), $P(y)$ is given by a MMSE power allocation similar to the one given by (5.50). Hence, the term $\int_0^\alpha P(y)\left|h(y)\right|^2 dy$ in (7.20) does not depend on the actual realizations of the channels. Replacing $\beta^\star$ by β^+ in (5.49), we obtain that $\int_0^\alpha P(y)\left|h(y)\right|^2 dy = \frac{\alpha\sigma^2\beta^+}{1-\alpha\frac{\beta^+}{1+\beta^+}}$, which gives us (5.52). Replacing $\beta^\star$ by β^+ in (5.50), we obtain the power allocation (5.51).

Expectation of the random variable (5.55)

For each user j, there are $L > 1$ paths with respective attenuations $h_\ell\left(\frac{j}{N}\right)$, $\ell = 1,\ldots,L$, which are i.i.d. complex random variables with mean zero and even distributions of the real and imaginary parts. The Fourier transform of those attenuations is $h_{nj} = h\left(\frac{n}{N},\frac{j}{N}\right) = \sum_{\ell=1}^L h_\ell\left(\frac{j}{N}\right)e^{-2\pi i\frac{n}{N}(\ell-1)}$. The total energy of the paths is $E_j = \sum_{\ell=1}^L \left|h_\ell\left(\frac{j}{N}\right)\right|^2$.

We want to show that the expectation of the random variable $\frac{1}{K}\sum_{j=1}^K\frac{|h_{nj}|^2}{E_j}$ is equal to 1. By expanding the expression of h_{nj}, this is equivalent to showing that the expectation of the random variable

$$\frac{h_\ell\left(\frac{j}{N}\right)h_{\ell'}\left(\frac{j'}{N}\right)}{E_j}$$

is equal to 0. Denoting by $p(\cdot)$ the distribution of $h_\ell = h_\ell\left(\frac{j}{N}\right)$, this expectation is equal to the L-dimensional integral of

$$\frac{h_\ell h_{\ell'}}{\left|h_\ell\right|^2+\left|h_{\ell'}\right|^2+\sum_{k\neq\ell,\ell'}\left|h_k\right|^2}p\left(h_\ell\right)p\left(h_{\ell'}\right)\prod_{k\neq\ell,\ell'}p\left(h_k\right)$$

which is an odd function of h_ℓ. Its integral is therefore 0, which proves the desired result.

Proof of (5.67) **and** (5.68)

Denote $m_k = P_{K-k}\left|h_{K-k}\right|$. From (5.64), with flat fading, the sequence $\{m_k\}_{k\in S^K}$ satisfies $m_0 = \beta^\star\sigma^2$ and $m_{k+1} = \beta^\star\sigma^2 + \frac{\beta^\star}{N}\sum_{j=0}^k m_j$. Using the fact that $\sum_{i=j}^k\binom{i}{j} = \binom{k+1}{j+1}$, it is immediate to prove by recurrence that

$$m_k = \beta^\star\sigma^2\sum_{j=0}^k\binom{k}{j}\frac{1}{N^j}\beta^{\star j} = \beta^\star\sigma^2\left(1+\frac{1}{N}\beta^\star\right)^k.$$

Hence formula (5.67). The demonstration is exactly similar for (5.68) from the recursion $m_0 = \beta^\star \sigma^2$ and $m_{k+1} = \beta^\star \sigma^2 + \frac{\beta^\star}{(1+\beta^\star)N} \sum_{j=0}^{k} m_j$.

Optimal Ordering of Users

We determine the ordering that makes use of the least total power for equilibrium power allocation (5.67) (the case is similar for (5.68), (5.69) and (5.70)). Let the ordering of the users be such as $|h_1|^2 < \cdots < |h_K|^2$. Let π be any permutation of $\{1, \ldots, K\}$. Let $a_{ij} = \left(1 + \frac{1}{N}\beta^\star\right)^{K-i} - \left(1 + \frac{1}{N}\beta^\star\right)^{K-j}$.

Then showing that the optimal ordering is such as $|h_1|^2 < \cdots < |h_K|^2$ is equivalent to showing that for any π

$$\sum_{k=1}^{K} \frac{1}{|h_k|^2} a_{k\pi(k)} > 0. \tag{7.21}$$

Consider first a cyclic permutation. By the definition of a_{ij}, the sum of the $a_{k\pi(k)}$ is equal to zero: $\sum_{k=1}^{K} a_{k\pi(k)} = 0$. The first coefficient $a_{1\pi(1)}$ is positive. It is affected coefficient $\frac{1}{|h_1|^2}$, which is the greatest coefficient in the sum in (7.21). Hence the sum in (7.21) is positive in this case.

Permutation π can be decomposed as a product of disjoint permutation cycles. Each cycle determines a subset of indexes k, these subsets form a partition of $\{1, \ldots, K\}$. With a similar reasoning as precedently, replacing index 1 with the smallest index in the cycle, the sum over the indexes k pertaining to a cycle of $\frac{1}{|h_k|^2} a_{k\pi(k)}$ is positive. Hence the global sum of (7.21) is also positive.

It can be proven in a similar way that the same ordering maximizes the sum of inverse powers of the users.

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