Bounds on the Distortion for Distributed Sensing of Slowly-Varying Random Fields

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***Abstract*— We consider a wireless sensor network deployed in an area to measure the realization of a finite multi-dimensional, slowly time-varying physical random field. Each sensor observes one noisy realization of the field, maps it linearly into a signal with a signature and sends it across a white Gaussian multiple access channel, under a constraint on the total energy given to all the sensors per field realization. The receiver or the 'collector node' receives all the signals and tries to construct an estimate of the field within a certain mean distortion based on the MSE fidelity criterion. We derive, under the total energy constraint, a lower-bound on the distortion, an achievable one, and another lower-bound under a TDMA transmission scheme. In the case of the non-existence of the observation noise, we find the asymptotic decreasing behavior of the achievable distortion as a function of the number of sensors. Moreover, we derive a lower-bound on the distortion over all possible encoding techniques, assuming a free collaboration and information exchange between the sensors. We compare these bounds for a particular example with another bound on the achievable distortion [1].**

I. Introduction

Wireless sensor networks are typically used to monitor some spatial characteristics of a field in the area over which the network is deployed. Examples of fields include temperature, electromagnetic radiation, natural or induced vibration, or auditory levels. In such networks, sensors make measurements of the field, process them locally, potentially with the help of neighboring nodes, and then collectively transmit the measurements over a wireless channel to one or more collector nodes. The collector nodes process the received measurement data further in order to extract and analyze the spatial characteristics of the field. The range of applications of such networks is becoming very large ; including environmental and habitat monitoring to military surveillance, security and civil protection applications. One of the more critical issues in these networks is the lifetime of the sensing nodes which is especially true in the case of applications which require small autonomous sensor devices, and thus small long-life energy sources. Energy efficiency, therefore, quickly becomes a critical factor.

While a sensor network is application-dependent, we restrict our work to an application where the sensors have to track a slowly time-varying random field. After sensing and coding the local data, the sensors have to send their information to a collector node through a white Gaussian multiple access channel. For achievable schemes, we assume a linear encoder in each sensor that maps the sensed value into the amplitude of a *signature* waveform which is transmitted across the channel. As is common in the literature, we consider that the sensing process is imperfect, so that the sensed values are subject to additive Gaussian observation noise. The choice of linear encoder is motivated firstly by its simplicity, and secondly by its optimality in the point-to-point communication model where a Gaussian source is sent over an AWGN channel [2], [3]. The latter is true only in the case where the number of channel uses per source symbol is one. Moreover, here we are confronted with a problem of correlated sources, since the sensed values are generally correlated, over a multiple-access channel. The source-channel separation theorem does not hold in this case, and therefore, separate coding strategies are in general sub-optimal [4]. In addition, joint source-channel encoders used to code long source sequences as described in [5] will not be appropriate in the case of a slowly time-varying random field due to the incurred delay. Recent results in [6] show the optimality of linear encoders for a simple sensor network model where a single Gaussian source is observed by multiple noisy Gaussian sensors, and these observations have to be transmitted via the standard Gaussian multiple access channel. This is again for the case where a single source letter is available per channel use.

Here we consider sensor networks with a constraint on the total radiated signal energy where we seek to minimize the distortion between the true field and its reconstructed estimate at the collector node. A similar model has been studied by Gastpar and Vetterli in [1], [7], and, under certain field configurations, the achievable distortion we find here can be compared to the scheme in [1] and clearly outperforms it. Other than demonstrating an achievable scheme, we derive a lower-bound on the distortion over all possible total energy distributions and all signatures, and another more general lower-bound which is not limited to linear encoders. This latter bound assumes that the sensors can communicate with each other freely in order to exchange information, a problem which was studied extensively in [8]. Under a TDMA transmission scheme, a lower-bound on the distortion is derived, which we find to be independent of the number of sensors. This result shows the sub-optimality of TDMA in the ideal case where the sensor observations are not corrupted by noise, since the other schemes exhibit decreasing distortion with the number of sensors.

The paper is organized as follows. In section II, we describe the sensor network model. An achievable distortion and a lower-bound on any linear encoding scheme is found in section III. In section IV, we derive the general lower-bound over all possible encoders. In section V, we calculate the asymptotic behavior of the distortion under an ideal sensor network model and derive a lower-bound for a TDMA-based transmission scheme. Numerical results, comparisons, and discussions are found in section VI.

Concerning the notations used in this paper, a bold letter (eg : $\mathbf{a}$) denotes a vector, while bold and underlined letter (eg : $\underline{\mathbf{a}}$) denotes a matrix. The i^{th} singular value and the i^{th} eigenvalue of a matrix $\underline{\mathbf{a}}$ are denoted respectively by $\sigma_i(\underline{\mathbf{a}})$ and $\mu_i(\underline{\mathbf{a}})$. $E[.]$ denotes the mean value over all random variables inside the brackets.

II. Model

The sensor network model is depicted in Fig.1. We consider a field $F(\mathbf{x})$ occupying a certain area $\mathcal{A}$ and depending on the spatial-coordinate vector $\mathbf{x}$. We assume that the field $F(\mathbf{x})$ can be represented in a finite-dimensional orthonormal basis of space functions $\phi_i(\mathbf{x})$ for $i = 1, \dots, N'$, by considering that the energy of the field lying outside the basis is too small and could be neglected. Then

$$F(\mathbf{x}) = \sum_{i=1}^{N'} \sqrt{\lambda_i} U_i \phi_i(\mathbf{x}) \tag{1}$$

where each λ_i is a constant representing the energy of the field in the i^{th} dimension and $\mathbf{U} = (U_1, \dots, U_{N'})^t$ is a Gaussian random vector with mean zero and identity covariance matrix. In the area $\mathcal{A}$, M sensors are randomly deployed, having $\mathbf{x}_1, \dots, \mathbf{x}_M$ as space coordinates. The sensor k senses the value $R(\mathbf{x}_k)$, a noisy version of the field at position $\mathbf{x}_k$:

$$R(\mathbf{x}_k) = F(\mathbf{x}_k) + W_k \tag{2}$$

where W_k for $k = 1, \dots, M$ are i.i.d Gaussian observation noise with zero-mean and variance σ_W^2 ; this value is mapped onto the signal

$$S_k\big(R(\mathbf{x}_k), t\big) = \sum_{i=1}^{N} S_{ki}\gamma_i(t) \qquad \text{for } t \in [0, T]$$

where the set $\{\gamma_1(t), \gamma_2(t), \dots, \gamma_N(t)\}$ forms an orthonormal basis for the signal space and S_{ki} is the projection of the signal on $\gamma_i(t)$. The vector $\mathbf{S}_k$ representing the signal components is taken equal to

$$\mathbf{S}_k = \begin{pmatrix} S_{k1} \\ S_{k2} \\ \vdots \\ S_{kN} \end{pmatrix} = \sqrt{\frac{E_k}{E[R^2(\mathbf{x}_k)]}} R(\mathbf{x}_k) \boldsymbol{\psi}_k \tag{3}$$

where E_k is the mean energy attributed to the sensor k and $\boldsymbol{\psi}_k$ a normalized vector representing a signature. A mean total energy E_T being dedicated to all the sensors in order that each one transmit one signal, the energy constraint could be written as

$$\sum_{k=1}^{M} E_k \le E_T. \tag{4}$$

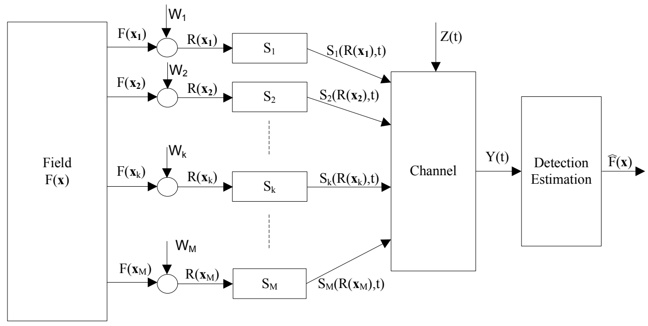


Fig. 1. The scheme of the considered wireless sensor network model

After the processing stage, the sensors send simultaneously their signals to the collector node through a Gaussian multiple access channel. The output $Y(t)$ of the channel can be written like

$$Y(t) = \sum_{k=1}^{M} \sqrt{\alpha_k} S_k\big(R(\mathbf{x}_k), t\big) + Z(t)$$

with α_k representing an attenuation factor proportional to the distance between the sensor k and the collector node, and $Z(t)$ the white Gaussian noise with zero mean and σ_Z^2 as power spectral density. The baseband expression of $Y(t)$ implies an adjustement at the sensor transmitters of the phases induced by the channel.

At the detection, we calculate $\widehat{F}(\mathbf{x})$, the estimate of $F(\mathbf{x})$ for all $\mathbf{x} \in \mathcal{A}$. Here, we assume that $\alpha_1, \dots, \alpha_M$ and $\mathbf{x}_1, \dots, \mathbf{x}_M$ are perfectly known to the collector node. The distortion measure is the mean squared error, and the total mean distortion that we want to minimize is equal to

$$D = \int_{\mathbf{x} \in \mathcal{A}} E\left[\big(F(\mathbf{x}) - \widehat{F}(\mathbf{x})\big)^2\right] d\mathbf{x} \tag{5}$$

III. Performance Limits Of Linear Coding

In this section, we'll focus on the linear mapping scheme that we have presented in our model, in order to test its performance in terms of minimal achievable distortion. In other words, with a such linear coding done by the sensors in the processing stage and under the sum-energy constraint, we aim to find the total energy distribution $E_1, \dots, E_M$ over the sensors and the appropriate choice of the signatures $\boldsymbol{\psi}_1, \dots, \boldsymbol{\psi}_M$ that will minimize the total mean distortion. We put all these variables in a matrix $\underline{\mathbf{A}} \triangleq (\sqrt{E_1}\boldsymbol{\psi}_1, \dots, \sqrt{E_M}\boldsymbol{\psi}_M)$ and let $\underline{\mathbf{A}} = \underline{\mathbf{U}}_A \underline{\boldsymbol{\Sigma}}_A \underline{\mathbf{V}}_A^t$ be the singular value decomposition of $\underline{\mathbf{A}}$. Then, the total energy constraint could be written as

$$\sum_{i=1}^{p} \sigma_i^2(\underline{\mathbf{A}}) \le E_T \tag{6}$$

where $p = \min(M, N)$. Consequently, our goal will be to find the minimal achievable distortion corresponding to a certain matrix $\underline{\mathbf{A}}$ satisfying (6). Unfortunately, this problem is quite hard to resolve, therefore, we limit ourselves to a lower-bound and the resulting distortion of a particular encoding scheme which yields an upper-bound on any optimal scheme.

A. Achievable Distortion

Developing (5), the distortion could be written as

$$D = \sum_{i=1}^{N'} \lambda_i E\left[(U_i - \widehat{U_i})^2\right] \quad (7)$$

where $\widehat{\mathbf{U}}$ is the estimate of $\mathbf{U}$. In order to minimize the distortion, the best estimator to be chosen is the minimum mean squared error (MMSE) estimator. This latter is equal to

$$\widehat{\mathbf{U}} = E[\mathbf{U}\mathbf{Y}^t]\left(E[\mathbf{Y}\mathbf{Y}^t]\right)^{-1}\mathbf{Y}, \quad (8)$$

$\mathbf{Y}$ being the projection of $Y(t)$ on the signal space basis. let

$$\underline{\gamma} = \begin{pmatrix} \frac{\alpha_1}{E[R^2(\mathbf{x}_1)]} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \frac{\alpha_M}{E[R^2(\mathbf{x}_M)]} \end{pmatrix}, \quad (9)$$

$$\underline{\mathbf{B}} = \begin{pmatrix} \sqrt{\frac{\lambda_1\alpha_1}{E[R^2(\mathbf{x}_1)]}}\phi_1(\mathbf{x}_1) & \dots & \sqrt{\frac{\lambda_{N'}\alpha_1}{E[R^2(\mathbf{x}_1)]}}\phi_{N'}(\mathbf{x}_1) \\ \vdots & & \vdots \\ \sqrt{\frac{\lambda_1\alpha_M}{E[R^2(\mathbf{x}_M)]}}\phi_1(\mathbf{x}_M) & \dots & \sqrt{\frac{\lambda_{N'}\alpha_M}{E[R^2(\mathbf{x}_M)]}}\phi_{N'}(\mathbf{x}_M) \end{pmatrix};$$

therefore, (7) gives us

$$D = \sum_{i=1}^{N'} \lambda_i \left[1 - \mathbf{\Gamma}_i^t \underline{\mathbf{h}}^t \left(\sigma_Z^2 \mathbf{I}_N + \underline{\mathbf{A}}\underline{\mathbf{C}}\underline{\mathbf{A}}^t\right)^{-1} \underline{\mathbf{h}}\mathbf{\Gamma}_i\right], \quad (10)$$

where $\mathbf{\Gamma}_i = (0, \dots, 0, 1, 0, \dots, 0)^t$ is the vector that has the i^{th} component equal to 1 and all other components equal to zero, $\underline{\mathbf{h}} = \underline{\mathbf{A}}\underline{\mathbf{B}}$ and $\underline{\mathbf{C}} = \sigma_W^2 \underline{\gamma} + \underline{\mathbf{B}}\underline{\mathbf{B}}^t$. Being symmetric, the eigenvalue decomposition of the matrix $\underline{\mathbf{C}}$ could be written as $\underline{\mathbf{C}} = \underline{\mathbf{U}}_C \underline{\boldsymbol{\mu}}_C \underline{\mathbf{U}}_C^t$ where

$$\underline{\boldsymbol{\mu}}_C = \textbf{diag}\left(\mu_1(\underline{\mathbf{C}}), \dots, \mu_M(\underline{\mathbf{C}})\right) \quad (11)$$

with $\mu_1(\underline{\mathbf{C}}) \geq \dots \geq \mu_M(\underline{\mathbf{C}})$, and $\underline{\mathbf{U}}_C$ the matrix of the corresponding eigenvectors.

Hence, by choosing $\underline{\mathbf{V}}_A = \underline{\mathbf{U}}_C$, and then, using Lagrange multipliers [9] in order to optimize the resultant distortion with respect to the singular values of $\underline{\mathbf{A}}$ subject to (6), we obtain

$$D_{ach} = \sum_{i=1}^{N'} \lambda_i - \sum_{i=1}^{N'} \sum_{j=1}^{p} \frac{\lambda_i l_{ij}^2 \gamma_j^+}{\mu_j(\underline{\mathbf{C}})[\sigma_Z^2 + \gamma_j^+]} \quad (12)$$

where $\mathbf{l}_i = (l_{i,1}, \dots, l_{i,M}) = \mathbf{\Gamma}_i^t \underline{\mathbf{B}}^t \underline{\mathbf{U}}_C$,

$$\gamma_j^+ = \max\left\{0, \sqrt{\frac{\sigma_Z^2 \sum_{k=1}^{N'} \lambda_k l_{kj}^2}{\delta}} - \sigma_Z^2\right\} \quad (13)$$

and δ is such that

$$\sum_{j=1}^{p} \frac{\gamma_j^+}{\mu_j(\underline{\mathbf{C}})} = E_T \quad (14)$$

B. Lower-Bound

We will derive here a lower-bound over all achievable distortions while the sensors are performing the linear mapping described in the previous section. Let q be the rank of $\underline{\mathbf{B}}$, $\underline{\mathbf{U}}_B \underline{\mathbf{\Sigma}}_B \underline{\mathbf{V}}_B^t$ its ordered ($\sigma_1(\underline{\mathbf{B}}) \geq \dots \geq \sigma_q(\underline{\mathbf{B}})$) singular value decomposition and $r = \min(p, q)$. Let also $\underline{\mathbf{h}}^t\underline{\mathbf{h}} = \underline{\mathbf{V}}\underline{\boldsymbol{\mu}}\underline{\mathbf{V}}^t$ be the ordered eigenvalue decomposition ($\mu_1(\underline{\mathbf{h}}^t\underline{\mathbf{h}}) \geq \dots \geq \mu_{N'}(\underline{\mathbf{h}}^t\underline{\mathbf{h}})$) of $\underline{\mathbf{h}}^t\underline{\mathbf{h}}$, v_{ij} the entries of $\underline{\mathbf{V}}$ and $\lambda_{\min}$ the minimum of $\{\lambda_1, \dots, \lambda_N'\}$. Any achievable distortion will be lowered by canceling the effect of the observation noise or equivalently by taking $\sigma_W = 0$. Therefore, we have the following inequalities

$$\begin{aligned} D &\geq \sum_{i=1}^{N'} \sum_{j=1}^{N'} \lambda_i \frac{\sigma_Z^2 v_{ij}^2}{\sigma_Z^2 + \mu_j(\underline{\mathbf{h}}^t\underline{\mathbf{h}})} && (15) \\ &\geq \lambda_{\min} \sum_{j=1}^{N'} \frac{\sigma_Z^2}{\sigma_Z^2 + \mu_j(\underline{\mathbf{h}}^t\underline{\mathbf{h}})} && (16) \\ &= \lambda_{\min} \sum_{j=1}^{r} \frac{\sigma_Z^2}{\sigma_Z^2 + \mu_j(\underline{\mathbf{h}}^t\underline{\mathbf{h}})} + \lambda_{\min}(N' - r) && (17) \end{aligned}$$

The eigenvalues of $\underline{\mathbf{h}}^t\underline{\mathbf{h}}$ are constrained by (see [13] p.171)

$$\prod_{i=1}^{r} \mu_i(\underline{\mathbf{h}}^t\underline{\mathbf{h}}) \leq \left(\prod_{i=1}^{r} \sigma_i^2(\underline{\mathbf{A}})\right)\left(\prod_{i=1}^{r} \sigma_i^2(\underline{\mathbf{B}})\right) \quad (18)$$

Maximizing the right hand side of (18) subject to the sum energy constraint, gives that for every matrix $\underline{\mathbf{A}}$ satisfying (6),

$$\prod_{i=1}^{r} \mu_i(\underline{\mathbf{h}}^t\underline{\mathbf{h}}) \leq \left(\frac{E_T}{r}\right)^r \left(\prod_{i=1}^{r} \sigma_i^2(\underline{\mathbf{B}})\right) \quad (19)$$

Then, minimizing (17) over (19) gives the following result (the proof has been omitted due to space restrictions)

$$D_{lower} = \lambda_{\min} \frac{r^2 \sigma_Z^2}{r\sigma_Z^2 + E_T \sqrt[r]{\prod_{i=1}^{r} \sigma_i^2(\underline{\mathbf{B}})}} + \lambda_{\min}(N' - r) \quad (20)$$

$$\text{for} \qquad \frac{E_T}{r} \sqrt[r]{\prod_{i=1}^{r} \sigma_i^2(\underline{\mathbf{B}})} \geq (2r - 1)\sigma_Z^2. \quad (21)$$

IV. General Lower-Bound

Until now, we have dealt with linear encoders that just forward their observed values across the channel. Such encoders are known to have some advantages in terms of low complexity and delay. It is natural to consider the distortion achievable with more general encoders. In this section, we derive a more general lower-bound on the distortion over all possible encoders, all total energy distributions and all signatures ; note that, as said previously, the separation theorem does not hold because of having to code correlated observations over a multiple access channel. Thus, doing multi-terminal source coding, then, using capacity-achieving channel encoders does not lead to an optimal distortion and consequently to a lower-bound. Therefore, we will assume that the sensors can communicate freely with each other, an assumption that will render our model equivalent to a point-to-point communication model on which the separation theorem holds and an achievable lower-bound is well-known. A similar lower-bound has been found in [1] (see also [10], [11]) and can be applied for the special case of our random field where $\lambda_1 = \ldots = \lambda_{N'}$; we generalise this result for general $\lambda_1, \ldots, \lambda_{N'}$ in order to obtain the lower-bound that we seek. Since the observations are noisy versions of the field, the lower-bound is equal to $D_{remote}(C)$, where

$$D_{remote}(R) = \min_{p(\hat{\mathbf{u}}/\mathbf{r}):I(\widehat{\mathbf{U}};\mathbf{R})\leq R} \sum_{i=1}^{N'} \lambda_i E[(U_i - \widehat{U}_i)^2] \quad (22)$$

is the remote distortion rate function of the source vector $\mathbf{U}$, $\mathbf{R} = (R(\mathbf{x}_1), \ldots, R(\mathbf{x}_M))$ and C is the capacity of the N uses of the multiple input one output channel. Let

$$\underline{\mathbf{\Phi}} = \begin{pmatrix} \sqrt{\lambda_1}\phi_1(\mathbf{x}_1) & \ldots & \sqrt{\lambda_{N'}}\phi_{N'}(\mathbf{x}_1) \\ \vdots & & \vdots \\ \sqrt{\lambda_1}\phi_1(\mathbf{x}_M) & \ldots & \sqrt{\lambda_{N'}}\phi_{N'}(\mathbf{x}_M) \end{pmatrix} \quad (23)$$

and

$$\underline{\mathbf{\Phi}}' = \underline{\mathbf{\Phi}}\mathbf{diag}(\sqrt{\lambda_1}, \ldots, \sqrt{\lambda_{N'}}). \quad (24)$$

We find that

$$D_{remote}(C) = \sum_{i=1}^{N'} \left[\lambda_i - \sigma_i^2 + \min\{\sigma_i^2, \delta\}\right] \quad (25)$$

with

$$\frac{1}{2}\sum_{i=1}^{N'} \left(\log \frac{\sigma_i^2}{\delta}\right)^+ = C \quad (26)$$

where $\sigma_1^2, \ldots, \sigma_{N'}^2$ are the eigenvalues of the matrix $\underline{\mathbf{\Phi}}'^t \left(\underline{\mathbf{\Phi}}\underline{\mathbf{\Phi}}^t + \sigma_W^2 \underline{\mathbf{I}}_M\right)^{-1} \underline{\mathbf{\Phi}}'$ and

$$C = \frac{N}{2}\log\left(1 + \frac{E_T \sum_{i=1}^M \alpha_i}{N\sigma_Z^2}\right) \quad (27)$$

V. Ideal Model

Consider an ideal sensor network model in which the node's sensing ability is perfect, in the sense that the observations collected by the sensors are no longer corrupted by observation noise. Assume that the signal space dimension is larger or equal to the field dimension and that $\alpha_i < \infty$ for $i = 1, \ldots, M$. In that case, by choosing $\underline{\mathbf{V}}_A = \underline{\mathbf{U}}_B$ and $\sigma_1(\underline{\mathbf{A}}) = \ldots = \sigma_{N'}(\underline{\mathbf{A}}) = \sqrt{\frac{E_T}{N'}}$,

$$D = \sum_{i=1}^{N'}\sum_{j=1}^{N'} \lambda_i \frac{\sigma_Z^2 v_{B_{ij}}^2}{\sigma_Z^2 + \frac{E_T}{N'}\sigma_j^2(\underline{\mathbf{B}})} \quad (28)$$

is achievable where $v_{B_{ij}}$ are the entries of $\underline{\mathbf{V}}_B$.

A. Scaling Law

Let $\mathbf{b}(\mathbf{x_i})$ be the i^{th} line vector in $\underline{\mathbf{B}}$ corresponding to a sensor at position $\mathbf{x}_i$. Here, we assume that $\phi_i(\mathbf{x}_j) < \infty$ for $i = 1, \ldots, N'$ and $j = 1, \ldots, M$. Suppose that there exists N' areas $\mathcal{A}_1, \ldots, \mathcal{A}_{N'} \subset \mathcal{A}$ such that $\forall \mathbf{x_1} \in \mathcal{A}_1, \ldots, \forall \mathbf{x}_{N'} \in \mathcal{A}_{N'}$, the vectors $\mathbf{b}(\mathbf{x_1}), \ldots, \mathbf{b}(\mathbf{x_{N'}})$ are linearly independent. Suppose also, that the position of every sensor, which is random, follows a certain probability density function $p(\mathbf{x})$ defined on $\mathcal{A}$ and is independent from the positions of the other sensors. Thus, the probability that a sensor belongs to an area $\mathcal{A}_i$ is

$$c_i \triangleq \int_{\mathcal{A}_i} p(\mathbf{x})d\mathbf{x} \quad (29)$$

Denote by n_i the number of sensors in $\mathcal{A}_i$ after throwing M sensors and let $n = \min\{n_1, \ldots, n_{N'}\}$. Therefore, with the line vectors of $\underline{\mathbf{B}}$, it can be at least constructed n full ranked N'-dimensional square matrices denoted by $\underline{\mathbf{B}}_1, \ldots, \underline{\mathbf{B}}_n$. Then, for $i = 1, \ldots, N'$, we have the following result (see [12] p.176)

$$\frac{n}{M}d_{\min} \leq \frac{\sigma_i^2(\underline{\mathbf{B}})}{M} \leq d_{\max} \quad (30)$$

where

$$d_{\min} = \min_{i=1,\ldots,n}\Big[\min_{\mathbf{s}^t\mathbf{s}=1} ||\underline{\mathbf{B}}_i\mathbf{s}||^2\Big] \quad (31)$$

and

$$d_{\max} = \max_{i=1,\ldots,M}\Big[\max_{\mathbf{s}^t\mathbf{s}=1} (\mathbf{b}(\mathbf{x_i})\mathbf{s})^2\Big] \quad (32)$$

Due to the field and channel assumptions, $d_{\min}$ and $d_{\max}$ are strictly positive and bounded constants. Note that $\lim_{M\to\infty} \frac{n}{M} = c$ where $c = \min\{c_1, \ldots, c_{N'}\}$. Hence, the achievable distortion in (28) scales asymptotically as $\frac{1}{M}$. This scaling behavior is under investigation in the case where the sensors have no information about the channel.

B. Comparison With a TDMA Scheme

In a TDMA transmission scheme, the number of sensors is equal to the signal space dimension and the matrix $\underline{\boldsymbol{\psi}} \triangleq (\boldsymbol{\psi}_1, \ldots, \boldsymbol{\psi}_M)$ is unitary. Therefore,

$$\sum_{i=1}^{r} \sigma_i^2(\underline{\mathbf{h}}) = \sum_{i=1}^{M} \alpha_i E_i \leq E_T \alpha_{\max} \quad (33)$$

with $\alpha_{\max} = \max_{i=1,\ldots,M} \alpha_i$. Then, an easily lower-bound on the achievable distortion could be found. In fact, minimizing (17) under (33) leads to

$$D_{TDMA} \geq \frac{\lambda_{\min}\sigma_Z^2 r^2}{\sigma_Z^2 r + \alpha_{\max}E_T} + \lambda_{\min}(N' - r). \quad (34)$$

Compared to the achievable distortion in (28) which scales asymptotically like $1/M$, the left-hand side term in (34) does not depend on the number of sensors. This leads to the conclusion of the sub-optimality of such a transmission scheme especially when the number of sensors becomes large. Note that this lower-bound is also true when observation noise exists.

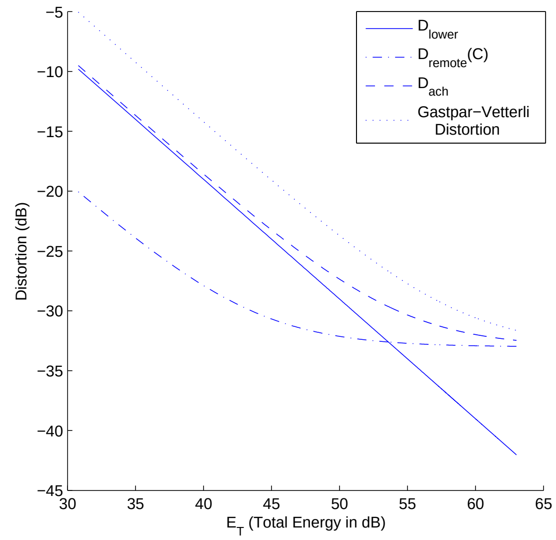
VI. Numerical Results

A simple sensor network model is considered to illustrate and compare some of the bounds derived in the above sections ; The area $\mathcal{A}$ is partitioned in ten smaller areas $\mathcal{A}_1, \ldots, \mathcal{A}_{10}$. These are put in a vector $\boldsymbol{\mathcal{A}} \triangleq (\mathcal{A}_1, \ldots, \mathcal{A}_{10})$. The space functions are taken equal to

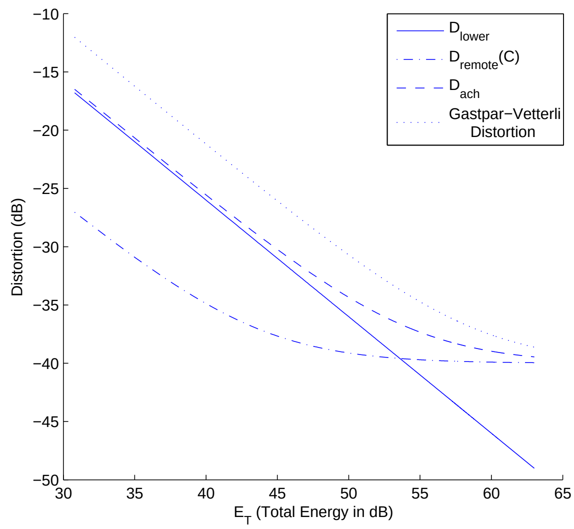
$$\phi_i(\mathbf{x}) = \begin{cases} \frac{1}{\sqrt{\mathcal{A}_i}} & \text{if } \mathbf{x} \in \mathcal{A}_i \\ 0 & \text{if } \mathbf{x} \notin \mathcal{A}_i \end{cases}$$

for $i = 1, \ldots, 10$.

In the figures 2(a) and 2(b), our achievable distortion outperforms the Gastpar-Vetterli result especially for relatevily small total energy. The distortion is due to the channel noise and to the observation noise, thus when the total energy increases, the influence of the channel noise on the distortion decreases. Therefore, it will be essentially caused by the observation noise, the influence of which being independant from the total energy. That's the reason why, the slope of the curves in 2(a) and 2(b) (except D_{lower} because it only depends on the channel noise) tends to zero when the total energy becomes relatively large. Comparing the curves corresponding to D_{ach} and D_{lower}, we see that the gap between them starts very small and then becomes larger which is due, as mentioned above, to the fact that the lower-bound does not take into account the observation noise. Note that the large difference between the two lower-bound curves for relatively small total energy reveals nothing on the efficiency of linear encoders, because of the free information exchange assumption taken in the calculation of the general lower bound ($D_{remote}(C)$) that render it non-achievable in general. In any case, $D_{remote}(C)$ still have the utility of giving us a lower-bound on the distortion over all possible encoders even if it is not achievable. At the end, comparing 2(a) to 2(b) reveals the decreasing behavior for all the distortion curves when the number of sensors increases.



(a) Number of sensors M=200



(b) Number of sensors M=1000

Fig. 2. Comparison between D_{lower}, D_{ach}, $D_{remote}(C)$ and Gastpar-Vetterli distortion : $\boldsymbol{\mathcal{A}} = [9,\ 15,\ 18,\ 12,\ 6,\ 7,\ 3,\ 13,\ 7,\ 10]$, $N = N' = 10$, $\lambda_1 = \ldots = \lambda_{10} = 10$, $\alpha_i = 1$ for $i = 1, \ldots, M$, $\sigma_Z = 1.5$, $\sigma_W = 0.01$

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