

# Capacity Maximizing Power Allocation for Interfering Wireless Links: A Distributed Approach

Saad G. Kiani  
Mobile Communications Department  
Eurécom Institute  
Sophia Antipolis, France F-06904  
Email: saad.kiani@eurecom.fr

David Gesbert  
Mobile Communications Department  
Eurécom Institute  
Sophia Antipolis, France F-06904  
Email: david.gesbert@eurecom.fr

**Abstract**—Recent results show that sum-rate maximizing multicell power allocation promises significant gains in interference-limited data networks. Finding practical, i.e. distributed, versions of this global optimization problem however remains a challenging task. In this work, we establish a general framework for the distributed power allocation problem for  $N$  mutually interfering links enabling us to derive a fully distributed power allocation algorithm. Although a gain for  $N = 2$  is observed, a performance gap is still observed compared to a centralized algorithm. As a way to fill that gap, we propose minimal information (in this case 1 bit) message passing between interfering links to improve performance. Numerical results show these algorithms to exploit a substantial amount of the capacity gain offered by centralized optimization.

## I. INTRODUCTION

Links operating on the same spectral resource are plagued by mutual interference which diminishes system capacity. Power control serves as a means to mitigate this effect and has been an extensively researched topic over the past 30 years. In traditional voice-centric wireless networks, power control was found to be an effective method to enhance the reliability of the system [1]–[4]. The key idea here is to balance the transmit powers to achieve a minimum acceptable level of signal-to-interference-plus-noise ratio (SINR) for each user.

In this work however, we investigate power control for future data wireless networks enabled with link adaptation protocols. These differ from voice networks due to the elastic nature of data traffic and thus guaranteeing a particular SINR requirement is not always the right strategy. Instead, we consider here the aggregate rate of the system as our metric and formulate the capacity maximizing power allocation problem. The optimal solution entails centralized processing of network-wide channel state information. Though this promises the maximum exploitable gain, from a practical point of view however, it is much too costly. Instead, we focus on distributed solutions to this problem.

As one avenue, game theoretic results have been explored to provide just that. Game theoretic algorithms represent the interfering links in the network as players of a non-cooperative game, where each tries to maximize its own utility function. Although the resulting power allocation strategies are very

The work reported herein was partially supported by the European Union FP 6 Project COOPCOM.

interesting and distributed by nature, such approaches do not always lead to globally (or "socially") optimum solutions. Pricing mechanisms have been looked at, which aim at penalizing the interference created to other links, in order to make the game outcome more socially optimum [5]–[9]. However, the pricing function itself needs to be optimized as well, and it typically depends on the particular system layout and environment [5], [7]. Some game theoretic approaches also require communication of information between links to compute the pricing function [9].

As an alternative to game theoretic approaches, Geometric Programming techniques can be applied in the high or low SINR regimes which render the power control problem convex [10], [11]. Finally, distributed capacity maximizing power control and scheduling algorithms were proposed in [12], which take advantage of a simplifying interference model. Such approaches [13] rely however, on statistical averaging properties of large random networks and thus are not applicable for all networks.

In this paper, we focus on distributed power allocation solutions, where the distributed nature of the optimization is formulated by means of statistical optimization. Our major contributions are listed below:

- We propose a statistical framework for rate maximizing power control in an arbitrary network with several interfering cells or links<sup>1</sup>. The key advantage of this framework is to allow for a fully distributed optimization of the power. The rate metric we are considering is the sum of rates achieved on each link under single user decoding, treating the multicell interference as noise.
- In the particular case of two links (say for a two cell network or a larger network with clusters of two cells), we develop a distributed algorithm based on the above framework, which leads to simple activation conditions for a link.
- Finally, by allowing a 1-bit message passing between interfering links, substantial improvement in the capacity performance can be obtained through a simple modification of the fully distributed algorithm.

Our numerical results show that the distributed and near

<sup>1</sup>In this work we will use the words link(s) and cell(s) interchangeably

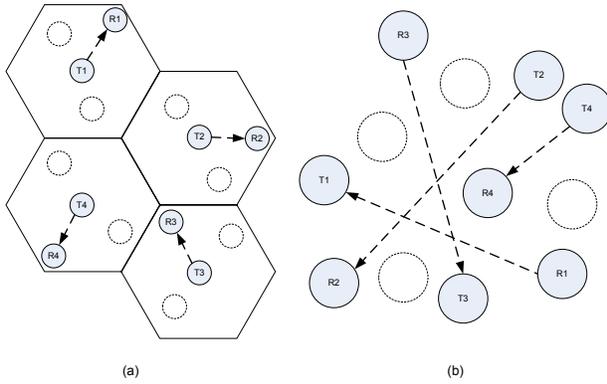


Fig. 1. Snapshot of network model, with  $N = 4$  interfering pairs of transmitters and receivers. The cellular (a) and ad-hoc (b) scenarios give rise to equivalent mathematical models. Dashed circles refer to silent users while solid circles refer to access points or users selected by the scheduler.

distributed power allocation algorithms largely outperform a system with fixed (or no) power control and are close to the performance given by centralized power control.

## II. SYSTEM MODEL

Consider a wireless network with a collection of nodes, which can be both transmitters and receivers. By virtue of a scheduling protocol,  $N$  transmit-receive active pairs are simultaneously selected from these nodes to communicate on any given *spectral resource slot* (time or frequency slots in TDMA/FDMA, or code in orthogonal CDMA), while others remain silent. In this paper, we do not worry about how or which links are activated as we only focus on power control. Note however that scheduling may be jointly optimized with power allocation, e.g. [12]. In this network, the transmitter sends a message to its intended receiver only. However, due to full spectral resource reuse, the receiver is interfered by all other active links. We assume single user decoding and thus interference from other links is treated as noise. This setup can be seen as an instance of the interference channel, the analysis of which is a famously difficult problem in information theory [14]. In practical terms, the situation depicted above can be that of a cellular network with reuse factor one (say e.g. the downlink with transmitters being access points (AP) or base stations) or, it can also depict a snapshot of an ad-hoc network (Fig.1).

### A. Signal Model

Denoting the random channel gain between any arbitrary transmitter  $i$  and receiver  $n$  by  $G_{n,i} \in \mathbb{R}^+$ , the received signal  $Y_n$  can be written as

$$Y_n = \sqrt{G_{n,n}}X_n + \sum_{i \neq n} \sqrt{G_{n,i}}X_i + Z_n,$$

where  $X_n$  is the intended signal from the transmitter,  $\sum_{i \neq n} \sqrt{G_{n,i}}X_i$  is the sum of interfering signals from other transmitters and  $Z_n$  is the noise. For convenience,  $Z_n$  is modeled as additive white Gaussian with power  $\mathbb{E}|Z_n|^2 = \sigma^2$ .

## III. POWER ALLOCATION FOR SUM-CAPACITY MAXIMIZATION

We now formulate the power allocation problem for sum-capacity maximization. We define the *transmit power vector*  $\mathbf{P} = [P_1 P_2 \cdots P_n \cdots P_N]$ , which contains transmit powers used by each transmitter to communicate with its respective receiver, where  $[\mathbf{P}]_n = P_n$ . As in all realistic networks, we impose a power constraint on each transmitter such that  $P_{\min} \leq P_n \leq P_{\max}$ . We assume from here on that  $P_{\min} = 0$ . We can then write the feasible set of transmit power vectors as  $\Omega = \{\mathbf{P} \mid 0 \leq P_n \leq P_{\max} \forall n = 1, \dots, N\}$ . Taking  $P_n = \mathbb{E}|X_n|^2$ , the signal to interference-plus-noise ratio (SINR) at the receiver of link  $n$  is then given by

$$\Gamma_n(\mathbf{P}) = \frac{G_{n,n}P_n}{\sigma^2 + \sum_{i \neq n} G_{n,i}P_i}. \quad (1)$$

### A. Objective Function

We see clearly that the SINR of each link is dependent on the complete transmit power vector and thus so will be the individual link capacities. Assuming an ideal link adaptation protocol and perfect CSI at the transmitter, we define the objective function as the sum of single user rates achieved over each link. This can be expressed in bits/sec/Hz using the Shannon capacity as

$$\mathcal{C}(\mathbf{P}) \triangleq \sum_{n=1}^N \log_2 \left( 1 + \Gamma_n(\mathbf{P}) \right). \quad (2)$$

With a slight abuse of terminology and in order to ease exposition, we shall refer to the expression above as the *network capacity*<sup>2</sup>.

### B. Optimal Power Allocation Problem

Taking (2) as the objective function we want to maximize, the optimal power allocation problem can be stated as

$$\mathbf{P}^* = \arg \max_{\mathbf{P} \in \Omega} \mathcal{C}(\mathbf{P}). \quad (3)$$

This problem is known to be non-convex and an optimal solution would require an exhaustive search over the feasible set of transmit powers which entails high complexity as well as centralized processing. However, an interesting result is presented in the next section which enables us to significantly reduce the complexity of this problem.

### C. Optimal Power Allocation for $N = 2$

An interesting result pertaining to problem (3) for  $N = 2$  is presented in [15], [16]. For convenience we restate it here:

*Lemma 1:* The optimal sum-capacity maximizing power allocation for 2 interfering links lies in the binary feasible set

$$\Omega^B = \{\mathbf{P} \mid P_n = 0 \text{ or } P_n = P_{\max}\}. \quad (4)$$

<sup>2</sup>We use capacity to refer to the sum of single user rates rather than capacity in the information-theoretic sense

*Proof:* See [15], [16]<sup>3</sup> ■

This result substantially decreases the optimization complexity from a continuous search space to 2 discrete values for each link. Moreover, numerical results suggest that with a greater number of links this binary allocation, although not strictly globally capacity-optimal in the Shannon sense, is close to the optimal power allocation [15], [16]. However, as the network capacity is coupled with the network-wide transmit powers, finding the optimal power allocation vector for given instantaneous channel realizations still requires a centralized solution. From a practical point of view this is not feasible and thus we turn our attention now to the distributed power allocation problem formulation.

#### IV. DISTRIBUTED POWER ALLOCATION

Distributed control can have many meanings depending on the availability of underlying information. In the most ideal setting, each link would make a decision based on local information i.e. information available at the transmitter. This would indeed be sub-optimal, as an assumption would have to be made about unknown information. None the less, this is the most practical form of distributed control in terms of both complexity and information exchange. In what follows, we formulate the distributed power allocation problem under statistical knowledge of unknown information. Note that this statistical knowledge can be acquired a priori during a network calibration phase.

##### A. Network Capacity Maximization Under Statistical Knowledge

As stated, we assume that each transmitter has only local knowledge. Let us declare the set containing all network information as  $\mathcal{G} = \{G_{i,j}\} \forall i, j$ . This means that the transmitter  $n$  only knows  $\mathcal{G}_n^{\text{local}}$ . Thus the unknown information at the transmitter can be represented by  $\tilde{\mathcal{G}}_n = \mathcal{G} \setminus \mathcal{G}_n^{\text{local}}$ . A transmitter  $n$  then tries to maximize the expected network capacity defined as

$$\bar{C}_n(\mathbf{P}) \triangleq \mathbb{E}_{\tilde{\mathcal{G}}_n} \left\{ \sum_{m=1}^N \log_2 \left( 1 + \frac{G_{m,m}P_m}{\sigma^2 + \sum_{i \neq m} G_{m,i}P_i} \right) \right\}. \quad (5)$$

$\mathbb{E}_{\tilde{\mathcal{G}}_n} \{\cdot\}$  is the expectation operator averaging the capacity over all realizations of  $\tilde{\mathcal{G}}_n$ , while keeping  $\mathcal{G}_n^{\text{local}}$  constant. The distributed power allocation problem under this framework can thus be written as

$$\text{for link } n \quad \mathbf{P}_n^* = \left[ \arg \max_{\mathbf{P} \in \Omega} \bar{C}_n(\mathbf{P}) \right]_n. \quad (6)$$

In what follows we assume local information to be  $\mathcal{G}_n^{\text{local}} = \{G_{n,i} \forall i\}$ , which means that a transmitter has knowledge of the direct channel and the interference from other cells to its intended receiver. This is a natural choice for local information, as these values can be measured at the receiver

<sup>3</sup>Note that this result was also reported independently in [17].

and fed back to the transmitter. Moreover, simulation results show this choice to give good performance in terms of network capacity gain. Under this knowledge, the expected network capacity that transmitter  $n$  tries to maximize is given by

$$\begin{aligned} \bar{C}_n(\mathbf{P}) \triangleq & \log_2 \left( 1 + \frac{G_{n,n}P_n}{\sigma^2 + \sum_{i \neq n} G_{n,i}P_i} \right) \\ & + \mathbb{E}_{\tilde{\mathcal{G}}_n} \left\{ \sum_{m \neq n}^N \log_2 \left( 1 + \frac{G_{m,m}P_m}{\sigma^2 + \sum_{i \neq m} G_{m,i}P_i} \right) \right\}. \quad (7) \end{aligned}$$

In the next section, we focus on the 2 link case which offers insight into the gain offered by this distributed approach. We propose a simple distributed algorithm to solve this problem as well as a modified version of this algorithm incorporating 1-bit message passing between links to enhance performance.

#### V. DISTRIBUTED POWER ALLOCATION FOR 2 LINKS

The case of problem (6) for 2 links is particular. However, the algorithm developed here can be used in a wider network with more links, where links are previously paired up in clusters of two links. Forming of the clusters should favor strongly interfering links, for which a distributed resource allocation technique will exhibit the largest benefits.

Focusing on link 1, we can write the expected network capacity as a function of the transmit powers as

$$\begin{aligned} \bar{C}_1(P_1, P_2) = & \log_2 \left( 1 + \frac{G_{1,1}P_1}{\sigma^2 + G_{1,2}P_2} \right) \\ & + \mathbb{E} \left\{ \log_2 \left( 1 + \frac{G_{2,2}P_2}{\sigma^2 + G_{2,1}P_1} \right) \right\}, \quad (8) \end{aligned}$$

where the expectation is taken over the other link channel gains, namely  $G_{2,2}$  and  $G_{2,1}$ . The expected capacity for link 2 can be expressed similarly by inverting the indices. Notice that the expectation will be different for different power allocation strategies; a point which will be touched upon in the next section. Finally, it has been shown that for  $N = 2$ , the optimal power allocation under complete channel knowledge is binary [15], that is,  $\mathbf{P}^* \in \Omega^B$  defined in (4). Thus, motivated by the optimality of binary power control for the centralized problem, we adopt the binary feasible set for the distributed problem as well. We point out here that binary power control is not necessarily optimal for the distributed problem. However, employing this power allocation strategy greatly simplifies the complexity of the resulting algorithm. As a result, we can formally write the distributed optimization problem based on statistical knowledge as

$$\mathbf{P}_i^* = \left[ \arg \max_{(P_1, P_2) \in \Omega^B} \bar{C}_i(P_1, P_2) \right]_i \quad \forall i = 1, 2 \quad (11)$$

$$\bar{R}_2(0,1) = \int_0^{+\infty} \log_2 \left( 1 + \frac{G_{2,2}}{\sigma^2} \right) f(G_{2,2}) dG_{2,2} \quad (9)$$

$$\bar{R}_2(1,1) = \int \int_0^{+\infty} \log_2 \left( 1 + \frac{G_{2,2}}{\sigma^2 + G_{2,1}} \right) f(G_{2,2}) f(G_{2,1}) dG_{2,2} dG_{2,1} \quad (10)$$

### A. Fully Distributed Power Allocation

By adopting binary power control a link will either transmit at  $P_{\max}$  (assumed as 1) or remain inactive (i.e. link power will be 0). Thus, solving problem (11) is equivalent to a link determining if it should be active or not depending on knowledge of local information.

A cell  $i$  needs to consider the following cases to determine which power allocation maximizes the expected capacity defined in (8):

- 1) Expected capacity of both cells being active:  $\bar{C}(1,1)$ .
- 2) Capacity of only cell  $i$ :  $\bar{C}(0,1)$  or  $\bar{C}(1,0)$ .

Focusing on link 1, the activity conditions can thus be summarized as follows:

$$P_1 = \begin{cases} 1 & \text{if } \bar{C}(1,1) \geq \bar{C}(0,1) \\ 1 & \text{if } \bar{C}(1,0) \geq \bar{C}(0,1) \\ 0 & \text{otherwise} \end{cases}$$

Note that there is no need to compare the expected capacity of both cells being active and only cell 1 being active, as cell 1 will be active in either case. By simple manipulation of the above conditions, link 1 will be active if either

$$\text{SINR}_1 \geq 2^{[\bar{R}_2(0,1) - \bar{R}_2(1,1)]} - 1 \quad (12)$$

or

$$\text{SNR}_1 \geq 2^{\bar{R}_2(0,1)} - 1 \quad (13)$$

where  $\bar{R}_2(0,1)$  and  $\bar{R}_2(1,1)$  are the expected capacities of the other link under the respective power allocations (shown above in (9) and (10)).  $f(G_{2,2}), f(G_{2,1})$  are respectively, the pdfs of the direct channel and interfering channel gains (assumed independent) for link 2. Due to symmetry, conditions for link 2 can be expressed in a similar way as (12) and (13).

Practically,  $\bar{R}_2(0,1)$  and  $\bar{R}_2(1,1)$  can be calculated offline by generation of a sufficient number of channel realizations and plugged into conditions (12) and (13) to determine if the cell should be active. Thus, based on simple conditions and in a fully distributed way, each link decides based on local channel information whether it transmits or not. We call this algorithm *Fully Distributed Power Allocation* (FDPA).

### B. Capacity Enhancement with 1-bit Message Passing

The FDPA algorithm presented in the previous section is completely distributed in that it requires no real-time information exchange from other links. It is interesting however to explore how a minimum amount of information exchange could be used to enhance performance. We let this amount of information be 1-bit. More precisely, a link is allowed to send 1-bit worth of information to the other link. The most

natural choice of information to send would be the result of its optimization solution.

We call this algorithm *1-Bit Distributed Power Allocation* (1-BDPA) and describe it as follows:

- 1) Link 1 performs the optimization (11) and sends a message (1-bit) to the other link to indicate whether it is active or not.
- 2) Link 2 then performs the optimization (11) to calculate  $P_2$  under the knowledge of  $P_1$ .

We deem this algorithm to enhance performance as with the 1-bit signal from link 1, a more *informed* decision can be made by link 2. Clearly, if link 1 sends a 0 then link 2 will be active. If a 1 is sent then link 2 needs only to consider if both cells being active gives better performance than the expected capacity of the other link.

## VI. NUMERICAL RESULTS

As stated, the formulation of the distributed power allocation is independent of the system layout. Thus for ease of simulation, we consider the downlink of a cellular network where the AP transmits to a user terminal (UT). Monte-Carlo simulations over random UT positions and channel realizations are carried out for 2 cells operating at 1.8 GHz, each with a radius  $r = 1$  Km and  $P_{\max} = 1$  Watt. Random UT positions are drawn from a uniform distribution over the cell area. Gains for *all inter-cell and intra-cell* AP-UT links are based on the COST-231 [18] path loss model including log-normal shadowing with standard deviation of 10 dB, as well as i.i.d. fast fading  $\sim \mathcal{CN}(0,1)$ . Expected capacity terms of the other cell are calculated offline based on an adequate number of channel gain realizations.

The performance of FDPA and 1-BDPA is compared with the no power control and centralized optimal allocation. To gain insight into the effects of power allocation we vary the distance between the 2 cells. Denoting the distance between APs by  $d$ , we vary the ratio  $\frac{d}{2r}$ . When  $\frac{d}{2r} < 1$ , the cells overlap and this results in severe interference, akin to that in ad-hoc networks. When  $\frac{d}{2r} > 1$ , the cells are further apart (equivalent to increasing spectral reuse) and thus the effects of interference diminish.

In fig. 2 we plot the average per cell network capacity versus  $\frac{d}{2r}$ . It can be seen that power allocation provides the most benefit when  $\frac{d}{2r}$  is small i.e. the links experience strong interference. The FDPA algorithm achieves nearly 50% of the gain offered by optimal power allocation whereas with 1-BDPA a substantial amount of the gain is exploited. As  $\frac{d}{2r}$  increases, the gain from power allocation decreases and all the schemes converge to the same capacity. This is quite

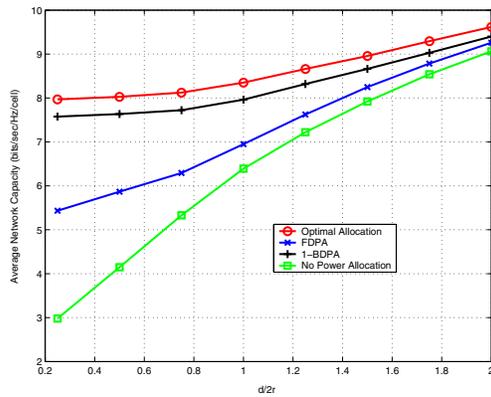


Fig. 2. Comparison of average network capacity for the distributed algorithm and 1-bit exchange approach with Optimal Power Allocation.

straightforward due to the fact that increasing the distance between the cells effectively decreases spectral reuse and both cells are more or less “shielded” from interference. Thus, from a network capacity maximization point of view, both should transmit at full power.

In fig. 3 we depict the percentage of errors made in the power allocation by each algorithm as compared to the optimal solution. FDPA makes a significant amount of errors in the high interference case. This is due to that fact that under severe interference both cells can be inactive as each cell comes to the conclusion that it will not contribute enough capacity to outweigh the interference caused. Clearly at least one cell should be active in this scenario. This error decreases in the low interference case as each cell deems it will offer enough capacity without causing too much interference and thus both cells being active becomes the optimal thing to do. We see that with 1-BDPA, in the high interference scenario the number of errors are relatively smaller. This is due to the fact that it can exploit the 1-bit information to make a better decision, which, in the severe interference case is to keep one, but not both of the cells active at the same time. At the other extreme when cells are far apart the error is small, again due to the fact that both cells are kept active in the presence of low interference.

## VII. CONCLUSIONS

In this work, we studied distributed power allocation for mutually interfering links. We proposed a framework for distributed capacity maximizing power control by exploiting statistical knowledge of non-locally available information. Based on binary power control, a computationally simple and completely distributed algorithm was proposed which provided significant performance gain. With the help of 1-bit message passing a near-distributed algorithm was shown to exploit a major part of the gain offered by the centralized optimal power allocation.

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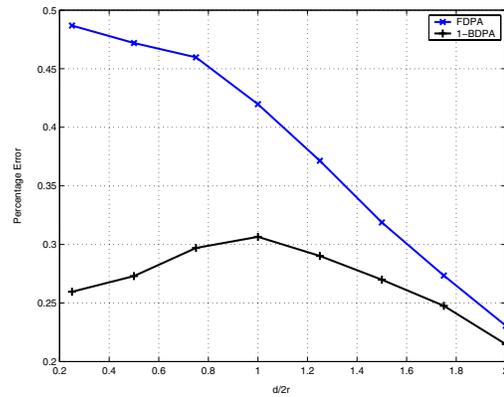


Fig. 3. Percentage Error of FDPA and 1-BDPA. Allowing 1-bit signaling reduces the number of errors made and thus 1-BDPA outperforms FDPA.

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