ELSEVIER

Signal Processing, vol.68, no.1, 1998, pp.xx–xx

SIGNAL PROCESSING

Estimation of Time Delays between unknown colored Signals

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Abstract

A number of papers have been dealing with the problem of estimating the differential delay of an unknown signal impinging on two sensors. Here, the case of L sensors and L sources is considered with $L(L-1)$ delays to be estimated. The solution resorts to blind multichannel MA identification in the time domain. All delays are estimated independently of each other using a relation between MA coefficients.

Résumé

De nombreux articles traitent le problème de l'estimation de temps de retards différentiels avec un seul signal arrivant sur deux capteurs. Ici, L sources et L capteurs sont considérés, avec $L(L-1)$ retards à estimer. Les retards sont obtenus en identifiant un modèle MA multi-variable dans le domaine temporel. Chaque retard est estimé indépendamment grâce aux relations entre les coefficients du modèle MA.

Keywords: Time Delay estimation, Blind identification, MA model, Independent Component Analysis, Cumulants.

[1]Submitted to *Signal Processing, Eurasip*, November, 1995. Revised June 1997, December, 1997.

1 Introduction

The estimation of time delays between several source signals is an important problem in signal processing: seismics, biomedecine, sonar, radar, and communications. In passive sonar (our main concern) for example, the delays are used to estimate the velocity and the location of an acoustic source.

It is assumed that L signals, $s_j(n)$, (n denoting the discrete time) are received on K sensors, $r_i(n)$, satisfying the model below:

$$r_i(n) = \sum_{j=1}^{L} s_j(n + \tau_{i,j}) + v_i(n), \tag{1}$$

where $\tau_{i,j}$ denotes the relative delay of arrival of the j^{th} signal on the i^{th} sensor. The first sensor is taken as time origin ($\tau_{1,j} = 1$). Delays are supposed to be non integer multiples of the sampling period, and $v_i(n)$ is an unknown noise independent of the signals.

The problem is the following: from a finite extent observation, delays $\tau_{i,j}$ and, if possible, source signals s_i, have to be identified. It is assumed that nothing is known about the statistics of the sources but their non Gaussian character and their statistical independence.

The identification of a differential delay between two signals is an old problem in signal processing; see for instance the June 1981 special issue of *IEEE Transaction on ASSP*. New methods have been proposed in [2], [9], [12]. See also the approaches based on MUSIC-like algorithms [13] [10], or based on the cyclostationarity of the source signals [7]. All these works are either dealing with the case of a single signal, *i.e.* , $s_j = 0, j \geq 2$, or with signals exhibiting specific properties, *e.g.* , cyclostationarity. Therefore, none of the above mentioned method can be used to address the problem we have stated, in particular in the passive sonar context.

The only works that have tackled simultaneous blind identification of more than one delay for general signals (*i.e.* neither signals $s_i(t)$ nor their spectra are known) are to our knowledge [3], [1], [6]. In [3], [1], a spectral approach is assumed, and adaptations are mandatory in order to take fully advantage of the wide-band character of the sources. In [6], the case of $K = 2$ sensors is considered. In this paper, a time domain approach is considered with $L \geq 2$ wide-band unknown sources.

The paper is organized as follows: we first write the problem as an MA identification (section 2) and show how to find the delays from the estimated MA coefficients (section 3). Finally, a blind MA identification method is considered (section 4), coupled with the Independent Component Analysis (ICA) method (section 5).

2 Problem formulation

Suppose we are given K sensors. The algorithm that will be subsequently described needs the number of sources, L, to be at most equal to the number of sensors. So the least restrictive condition consists of assuming that $L = K$, since the number of sources cannot be controlled, being understood that it is always possible to reduce the dimension of the sensor space, for instance by projection onto the dominant principal components.

Suppose from now that $L = K$. The first goal is to estimate the time delays by fully exploiting the wide-band character of the sources; this task will be carried out in the time domain. Source signals are supposed to be MA processes driven by zero-mean, non-Gaussian, independent signals, $\epsilon_j(t)$, each of them being an independent and identically distributed (i.i.d.) sequence:

$$s_j(t) = \sum_{k=0}^{q} a_{j,k} \epsilon_j(t-k), \tag{2}$$

where the coefficients $a_{j,l}$ are unknowm. The noise v_i is assumed to be Gaussian. Because of the use of fourth order cumulants, its effect asymptotically vanishes. Considering for the moment the noiseless case, the system is now as follows:

$$\begin{array}{l} r_1(t) = \sum_{j=1}^{L} \sum_{k=0}^{q} a_{j,k} \epsilon_j(t-k), \\ r_2(t) = \sum_{j=1}^{L} \sum_{k=0}^{q} a_{j,k} \epsilon_j(t + \tau_{2,j} - k), \\ \dots \\ r_L(t) = \sum_{j=1}^{L} \sum_{k=0}^{q} a_{j,k} \epsilon_j(t + \tau_{L,j} - k). \end{array} \tag{3}$$

The delays are not necessarily integer multiples of the sampling period. So, delayed source signals, ϵ_i, must be interpolated, *e.g.* using the sampling theorem, assuming all signals are band-limited:

$$\epsilon_j(n+\tau_{i,j}-k) = \sum_{r=-\infty}^{+\infty} \epsilon_j(n-k-r)sinc(r+\tau_{i,j}).$$

$sinc$ represents the cardinal sine function. Since the summation has to be truncated, denote $-m$ and m its limits:

$$\epsilon_j(n+\tau_{i,j}-k) = \sum_{r=-m}^{+m} \epsilon_j(n-k-r)sinc(r+\tau_{i,j}). \quad (4)$$

Inserting the previous relation (4) in equation (3), and performing the variable change $l = r + k$, yields:

$$r_i(n) = \sum_{j=1}^{L} \sum_{r=-m}^{m} \sum_{l=r}^{q+r} a_{j,l-r}\epsilon_j(n-l)sinc(r+\tau_{i,j}). \quad (5)$$

Since for $l < 0$ and $l > q$, $a_{j,l} = 0$, one can get rid of index r by extending the summation on l:

$$r_i(n) = \sum_{j=1}^{L} \sum_{r=-m}^{m} \sum_{l=-m}^{q+m} a_{j,l-r}\epsilon_j(n-l)sinc(r+\tau_{i,j}). \quad (6)$$

A non-causal multichannel MA model is then obtained:

$$r_i(n) = \sum_{j=1}^{L} \sum_{l=-m}^{q+m} [c_{i,j,l}\epsilon_j(n-l)] \qquad i \in \{1, ..., K\}, \quad (7)$$

whose MA coefficients $c_{i,j,l}$ are linked to each other by the relations:

$$c_{i,j,l} = \sum_{r=-m}^{m} a_{j,l-r} sinc(r+\tau_{i,j}), \quad i > 1 \quad (8)$$

$$c_{1,j,l} = a_{j,l}. \quad (9)$$

Relations (8) are considered for $-m \le l \le q+m$ knowing that for $l < 0$ and $l > q$, $c_{1,j,l} = 0$.

3 Delays estimation

From equation (7), it is clear that the observations are following a multichannel non causal and non monic MA model. The first step of the algorithm is to estimate its matrix coefficients. The proposed procedure is described in section 4. Assume now the model coefficients are known, and concentrate on the estimation of the differential delays.

3.1 Interpolation solution

Once the coefficients $c_{i,j,l}$, and also $a_{j,k}$ from equation (9), have been estimated (see section 4), relations (8) and (9) are utilized to estimate $\alpha_{i,j,p} = sinc(p+\tau_{i,j})$ by solving the following overdeterminated system for one delay ($\tau_{i,j}, i > 1$):

$$\begin{pmatrix} c_{i,j,-m} \\ \vdots \\ c_{i,j,q+m} \end{pmatrix} = \begin{pmatrix} a_{j,0} & \cdots & a_{j,-2m} \\ \vdots & & \\ a_{j,q+2m} & \cdots & a_{j,q} \end{pmatrix} \begin{pmatrix} \alpha_{i,j,-m} \\ \vdots \\ \alpha_{i,j,m} \end{pmatrix} \quad (10)$$

with, $a_{j,l} = 0$ for $l < 0$ et $l > q$.

In compact notation, the relation (10) becomes: $C_{-m,q+m} = A\alpha_{-m,m}$, that has as least squares solution:

$$\alpha_{-m,m} = (A^T A)^{-1} A^T C_{-m,q+m}.$$

The location of the maximum of the function below provides an estimate of the delay:

$$f(t) = \sum_{l=-m}^{m} \alpha_{i,j,l} sinc(t-l) = \sum_{l=-m}^{m} sinc(l+\tau_{i,j}) sinc(t-l).$$

Indeed, for $m = \infty$, $f(t) = sinc(t + \tau_{i,j})$. This function reaches its maximum for $t = -\tau_{i,j}$; see also [2]. The same procedure can be repeated independently for every delay $\tau_{i,j}$.

Remark: if the delayed path is attenuated, this method can also be used to estimate attenuations, the amplitude of f at $-\tau_{i,j}$ accounting for the attenuation. See also [6].

3.2 Direct solution

Another solution based on a direct estimation of delays without optimisation could be introduced (see [2]):

$$\tau_{i,j} = -n + \frac{\alpha_{i,j,n-1}}{\alpha_{i,j,n} + \alpha_{i,j,n-1}} \tag{11}$$

where $n = \max_r(\alpha_{i,j,r})$. This solution is obtained by writing $\alpha_{i,j,l}$ as:

$$\alpha_{i,j,k} = \frac{(-1)^{k+e} sinc(\pi f)}{\pi(k + \tau_{i,j})} \tag{12}$$

where $\tau_{i,j} = e + f$ with $0 < f < 1$.

3.3 Fourier transform solution

Denote $\bar{a}_j(\omega)$ and $\bar{c}_{i,j}(\omega)$ the Fourier transform of the samples $a_{j,l}$ and $c_{i,j,l}$. Because of the equation (8), the ratio $\frac{\bar{c}_{i,j}}{\bar{a}_j}$ gives a complex exponential which contains the delay, $e^{i\omega\tau_{i,j}}$, since $\bar{c}_{i,j} = \bar{a}_j e^{i\omega\tau_{i,j}}$. The inverse Fourier transform of $\bar{c}_{i,j}/\bar{a}_j$ exhibits a maximum at the value [9]. The location of the maximum is used afterwards to estimate $\tau_{i,j}$ via interpolation. Indeed, if delays are interger multiples of the sampling period, this maximum is a Dirac delta function.

4 MA identification

The identification of the coefficients $c_{i,j,k}$ is now addressed. The equation to be solved is (7), or in compact form:

$$r(t) = \sum_{k=-m}^{q+m} B_k \epsilon(t-k) \tag{13}$$

where: $B_k = \left[\ c_{i,j,k}\ \right]_{i,j}$, $r(t) = \left[\ r_i(t)\ \right]_i$, $\epsilon(t) = \left[\ \epsilon_i(t)\ \right]_i$, $i,j \in \{1, ..., L\}$. As already pointed out, this is a non causal multichannel MA model. Its solution is based on the results found in [4], extended to the non causal case. Non-causal MA models have also been studied in [11]. The only known terms in the model are the signals received on the sensors, $r_i(t)$. Thus, this is a blind multichannel MA identification problem.

Standard MA identification methods assume the model is monic, viz $B_0 = I$. In order to cope with this difficulty, the auxiliary model below is identified in a first stage:

$$r(t) = \sum_{k=-m}^{q+m} A_k w(t-k),\ w(t) = B_0 \epsilon(t), \tag{14}$$

where $A_0 = I_L$, the $L \times L$ identity matrix. For this approach to be valid, it is necessary that B_0 be invertible, which is satisfied with probability one. Note that this assumption is weaker than assuming that the matrix polynomial $B(z) = \sum_k B_k z^{-k}$ is full rank for all values of z, as in most papers dealing with multivariate blind deconvolution (*e.g.* see [8] and references therein). In addition, since causality is not mandatory in our approach, it is actually sufficient that one matrix B_k be invertible for our approach to be valid, not necessarily the first one, B_0.

In a second stage, an ICA can be performed on the residual $w(t)$ in order to identify coefficient B_0 [3], as explained in section 5. The process $w(t)$ is still temporally white, but is spatially correlated, because $w(t) = B_0\epsilon(t)$. So, only its cumulant at zero lag is non-zero, and is denoted $C_w = cum(w(t)\otimes w(t)\otimes w(t)\otimes w(t))$, where $\otimes$ denotes the Kronecker product.

By analogy with the work made in [3] [4], we can define the cumulants of sensors processes as:

$$C_{u,v,q,m} = cum\left(\begin{pmatrix} r_1(t+u) \\ r_2(t+u) \\ \cdots \\ r_p(t+u) \end{pmatrix} \otimes \begin{pmatrix} r_1(t+v) \\ r_2(t+v) \\ \cdots \\ r_p(t+v) \end{pmatrix} \otimes \begin{pmatrix} r_1(t+q) \\ r_2(t+q+m) \\ \cdots \\ r_p(t+q+m) \end{pmatrix} \otimes \begin{pmatrix} r_1(t) \\ r_2(t-m) \\ \cdots \\ r_p(t-m) \end{pmatrix}\right).$$

The process $w(t)$ is temporally white and if we denote $c'_{i,j,k}$ the entries of matrices A_k, the previous equation becomes:

$$C_{u,v,q,m} = \left[A_u \otimes A_v \otimes \begin{pmatrix} c'_{1,1,q} & \cdots & c'_{1,N,q} \\ c'_{2,1,q+m} & \cdots & c'_{2,N,q+m} \\ \cdots & & \\ c'_{N,1,q+m} & \cdots & c'_{N,N,q+m} \end{pmatrix} \otimes \begin{pmatrix} c'_{1,1,0} & \cdots & c'_{1,N,0} \\ c'_{2,1,-m} & \cdots & c'_{2,N,-m} \\ \cdots & & \\ c'_{N,1,-m} & \cdots & c'_{N,N,-m} \end{pmatrix}\right] C_w. \tag{15}$$

The operator vec is a linear operator mapping any matrix of size $p * q$ to a vector of dimension pq by stacking its columns one after the other. Conversely $unvec_p$ defines the inverse operator, p indicating the number of columns. Denoting $\bar{C} = unvec_p(C)$, and using the property $[F^T \otimes G]vec(X) = vec(GXF)$ with:

$$F^T = A_u,$$

$$G = A_v \otimes \begin{pmatrix} c'_{1,1,q} & \cdots & c'_{1,N,q} \\ c'_{2,1,q+m} & \cdots & c'_{2,N,q+m} \\ \cdots & & \\ c'_{N,1,q+m} & \cdots & c'_{N,N,q+m} \end{pmatrix} \otimes \begin{pmatrix} c'_{1,1,0} & \cdots & c'_{1,N,0} \\ c'_{2,1,-m} & \cdots & c'_{2,N,-m} \\ \cdots & & \\ c'_{N,1,-m} & \cdots & c'_{N,N,-m} \end{pmatrix},$$

$$vec\{X\} = C_w,$$

the previous equation becomes:

$$\bar{C}_{u,v,q,m} = \left[A_v \otimes \begin{pmatrix} c'_{1,1,q} & \cdots & c'_{1,N,q} \\ c'_{2,1,q+m} & \cdots & c'_{2,N,q+m} \\ \cdots & & \\ c'_{N,1,q+m} & \cdots & c'_{N,N,q+m} \end{pmatrix} \otimes \begin{pmatrix} c'_{1,1,0} & \cdots & c'_{1,N,0} \\ c'_{2,1,-m} & \cdots & c'_{2,N,-m} \\ \cdots & & \\ c'_{N,1,-m} & \cdots & c'_{N,N,-m} \end{pmatrix}\right] \bar{C}_w A_u^T.$$

The same relation exists if $u = 0$. The matrices A_u can be obtained in a Least Squares (LS) sense. To see this more clearly, eliminating $\bar{C}_w$ and the terms inside the brackets for $u \neq 0$ and $u = 0$ yields:

$$\bar{C}_{u,v,q,m} = \bar{C}_{0,v,q,m} A_0^{-T} A_u^T, \tag{16}$$

where $-m \leq u, v \leq q$ and $u \neq 0$. Note that the relation (16) could be used to estimate coefficient A_u, but a LS solution is preferred.

A linear system can be built by gathering all the values of v, in the range $-m \leq v \leq q$:

$$\begin{pmatrix} \bar{C}_{u,-m,q,m} \\ \vdots \\ \bar{C}_{u,q,q,m} \end{pmatrix} A_u^{-T} = \begin{pmatrix} \bar{C}_{0,-m,q,m} \\ \vdots \\ \bar{C}_{0,q,q,m} \end{pmatrix} A_0^{-T}, \tag{17}$$

or in compact notation:

$$C_u A_u^{-T} = C_0 A_0^{-T}. \tag{18}$$

Using the property that if $GX = C$ then $[I_p \otimes G]vec(X) = vec(C)$, a linear system is constructed:

$$[I_p \otimes (C_0 A_0^{-T})]vec(A_u^T) = vec(C_u) \tag{19}$$

Relation (19) gives an estimation of the MA coefficients of the model (14).

Remark

If the filter length m is unknown, or very different for each source, a new notation for cumulants is assumed: $C_{u,v,q} = cum\{r(t+u) \otimes r(t+v) \otimes r_1(t+q) \otimes r_1(t)\}$ instead of $C_{u,v,q,m}$. The solution is obtained with the same procedure than before with fewer equations because $r_1(t)$ and $r_1(t+q)$ are now scalars:

$$C_{u,v,q} = [A_u \otimes A_v]\, c'_{1,1,0}\, c'_{1,1,q}\, C_w.$$

Using the same notation than before, the previous equation becomes:

$$\bar{C}_{u,v,q} = A_v\, c'_{1,1,0}\, c'_{1,1,q}\, \bar{C}_w\, A_u.$$

Eliminating the term $A_v c'_{1,1,0} c'_{1,1,q} \bar{C}_w$ between the equations obtained for $u \neq 0$ and $u = 0$ yields:

$$\bar{C}_{u,v,q} = \bar{C}_{0,v,q} A_0^{-T} A_u^T.$$

This relation has the same form as equation (16), so that A_u can be obtained by using the same procedure than the one described in section 4.

5 Independent Component Analysis (ICA)

As shown in section 4, because $w(t) = B_0 \epsilon(t)$, the relation $B_k = A_k B_0$ follows. So, once every A_k has been identified, only B_0 is needed to obtain every B_k. The identification of B_0 is based on

$$w(t) = B_0 \epsilon(t)$$

where $\epsilon(t)$ is white and spatially uncorrelated, and on the estimation of the cumulants of $w(t)$. The ICA algorithm presented in [5] is utilised in this paper for this purpose.

It is irrelevant to reproduce here a complete theory of the ICA method, already described in details in [5]. But it is useful to briefly recall the basic steps of the algorithm. Because of an undetermination inherent in the problem, we can only pretend to estimate $z = P\epsilon$, where P is a permutation. So the system to identify is actually the following: $w(t) = H\, z(t)$. The first step according to [5] is to decorrelate the observations by filtering them with L^{-1}, where L is any square root of the covariance of $w(t)$; denote $\bar{w} = L^{-1}w$. In a second step, the orthogonal component of the mixture is identified and yields the variables $s = Q^H \bar{w}$, that are ideally independent of each other (in the noiseless case). Finally a normalisation can be performed by post-multiplication by a diagonal matrix, Δ. As a summary, we end up with: $w = LQ\Delta^{-1}z$.

The main problem consists of estimating Q. When $L = 2$, the matrix Q is a plane rotation, and the solution is given directly by rooting a fourth degree polynomial. When $L > 2$, the fact that matrix Q can be decomposed into the product of $L(L-1)/2$ plane rotations is exploited.

6 Computer simulations

We first consider the case of two sensors. The two sources (s_1 and s_2) are two MA processes of order two:

$$s_i(t) = v_i(t) + a_{1,i}\epsilon_i(t-1) + a_{2,i}\epsilon_i(t-2),$$

where $\epsilon_i(t)$ are uniformly distributed independent i.i.d. sequences, with zero-mean and unit variance; parameters are $a_{1,1} = -.86, a_{2,1} = .74$, $a_{1,2} = -.77, a_{2,2} = .59$. The two delays are $\tau_1 = 0.8$ and $\tau_2 = 0.2$ so that $m = 2$ can be a good choice. All results are obtained over 100 independent trials, each of sample size 5000.

Every A_i is estimated for $0 < i \leq q$ with the method presented in section 4, but all equations containing m are dropped; so the value of m in not needed. Matrix B_0 is estimated with the ICA method, then matrices B_i can be estimated (see section 5). We come to the delays by using the interpolation method (section 3.1); we maximize the two cardinal sine functions (figure 1 and table 1).

τ_1		τ_2	
mean	std	mean	std
0.76	0.14	0.24	0.096

Table 1 *Mean and standard deviation for 100 independent trials. True delays are:* $\tau_1 = 0.8$ *et* $\tau_2 = 0.2$.

The same approach (table 1) is presented with v_1 and v_2 being two independent and uniformly distributed noises. The signal to noise ratio (SNR) is defined as follows: $SNR = 20log(std(s_1 + s_2)/std(v1))$. Results are presented in table 2. Even the highest SNR value is realistic in an interception environment.

	τ_1		τ_2	
SNR	mean	std	mean	std
27 dB	0.85	0.15	0.13	0.32
13 dB	0.93	0.23	0.09	0.35
0 dB	1.2	0.48	0.33	0.28
-13 dB	1.47	0.55	0.6	0.53

Table 2 *Mean and standard deviation for 100 independent trial in a noisy context. True delays are:* $\tau_1 = 0.8$ *et* $\tau_2 = 0.2$.

On the other hand with three sources: $a_{1,1} = -.86, a_{2,1} = .74$, $a_{1,2} = -.77, a_{2,2} = .59$, $a_{3,1} = -0.68, a_{3,2} = 0.67$, we have 6 functions to maximize. (figure 2 and table 3).

τ_{ij} j	1		2		3	
i	mean	std	mean	std	mean	std
2	0.78	0.24	0.28	0.18	0.17	0.09
3	0.84	0.32	0.61	0.15	0.23	0.16

Table 3 *Mean and standard deviation of six delays. True delays are:* $\tau_{21} = 0.8, \tau_{22} = 0.4, \tau_{23} = 0.2, \tau_{31} = 0.9, \tau_{32} = 0.6, \tau_{33} = 0.3$.

7 Conclusion

The algorithm described in this paper allows to identify delays of arrival of sources, that are as numerous as the sensors. Sources are assumed to be wide-band, and the identification is carried out in the time domain, in order to avoid the difficulties encountered when trying to perform a fusion between successive bands, if a spectral appraoch had been used.

The solution resorts to the identification of a non-monic and non-causal multichannel MA model. A relation between the MA coefficients is then used to estimate the delays. All delays are estimated in two steps, namely computing an analytical expression followed either by an iterative optimisation or by a Fourier transform, in order to estimate their exact values (that are a priori fractions of the sampling period).

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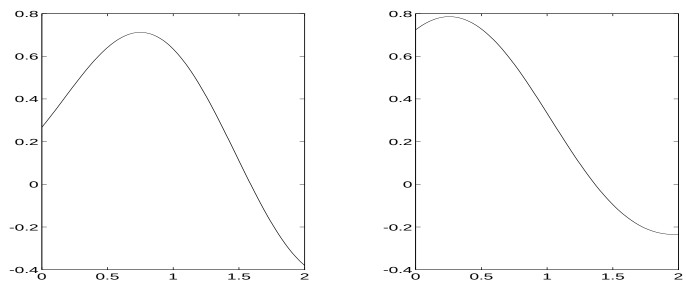

Figure 1: Functions whose maximum corresponds to the delay to be estimated (0.2 and 0.8).

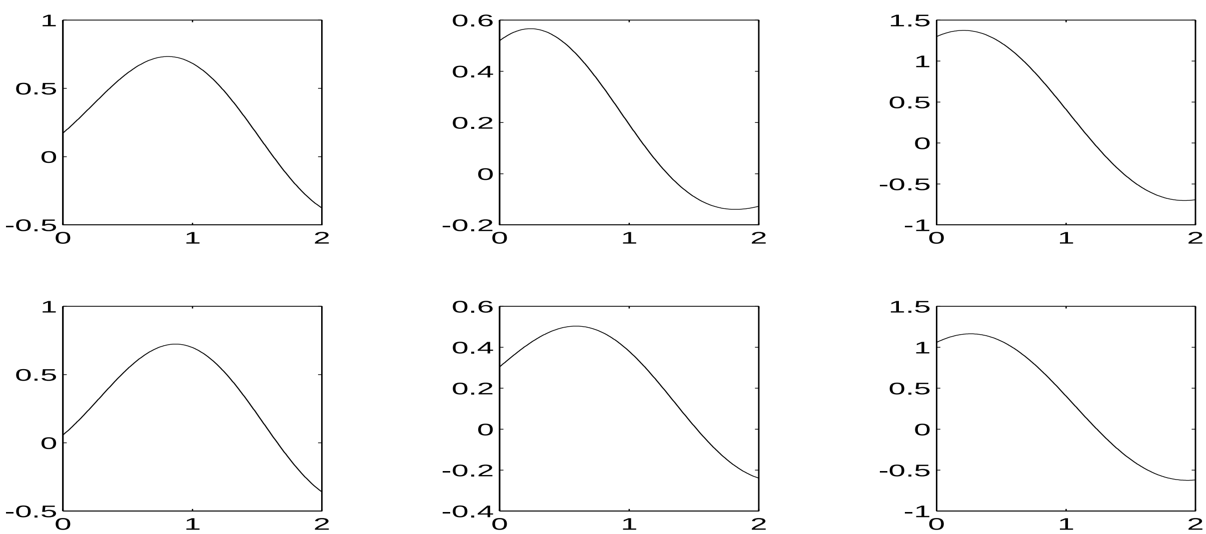

Figure 2: Functions whose maximum corresponds to the delay to be estimated (0.8, 0.4, 0.2, 0.9, 0.6, and 0.3)